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Multi-innovation stochastic gradient algorithms for dual-rate sampled systems with preload nonlinearity[☆]

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ABSTRACT

Since the stochastic gradient algorithm has a slower convergence rate, this letter presents a multi-innovation stochastic gradient algorithm for a class of dual-rate sampled systems with preload nonlinearity. The basic idea is to transform the dual-rate system model into an identification model which can use dual-rate data by using the polynomial transformation technique. A simulation example is provided to verify the effectiveness of the proposed method.

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1. Introduction

System identification has been widely used in many areas, e.g., chemical processes, aerospace engineering, mechanical systems and biological systems. There exist many system identification methods, such as the iterative methods [1–6], the least squares methods [7–11], the stochastic gradient (SG) methods [12–15]. The systems to be identified can be divided into the continuous-time systems and the discrete-time systems, where the latter includes single-rate systems and multirate systems. The multirate sampled data systems whose input and output signals have different sampled rates are abundant in industrial processes [16,17]. Recently, a lot of attention has been paid to the identification of dual-rate/multirate systems [18–24].

The Hammerstein system with the block structure oriented nonlinearity consists of a static nonlinear block followed by a linear dynamic block. Much work has been performed on the parametric model identification of Hammerstein systems. Some work assumed that the nonlinearity is the polynomial nonlinearity [11,25] and the other assumed that the nonlinearity is the hard nonlinearity [26–31]. The feature of the hard nonlinearity is that the parameters of the nonlinear part are coupled with the linear part, thus identification of the Hammerstein system with hard nonlinearity is difficult. Recently, Chen et al. have proposed a modified SG algorithm and a forgetting factor SG algorithm for a dual-rate Hammerstein system with preload nonlinearity [15]. Bai used a deterministic correlation analysis method to estimate the parameters of systems with hard nonlinearity [28]. Vörös studied an appropriate switching function to model and identify a Hammerstein system with multi-segment piecewise-linear characteristics [29] and with backlash [30].

On the basis of the work in [15], this letter uses two switching functions and the polynomial transformation technique to transform the model of the dual-rate nonlinear system with preload nonlinearity into an identification model which can

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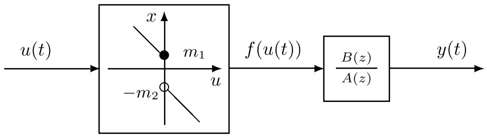

Fig. 1. The input nonlinear system with the preload nonlinearity.

use the dual-rate data, and then propose a multi-innovation SG algorithm to estimate the unknown parameters of a class of input nonlinear systems.

Briefly, the letter is organized as follows. Section 2 describes the problem formulation of the dual-rate systems with preload nonlinearity and derives a suitable model. Section 3 discusses an SG algorithm and a multi-innovation stochastic gradient algorithm for the obtained dual-rate model. Section 4 provides an illustrative example. Finally, concluding remarks are given in Section 5.

2. Problem description

Consider the following dual-rate nonlinear system, shown in Fig. 1:

$$A(z)y(t) = B(z)f(u(t)), \tag{1}$$
$$A(z) := 1 + a_1z^{-1} + a_2z^{-2} + \cdots + a_nz^{-n},$$
$$B(z) := b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + \cdots + b_nz^{-n},$$

where $y(t)$ is the system output, $u(t)$ is the system input, and z^{-1} is a unit backward shift operator: $z^{-1}y(t) = y(t-1)$, the function $f(u(t))$ is a preload nonlinearity and can be expressed as

$$f(u(t)) = \begin{cases} -u(t) + m_1, & u(t) \geqslant 0, \\ -u(t) - m_2, & u(t) < 0. \end{cases}$$

The numbers m_1 and $-m_2$ are two preload points.

The dual-rate sampled-data system under consideration in this letter assumes that all input data $\{u(t),\ t = 0, 1, 2, \ldots\}$ are available and so are only the scarce output data $y(qt), q \geqslant 2,\ t = 0, 1, 2, \ldots$. The intersample outputs or missing outputs $y(qt + j),\ j = 1, 2, \ldots, q-1$ are unavailable. The following uses the polynomial transformation technique to generate a new dual-rate model which works on the dual-rate sampled-data [19]. Let

$$A(z) = (1 - z_1z^{-1})(1 - z_2z^{-1}) \cdots (1 - z_nz^{-1}),$$

where $z_i,\ i = 1, 2, \ldots, n$, are the roots of $A(z)$ [15]. Define the polynomials,

$$\begin{aligned} r(z) &:= \prod_{i=1}^{n}(1 + z_iz^{-1} + z_i^2z^{-2} + \cdots + z_i^{q-1}z^{-q+1}) \\ &= 1 + r_1z^{-1} + r_2z^{-2} + \cdots + r_mz^{-m}, \quad m := n(q-1), \end{aligned}$$
$$\alpha(z) := r(z)A(z) = 1 + \alpha_1z^{-q} + \alpha_2z^{-2q} + \cdots + \alpha_nz^{-nq}, \tag{2}$$
$$\beta(z) := r(z)B(z) = \beta_1z^{-1} + \beta_2z^{-2} + \cdots + \beta_{nq}z^{-nq}. \tag{3}$$

Multiplying both sides of (1) by $r(z)$ yields $\alpha(z)y(t) = \beta(z)f(u(t))$. Consider a disturbance in a physical system, introducing a noise term $v(t)$ gives

$$\alpha(z)y(t) = \beta(z)f(u(t)) + v(t). \tag{4}$$

For simplification, we introduce two switching functions:

$$h_1[u(t)] = \begin{cases} 1, & \text{if } u(t) \geqslant 0, \\ 0, & \text{if } u(t) < 0, \end{cases} \qquad h_2[u(t)] = \begin{cases} 0, & \text{if } u(t) \geqslant 0, \\ -1, & \text{if } u(t) < 0. \end{cases}$$

Then the preload nonlinearity $f(u(t))$ can be written as

$$f(u(t)) = -u(t) + m_1h_1(u(t)) + m_2h_2(u(t)), \tag{5}$$

and Eq. (4) can be written as

$$\alpha(z)y(t) = \beta(z)(-u(t) + m_1h_1(u(t)) + m_2h_2(u(t))) + v(t). \tag{6}$$

3. The multi-innovation estimation algorithm

Define the parameter vector $\boldsymbol{\theta}$ and the information vector $\boldsymbol{\varphi}(t)$ as

$$\begin{aligned}\boldsymbol{\theta} &:= [\beta_1, \beta_2, \beta_3, \ldots, \beta_{nq}, \beta_1 m_1, \beta_2 m_1, \beta_3 m_1, \ldots, \beta_{nq} m_1,\\ &\quad \beta_1 m_2, \beta_2 m_2, \beta_3 m_2, \ldots, \beta_{nq} m_2, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n]^{\mathrm{T}} \in \mathbb{R}^{3nq+n},\\ \boldsymbol{\varphi}(t) &:= [-u(t-1), -u(t-2), -u(t-3), \ldots, -u(t-nq), h_1(u(t-1)), h_1(u(t-2)),\\ &\quad h_1(u(t-3)), \ldots, h_1(u(t-nq)), h_2(u(t-1)), h_2(u(t-2)), h_2(u(t-3)), \ldots,\\ &\quad h_2(u(t-nq)), -y(t-q), -y(t-2q), \ldots, -y(t-nq)]^{\mathrm{T}} \in \mathbb{R}^{3nq+n}.\end{aligned}$$

From (6), we have the identification model,

$$y(t) = \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta} + v(t) \quad \text{or} \quad y(qt) = \boldsymbol{\varphi}^{\mathrm{T}}(qt)\boldsymbol{\theta} + v(qt). \tag{7}$$

The vector $\boldsymbol{\varphi}(qt)$ contains only the available measurement outputs and inputs.

Let $\hat{\boldsymbol{\theta}}(qt)$ be the estimate of $\boldsymbol{\theta}$ at time qt. The following SG algorithm can estimate the parameter vector $\boldsymbol{\theta}$ in (7) [15,19]:

$$\hat{\boldsymbol{\theta}}(qt) = \hat{\boldsymbol{\theta}}(qt-q) + \frac{\boldsymbol{\varphi}(qt)}{r(qt)} e(qt), \tag{8}$$

$$\hat{\boldsymbol{\theta}}(qt+i) = \hat{\boldsymbol{\theta}}(qt), \quad i = 0, 1, 2, \ldots, q-1, \tag{9}$$

$$e(qt) = y(qt) - \boldsymbol{\varphi}^{\mathrm{T}}(qt)\hat{\boldsymbol{\theta}}(qt-q), \tag{10}$$

$$r(qt) = r(qt-q) + \|\boldsymbol{\varphi}(qt)\|^2, \qquad r(0) = 1, \tag{11}$$

where $1/r(qt)$ is the step-size and the norm of matrix $\boldsymbol{X}$ is defined by $\|\boldsymbol{X}\|^2 := \mathrm{tr}[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}]$.

In order to enhance the convergence rate of the SG algorithm, we extend the SG algorithm such that the parameter estimation accuracy can be improved. Such an algorithm is derived from the multi-innovation identification theory [32]. At time qt, the SG algorithm only uses the current data $y(qt)$ and $\boldsymbol{\varphi}(qt)$ thus has slow convergence rate. Referring to [32], we derive a new algorithm by expanding the single innovation $e(qt)$ to an innovation vector:

$$\begin{aligned}\boldsymbol{E}(p, qt) &= [e(qt), e(qt-q), e(qt-2q), \ldots, e(qt-(p-1)q)]^{\mathrm{T}}\\ &\approx [y(qt) - \hat{\boldsymbol{\theta}}^{\mathrm{T}}(qt-q)\boldsymbol{\varphi}(qt), y(qt-q) - \hat{\boldsymbol{\theta}}^{\mathrm{T}}(qt-q)\boldsymbol{\varphi}(qt-q), \ldots,\\ &\qquad y(qt-(p-1)q) - \hat{\boldsymbol{\theta}}^{\mathrm{T}}(qt-q)\boldsymbol{\varphi}(qt-(p-1)q)]^{\mathrm{T}} \in \mathbb{R}^p,\end{aligned}$$

which uses the past data $\{y(qt-iq), \hat{\boldsymbol{\varphi}}(qt-iq) \colon i = 1, 2, \ldots, p-1\}$, where p represents the innovation length.

Define the information matrix $\boldsymbol{\Phi}(p, qt)$ and the stacked output vector $\boldsymbol{Y}(p, qt)$ as

$$\begin{aligned}\boldsymbol{\Phi}(p, qt) &:= [\boldsymbol{\varphi}(qt), \boldsymbol{\varphi}(qt-q), \boldsymbol{\varphi}(qt-2q), \ldots, \boldsymbol{\varphi}(qt-(p-1)q)]^{\mathrm{T}} \in \mathbb{R}^{p\times(3nq+n)},\\ \boldsymbol{Y}(p, qt) &:= [y(qt), y(qt-q), y(qt-2q), \ldots, y(qt-(p-1)q)]^{\mathrm{T}} \in \mathbb{R}^p.\end{aligned}$$

The innovation vector $\boldsymbol{E}(p, qt)$ can be expressed as

$$\boldsymbol{E}(p, qt) = \boldsymbol{Y}(p, qt) - \boldsymbol{\Phi}(p, qt)\hat{\boldsymbol{\theta}}(qt-q).$$

Referring to the multi-innovation stochastic gradient method for linear regression models [32–41], we can obtain the following multi-innovation stochastic gradient (MISG) algorithm for the input nonlinear system:

$$\hat{\boldsymbol{\theta}}(qt) = \hat{\boldsymbol{\theta}}(qt-q) + \frac{\boldsymbol{\Phi}^{\mathrm{T}}(p, qt)}{\boldsymbol{r}(qt)} \boldsymbol{E}(p, qt), \tag{12}$$

$$\hat{\boldsymbol{\theta}}(qt+i) = \hat{\boldsymbol{\theta}}(qt), \quad i = 0, 1, 2, \ldots, q-1, \tag{13}$$

$$\boldsymbol{E}(p, qt) = \boldsymbol{Y}(p, qt) - \boldsymbol{\Phi}(p, qt)\hat{\boldsymbol{\theta}}(qt-q), \tag{14}$$

$$\boldsymbol{Y}(p, qt) = [y(t), y(qt-q), y(qt-2q), \ldots, y(qt-(p-1)q)]^{\mathrm{T}}, \tag{15}$$

$$\boldsymbol{\Phi}(p, qt) = [\boldsymbol{\varphi}(qt), \boldsymbol{\varphi}(qt-q), \boldsymbol{\varphi}(qt-2q), \ldots, \boldsymbol{\varphi}(qt-(p-1)q)]^{\mathrm{T}}, \tag{16}$$

$$\begin{aligned}\boldsymbol{\varphi}(qt) &= [-u(qt-1), -u(qt-2), -u(qt-3), \ldots, -u(qt-nq), h_1(u(qt-1)), h_1(u(qt-2)),\\ &\quad h_1(u(qt-3)), \ldots, h_1(u(qt-nq)), h_2(u(qt-1)), h_2(u(qt-2)), h_2(u(qt-3)), \ldots,\\ &\quad h_2(u(qt-nq)), -y(qt-q), -y(qt-2q), \ldots, -y(qt-nq)]^{\mathrm{T}},\end{aligned} \tag{17}$$

$$r(qt) = r(qt-q) + \|\boldsymbol{\varphi}(qt)\|^2, \qquad r(0) = 1. \tag{18}$$

Because $\boldsymbol{E}(p, qt) \in \mathbb{R}^p$ is an innovation vector, namely, the multi-innovation, the algorithm in (12)–(18) is called the multi-innovation identification algorithm. As $p = 1$, the MISG algorithm reduces to the SG algorithm in (8)–(11).

Table 1
The SG estimates and errors.

t	100	200	300	500	1000	1500	2000	2500	3000	True values
α_1	0.26952	0.28334	0.29128	0.30112	0.31416	0.32162	0.32683	0.33082	0.33406	0.95000
α_2	−0.00894	−0.00198	0.00176	0.00619	0.01180	0.01490	0.01703	0.01864	0.01994	0.36000
β_1	0.45187	0.44961	0.44820	0.44635	0.44378	0.44225	0.44117	0.44033	0.43964	0.40000
β_2	0.04488	0.04499	0.04512	0.04535	0.04574	0.04599	0.04619	0.04635	0.04648	0.10000
β_3	−0.12242	−0.11634	−0.11287	−0.10859	−0.10297	−0.09978	−0.09756	−0.09587	−0.09450	0.09000
β_4	0.17240	0.17265	0.17272	0.17274	0.17268	0.17262	0.17257	0.17253	0.17250	0.18000
$\beta_1 m_1$	0.10862	0.11389	0.11696	0.12079	0.12590	0.12884	0.13090	0.13248	0.13376	0.08000
$\beta_2 m_1$	0.08160	0.08274	0.08340	0.08421	0.08527	0.08586	0.08627	0.08657	0.08681	0.02000
$\beta_3 m_1$	−0.05231	−0.05641	−0.05855	−0.06101	−0.06398	−0.06555	−0.06659	−0.06736	−0.06797	0.01800
$\beta_4 m_1$	0.03158	0.03275	0.03346	0.03436	0.03557	0.03625	0.03673	0.03709	0.03738	0.03600
$\beta_1 m_2$	0.10997	0.11638	0.11990	0.12413	0.12956	0.13259	0.13468	0.13627	0.13755	0.04000
$\beta_2 m_2$	0.08296	0.08523	0.08634	0.08755	0.08893	0.08961	0.09005	0.09036	0.09060	0.01000
$\beta_3 m_2$	−0.05095	−0.05392	−0.05561	−0.05767	−0.06033	−0.06181	−0.06281	−0.06358	−0.06418	0.00900
$\beta_4 m_2$	0.03293	0.03524	0.03640	0.03770	0.03922	0.04000	0.04051	0.04088	0.04117	0.01800
δ (%)	73.32206	72.01943	71.28427	70.38527	69.21194	68.54941	68.08961	67.73869	67.45562	

Table 2
The MISG estimates and errors with $p = 10$.

t	100	200	300	500	1000	1500	2000	2500	3000	True values
α_1	0.92857	0.93510	0.93788	0.94061	0.94330	0.94447	0.94516	0.94564	0.94599	0.95000
α_2	0.34751	0.35133	0.35296	0.35455	0.35611	0.35680	0.35720	0.35748	0.35768	0.36000
β_1	0.40053	0.40038	0.40031	0.40024	0.40017	0.40013	0.40012	0.40010	0.40009	0.40000
β_2	0.10148	0.10115	0.10098	0.10080	0.10060	0.10051	0.10045	0.10041	0.10038	0.10000
β_3	0.07593	0.07963	0.08130	0.08301	0.08477	0.08558	0.08607	0.08641	0.08667	0.09000
β_4	0.17865	0.17903	0.17918	0.17933	0.17948	0.17954	0.17958	0.17961	0.17963	0.18000
$\beta_1 m_1$	0.06549	0.06661	0.06713	0.06765	0.06820	0.06844	0.06859	0.06870	0.06877	0.08000
$\beta_2 m_1$	0.01912	0.02051	0.02116	0.02184	0.02257	0.02290	0.02311	0.02325	0.02335	0.02000
$\beta_3 m_1$	0.04511	0.04063	0.03840	0.03595	0.03322	0.03189	0.03104	0.03045	0.02999	0.01800
$\beta_4 m_1$	0.03443	0.03538	0.03578	0.03616	0.03652	0.03665	0.03673	0.03677	0.03681	0.03600
$\beta_1 m_2$	0.04564	0.04679	0.04733	0.04790	0.04850	0.04877	0.04894	0.04906	0.04915	0.04000
$\beta_2 m_2$	−0.00038	0.00104	0.00172	0.00244	0.00322	0.00358	0.00381	0.00397	0.00408	0.01000
$\beta_3 m_2$	0.02514	0.02070	0.01849	0.01609	0.01340	0.01210	0.01128	0.01070	0.01026	0.00900
$\beta_4 m_2$	0.01475	0.01573	0.01616	0.01658	0.01699	0.01716	0.01725	0.01731	0.01736	0.01800
δ (%)	4.16769	3.30340	2.92591	2.55473	2.19616	2.04601	1.96042	1.90417	1.86395	

4. Example

Consider the following system,

$$A(z^{-1})y(t) = B(z^{-1})f(u(t)) + v(t),$$
$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} = 1 + 0.5z^{-1} + 0.6z^{-2},$$
$$B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} = 0.4z^{-1} + 0.3z^{-2},$$
$$f(u(t)) = -u(t) + 0.2h_1(u(t)) + 0.1h_2(u(t)),$$

in simulation, $\{u(t)\}$ is taken as a persistently excited signal sequence with zero mean and unit variance, and $\{v(t)\}$ as a white noise sequence with zero mean and variance $\sigma^2 = 0.10^2$. Taking $q = 2$ and $r(z) = 1 - 0.5z^{-1} + 0.6z^{-2}$, we have

$$\alpha(z) = r(z)A(z) = 1 + 0.95z^{-2} + 0.36z^{-4},$$
$$\beta(z) = r(z)B(z) = 0.4z^{-1} + 0.1z^{-2} + 0.09z^{-3} + 0.18z^{-4},$$
$$\begin{aligned}\boldsymbol{\theta} &= [\beta_1, \beta_2, \beta_3, \beta_4, \beta_1 m_1, \beta_2 m_1, \beta_3 m_1, \beta_4 m_1, \beta_1 m_2, \beta_2 m_2, \beta_3 m_2, \beta_4 m_2, \alpha_1, \alpha_2]^{\mathrm{T}} \\ &= [0.4, 0.1, 0.09, 0.18, 0.08, 0.02, 0.018, 0.036, 0.04, 0.01, 0.009, 0.018, 0.95, 0.36]^{\mathrm{T}}.\end{aligned}$$

Applying the SG algorithm and the MISG algorithm to estimate the parameters of this system, the parameter estimates and their errors are shown in Tables 1–2 and the parameter estimation errors $\delta := \|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\|/\|\boldsymbol{\theta}\|$ versus t are shown in Fig. 2.

From Tables 1–2 and Fig. 2, we can see that the convergence rate of the MISG algorithm is faster than the SG algorithm and the parameter estimates given by the MISG algorithm converge their true values with the increasing of t.

5. Conclusions

This letter uses the polynomial transformation technique to study identification problems for a class of dual-rate input nonlinear systems and presents a multi-innovation stochastic gradient algorithm by expanding the scalar innovations to the

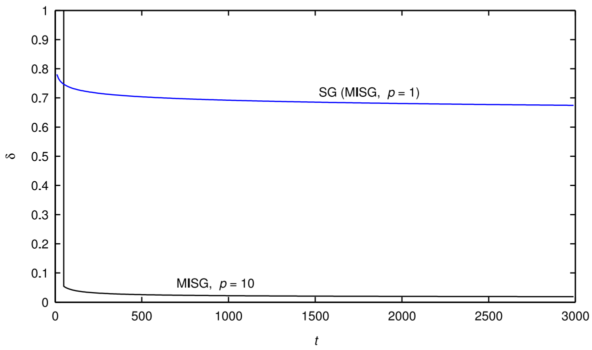

Fig. 2. The parameter estimation errors δ versus t.

innovation vector. The convergence of the proposed algorithm can be analyzed in a similar method in [32]. The proposed method can be extended to other nonlinear systems, e.g., non-uniformly sampled-data nonlinear systems [42–48].

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