International Journal of Solids and Structures 50 (2013) 1193–1202

Contents lists available at SciVerse ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

The effect of small scale on the pull-in instability of nano-switches using DQM

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ARTICLE INFO

Article history:
Received 2 March 2012
Received in revised form 22 October 2012
Available online 5 January 2013

Keywords:
Nano-switches
Pull-in instability
Nonlocal effect
Electrostatic force
Intermolecular force
DQM

ABSTRACT

This paper deals with the study of the small scale effect on the pull-in instability of nano-switches subjected to electrostatic and intermolecular forces. Using Eringen's nonlocal elasticity theory, the nonlocal Euler–Bernoulli beam model is derived through virtual displacement principle. The static governing equation which is extremely nonlinear due to the intermolecular and electrostatic attraction forces is solved numerically by differential quadrature method. The accuracy of the present method is verified by comparing the obtained results with the finite difference method and those in the literatures and very good agreement is obtained. Finally a comprehensive study is carried out to determine the influence of nonlocal parameter on the pull-in instability characteristics of cantilever and clamped–clamped nano-beam and some conclusions are drawn.

1. Introduction

Since their astonishing discovery by Iijima (1991), carbon nanotubes have become a subject of extensive research due to their remarkable mechanical, electrical, thermal and electro-mechanical properties. These exceptional features have opened new era in science and technology and have motivated many researchers of all kind of studies including chemistry, physics, material science and other natural science to engineering ones and invite them to challenge. The growing trend of scientific and engineering publications reported annually and the investment of governments and private institutions apparently confirm this claim. Nanotechnology concerns to the study of observing, measuring, controlling, manufacturing and manipulation typically with dimensions smaller than 100 nm to create novel materials and devices.

Because of distinguished properties, carbons nanotubes have played a significant role in the development of nanotechnology. The small size, low mass, high stiffness, high integration density and transistor aspects make them ideal to use as a Nano-electro-mechanical systems (NEMS) such as switches, actuators, tweezers and atomic force microscopy (AFM). They also have the capability to withstand the extreme conditions because of high mechanical strength and their high frequency attribute is the key feature for designing super sensitive sensors and high speed actuators. Regarding to this subject, Li et al. (2008) reviewed some of the recent advances in nanotube and nanotube-based composite sensors and actuators, with a particular emphasis on the electromechanical behavior and introduced its application towards the development of nano-scale sensor and actuator systems. The review of articles about the application of nanotechnology in designing biosensors are presented by Vo-Dinh et al. (2001) and Jianrong et al. (2004).

Typically, an electrostatic switch consists of a flexible electrode suspended above a stationary ground electrode. Two electrodes are conductive and a dielectric spacer fills the gap between them. Applying electric voltage between two parts results in deformation of the flexible electrode. When the applied voltage increases beyond a certain value which is known as pull in voltage, the instability occurs and the upper electrode sticks to the ground. This phenomenon is known as pull-in instability and the critical value of voltage and the corresponding beam displacement at the instability condition are called pull-in parameters.

Before advent of NEMS, most of electrostatically actuated devices include components that constructed in micro-scale called micro-electro-mechanical systems (MEMS). Last few decades have given much attention in this area of study and a large deal of researches in the field of mini-structures are devoted to micro-structures. Batra et al. (2008c) studied the vibration of fixed–fixed narrow micro-beams pre-deformed by an electric field and described the variation of fundamental frequency versus applied DC voltage and shown the strain hardening and electrostatic softening effect. Batra et al. (2006) applied the meshless local Petrov–Galerkin method to investigate the electrostatic deformation of different shape MEMS and extracted the pull-in parameters in each case. Chaterjee and Pohit (2009) used the large deflection nonlinear beam model to investigate the static and dynamic pull-in instability analysis of micro-cantilever beam. Their study indicated that although electrostatic forces cause softening effect

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http://dx.doi.org/10.1016/j.ijsolstr.2012.11.024

whereas geometric nonlinearity leads to stiffening effect on the microstructure. The static and dynamic behavior of electrically actuated clamped–clamped micro-beams under applied axial load is studied by Abdel-Rahman et al. (2002). In this research the eigenvalue problem associated to vibration around its statically deflected position is solved numerically and excellent agreement was obtained in comparison to experimental results. Pamidighantam et al. (2002) derived a closed-form solution for the pull-in voltage of fixed–fixed and cantilever beams based on simplified lump spring-mass system and verified the results with empirical and analytical models as well as finite element results obtained by COVENTORWARE software. The generalized differential quadrature method (DQM) is used by Sadeghian et al. (2007) to study the pull-in instability of cantilever and fixed–fixed beam-type MEMS switches incorporating the fringing field effect.

In recent years, the distance between movable electrode and rigid ground electrode reduced to the range of hundreds or even tens of nanometers. Decreasing the size of MEMS makes surface traction due to intermolecular interaction an important force in designing micro and nano-structures. At such separation, the effect of intermolecular forces (dispersion forces) become important and may extremely influence the function of NEMS/MEMS devices and change their pull-in instability parameters. Gusso and Delben (2008) calculated the dispersion forces between two semi spaces using the Lifshitz theory for different materials relevant for micro and nano devices fabrication. The van der Waals (vdW) and Casimir forces can be treated in unified form resulting from the electromagnetic quantum vacuum fluctuations existing between two separated dielectric bodies. A detailed review of the theory of the Casimir force which is comprised experiments and application history and its corrections for real material and finite temperature is presented by Lamoreaux (2005). Based on the latter, it can be found that when the separation is much less than the plasma wavelength (for a metal) or the absorption wavelength (for a dielectric) of the constitute material of the surfaces (typically below 20 nm) the retardation is not significant and the intermolecular force is simplified as the vdW attraction. However, when the separation is large sufficient (typically above 20 nm) the retardation is dominant and the intermolecular interaction is described by the Casimir force.

There exist many publications in literatures about microstructures which are considered the intermolecular interaction. Batra et al. (2007a) analyzed the pull in instability of micro-membrane and studied the effect of the Casimir force on the pull in parameters. They found that when the size of device is reduced, the magnitude of the Casimir force is comparable with that of the Coloumb or electrostatic force and it substantially change the pull in parameters. Batra et al. (2008a), Batra et al. (2008b) investigated the application of reduced order model for studying the pull-in behavior of micro-plates in combination with the vdW and Casimir forces. Zhang and Zhao (2006) studied the pull-in behavior of micro-structure under electrostatic loading numerically and analytically. Bending and vibration of electrically actuated circular microplate were investigated by Wang et al. (2011b) in the presence of the Casimir forces based on von Karman's nonlinear bending theory of thin plate. Wang et al. (2011a) also analyzed the static pull-in instability and dynamic behavior of multi-layer micro-beams actuated electrically based on the geometrically nonlinear Euler–Bernoulli beam equation. A comprehensive review of modeling of electrostatically actuated MEMS can be found in a work by Batra et al. (2007b).

Because of the smaller scale on which they can function, NEMS are expected to significantly impact many areas of science and technology and eventually replaced MEMS. Recently, many researchers have been in contact with nano-devices more than ever and have shown much interest for studying nano-structures. Dequesnes et al. (2002) analyzed the pull-in instability of carbon nanotube based nano-switches. They proposed continuum model and compared the accuracy of the continuum theory with the results obtained by molecular dynamic simulation and also reported good agreement between experimental results and numerical simulation data for nano-tweezers. They also developed analytical expression for pull-in voltage of cantilever and doubly clamped switches. The instability behavior of nano-switches and nano-tweezers under the vdW and Casimir attractions and with consideration of large deformation nonlinearity were presented by Lin and Zhao (2003, 2005a,b), and Wang et al. (2004). Ke et al. (2005b) tested the nanotube-based NEMS and reported the in situ scanning electron microscopy measurement of the deflection of nano-cantilever actuated electrically. Their article includes analytical model based on energy method in order to predict the structural behavior and instability of the nanotube and the accuracy of the model is assessed by experimental data. The role of finite deformation, stretching and charge concentrations nanotube devices are investigated by Pugno et al. (2005) and Ke et al. (2005a). Their results show significant effect of finite kinematics to increasing the pull-in voltage of doubly clamped devices, but it has negligible effect in singly clamped devices. Likewise, it was demonstrated the reduction of pull-in voltage in singly clamped devices due to charge concentration. Siddique et al. (2011) performed new experimental electrostatic deflection and compared with theoretical results in the literatures. They revealed the differences between theory and experiment results and indicated the direction of future modeling and analysis. Ramezani et al. (2007) used a distributed parameter model to obtain the closed form solution for pull-in instability of nano-cantilever beam. Ramezani (2011) also studied the instability analysis of nano-tweezers under electrostatic loading considering dispersion forces using distributed and lumped parameter models. Analytical solution of nonlinear differential equation of nano-actuator under electrostatic and dispersion forces is presented by Soroush et al. (2010) using the Adomian decomposition method.

Since the classical continuum theory has not capability to capture the small scale effect or the size dependent behavior of the structures and components in micro and nano dimensions, non-classical continuum theories have appealed much attention in simulating nano-structures, recently. Differences between continuum model and experimental results as well as molecular dynamic simulation are often caused by the lack of small scale considerations. Coupled stress theory (Kishida et al., 1990) and non-local elasticity theory proposed by Eringen (2001) are two well-known non-classical continuum theories which are able to model the small scale effect. There are a few studies that investigate the pull-in instability incorporating the small scale effect. Abdi et al. (2011) used modified couple stress theory (Yang et al., 2002) to study the size effect on the static pull-in instability of nano-cantilever in the presence of dispersion forces by employing monotonically iterative and homotopy perturbation methods as well as numerical one. Their results demonstrate that the size effect can increase pull-in parameters. Rahaeifard et al. (2011) investigated the pull-in analysis of micro-cantilever based on modified couple stress theory and compared the results with experimental observations and obtained excellent agreement. Yang et al. (2008) concluded the pull-in instability analysis of nano-switches subjected to electric voltage and intermolecular interaction within the framework of Eringen's nonlocal elasticity theory. They proposed a linear distributed load model to linearize the intermolecular and electrostatic forces and simplified the complex nonlinear differential equation to obtain a closed-form solution for cantilever and fixed–fixed nano-beam. They found that the pull-in voltage of the cantilever nano-beam increases whereas that of fixed–fixed nano-beam decreases as the small scale factor increases.

In the present work, the nonlocal Euler–Bernoulli beam model is used to investigate the pull-in instability of beam-type NEMS. Using Eringen's nonlocal elasticity theory, the static governing equation is derived under the nonlinear dispersion and electrostatic forces. Then, the nonlinear governing equation is discretized and solved by DQM. Finally, the influence of small scale effect on the pull-in parameters of cantilever and doubly clamped nanobeam are concluded via a comprehensive survey.

2. Nonlocal nonlinear governing equation

The nonlocal elasticity theory for the Euler–Bernoulli, Timoshenko, Reddy and Levinson beams are developed by Reddy (2007, 2010) and analytical solution of bending deflection, buckling loads and natural frequencies of vibration were obtained to demonstrate the effect of nonlocal features. This section is served for deriving the nonlocal static governing equation of nano-switch based on the developed nonlocal Euler–Bernoulli beam theory. Neglecting axial displacement and finite kinematics, Dequesnes et al. (2002) obtained closely matched results with experimental data.

2.1. Euler–Bernoulli beam theory (EBT)

The classical hypothesis for displacement field in EBT is of the form

$$u_1(x,z) = -z\frac{du}{dx}, \quad u_2 = 0, \quad u_3 = u(x) \tag{1}$$

where u is the mid-plane displacements in which $z = 0$ in the transverse direction. The only nonzero strain of EBT is given by

$$\varepsilon_{xx} = -z\frac{d^2u}{dx^2} \tag{2}$$

Since the principal of virtual displacement is independent of constitutive equation, the governing equation which is expressed in terms of stress resultant is valid for local and nonlocal theories. Principal of virtual displacement states that for an object in equilibrium, the total virtual work done by the all forces (internal and external) due to an arbitrary small virtual displacement compatible to boundary conditions, is zero.

$$\int_0^L (f\delta u - M\delta\varepsilon)dx = 0 \tag{3}$$

Substituting the small virtual axial strain in Eq. (3) and setting the coefficient of δu to zero leads to the Euler–Lagrange equation in the following form

$$\frac{d^2M}{dx^2} + f = 0 \tag{4}$$

where $f(x)$ is the external transverse distributed force (measured per unit length) and M is the stress resultant

$$M = \int_A z\sigma_{xx}dA \tag{5}$$

The boundary conditions involve specifying one element of each of the following three pairs at $x = 0$ and $x = L$

$$\begin{aligned} &u = 0, \quad M = 0 \\ &\frac{du}{dx} = 0, \quad Q = \frac{dM}{dx} = 0 \end{aligned} \tag{6}$$

2.2. Nonlocal theory

According to the nonlocal elasticity theory by Eringen (2001), the stress field at an arbitrary point in a continuum body not only depends on the strain field at that point but also on strains at all of the other points of the body. In other words, each point in a continuum media generates a strain field around itself through the long-range forces among atoms which influence other parts of the media. Thus, the stress state at a point is affected by the contribution of all strains from the whole surrounding area.

The nonlocal constitutive equation for a homogeneous isotropic beam neglecting the nonlocal effect in the thickness direction, can be expressed in one dimensional form as

$$£(\sigma_{xx}) = E\varepsilon_{xx}, \quad £ = 1 - \mu\frac{d^2}{dx^2}, \quad \mu = (e_0a)^2 \tag{7}$$

where e and a are material constant and the internal characteristic length, respectively. E stands for Young's modulus and £ is nonlocal linear differential operator. The local or classic constitutive equation can be obtained by setting $\mu = 0$.

Multiplying Eq. (7) by z and then taking integration of both sides, yields

$$£(M) = -EI\frac{d^2u}{dx^2} \tag{8}$$

where I is the second moment of area about the I-axis and defines as

$$I = \int_A z^2 dA \tag{9}$$

By substitution of the second derivative of M from Eq. (4) into Eq. (8) gives the bending moment in terms of displacement and external loads and the resulting shear force take the form as

$$\begin{aligned} M &= -EI\frac{d^2u}{dx^2} - \mu f \\ Q &= -EI\frac{d^3u}{dx^3} - \mu\frac{df}{dx} \end{aligned} \tag{10}$$

By putting bending moment from Eq. (10) into Eq. (4), the equilibrium equation can be written as

$$\frac{d^2}{dx^2}\left(EI\frac{d^2u}{dx^2}\right) = £(f) \tag{11}$$

The classical Euler–Bernoulli beam theory and the related stress resultants are obtained by setting $\mu = 0$ and $£ = 1$ in Eqs. (10) and (11).

2.3. Electrostatic and dispersion forces

Fig. 1 shows a typical cantilever nano-switch which consists of a movable beam suspended above a fixed ground plane. Applying the electric potential difference between the movable and fixed parts inducing electrostatic charge to switch and cause to function as a capacitor. The electric field between two parts result in an attractive electrostatic force and deflect the beam downward. In addition to electrostatic force, the intermolecular forces also act on the switch and enforce it to deflect moreover. Due to deflection, strain energy is stored in the switch and the elastic restoring force acts at apposition to electrostatic and intermolecular forces and try to return the beam to the original position. At a certain value of voltage, the beam becomes unstable and sticks to the ground plane. This instability is a kind of buckling and called pull-in instability and the corresponding critical voltage and deflection named pull-in parameters. Therefore, there are two sources which applied the external forces to nano-switch. First one is electrostatic force arising from inducing voltage and the second one is dispersion force which comes from intermolecular interaction of fixed and movable parts. Incorporation the fringing field effect by first order correction assumption and neglecting charge concentration due to

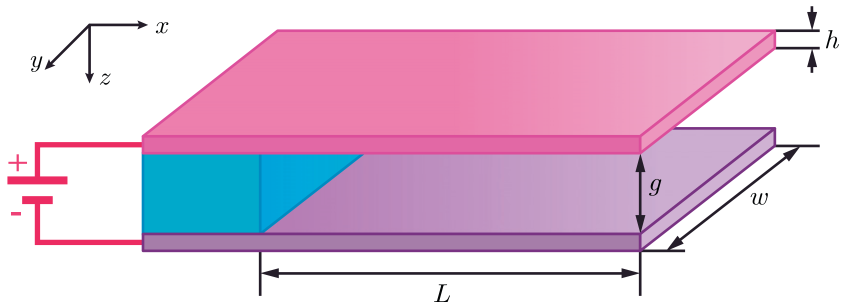

Fig. 1. Schematic of a nano-switch.

finite length, the electrostatic force per unit length of the beam is given by Huang et al. (2003)

$$f_{elec} = \frac{\varepsilon_0 w V^2}{2(g-u)^2}\left(1 + 0.65\frac{g-u}{h}\right) \tag{12}$$

where $\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2\,N^{-1}\,m^{-2}}$ is the permittivity of vacuum, V is the applied voltage and g is the initial gap between the movable part and the ground plane.

The dispersion forces appear in two forms, vdW and Casimir. The vdW force per unit length of the beam is expressed as (Gusso and Delben, 2008)

$$f_{vdW} = \frac{Aw}{6\pi(g-u)^3} \tag{13}$$

While the Casimir force per unit length of the beam can be expressed as (Gusso and Delben, 2008)

$$f_{Casimir} = \frac{\pi^2 \hbar c w}{240(g-u)^4} \tag{14}$$

where A is the Hamaker constant, $\hbar = 1.055 \times 10^{-34}$ Js is the Plank's constant divided by 2π and $c = 3 \times 10^8\ \mathrm{m\,s^{-1}}$ is the speed of light.

2.4. Non-dimensional nonlocal governing equation

For convenience, the governing equation is reformulated in non-dimensional form. Introducing the non-dimensional variables as

$$\begin{aligned} &y = 1 - \frac{u}{g}, \quad \xi = \frac{x}{L} \\ &\alpha_3 = \frac{AwL^4}{6\pi g^4 EI}, \quad \alpha_4 = \frac{\pi^2 \hbar c w L^4}{240 g^5 EI} \\ &\beta = \frac{\varepsilon_0 w V^2 L^4}{2g^3 EI}, \quad \gamma = 0.65\frac{g}{w} \end{aligned} \tag{15}$$

Substituting the electrostatic and intermolecular forces into Eq. (11) and using the non-dimensional variables from Eq. (15), we obtain the non-dimensional nonlocal governing equation which is expressed as

$$y^{(4)} + \bar{\pounds}(\bar{f}) = 0 \tag{16}$$

in which

$$\bar{f} = \frac{\alpha_n}{y^n} + \frac{\beta}{y^2} + \frac{\beta\gamma}{y}, \quad \bar{\pounds} = 1 - e^2\frac{d^2}{d\xi^2}, \quad e = \frac{e_0 a}{L} \tag{17}$$

The index n is 3 for the vdW force and 4 for the Casimir force. $\bar{\pounds}$ stands for dimensionless form of the nonlocal differential operator. The non-dimensional shear force and bending moment are written as

$$\begin{aligned} &\bar{M} = \frac{ML^2}{EIg} = \frac{d^2y}{d\xi^2} - e^2\bar{f} \\ &\bar{Q} = \frac{QL^3}{EIg} = \frac{d^3y}{d\xi^3} - e^2\frac{d\bar{f}}{d\xi} \end{aligned} \tag{18}$$

The boundary conditions involve specifying one element of each of the following pairs at $\xi = 0$ and $\xi = 1$:

$$\begin{aligned} &y = 1, \quad \bar{M} = 0 \\ &\frac{dy}{d\xi} = 0, \quad \bar{Q} = 0 \end{aligned} \tag{19}$$

3. Differential quadrature method (DQM)

DQM is a higher order numerical method which provide a cost-effective tool for solving many nonlinear differential equations and was introduced by Richard Bellman and his associates in the early 1970s (Shu, 2000). The rudimentary concept of DQM is presented here and readers are referred to book by Zong (2009) and Shu (2000) for detailed information. The basic idea of DQM comes from the Gauss quadrature, a useful numerical integration method which is approximated a definite integral by a weighted sum of integrand values at a group of the so-called Gauss points. Extending this idea, the partial derivatives of a function with respect to a space variable at a given discrete point can be approximated by a weighted linear sum of the function values at all discrete points. The approximation value of kth derivative at the ith discrete points for a one dimensional continuous function $f(x)$ can be presented by

$$\left.\frac{d^k f}{dx^k}\right|_{x=x_i} = \sum_{j=1}^{N} C_{ij}^{(k)} f(x_j) \tag{20}$$

where $x_i = 1, 2, \ldots, N$ are the discrete points and often introduced in uniform and non-uniform patterns and $C_{ij}^{(k)}$ are weighting functions. The non-uniform Chebyshev–Gauss–Lobatto (Shu, 2000) nodal distribution is most frequently used pattern in literatures that ensures the solution convergence and is expressed as

$$x(i) = \frac{1}{2}\left[1 - \cos\left(\pi\frac{i-1}{N-1}\right)\right], \quad i = 1, 2, \ldots, N \tag{21}$$

The weighting function associate to first order derivative of $f(x)$ based on the Lagrange interpolation functions are formulated as followsflip

$$C_{ij}^{(1)} = \begin{cases} \prod\limits_{k=1,k\neq i,j}^{N}(x_i - x_k) \Big/ \prod\limits_{k=1,k\neq i,j}^{N}(x_i - x_k) \prod\limits_{k=1,k\neq j}^{N}(x_j - x_k) \prod\limits_{k=1,k\neq j}^{N}(x_j - x_k) & (i \neq j) \\ \sum\limits_{k=1,k\neq i}^{N} \frac{1}{(x_i - x_k)} & (i = j) \end{cases} \quad i,j = 1,2,\ldots,N \tag{22}$$

The higher order weighting coefficients associate to higher order derivatives can be calculated by matrix product of the weighting coefficients of the first and lower order ones and therefore any linear differential operators such as nonlocal differential operator $\bar{\pounds}$ can be treated in the same way. This is the main feature and most important advantage of DQM which has made it a simple numerical method as well as powerful one.

$$C_{ij}^{(2)} = C_{ik}^{(1)}C_{kj}^{(1)}, \quad C_{ij}^{(3)} = C_{ik}^{(1)}C_{kj}^{(2)}, \quad C_{ij}^{(4)} = C_{ik}^{(1)}C_{kj}^{(3)} \tag{23}$$

Approximating the spatial derivative in Eq. (16), one obtains

$$C_{ij}^{(4)}y_j + \left(\delta_{ij} - e^2C_{ij}^{(2)}\right)\bar{f}_j = 0 \tag{24}$$

Evaluating Eq. (24) in all of the Gauss points result in a set of nonlinear algebraic equations which can be shown in the following matrix form.

$$\mathbf{C}^{(4)}\mathbf{y} + \mathbf{T}(\bar{\mathbf{f}}) = 0 \tag{25}$$

in which $\mathbf{T} = \mathbf{I} - e^2\mathbf{C}^{(2)}$.

$\mathbf{I}$ is the identity matrix and the vector $\bar{\mathbf{f}}$ contains the nonlinear terms of electrostatic and intermolecular forces. The iterative techniques such as Newton–Raphson algorithm can be used to solve the final nonlinear algebraic equation (Eq. 25). The shear force and the bending moment in all points can be calculated by replacing the beam deflection obtained from Eq. (25), which are expressed as

$$\begin{aligned}\overline{\mathbf{M}} &= \mathbf{C}^{(2)}\mathbf{y} + e^2\bar{\mathbf{f}}\\ \overline{\mathbf{Q}} &= \mathbf{C}^{(1)}\overline{\mathbf{M}} = \mathbf{C}^{(3)}\mathbf{y} + e^2\mathbf{C}^{(1)}\bar{\mathbf{f}}\end{aligned} \tag{26}$$

4. Results and discussion

In this section based on the developed model the instability of nano-switches is investigated in two subsections. The pull-in instability behavior due to the applying electric voltage is presented at first and then the freestanding behavior is studied. The procedure to find the instability characteristics is explained at following. When the electric voltage increases, both the electrostatic and elastic restoring forces increase. Before instability, the forces balance each other and the beam remains in static equilibrium position. At a certain value of voltage which is called pull-in voltage, the beam cannot sustain the electrostatic force and becomes unstable. Further increasing the voltage will cause the beam to deflect dramatically and finally buckled. Near instability region the slope of u–β curve approaches to infinity because the beam takes large displacement due to tiny applied voltage. Therefore, the pull-in parameters for any given value of α and g/w can be obtained by setting $du/d\beta \to \infty$. This goal will be achieved easily by analytical methods as a closed form solution. In numerical methods the voltage increases from zero to the unstable value in a step by step manner and in each step a set of nonlinear algebraic equation is solved by the iterative procedures such as the Newton–Raphson algorithm. The step before whom the solution of the nonlinear equation set is not available gives the pull-in parameters. For cantilever beam the displacement at the tip position (u_{tip}) is plotted

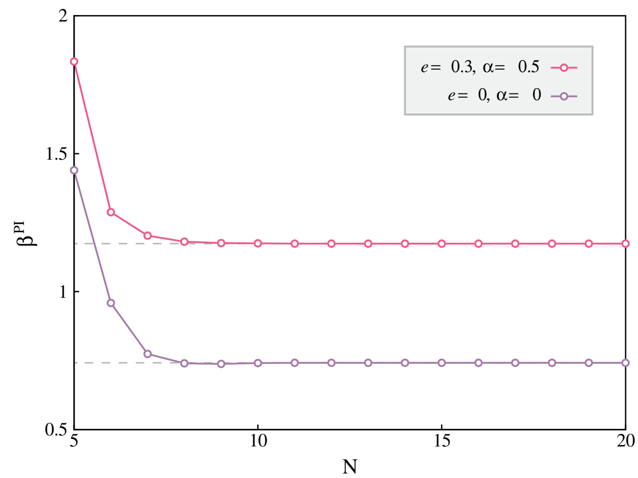

Fig. 2. Convergence of results versus the number of node in DQM ($n = 3$).

versus $\beta$to determine the pull in voltage β^{PI} and the pull-in tip deflection which is denoted by u_{tip}^{PI}. In clamped–clamped beam, the displacement at the middle point of the beam (u_{mid}) is plotted versus $\beta$to calculate β^{PI} and the pull-in mid-point deflection u_{mid}^{PI}. Similarly, the critical value of intermolecular force parameter α_n^{cr} and the maximum deflection associated to the instability point in freestanding behavior can be calculated by plotting u versus α_n and by setting $du/d\alpha_n \to \infty$.

4.1. The electrostatic behavior

In order to validate the obtained results, the proposed model is compared with those of numerical and analytical methods such as lumped parameter model (Lin and Zhao, 2003, 2005a), distributed parameter model (Ramezani et al., 2007) and linear distributed load (LDL) model used by Yang et al. (2008). The comparison of the results is performed in critically pull-in condition with and without fringing field effect and are presented in Table 1 for cantilever nano-beam when intermolecular forces and the small scale effect are neglected. As can be observed, the present results are quite coinciding with the numerical results which are obtained by MAPLE commercial software. The numerical algorithm used by MAPLE to solve nonlinear differential equation is based on the finite difference method (FDM). To compare with analytical ones, the results are close to the distributed parameter model and also with LDL model because of the same modeling of the nano-beam as a continuous system. But, the result of the lumped model which considers the system with single degree of freedom and neglects the bending deformation of the beam significantly differ with other results and underestimates the pull-in parameters. It can be found that the fringing field effect has a remarkable influence on the pull-in characteristics and could decrease the pull-in voltage. The sensitivity analysis of the results based on the number of the chosen Gauss nodes used in DQM must be established. For this purpose

Table 1
Comparison of results for cantilever nano-beam when nonlocal effect and intermolecular forces are neglected.

Model	u^{PI}		β^{PI}	
	$g/w = 0$	$g/w = 1$	$g/w = 0$	$g/w = 1$
Numerical (FDM)	0.436	0.478	1.680	1.174
DQM (present)	0.436	0.478	1.680	1.174
Distributed parameter (Ramezani et al., 2007)	0.472	0.517	1.827	1.274
Lumped parameter (Lin and Zhao, 2003, 2005a)	0.333	0.369	1.185	0.834
LDL (Yang et al., 2008)	0.333	0.369	1.616	1.136

Table 2
Comparison of dimensionless pull-in voltage β^{PI} for various value of nonlocal parameters when the Casimir force is considered.

Model	Cantilever			Clamped–Clamped		
	$e = 0.1$	$e = 0.2$	$e = 0.3$	$e = 0.1$	$e = 0.2$	$e = 0.3$
Numerical (FDM)	0.497	0.742	2.780	45	30.4	19.4
Present (DQM)	0.497	0.742	2.780	45	30.5	19.4
LDL (Yang et al., 2008)	0.452	0.637	1.322	50.41	27.29	13.95

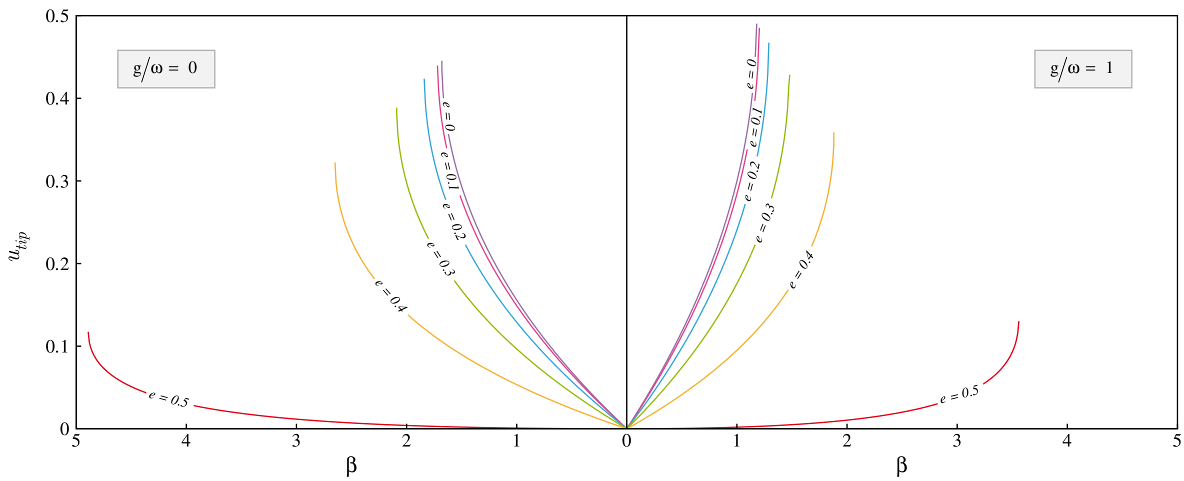

Fig. 3. Cantilever tip deflection as a function of parameter β for different nonlocal parameters when $\alpha = 0$, $\frac{g}{w} = 0, 1$.

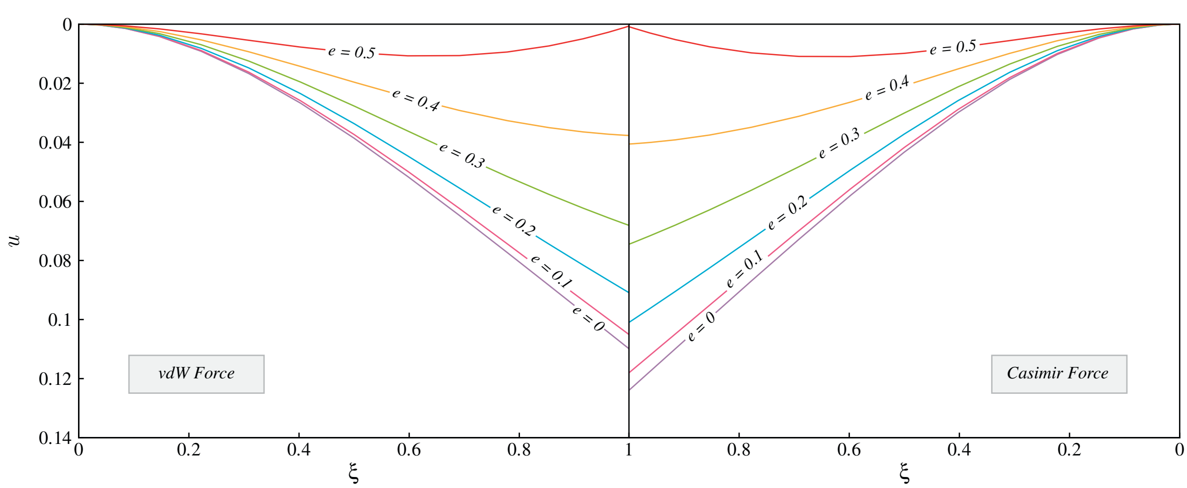

Fig. 4. Effect of nonlocal parameter and intermolecular forces on cantilever beam deflection for $\beta = 0$, $\alpha = 0.7$.

the convergence of dimensionless pull-in voltage for cantilever beam versus to variation of number of the Gauss nodes is plotted in Fig. 2 when $n = 4$. It can be seen that the accuracy of results is improved by increasing the number of nodes N and the convergence is obtained for $N \geqslant 9$. But, $N = 17$ has been exhibited high efficiency in the wide range of variation in parameters and is used cautiously for all of the following results due to inherent nonlinearity of governing equation and the associated boundary conditions.

Before investigating the influence of the small scale on the pull-in instability, it should be better to assure the accuracy and efficiency of the present model and DQM in such an extreme nonlinear equation and their complicated boundary conditions when the electrostatic and dispersion forces are considered with nonlocal effect. Therefore, it is possible to study all of the consideration, simultaneously. The results list in Table 2 for cantilever and clamped–clamped nano-beam and are compared with numerical and LDL model. The results demonstrate perfect agreement with numerical but LDL model still underestimate the pull-in parameters due to linearization of the external forces. The variation of tip deflection u_{tip} with respect to $\beta$for cantilever nano-beam is shown in Fig. 3 for different values of nonlocal parameter when the intermolecular parameter α is set to value 0. It can be found apparently from the figure that the nonlocal parameter increases the pull-in voltage and decreases the pull-in tip deflection. Therefore, the small scale effect causes the beam to become stiffen and delays the pull-in instability, consequently the beam collapse at higher value of β. Furthermore, Fig. 3 presents that the fringing field effect decreases the pull-in voltage and slightly increases the pull-in tip deflection.

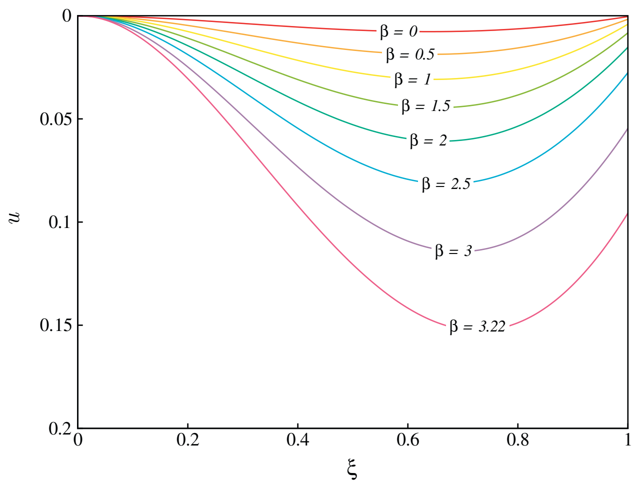

Fig. 5. Cantilever beam deflection with increasing voltage for $\alpha = 0.7,\ \frac{g}{w} = 0.5,\ n = 3$ and $e = 0.5$.

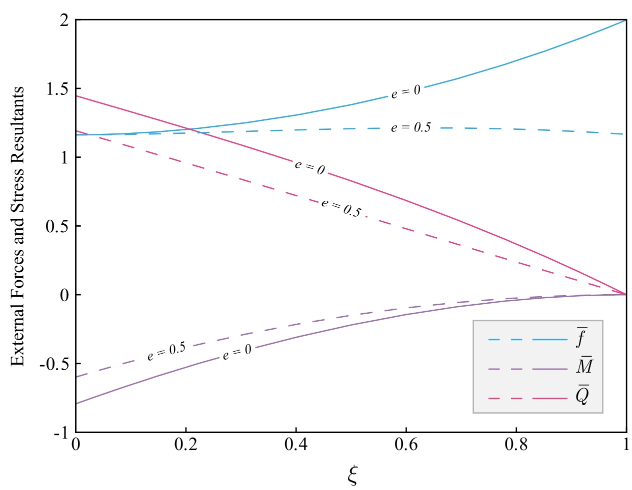

Fig. 6. External forces and stress resultant distribution for cantilever nano-beam with and without nonlocal effect when $\alpha = 0.5,\ \beta = 0.5,\ \frac{g}{w} = 0.5,\ n = 3$.

Fig. 4 illustrates the beam deflection for different nonlocal parameters and compares the result of the vdW and Casimir forces in the absence of electric voltage for cantilever nano-beam. As expected, the stiffening effect of the nonlocal beam can be found from the figure as well and the beam treated differently compared with classical beam (without nonlocal effect) especially in higher values of nonlocal parameter. When the nonlocal parameter increases, the beam tends to become stiffer and the work done by the electrostatic load enforces the beam to deform rather than to deflect, so that the beam experiences small displacement and large deformation. In addition, the beam deflection predicted by the Casimir force is greater than vdW force at the same value of e. It should be noted that the Casimir force is proportional to the inverse fourth power of the separation gap which led to act higher force on the beam in comparison with the vdW force which is proportional to the inverse third power of the separation. Thus, the beam under the Casimir force pulls-in on the voltage lower than the vdW force.

Because of the distinct behavior of the cantilever nano-beam in the presence of nonlocal effect as mentioned above in Fig. 4, the beam deflection is depicted in Fig. 5 when the electric voltage approaches from zero to pull-in value for $e = 0.5$. When the applied voltage increases, the beam not only deflects naturally but also prefers to deform as well. It is not possible to easily clarify this behavior unless to study the total external force which includes the electrostatic and the intermolecular attractions. That is why; the total external force and the resulting moment and shear forces in pull-in condition are drawn in Fig. 6. As can be observed, the total external force distribution which is dependent on the beam deflection, increases from the clamped end and then decreases to the free end of the beam due to the small scale effect and finally causes the beam to undergo deformation as well as deflection.

To go through in details and obtain deep understanding of cantilever nano-switches behavior, Fig. 7 shows how the intermolecular and fringing field effects in combination with nonlocal parameter affect the pull-in characteristics. As seen, when the intermolecular forces increase due to the small initial gap and the long narrow beam, the pull-in voltage decreases. In addition, increasing the nonlocal parameter results in increasing the pull-in voltage while the fringing field effect shifts instability to the lower value of voltage which also was shown previously in Fig. 3.

The same survey is performed for clamped–clamped nano-beam to study the electrostatic instability and extract the related pull-in parameters. The main features of doubly clamped nano-beam is investigated and briefly discussed via some figures. The variation of the beam deflection associated to middle point u_{mid} with respect to dimensionless voltage parameter $\beta$is displayed in

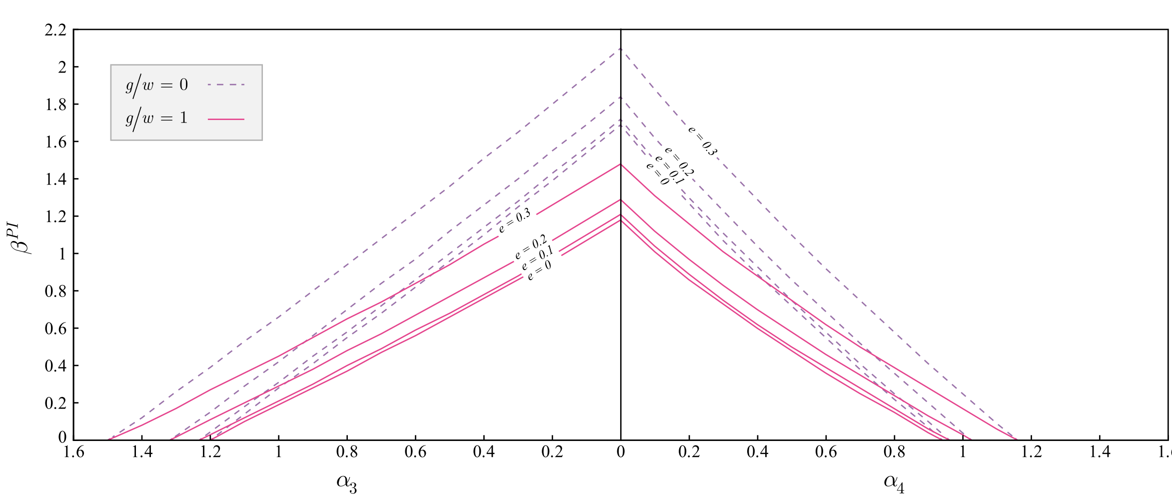

Fig. 7. Effect of intermolecular forces and nonlocal parameter on pull-in parameter β^{PI} for nano-cantilever beam.

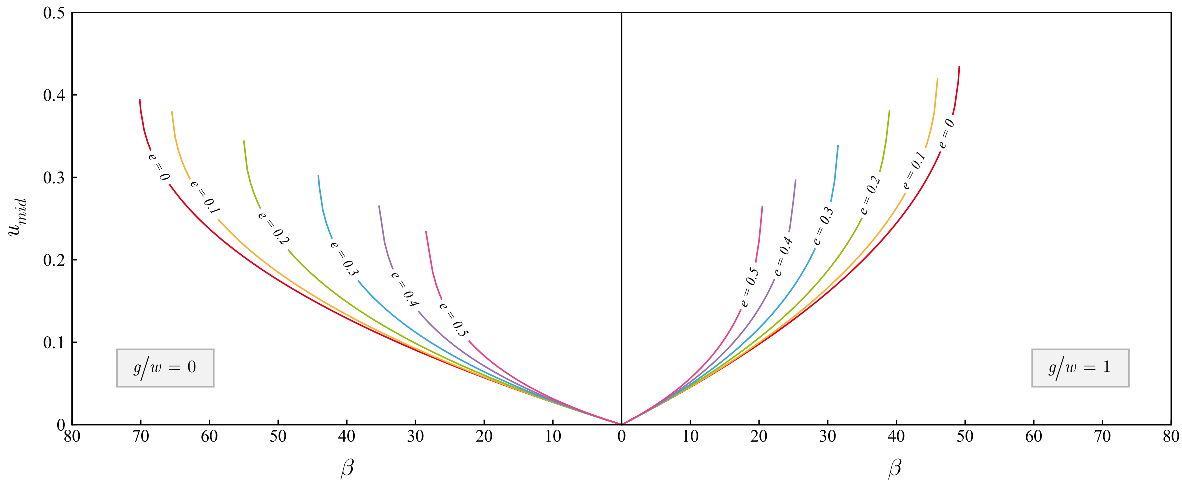

Fig. 8. Clamped–clamped mid-deflection as a function of parameter β for different nonlocal parameters when $\alpha = 0$, $\frac{g}{w} = 0, 1$.

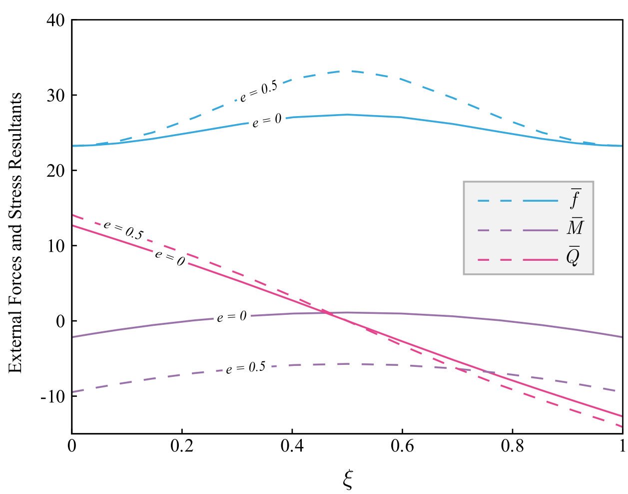

Fig. 9. External forces and stress resultant distribution for clamped–clamped nano-beam with and without nonlocal effect when $\alpha = 10$, $\beta = 10$, $\frac{g}{w} = 0.5$, $n = 3$.

Fig. 8 and the effect of g/w is also shown. The results demonstrate that the nonlocal parameter decreases the pull-in voltage β^{PI} and the pull-in deflection u^{PI}_{mid}. So, the doubly clamped nano-beam becomes soften due to the nonlocal effect in contrast to the nano-cantilever beam and because of such strong fixed end boundary conditions, the instability of doubly clamped nano-beam occurs in the voltage much higher than the cantilever type beam. Furthermore, the fringing field effect decreases β^{PI} and increases u^{PI}_{mid}. The external forces and the resulting stress resultants are depicted in Fig. 9 when the vdW force is considered. Comparing with nano-cantilever beam, there is not any strange behavior under small scale effect and the distributions are as usual as local beam. Fig. 10 shows the effect of intermolecular forces on pull-in voltage for different values of nonlocal parameter with and without fringing filed effect. The intermolecular attraction like fringing field effect decreases β^{PI} and as predicted before, the beam under the Casimir force collapses in the voltage lower than the vdW force.

4.2. The freestanding behavior

This subsection is devoted to study the freestanding behavior of nano-switches. The nano-switch may collapse onto the ground electrode due to intermolecular interactions even if no voltage is applied. When the length of nano-devices (nano-beam) is long or the gap between surfaces (initial gap) is small enough, the intermolecular forces is significant in the absence of electric voltage. This behavior is substantially important in designing NEMS for pre-

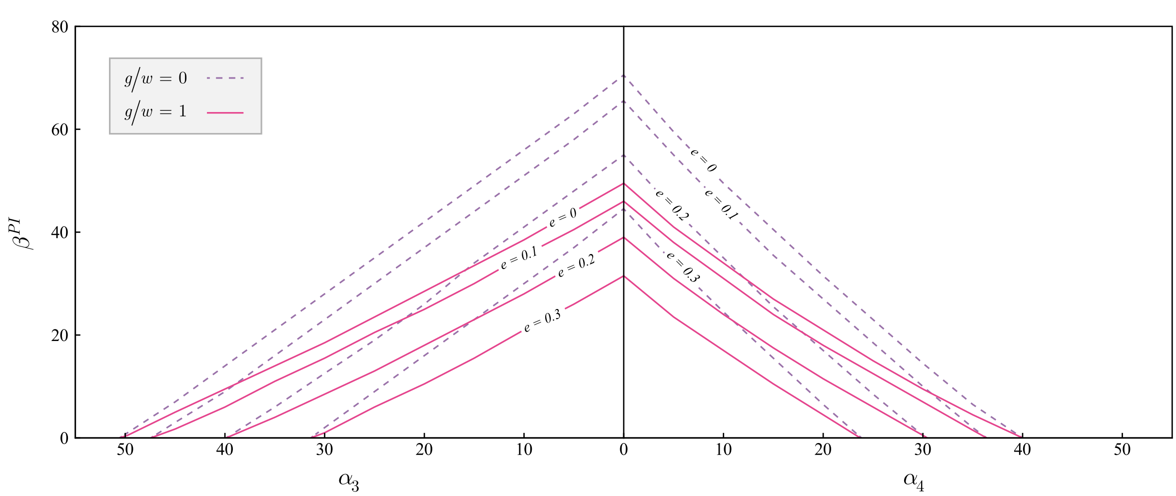

Fig. 10. Effect of intermolecular forces and nonlocal parameter on pull-in parameter β^{PI} for clamped–clamped nano-beam.

Table 3
Comparison of results for freestanding behavior of cantilever nano-beam when nonlocal effect is ignored.

Model	Van der Waals force		Casimir force	
	u_{tip}^{cr}	α_3^{cr}	u_{tip}^{cr}	α_4^{cr}
Numerical (FDM)	0.328	1.204	0.267	0.939
DQM (Present)	0.328	1.204	0.267	0.939
Distributed parameter (Ramezani et al., 2007)	0.359	1.313	0.289	1.025
Lumped parameter (Lin and Zhao, 2003, 2005a)	0.250	0.844	0.200	0.655
LDL (Yang et al., 2008)	0.250	1.151	0.200	0.894

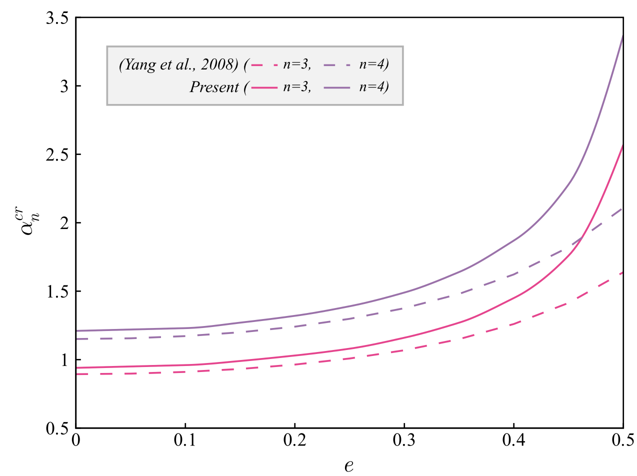

Fig. 11. Effect of nonlocal parameter on critical freestanding parameter α_n^{cr} for cantilever nano-beam.

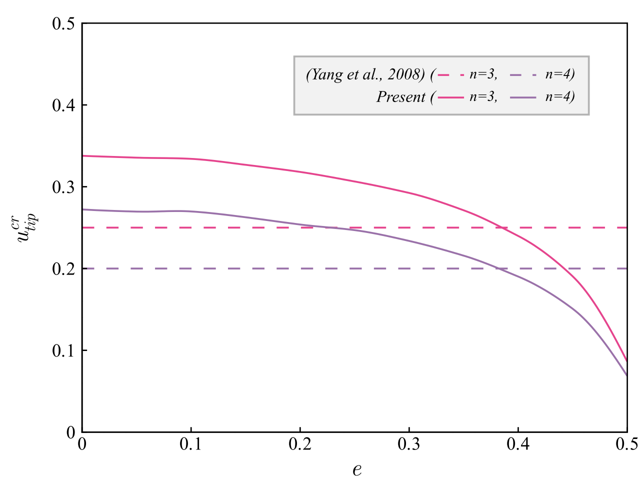

Fig. 12. Effect of nonlocal parameter on critical freestanding tip deflection u_{tip}^{cr} for cantilever nano-beam.

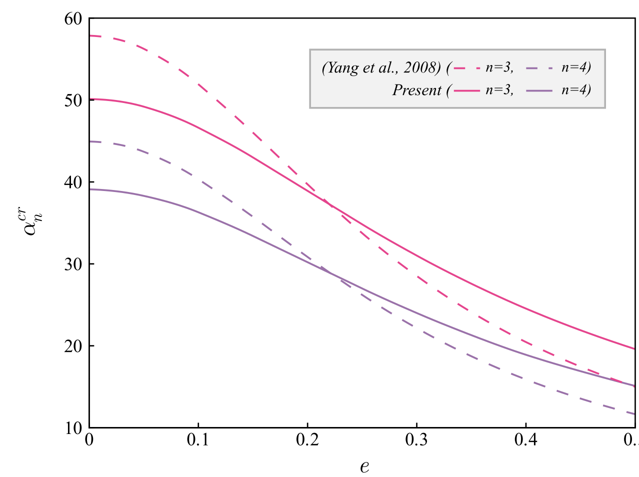

Fig. 13. Effect of nonlocal parameter on critical freestanding parameter α_n^{cr} for clamped–clamped nano-beam.

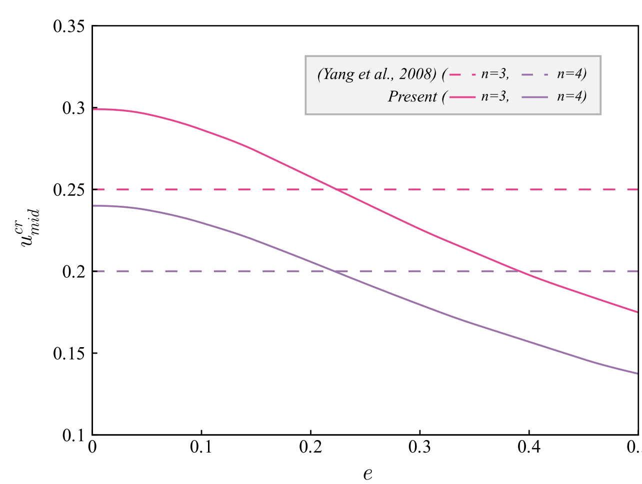

Fig. 14. Effect of nonlocal parameter on critical freestanding mid-deflection u_{mid}^{cr} for clamped–clamped nano-beam.

dicting the minimum initial gap and also maximum freestanding length of nano-beam which is known as detachment length to prohibit self-buckling. Knowing these parameters help designers to ensures that the switch does not stick to the ground automatically. The detachment length and the minimum initial gap are summarized in dimensionless parameter α and can be computed easily.

Table 3 compares the results for critical values of dimensionless intermolecular parameter α_n^{cr} and the corresponding beam displacement at the tip position u_{tip}^{cr} for the cantilever nano-beam when no voltage is applied and the nonlocal effect is ignored. The comparison is concluded for the vdW and Casimir effects and again the same agreement with other methods is obtained. Fig. 11 gives the critical intermolecular force parameter α_n^{cr} as a function of nonlocal parameter for cantilever nano-beam and the critical tip deflection u_{tip}^{cr} versus nonlocal parameter is shown in Fig. 12 for the vdW and Casimir forces. The figures also reveal the comparisons of the present DQM results with LDL results. The results in Fig. 11 shows good agreement between two methods and indicates that α_n^{cr} increases with an increasing in nonlocal parameter. As can be observed in Fig. 12, based on LDL results the nonlocal parameter does not affect u_{tip}^{cr} due to linearization of intermolecular forces whereas our result shown considerably decreasing. Based on the obtained results, the stiffening behavior of nano-cantilever beam can also be found once again due to the small scale effect.

The same approach is used to investigate the freestanding behavior of clamped–clamped nano-beam. Fig. 13 shows α_n^{cr} versus nonlocal parameter for the Casimir and vdW forces and the critical mid-point deflection u_{mid}^{cr} as a function of nonlocal parameter is illustrated in Fig. 14. In contrast to cantilever beam, the nonlocal parameter causes to decrease α_n^{cr} due to softening effect of nonlocal aspects and also decreases the critical beam deflection.

5. Conclusion remarks

Based on Eringen's nonlocal elasticity theory, the nonlinear static governing equation of beam-typed NEMS subjected to electrostatic voltage and intermolecular attractive forces is derived. The DQM is used as a solution method to numerically solve the governing equation and the associated boundary conditions. According to the present model, the effect of nonlocal parameter on the pull-in instability and freestanding behavior of nano-switches are studied in details.

It is observed that the cantilever and the doubly clamped nano-beam treated differently under the small scale effect. For cantilever nano-beam, the nonlocal effect increases the pull-in voltage while the corresponding critical deflection decreases. It is in contrast to clamped–clamped nano-beam which results in lower pull-in voltage and pull-in deflection. The nonlocal cantilever nano-beam shows stiffening effects due to the small scale effect and tends to become stiffer compared to local one. Furthermore, in the high values of the nonlocal parameter, the beam undergoes small deflection and large deformation and the work done by the electrostatic and intermolecular attractions enforce the beam to deform rather than to deflect whereas doubly clamped nano-beam deflects such as classical beam and displays softening effects. It is shown that the fringing field effect has the same influence on both cantilever and clamped–clamped nano-beam which causes to decrease the non-dimensional pull-in voltage.

The freestanding behavior is studied through some figures with respect to the variation of nonlocal parameter. The results show that in the presence of nonlocal effect, the critical intermolecular parameter which containing the minimum initial gap and the detachment length increases for cantilever nano-beam and decreases for clamped–clamped nano-beam due to stiffening and softening effects, respectively.

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