



Role of glueballs in non-perturbative quark–gluon plasma

Nikolai Kochelev^{a,b,*}, Dong-Pil Min^a

^a Department of Physics and Astronomy, Center for Theoretical Physics, Seoul National University, Seoul 151-747, Republic of Korea

^b Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Moscow region 141980, Russia

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Abstract

Discussed is how non-perturbative properties of quark gluon plasma, recently discovered in RHIC experiment, can be related to the change of properties of scalar and pseudoscalar glueballs. We set up a model with the Cornwall–Soni’s glueball–gluon interaction, which shows that the pseudoscalar glueball becomes massless above the critical temperature of deconfinement phase transition. This change of properties gives rise to the change of sign of the gluon condensate at $T > T_c$. We discuss the other physical consequences resulting from the drastic change of the pseudoscalar glueball mass above the critical temperature.

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1. Introduction

The results obtained recently at RHIC suggest the formation of a new phase of nuclear matter, i.e., the strongly interacting quark–gluon plasma, in high energy heavy ion collision [1,2]. The origin of such phase might be related to the survival of some strong non-perturbative QCD effects above the deconfinement temperature (see, for example, [3,4]). This conjecture is supported by the lattice result for the pure $SU(3)_c$ theory [5], which shows that the gluon condensate changes its sign and remains to be large till very high temperature as the system crosses over the deconfinement temperature.¹ This behavior of gluon condensate forms a clear contrast to that of quark which vanishes at the deconfinement transition [6]. Therefore, it is evi-

dent that the gluonic effect plays the leading role in the dynamic of quark–gluon plasma (QGP) above the deconfinement temperature and may give the clue to the unexpected properties of matter produced at RHIC. The change of sign of the gluon condensate at the finite temperature may give the influence on the glueball properties, because it is well known at zero temperature that the gluon condensate plays the role of fixing the mass scale of glueball and therefore its dynamics [7–9]. It is our goal of this work to investigate the influence at finite temperature.

We anticipate the change of the structure of QCD vacuum at $T > T_c$, which may lead to the strong modification of the properties of scalar and pseudoscalar glueballs above the deconfinement temperature. Some possible signatures of the change of properties are already discussed for the scalar glueball in QGP in the recent papers by Vento [10], but not for the pseudoscalar glueball. Indeed, glueball properties should be strongly modified along with the change of gluon condensate. In the normal QCD, however, the role of two types of glueball may not be important because of their large masses. In this Letter within effective Lagrangian approach, based on the specific non-perturbative gluon–glueball interaction, it is suggested that the pseudoscalar glueball can be much lighter above T_c .

* Corresponding author at: Department of Physics and Astronomy, Center for Theoretical Physics, Seoul National University, Seoul 151-747, Republic of Korea.

E-mail addresses: kochelev@ theor.jinr.ru (N. Kochelev), dpmin@snu.ac.kr (D.-P. Min).

¹ In the full QCD there is a shift of temperature where the sign change of the gluon condensate is taking place [5].

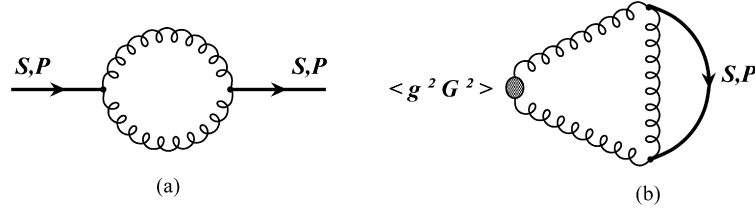


Fig. 1. (a) Diagrams for the contribution to the glueball mass, (b) and for the contribution to gluon condensate.

Therefore its role in such high temperature QGP could be substantially enhanced. This light glueball may lead the equation of state, and play the role of the mediator of the strong interaction between gluons in this environment, more than what the screening perturbative gluon does.

2. Glueball–gluon interaction and glueball mass above T_c

In [11] Cornwall and Soni proposed a simple form of the effective scalar and pseudoscalar glueball interaction with gluons,

$$\mathcal{L}_{Ggg} = \frac{b_0}{16\pi \langle S \rangle} \alpha_s G_{\mu\nu}^a G_{\mu\nu}^a S + \frac{\xi b_0}{16\pi \langle S \rangle} \alpha_s G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a P, \quad (1)$$

where $G_{\mu\nu}^a$ is gluon field strength and $\tilde{G}_{\mu\nu}^a = \epsilon_{\mu\nu\alpha\beta} G_{\alpha\beta}^a/2$. S and P are scalar and pseudoscalar glueball fields, respectively, and $b_0 = 11N_c/3$ for pure $SU(3)_c$, $\xi (\approx 1)$ is the parameter of the violation of S – P symmetry,² and $\langle S \rangle$ is the non-vanishing expectation value of scalar glueball field with respect to the vacuum. They incorporate the low-energy QCD theorems by identifying correlators with gluonic operators

$$\begin{aligned} J_S &= \alpha_s G_{\mu\nu}^a G_{\mu\nu}^a, \\ J_P &= \alpha_s G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a. \end{aligned} \quad (2)$$

Note that $\langle S \rangle$ is related to the gluon condensate $\langle g^2 G^2 \rangle$ with the strong coupling constant g as follows

$$\langle S \rangle^2 = \frac{b_0}{32\pi^2} \langle g^2 G^2 \rangle. \quad (3)$$

This equation can be rewritten in more convenient way with the mass of scalar glueball M_S at zero temperature by low energy theorem. (See [13] and references therein.)

$$f_S^2 M_S^2 \simeq \frac{8}{b_0} \langle g^2 G^2 \rangle, \quad (4)$$

where f_S is the residue

$$f_S M_S^2 = \langle 0 | J_S | S \rangle. \quad (5)$$

From the comparison of Eqs. (3) and (4) we get the simple relation

$$\langle S \rangle \simeq \frac{b_0}{16\pi} f_S. \quad (6)$$

² It can be shown within instanton model for QCD vacuum [12] that the effective scalar and pseudoscalar couplings in Eq. (7) should be identical due to the self-duality of instanton field.

Therefore, Eq. (1) can be rewritten for the zero temperature as

$$\mathcal{L}_{Ggg} = \frac{1}{f_S} (\alpha_s G_{\mu\nu}^a G_{\mu\nu}^a S + \xi \alpha_s G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a P). \quad (7)$$

The effective mass of glueball at $T > T_c$ for pure $SU(3)_c$ can be calculated by considering the contribution of the diagram presented in Fig. 1(a) to the mass operator $\Pi(p)$ by using standard methods of finite-temperature field theory [14]. The result is following

$$\begin{aligned} \Pi(p)_{S,P} &= i \frac{4\alpha_s^2}{\pi^4 f_S^2} \left[\frac{1}{2} \int_{-i\infty}^{i\infty} dk_0 d\vec{k} (D(k_0, \vec{k}) + D(-k_0, \vec{k})) \right. \\ &\quad \left. + \int_{-i\infty+\epsilon}^{i\infty+\epsilon} dk_0 d\vec{k} (D(k_0, \vec{k}) + D(-k_0, \vec{k})) \frac{1}{e^{k_0/T} - 1} \right], \end{aligned} \quad (8)$$

where

$$D(k_0, \vec{k}) = \frac{F_{S,P}(k_1, k_2)}{(k_1^2 - m_g^2)(k_2^2 - m_g^2)} F_{\text{cut}}(k_1^2, k_2^2), \quad (9)$$

and a simple form for non-perturbative gluon propagator as a free propagator with the effective mass m_g has been used. In Eq. (9), $k_1 = p + k$, $k_2 = k$, and function F_{cut} provide the ultraviolet cut-off in the Euclidean space. In Eq. (9), numerator for scalar and pseudoscalar glueballs can be rewritten as

$$\begin{aligned} F(k_1, k_2)_S &= 2(k_1 \cdot k_2)^2 + k_1^2 k_2^2 \\ &= p^2 k^2 + 6k^2 (p \cdot k) + 2(p \cdot k)^2 + 3k^4, \\ F(k_1, k_2)_P &= 2\xi^2 ((k_1 \cdot k_2)^2 - k_1^2 k_2^2) \\ &= 2\xi^2 ((p \cdot k)^2 - p^2 k^2). \end{aligned} \quad (10)$$

As usual, we define the effective mass of the glueball as a static infrared limit of mass operator

$$M_{S,P}^2 = \Pi_{S,P}(p_0 = 0, \vec{p} \rightarrow 0). \quad (11)$$

After Wick rotation to Euclidean space and with assumption about Gaussian form of cut-off function in this space

$$F_{\text{cut}}^E(k_1^2, k_2^2) = e^{-\Lambda^2(k_1^2 + k_2^2)}, \quad (12)$$

the calculation of the integrals in Eq. (9) leads to

$$\begin{aligned} M_S^2(T) &\approx \frac{72\alpha_s^2}{\pi^2 \Lambda^4 f_S^2} \int_0^\infty \frac{t dt}{(2+t)^4} e^{-tm_g^2(T)\Lambda^2}, \\ M_P^2(T) &= 0, \end{aligned} \quad (13)$$

where we assume that at $T_c < T < 2T_c$ the effective mass of gluon coincides with its thermal mass [15]

$$m_g = m_g(T) \approx 3T. \quad (14)$$

We will neglect the contribution coming from the second term in the right-hand side of Eq. (8), because, in our simple model with heavy effective gluon thermal mass such contribution should be suppressed by factor $T^2/m_g^2(T) \approx 1/10$.³

Most remarkable result of Eq. (13) is that the pseudoscalar glueball mass vanishes at $T > T_c$ due to the interaction Eq. (7). We should point out that below T_c one can expect that the glueball interaction with gluons has no such a simple form because confinement forces should also be included in the consideration. The zero effective mass of the pseudoscalar glueball in QGP follows from the specific Lorenz structure of pseudoscalar glueball–gluon interaction and, therefore, this result does not depend on some particular values of the our model parameters. Note that the mass of pseudoscalar glueball, arising from the interaction Eq. (7), is proportional to so-called topological susceptibility $\chi(T)$

$$\begin{aligned} M_P^2(T) &= -\frac{2i}{f_S^2} \int d^4x \langle T \alpha_s G_{\mu\nu}^a(x) \tilde{G}_{\mu\nu}^a(x) \alpha_s G_{\mu\nu}^a(0) \tilde{G}_{\mu\nu}^a(0) \rangle \\ &= \frac{128\pi^2}{f_S^2} \chi(T), \end{aligned} \quad (15)$$

which vanishes above T_c at the lowest order of our model. Zero topological susceptibility above deconfinement temperature is in good agreement with the recent lattice calculation for pure $SU(3)_c$ which shows a sharp drop of topological susceptibility across the deconfinement transition [16] and suggests much simpler topological structure of the strong interaction above T_c in comparison with the confinement regime. In contrast to the pseudoscalar glueball, the scalar glueball remains rather massive even for $T > T_c$. The temperature dependency of glueball masses is presented in Fig. 2. In the region $0 < T < T_c$ we assume that values of glueball masses are equal to their zero temperature values, which is consistent with the observation produced from lattice calculations of very small change of the gluon condensate in this temperature interval [5,6]. The value of residue $f_S = 0.35$ GeV has been fixed from Eq. (4) by using $\langle g^2 G^2 \rangle \approx 0.5$ GeV⁴ for the value of gluon condensate at zero temperature [17] and value of scalar glueball mass $M_S(0) \approx 1.7$ GeV from quenched lattice results [18]. We use also the recent lattice results for thermal mass of gluons above deconfinement temperature, $\alpha_s \approx 0.5$ and value of the deconfinement temperature for pure $SU(3)_c$, $T_c \approx 270$ MeV [19]. For the cut-off parameter Λ in Euclidean space in Eq. (12), the value $\Lambda \approx 1/M_S(0)$ has been taken.⁴ We should point out

³ To take into account of this effect is beyond the scope of our accuracy estimations involved in some values of glueball–gluon couplings, shape of cut-off function and their possible temperature dependency. Such contribution, however, could not change our conclusion of vanishing of pseudoscalar glueball mass above T_c .

⁴ We assume that f_S and Λ do not change their values over the deconfinement transition.

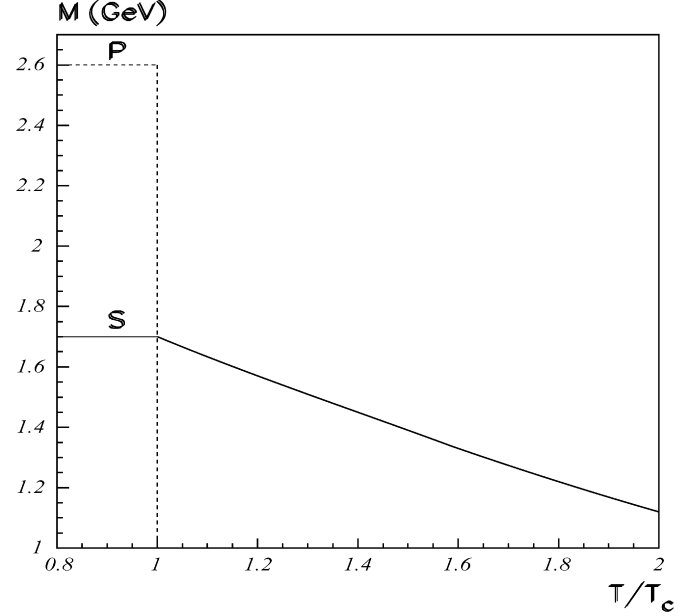


Fig. 2. The solid (dotted) line is the temperature dependence of scalar (pseudoscalar) glueball mass.

that at $T_c < T < 2T_c$, the mass of scalar glueball is large, $M_S(T) \gg T$, therefore, one can not expect significant contribution of this glueball to the bulk properties of QGP in this range of temperatures. On the other hand, the massless pseudoscalar glueball should play an important role in the thermodynamics of QGP at $T > T_c$. Indeed, the simple estimation shows that the ratio of the scattering amplitudes of gluon–gluon in the t -channel exchanged by massless glueball to that by massive gluon, Eq. (14), should be about $\alpha_s m_g^2(T)/f_S^2 \gg 1$ at $T > T_c$. Therefore, one might expect the dominance of glueball exchange over screening perturbative one-gluon exchange at $T > T_c$.

3. Gluon condensate

The contribution of glueballs to the gluon condensate in the lowest order of the effective coupling constant is presented in Fig. 1(b). The result of calculation is

$$\langle g^2 G^2(T) \rangle_{S,P} = \frac{24\alpha_s^3}{\pi^3 f_S^2 \Lambda^6} \int_0^1 dx \int_0^\infty dt \int_0^\infty dy \Phi_{S,P}(t, x, y), \quad (16)$$

where

$$\begin{aligned} \Phi_S(t, x, y) &= \frac{ty^2(1-x)}{(1+y)^6(t+m_g^2(T)\Lambda^2)} \\ &\times (3t(1+y)^2 + t^3x^4y^4 + 5t^2x^2y^2(1+y)) \\ &\times \exp\left[-\frac{t(1+y(1+x) + y^2x(1-x))}{1+y}\right. \\ &\left. - yxM_S(T)^2\Lambda^2 - y(1-x)m_g^2(T)\Lambda^2\right], \end{aligned}$$

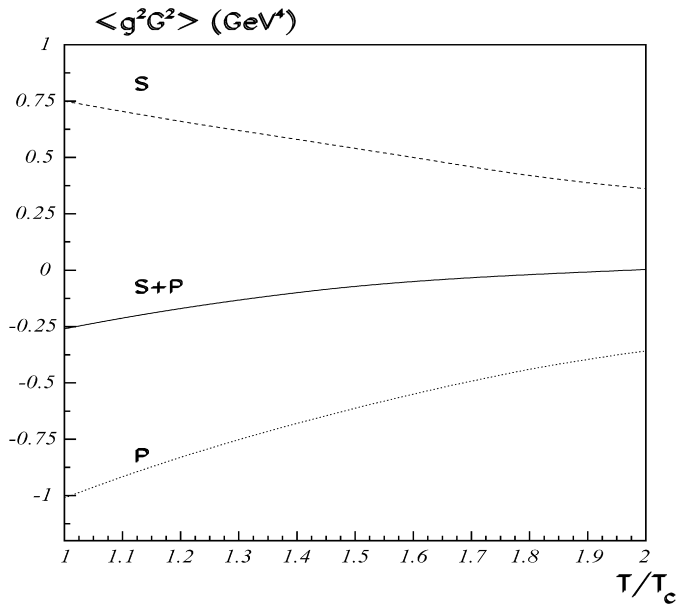


Fig. 3. The temperature dependency of the gluon condensate at $T > T_c$. The solid line is total condensate, the dashed (dotted) line is the scalar (pseudoscalar) glueball contributions.

$$\begin{aligned} \Phi_P(t, x, y) = & -\frac{\xi^2 t y^2 (1-x)}{(1+y)^5 (t + m_g^2(T) \Lambda^2)} \\ & \times (3t(1+y) + t^2 x^2 y^2) \\ & \times \exp \left[-\frac{t(1+y(1+x) + y^2 x(1-x))}{1+y} \right. \\ & \left. - yx M_P(T)^2 \Lambda^2 - y(1-x) m_g^2(T) \Lambda^2 \right]. \end{aligned}$$

In Fig. 3 the dependency of the gluon condensate on temperature for $T_c < T < 2T_c$ is shown. We should mention that the contribution of the pseudoscalar glueball to gluon condensate is proportional to ξ^2 . Therefore it is quite sensitive to the degree of violation of the S – P symmetry shown in Eq. (7). In our numerical estimation we assume that parameter $\xi \approx 1$ in the light of the Peccei–Quinn (PQ) mechanism [20] (see discussion in [11]). The decrease of the ξ value will lead to the decrease of the pseudoscalar glueball contribution to the condensate.⁵ At zero temperature by using Eq. (16) we get the following values of condensates

$$\begin{aligned} \langle g^2 G^2(0) \rangle_{\text{total}} &= 0.47 \text{ GeV}^4, & \langle g^2 G^2(0) \rangle_S &= 1.31 \text{ GeV}^4, \\ \langle g^2 G^2(0) \rangle_P &= -0.84 \text{ GeV}^4, \end{aligned} \quad (17)$$

where we use the quenching result $M_P = 2.6 \text{ GeV}$ for the mass of pseudoscalar glueball at zero temperature [18]. One can see that scalar glueball contribution to gluon condensate is positive and pseudoscalar glueball contribution is negative.

Total gluon condensate strongly depends on the masses of glueballs. Note that its value at zero temperature is taken to be in agreement with QCD sum rule result [17]. It is evident from Fig. 3 that just above T_c the negative contribution of massless pseudoscalar glueball to gluon condensate is greater than the positive contribution coming from massive scalar glueball and this is a reason of change of the sign of gluon condensate above the deconfinement temperature in our model based on the effective glueball–gluon interaction Eq. (7). Unfortunately the direct comparison of our result with lattice data [5] is not possible due to the necessity of the subtraction of the perturbative contribution to $\langle g^2 G^2(T) \rangle$ from the data.⁶ We would like to emphasize that in our calculation the effective dimensionless coupling for interaction Eq. (7) is very small, $g_{\text{eff}} = \alpha_s^2 / (16\pi^2 f_S^2 \Lambda^2) \ll 1$, due to the factor of $1/(16\pi^2)$ which comes from the loop momentum integration for the diagrams presented in Fig. 1(b). Therefore, one can safely neglect high order corrections coming from effective interaction Eq. (7).

4. Conclusion

In summary, we consider the properties of scalar and pseudoscalar glueballs in quark–gluon plasma. In the effective Lagrangian approach, based on the low-energy QCD theorems, it is suggested that scalar glueball remains massive above deconfinement temperature. At the same time, pseudoscalar glueball changes its properties in QGP in a drastic way. Indeed, this glueball become massless at $T > T_c$ and therefore it can contribute strongly to the bulk properties of QGP. We demonstrate that the disappearance of pseudoscalar glueball mass above the deconfinement temperature and its strong coupling to gluons gives the rise to the sign change of the gluon condensate in the pure $SU(3)_c$ gauge theory as observed in the lattice calculations at $T \approx T_c$. The strong non-perturbative coupling of the glueball to the gluons leads to the conjecture that one might expect that the role of very light pseudoscalar glueball in QGP must be quite similar to the role played by the massless pion in nuclear matter below deconfinement temperature. In spite of the rather simple form of non-perturbative glueball–gluon interaction and the neglect of high order non-perturbative and perturbative interactions, as well as the possible double-counting arising from the simultaneous consideration of hadronic and partonic degrees of freedom, the possibility of the existence of very light pseudoscalar glueball above T_c is worth being considered in realizing the survival of the strong coupling between gluons via pseudoscalar glueball in the region of the temperature above T_c . We like to mention on the lattice data in which bulk properties of QGP do not change much with the inclusion of light quarks [5]. This may suggest that the appropriate extension of our model to full QCD is possible [21].

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⁵ In our model the change of the sign of the total glueball contribution to the condensate at $T = T_c$ turns out to be possible only for the large value of that parameter, $\xi > 0.86$. Such large value of ξ for the full QCD could result from the restoration of $U(1)_A$ symmetry at $T > T_c$ and PQ mechanism which relates this symmetry to the S – P symmetry.

⁶ We are grateful to S.-H. Lee for the discussion of this problem.

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