D’Hainaut’s operationalization model in mathematics

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Abstract

The article is specific to the sciences of education and describes effective practices for students from the Faculty of Mathematics, “Vasile Alecsandri” University of Bacau. The aim was to investigate the efficiency of D’Hainaut’s operationalization model with applications in mathematics, the formation of concepts, theorem proving, problem solving. The hypothesis can be reduced to the claim that the use of the operationalization model in relation to an object, product, cognitive operation in the teaching of Mathematics will lead to the building of effective skills in solving Mathematical problems and the development of thinking.

Keywords: object, product, cognitive operation, problem solving, problematic situation.

1. Introduction

Mathematical education has a double effect: on the one hand, the learner acquires knowledge and, on the other hand, builds those skills which are engaged in work, develops the forces required by this type of education. Mathematical education builds thinking skills. Of course, other actions contribute to forming thought as well, but the role of mathematical education is paramount.

This study is concerned with aspects such as: the model object-product-cognitive operation through mathematical education; achieving a systematic observation regarding the initiation in the particularities of mathematical activity;

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gaining knowledge of the elements and priorities of certain fundamental notions and configurations; using concepts
and theorems in solving mathematical problems.

2. The model object-product-cognitive operation

More specifically, a cognitive operation is a mental activity, which in the context of an intellectual act, ensures
the correspondence between a given object and a certain product, possibly through the intervention of an operator.
The nature of the cognitive operation depends upon the operator’s degree of availability in relation to the student’s
cognitive repertoire.

The following categories of cognitive operations are defined:

a) **Reproduction** – The subject, placed before an object which is identical with the object from the learning
situation, provides the same product.

b) **Conceptualization or understanding** – The subject, placed in front of an object, provides an answer which is
valid for the class to which the object belongs, on condition that in the learning situation the product should not be
associated with the object.

c) **Application** – The activity through which the student provides a determined product for a given object,
belonging to another class, without this particular object having been associated with this specific answer during
training, given the fact that the class of the object has been associated with the class of the product.

d) **Exploitation** – Consists in extracting a determined piece of information, of content, out of a situation.

e) **Mobilization** – Consists in extracting from the cognitive repertoire one or several pieces of information (the
product), which respond to one or several precise conditions, without there being a previous association between
these conditions and this product.

f) **Problem solving** – Is the cognitive activity consisting in creating a product by starting from a certain object, on
condition that the product, object or solving procedure may present a certain degree of novelty.

Generally, one objective of mathematical education refers to describing a class of tasks (questions, exercises,
problems, problem situations) which we expect that the student may be able to solve by the end of a training unit
(group of lessons, intra-interdisciplinary module). Such a task may be characterized by the following aspects:

a) the task information (conditions, hypotheses) is what the student is being offered;

b) what should be demonstrated or found, is what the student should obtain, the answer which should be given;

c) the methods, procedures, knowledge used to obtain the solution.

The task information and the methodological indications regarding what should be found or demonstrated forms
the **object** upon which the student’s activity will be exerted.

The result obtained by solving the task, the solution itself, which may be a number, function, demonstration etc.,
represents the **product** of the activities.

The procedures used in providing the answer, which may be more or less obvious, with a higher or lower degree
of novelty, in relation to the learning situation, form the operational skills and abilities. The effective application of
these in obtaining the product, a certain object having been given, form the **cognitive operation**.

The object, as well as the product of the activity, belongs entirely to the domain of Mathematics. The object-
product perspective alone is not enough for the use of objectives in designing training. In the process of finding the
solution, the intellectual activity, which the students perform, is characterized by the relation established between
the **object-product** from the learning situation and the **object-product** from the evaluation situation. This process
involves various cognitive operations, elements which constitute intellectual qualities of the individual engaged in
**problem solving** as a distinct intellectual phenomenon. (D'Hainaut, 1981).

*Participants Initiation in the specific of mathematical activity*

The organization of training should make the student understand the following aspects:

1) The activity’s inherent particularity is the placing of a real situation within a model, which would replace
reality, and within which concepts would have a well-defined signification.

2) The activity implies the adaptation and acceptance of certain principles, so that the judgements made may refer
not only to these principles, concepts or other demonstrated propositions, by complete ignorance, at the moment of
the demonstration, of the modelled real situation.
Thus, to establish the truth of a statement regarding the object of the model, we can resort only to procedures that had been defined within the model, but not to the modelled real situation. The problem is not to completely exclude the real situation from the discussion, but to highlight the *distinction* between issuing a conjecture by inspecting some aspects of the real situation and establishing the respective statement, by reasoning, based on acknowledged or already demonstrated principles. The students should be made to understand the fact that making a drawing on a sheet of paper and performing a reasoning are two different activities.

**Knowing the elements and properties of certain fundamental notions and configurations**

a) To understand the statement of the sentences from Algebra, Analysis or Geometry, it is necessary to know the elements and properties of certain fundamental notions and configurations: segment, triangle, rectangle, circle, polyhedron etc. On coming across a certain term in a text, the student should be able to substitute a property of a certain notion or geometrical configuration, although this may not be clearly demanded by the statement.

b) To know the terms characteristic of Geometry (median, height, bisector, middle line, chord, tangent etc.), requires that the student should be able to perform the following actions: - being given the notion, indicate the properties, methods or solving stages; - being given the name, reproduce the definition; - being given the name, point out the respective object within a certain configuration; - being given the definition, indicate the name of the respective object; - being given an element of a geometrical configuration, indicate its name.

c) To understand a mathematical text, which may represent the statement of a task, or a set of statements which may constitute, together, a demonstration, relies, first of all, on knowledge of the concepts and theorems, as it had been previously methodologically defined. Besides knowledge of the disparate elements, it also implies a series of integrating skills, which may enable the student to: - make the drawing associated with a given configuration, introduce notations, transpose the hypothesis and the conclusion in the language of the introduced notations; - being given a statement, recognize or produce the properties which may be deduced, as conclusions, if the respective statement may be regarded as a hypothesis (by recalling a certain theorem, definition); - justify each step of a demonstration by referring to a hypothesis of the demonstrated sentence, a previously demonstrated theorem or other steps of the same demonstration; - identify that part of the hypothesis which has not been involved in the demonstration; - seize the formal logical structure of the statements and demonstrations (expressing geometrical sentences like an implication, replacing the expression “p if and only if q” within a conjunction of implications, formulating the converse or converses of a given sentence). (Lupu, 2011).

**Using concepts and theorems in solving problems**

The study of interrogation is undertaken by psychology (including the psychology of science), whereas the study of questions regarded as linguistic objects (namely, sentences ending with a question mark) belongs to linguistics. In our case, we shall designate by the term *problematic task* a statement which contains certain information with which the student is provided and in which the student is asked to demonstrate a mathematical fact or find the measure of a certain element of the configuration, on condition that the solving may involve a certain amount of initiative from the part of the solver.

According to D’Hainaut, problem solving is an essential thinking process, a phenomenon which is complex in terms of the processes involved as well as the diversity of the situations contained. In his view, problem solving is a cognitive activity which may be described through the following attributes:

1) – the subject is placed in an *initial situation*, which contains the *object* of its activity and a *problematic situation*; 2) – the subject deals with this object, meaning that he applies a process or a set of cognitive operations to it; 3) – the subject reaches the *final situation* which contains the *product* of his activity, meaning the very solution to the problem.

**3. Research Methodology**

D’Hainaut’s operationalization model is characteristic of the mathematical domain and allows the adaptation of the model object-product-cognitive operation, to the context of specifying the pedagogic intention associated with content elements. For description on an operational level, D’Hainaut proposes the model of the basic intellectual act,
according to which stating an objective should describe a student’s activity by referring to three aspects: the object subjected to the student’s activity; the product resulted at the end of the activity; the cognitive operation, characterized by the circumstances in which the activity is conducted.

**Participants**

The study was conducted in 2013 on a sample of 36 participants, 19 are feminine and masculine 9, third-year students from the Faculty of Mathematics, University "Vasile Alecsandri" of Bacau.

**Research Objectives**

Students have the following objectives: - Use of the operationalization D'Hainaut in mathematics leads to the formation of students' thinking and develop an appropriate course in mathematical problem solving approach; - To show that, regardless of the field, using the model to operationalization in solving problems should characterize every man, in many cases, school, family and society; - Establish a research study on the effectiveness of model effectiveness in problem solving methods and combining traditional methods with active-participative methods; - Promoting the idea that using model operationalization D'Hainaut in mathematical problem solving leads to the development of thought, creativity, feelings and attitude, intellectually competitive spirit.

**Research Methods and Techniques**

Research methods and techniques used were: - Observation teaching; - Communication; - Analysis of school documents and student products; - Interview; - Questionnaire; - Statistical techniques for data processing.

**The research hypothesis**

The research started from the assumption that: if the students will use to the operationalization model object - capacity - items proposed by D' Hainaut’s they will form and grow among middle school students, skills and abilities to solve problems and exercises and lessons will be more effective and better student outcomes.

**Results and their interpretation**

Through experiments conducted on a test proposed by students of mathematics faculty at the University "Vasile Alecsandri" of Bacau, activity analysis of 260 students from classes V-VIII National Pedagogical College "Stefan cel Mare" of Bacau, presented in:

<table>
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<th>Note</th>
<th>Ratings</th>
<th>Frequency</th>
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</thead>
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<tr>
<td>9 - 10</td>
<td>very good</td>
<td>100</td>
</tr>
<tr>
<td>7 - 8</td>
<td>well</td>
<td>90</td>
</tr>
<tr>
<td>5 - 6</td>
<td>enough</td>
<td>60</td>
</tr>
<tr>
<td>1 - 4</td>
<td>insufficient</td>
<td>10</td>
</tr>
</tbody>
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Test results that reflect student performance on the test determined the overall mean 7.50
4. Conclusions

The research was conducted during the second semester, from February to May 2013. It involved observation and identifying evidence in support of teaching practice lessons with 36 students in the third year from the specialization of Mathematics. It involved the systematic observation of 46 Math lessons and the teaching of 36 lessons, followed by testing a sample group of 260 middle-school students.

The analysis of the activity of the 260 students, grades 5-8, from “Ștefan cel Mare” Pedagogic National College from Bacau, has shown the fact that the students who had received D’Hainaut’s operationalization instructions for Mathematics: tend to learn more in a shorter time; have obtained 18% higher marks in Mathematics; were capable of using concepts and theorems efficiently in solving problems. The percentage of the students who declared themselves interested in the study of Mathematics following the initiation in the specific of the mathematical activity has risen from 73.1% to 89.5%. In middle school, 58% of the 8th graders believe that knowledge of the elements and properties of certain fundamental notions and configurations has helped them in achieving higher quality papers and improving their attitude towards school. The students had better results in the National Evaluation exam, in mathematical tests, compared to students from other schools in town.

The students from “Vasile Alecsandri” University of Bacau, the specialization of Mathematics, have discovered, through the lessons of pedagogic training, the fact that using D’Hainaut’s operationalization model in Mathematics leads to forming the students’ thinking and developing an appropriate conduct in solving mathematical problems, motivating apathetic students, whereas teachers have changed their view upon the activity of designing lessons.

References