Inflation at the TeV scale with a PNGB curvaton

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Abstract

We investigate a particular type of curvaton mechanism, under which inflation can occur at Hubble scale of order 1 TeV. The curvaton is a pseudo Nambu–Goldstone boson, whose order parameter increases after a phase transition during inflation, triggered by the gradual decrease of the Hubble scale. The mechanism is studied in the context of modular inflation, where the inflaton is a string axion. We show that the mechanism is successful for natural values of the model parameters, provided the phase transition occurs much earlier than the time when the cosmological scales exit the horizon. Also, it turns our that the radial mode for our curvaton must be a flaton field.
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Inflation is the only compelling theory to date for the solution of the horizon and flatness problems of the big bang cosmology as well as for explaining structure formation in the Universe. Recent precise observations have confirmed the basic predictions of the inflationary paradigm by ascertaining the spatial flatness of the Universe and the approximate scale invariance of the density perturbations, which give rise to the anisotropy of the Cosmic Microwave Background Radiation (CMBR) and seed structure formation. These exciting developments have rendered the inflationary paradigm a necessary extension to the hot big bang standard cosmology.

In the light of precision data, inflation model-building can be upgraded beyond the simple single-field stage of its early beginnings. Indeed, more complex and realistic models of inflation, with tighter connections to the theory, less fine tunning and enhanced predictability and falsifiability are now possible to construct, making use of the rich content of particle physics. A first such example is the well-known hybrid inflation model [1], which couples the inflaton field to the Higgs field of a Grand Unified Theory (GUT) in order to obtain without tunning the desired false vacuum energy scale [2]. In hybrid inflation the inflationary period is terminated through the dynamics of this other field.

In an analogous manner, one can attribute the generation of density perturbations during inflation to a field other than the inflaton [3]. This so-called curvaton field allows inflation to take place at a much lower energy scale than the typically required GUT-scale [4] and, in general, may relax a number of constraints regarding inflation model-building [5]. Low-scale inflation can revamp a number of inflation models that are well motivated on particle physics grounds [4]. It is important to stress here that the curvaton is *not* an ad hoc additional degree of freedom introduced "by hand", but it may be a realistic field, already present in simple extensions of the standard model. Indeed, many such examples exist in the literature [6,7].

However, even when a curvaton field is considered, there exists a lower bound for the inflationary scale, which, for generic curvaton models, can be quite tight [8]. This lower bound can be substantially relaxed for certain types of curvaton models [9], which enables inflation to be directly connected to realistic, beyond the standard model physics.

In this Letter I present a curvaton model which allows inflation at a Hubble scale as low as 1 TeV. The curvaton field is a pseudo Nambu–Goldstone boson (PNGB), whose order parameter is substantially increased after the cosmological scales exit the horizon during inflation. As shown in [9], the result of this increase is to amplify the curvaton's perturbations. This enables even low-scale inflation to generate density perturbations of the observed amplitude. In the curvaton model presented, the increase of the PNGB order parameter follows a phase transi-

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tion during inflation, which releases the radial mode from the top of the potential hill.

The use of a PNGB curvaton is highly motivated because such a curvaton can be naturally light during inflation, since its mass is protected by the global U(1) symmetry [7]. This dispenses with the danger imposed by supergravity corrections, which typically lift the flatness of the scalar potential [10]. We investigate the performance of the curvaton model in the context of modular inflation, which corresponds to Hubble scale of order 1 TeV. Modular inflation is a well motivated model, which uses a string axion as the inflaton [11].

Let us begin by presenting the amplification mechanism for the curvature perturbations. We discuss here the case of an PNGB curvaton, whose order parameter has a different (larger) expectation value in the vacuum than during inflation and, in particular, when the cosmological scales exit the horizon. Thus, the potential for the curvaton field σ is

$$V(\sigma) = (v\tilde{m}_\sigma)^2 \big[1 - \cos(\sigma/v)\big]$$
$$\Rightarrow \quad V\big(|\sigma| < v\big) \simeq \frac{1}{2}\tilde{m}_\sigma^2 \sigma^2, \tag{1}$$

where $v = v(t)$ is the order parameter determined by the expectation value of the radial field $|\phi|$ and $\tilde{m}_\sigma = \tilde{m}_\sigma(v)$ is the mass of the curvaton at a given moment. In the true vacuum we have $v = v_0$ and $\tilde{m}_\sigma = m_\sigma$ with v_0 being the vacuum expectation value (VEV) of the radial field and m_σ being the mass of the curvaton in the vacuum.

Let us demonstrate that the curvaton perturbations can be amplified by the non-trivial evolution of the radial field. We begin by using the fact that [3]:

$$\zeta \sim \Omega_{\rm dec}\zeta_\sigma, \tag{2}$$

where $\zeta \simeq \sqrt{\mathcal{P}_\zeta} = 2 \times 10^{-5}$ is the curvature perturbation of the Universe, $\Omega_{\rm dec} \leqslant 1$ is the density fraction of the curvaton density over the density of the Universe at the time of the decay of the curvaton:

$$\Omega_{\rm dec} \equiv \frac{\rho_\sigma}{\rho}\bigg|_{\rm dec} \leqslant 1 \tag{3}$$

and ζ_σ is the curvature perturbation of the curvaton field σ, which is given by

$$\zeta_\sigma \sim \frac{\delta\sigma}{\sigma}\bigg|_{\rm dec} \sim \frac{\delta\sigma}{\sigma}\bigg|_{\rm osc}, \tag{4}$$

where 'osc' denotes the time when the curvaton oscillations begin. Note that, non-Gaussianity constraints from the observations from the WMAP satellite [12] restrict the range of $\Omega_{\rm dec}$ as follows:

$$10^{-2} \leqslant \Omega_{\rm dec} \leqslant 1. \tag{5}$$

In this Letter we consider the inflationary Hubble scale to be comparable to the tachyonic mass of the radial field, which determines the value of the order parameter of our PNGB curvaton. This means that the evolution of the radial field ceases at (or soon after) the end of inflation. Therefore, at the end of inflation, $v \to v_0$ and the mass of the curvaton assumes its vacuum

value m_σ. Hence, in the following *we assume that the curvaton mass has already assumed its vacuum value before the onset of the curvaton oscillations*. Consequently, the curvaton oscillations begin when

$$H_{\rm osc} \sim m_\sigma. \tag{6}$$

Before the oscillations begin the curvaton is overdamped and remains frozen. This means that $\theta_{\rm osc} \simeq \theta_*$, where the '$*$' denotes the values of quantities at the time when the cosmological scales exit the horizon during inflation and

$$\theta \equiv \sigma/v, \tag{7}$$

with $\theta \in (-\pi, \pi]$. Hence, for the curvaton fractional perturbation we find

$$\frac{\delta\sigma}{\sigma}\bigg|_* = \frac{\delta\theta}{\theta}\bigg|_* \simeq \frac{\delta\sigma}{\sigma}\bigg|_{\rm osc}. \tag{8}$$

Now, for the perturbation of the curvaton we have

$$\delta\sigma_* = \frac{H_*}{2\pi}. \tag{9}$$

We assume that the expectation value of the radial field during inflation is smaller compared to its VEV by a factor

$$\varepsilon \equiv \frac{v_*}{v_0} \ll 1. \tag{10}$$

Combining Eqs. (8), (9) and (10), in view also of Eq. (7), we find

$$\delta\sigma_{\rm osc} \simeq \frac{H_*}{2\pi\varepsilon}, \tag{11}$$

which means that after the end of inflation, when the radial field assumes its VEV, *the curvaton perturbation is amplified by a factor ε^{-1}* (see Fig. 1). From Eqs. (2) and (4) we have

$$\sigma_{\rm osc} \sim (\Omega_{\rm dec}/\zeta)\delta\sigma_{\rm osc}. \tag{12}$$

Using Eqs. (9) and (11), we can recast the above as

$$\sigma_{\rm osc} \sim \frac{H_* \Omega_{\rm dec}}{\varepsilon\zeta}. \tag{13}$$

We may obtain a lower bound on ε as follows:

$$\frac{\delta\sigma_*}{\sigma_*} \leqslant 1 \quad \Rightarrow \quad \varepsilon \geqslant \varepsilon_{\rm min} \equiv \frac{H_*}{2\pi v_0}, \tag{14}$$

where we have used Eqs. (7), (9) and (10) and that $\sigma_{\rm osc} \lesssim v_0$. The above, however, is only the lowest ε in principle. The observational constraints on non-Gaussianity, impose a more stringent lower bound on ε. Indeed, from Eqs. (2), (4) and (5) we obtain

$$\frac{\varepsilon_{\rm min}}{\varepsilon} \lesssim \frac{\delta\sigma_{\rm osc}}{\sigma_{\rm osc}} \sim \zeta_\sigma \sim \frac{\zeta}{\Omega_{\rm dec}}$$
$$\Rightarrow \quad \varepsilon \gtrsim \frac{\Omega_{\rm dec}}{\zeta}\varepsilon_{\rm min} \geqslant 5 \times 10^2 \varepsilon_{\rm min}, \tag{15}$$

where we used that $\zeta = 2 \times 10^{-5}$.

Now, as is shown in [9], in the case when the curvaton oscillations begin after the radial field has attained its VEV, we
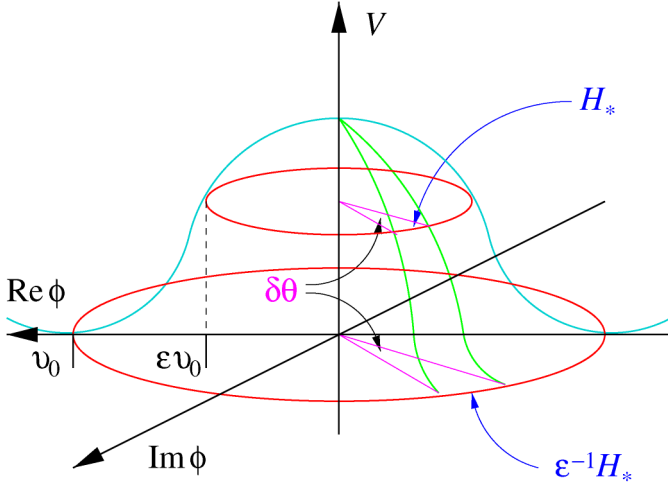
Fig. 1. Schematic representation of the amplification of the PNGB curvaton perturbation, when the order parameter v increases from the value it has when the cosmological scales exit the horizon $v_* = \varepsilon v_0$ to its vacuum value v_0. The perturbation at horizon crossing has amplitude $\delta\sigma_* \sim H_*$, which corresponds to a phase perturbation for the radial field $|\phi|$ of magnitude $\delta\theta = \delta\sigma_*/v_*$. As the order parameter grows $\delta\theta$ remains constant (the phase perturbation is frozen on superhorizon scales) but the amplitude of the curvaton perturbation is increased up to $\delta\sigma \sim \varepsilon^{-1} H_*$.

have[1]

$$H_* \sim \Omega_{\text{dec}}^{-2/5} \left(\frac{H_*}{\min\{m_\sigma, \Gamma_{\text{inf}}\}} \right)^{1/5} \left(\frac{\max\{H_{\text{dom}}, \Gamma_\sigma\}}{H_{\text{BBN}}} \right)^{1/5}$$
$$\times (\varepsilon\zeta)^{4/5} \left(T_{\text{BBN}}^2 m_{\text{P}}^3 \right)^{1/5}, \tag{16}$$

where Γ_{inf} and Γ_σ are the decay rates of the inflaton and the curvaton fields respectively, H_{dom} is the Hubble parameter at the time when the curvaton density dominates the Universe (if the curvaton does not decay earlier) and $H_{\text{BBN}} \sim T_{\text{BBN}}^2/m_{\text{P}}$ is the Hubble parameter at the time of big bang nucleosynthesis (BBN), with $T_{\text{BBN}} \sim 1$ MeV.

Now, we require that the curvaton decays before BBN, i.e. $\Gamma_\sigma > H_{\text{BBN}}$. We also have $\Gamma_{\text{inf}} \leqslant H_*$. Hence, Eq. (16) provides the following bound

$$H_* > \Omega_{\text{dec}}^{-2/5}(\varepsilon\zeta)^{4/5} \left(T_{\text{BBN}}^2 m_{\text{P}}^3 \right)^{1/5} \sim \left(\varepsilon^2/\Omega_{\text{dec}} \right)^{2/5} \times 10^7 \text{ GeV}. \tag{17}$$

Furthermore, we also note that

$$\Gamma_\sigma \geqslant \frac{m_\sigma^3}{m_{\text{P}}^2}, \tag{18}$$

where the equality corresponds to gravitational decay. The above can be shown [9] to imply that

$$H_* \geqslant \Omega_{\text{dec}}^{-1}(\varepsilon\zeta)^2 m_{\text{P}} \left(\frac{m_\sigma}{H_*} \right) \max\left\{ 1, \frac{m_\sigma}{\Gamma_{\text{inf}}} \right\}^{1/2}, \tag{19}$$

which results in the bound

$$H_* \geqslant \Omega_{\text{dec}}^{-1}(\varepsilon\zeta)^2 m_{\text{P}} \left(\frac{m_\sigma}{H_*} \right) \sim \left(\varepsilon^2/\Omega_{\text{dec}} \right) \times 10^{10} \text{ GeV} \left(\frac{m_\sigma}{H_*} \right). \tag{20}$$

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[1] We use natural units, where $c = \hbar = 1$ and Newton's gravitational constant is $G = 8\pi m_{\text{P}}^{-2}$ with $m_{\text{P}} = 2.44 \times 10^{18}$ GeV being the reduced Planck mass.

This bound may be relaxed if ε is small enough. In particular, for a PNGB curvaton we may have $m_\sigma < H_*$. Comparing the bound in Eq. (17) with the one in Eq. (20) we find that the former bound is more stringent if

$$\varepsilon < \frac{1}{\zeta\sqrt{\Omega_{\text{dec}}}} \left(\frac{T_{\text{BBN}}}{m_{\text{P}}} \right)^{1/3} \left(\frac{H_*}{m_\sigma} \right)^{5/6} \sim 10^{-3} \Omega_{\text{dec}}^{-1/2} (H_*/m_\sigma)^{5/6}. \tag{21}$$

Thus, for $\varepsilon \ll 1$, the second bound is typically less stringent than the first one.

During inflation, the evolution of the order parameter of the PNGB curvaton, is subject to an important constraint, which has to do with preserving the scale invariance of the spectrum of the curvature perturbations.

The amplitude of the density perturbations is determined by the magnitude of the perturbations of the curvaton field, which, in this scenario, apart from the scale of H_* is also determined by the amplification factor ε^{-1}. The latter is determined by the value of the order parameter v_* when the curvaton quantum fluctuations exit the horizon during inflation. A strong variation of $v(t)$ at that time results in a strong dependence of $\varepsilon(k)$ on the comoving momentum scale k, which would reflect itself on the perturbation spectrum threatening significant departure from scale invariance.

In Ref. [9] is was shown that, in order for this to be avoided, the rate of change of the radial field must be constrained as

$$|\dot{v}/v|_* = |\dot{\phi}/\phi|_* \ll H_*, \tag{22}$$

where $|\phi|$ is the radial field, which determines the value of the order parameter. In fact, the contribution to the spectral index due to the evolution of v is $\delta n_s = -2H_*^{-1}(\dot{v}/v)_*$. From the above it is evident that, in order not to violate the observational constraints regarding the scale invariance of the density perturbation spectrum, *the roll of the radial field has to be at most very slow when the cosmological scales exit the horizon*. However, this cannot remain so indefinitely because we need $v_0 \gg v_*$ to have substantial amplification of the perturbations (i.e. $\varepsilon \ll 1$). Consequently, v has to increase dramatically at some point *after* the exit of the cosmological scales from the horizon. This requirement is crucial for model-building.[2]

In our model, we will show that the evolution of v begins at a phase transition during inflation. Initially, the growth of v is very slow, but later, near the end of inflation, v grows substantially until it reaches its vacuum value v_0.

Let us now briefly describe the model of inflation. We are going to consider a PNGB curvaton σ whose radial field $|\phi|$ is of bare mass similar to the Hubble parameter during inflation, that is

$$m_\phi \sim H_*. \tag{23}$$

This has the advantage that the radial field rolls substantially by the end of inflation so that ε can be very small. In particular, we

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[2] The requirement in Eq. (22) may be even more fundamental in origin. Indeed, a PNGB with rapidly varying order parameter cannot be treated as an effectively free field. I would like to thank D.H. Lyth for pointing this out.

will assume that the tachyonic mass of the radial field is a soft mass generated by supersymmetry breaking and it is, therefore, roughly of the electroweak scale $m_{3/2}$. Hence, we consider inflation at the intermediate scale

$$V_*^{1/4} \sim \sqrt{m_{3/2} m_P} \sim 10^{10.5} \text{ GeV} \quad \Rightarrow \quad H_* \sim m_{3/2}. \quad (24)$$

A particular example of such an inflation model (but, by all means, not the only one) is modular inflation [11], where the inflaton field s is a string axion, whose flatness is lifted by gravity mediated supersymmetry breaking. In this model the inflationary potential is of the form:

$$V(s) = V_{\text{inf}} - \frac{1}{2} m_s^2 s^2 + \cdots, \quad (25)$$

where the ellipsis denotes terms, which are expected to stabilise the potential at $s_{\text{VEV}} \sim m_P$. Therefore, in the above we have

$$V_{\text{inf}} \sim (m_{3/2} m_P)^2 \quad \text{and} \quad m_s \sim H_{\text{inf}} \sim m_{3/2}, \quad (26)$$

where $H_{\text{inf}} \simeq \sqrt{V_{\text{inf}}}/3 m_P$.

This inflation model results in fast roll inflation [13], where

$$s = s_{\text{in}} \exp(F_s \Delta N) \quad \text{and} \quad F_s \equiv \frac{3}{2} \left(\sqrt{1 + 4c/9} - 1 \right)$$

$$\text{with} \quad c \equiv \left(\frac{m_s}{H_{\text{inf}}} \right)^2 \sim 1, \quad (27)$$

where ΔN is the number of the elapsed e-foldings. From the above one can easily obtain the inflation scale N e-foldings before the end of inflation, as

$$V(N) \simeq V_{\text{inf}} \left(1 - e^{-2F_s N} \right). \quad (28)$$

Even though fast-roll, modular inflation keeps the Hubble parameter H rather rigid. Indeed, it can be easily shown that

$$\epsilon = \frac{1}{2} F_s^2 \left(\frac{s}{m_P} \right)^2 \simeq \frac{1}{2} F_s^2 e^{-2F_s N} \ll 1, \quad (29)$$

because $F_s \sim 1$ and $s \ll m_P$ during inflation, with $\epsilon \ll 1$ being one of the so-called slow roll parameters defined as

$$\epsilon \equiv -\frac{\dot{H}}{H^2}. \quad (30)$$

For modular inflation the initial conditions for the inflaton field are determined by the quantum fluctuations, which send the field off the top of the potential hill. (The modulus can be considered to be originally placed at the local maximum because the latter can be thought of as a fixed point of the symmetries.) Hence, we expect that the initial value for the inflaton is

$$s_{\text{in}} \simeq H_{\text{inf}}/2\pi. \quad (31)$$

Using the above and considering that the final value is $s_{\text{VEV}} \sim m_P$, we can estimate, through the use of Eq. (27), the total number of e-foldings as

$$N_{\text{tot}} \simeq \frac{1}{F_s} \ln \left(\frac{m_P}{m_{3/2}} \right), \quad (32)$$

where we took into account Eq. (26).

Let us turn our attention to the curvaton model. Consider the superpotential[3]

$$W = \frac{\lambda}{n+3} \frac{\phi^{n+3}}{m_P^n}, \quad (33)$$

where $n \geqslant 0$ and the complex field ϕ can be thought to contain the curvaton phase field σ and one radial field $|\phi|$ as follows:

$$\phi \equiv |\phi| e^{i\theta} = |\phi| \exp(i\sigma/\sqrt{2}v). \quad (34)$$

Then the scalar potential can be written as

$$V = \left(C_\phi H^2 - m_\phi^2 \right) |\phi|^2 + \left[(C_A H + A) \frac{\lambda}{n+3} \frac{\phi^{n+3}}{m_P^n} + \text{h.c.} \right]$$

$$+ \lambda^2 \frac{|\phi|^{2n+4}}{m_P^{2n}}$$

$$= \left(C_\phi H^2 - m_\phi^2 \right) |\phi|^2 + \lambda^2 \frac{|\phi|^{2n+4}}{m_P^{2n}}$$

$$+ (C_A H + A) \frac{2\lambda}{n+3} \frac{|\phi|^{n+3}}{m_P^n} \cos \left[(n+3)\theta \right], \quad (35)$$

where m_ϕ and A are soft supersymmetry breaking mass-scales at zero temperature, both given by the electroweak scale $m_{3/2}$. Note that we have put negative mass-squared for the $|\phi|$ field at zero temperature to break the U(1) symmetry. We also considered corrections coming from supergravity, which provide effective mass terms of order H [10] (for their effect on curvaton physics see Ref. [15]). Absorbing the $(n+3)$ factor into θ (and shifting the latter by π)[4] we can write the curvaton potential as:

$$V(\sigma) \approx \lambda (C_A H + A) v^3 \left(\frac{v}{m_P} \right)^n \left[1 - \cos \left(\frac{\sigma}{v} \right) \right]. \quad (36)$$

We are going to assume that the U(1) symmetry is broken at some moment during inflation with $H_* \sim m_\phi \sim m_{3/2}$. Hence we take $C_\phi \sim +1$. After this moment the radial field $|\phi|$ begins to grow, which can result in $\varepsilon \ll 1$. In time, after the symmetry breaking, the tachyonic effective mass of the radial field approaches its vacuum value m_ϕ as the supergravity correction diminishes due to the gradual decrease of the Hubble parameter.

After the phase transition, the time-dependent minimum of the potential of the radial field is given by

$$|\phi|_{\text{min}} = \left(\lambda^{-1} m_P^n \sqrt{m_\phi^2 - C_\phi H^2} \right)^{\frac{1}{n+1}}, \quad (37)$$

which gradually grows. Soon $|\phi|_{\text{min}}$ assumes its vacuum value:

$$v_0 \sim \left(\lambda^{-1} m_P^n m_\phi \right)^{\frac{1}{n+1}}. \quad (38)$$

From the above and also in view of Eqs. (1) and (36) we find

$$\tilde{m}_\sigma^2 \approx \lambda (C_A H + A) v \left(\frac{v}{m_P} \right)^n. \quad (39)$$

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[3] Such type of superpotential is reminiscent of supersymmetric realisations of the Peccei–Quinn symmetry in which the Peccei–Quinn scale is generated dynamically [14].

[4] This, in effect, means considering the range: $-\frac{\pi}{n+3} < \theta - \pi \leqslant \frac{\pi}{n+3}$.

Evaluating the above after the order parameter assumes its vacuum value $v \to v_0$ we obtain

$$m_\sigma^2 \approx (C_A H + A) m_\phi, \tag{40}$$

where we used Eq. (38). Since $m_\phi \sim A \sim m_{3/2}$, $C_A \sim 1$ and $H \leqslant H_*$ we find that

$$m_\sigma \sim m_{3/2} \sim H_*. \tag{41}$$

However, during inflation the effective mass of the curvaton is much smaller. Indeed, in view of Eq. (39), we get

$$\frac{\tilde{m}_\sigma^2}{m_\sigma^2} \sim \left(\frac{v}{v_0}\right)^{n+1} \quad \Rightarrow \quad \tilde{m}_\sigma(v_*) \sim \varepsilon^{\frac{n+1}{2}} m_\sigma, \tag{42}$$

where we used Eq. (10). Therefore, since $\varepsilon \ll 1$ and $m_\sigma \sim m_{3/2} \sim H_*$ we see that, during inflation $\tilde{m}_\sigma \ll H_*$, i.e. the PNGB is appropriately light and can act as a curvaton field.

Let us now calculate the value of ε required so that the scenario works. Firstly, we note that, in our case, the curvaton assumes a random value at the phase transition, which typically is $\sigma \sim v$. After the end of inflation and before the onset of the oscillations the field is overdamped and remains frozen. Hence, we expect that at the onset of the oscillations we have:

$$\sigma_{\mathrm{osc}} \sim \theta v_0, \tag{43}$$

where, typically, $\theta \sim \mathcal{O}(1)$ and we took into account that the radial field assumes its VEV very soon after the end of inflation. Combining Eqs. (13) and (43), we find

$$\varepsilon \sim \frac{\Omega_{\mathrm{dec}}}{\zeta\theta} \left(\frac{m_{3/2}}{m_{\mathrm{P}}}\right)^{\frac{n}{n+1}}, \tag{44}$$

where we also used Eq. (38) taking $m_\phi \sim m_{3/2}$ and $\lambda \sim 1$. The above is always larger than $\varepsilon_{\min}$, where

$$\varepsilon_{\min} \sim \left(\frac{m_{3/2}}{m_{\mathrm{P}}}\right)^{\frac{n}{n+1}}, \tag{45}$$

where we also used Eq. (14) with $H_* \sim m_{3/2}$.

Let us now enforce the constraint in Eq. (17), which, for $H_* \sim m_{3/2}$, reads

$$\varepsilon < \frac{\sqrt{\Omega_{\mathrm{dec}}}}{\zeta} \left(\frac{m_{\mathrm{P}}}{T_{\mathrm{BBN}}}\right)^{1/2} \left(\frac{m_{3/2}}{m_{\mathrm{P}}}\right)^{5/4} \sim 10^{-4}\sqrt{\Omega_{\mathrm{dec}}}. \tag{46}$$

From Eqs. (44) and (46) it is easy to find that the above bound can be satisfied only if n is large enough:

$$n > \frac{8 + \log(\sqrt{\Omega_{\mathrm{dec}}}/\theta)}{7 - \log(\sqrt{\Omega_{\mathrm{dec}}}/\theta)}. \tag{47}$$

According to Eq. (5), we see that, at the best of cases, (when $\Omega_{\mathrm{dec}} \sim 10^{-2}$ and $\theta \sim 1$) we have $n \geqslant 1$. Hence, we see that *the radial field must correspond to a flaton field, stabilised by non-renormalisable terms*. An upper bound on n can be obtained by requiring that the curvaton decays before BBN.

The decay of the curvaton depends on its coupling to other particles. The lowest decay rate corresponds to gravitational decay with $\Gamma_\sigma \sim m_\sigma^3/m_{\mathrm{P}}^2$. However, if ϕ is part of a supersymmetric theory we may expect a much larger value for Γ_σ. An

interesting possibility is realised by introducing the following coupling between ϕ and the Higgses:

$$\Delta W = \lambda_h \frac{\phi^{n+1}}{m_{\mathrm{P}}^n} h^2. \tag{48}$$

In this case, as is evident from Eq. (38), our curvaton model also solves the μ-problem for $\lambda_h/\lambda \sim 1$.

Now, the interaction of σ with ordinary particles is governed by the effective μ-term in Eq. (48), which results into the following decay rate of σ into two Higgs particles:

$$\Gamma_\sigma \simeq \frac{(n+1)^2}{4\pi} \frac{m_\sigma^3}{v_0^2}. \tag{49}$$

Demanding that $\Gamma_\sigma \geqslant H_{\mathrm{BBN}}$ results in the bound

$$\Gamma_\sigma \sim 10^{-\frac{30n}{n+1}} \left(\frac{m_\sigma}{\mathrm{TeV}}\right)^3 \mathrm{TeV} \geqslant H_{\mathrm{BBN}} \sim 10^{-27}\,\mathrm{TeV}$$

$$\Rightarrow \quad m_\sigma \geqslant 10^{\frac{n-9}{n+1}}\,\mathrm{TeV}, \tag{50}$$

where we used Eq. (38). Since we consider $m_\sigma \sim m_{3/2} \lesssim \mathrm{TeV}$ we see that there is a mild upper bound on n which, roughly, demands $n \lesssim 9$.

To proceed further, we have to consider separately the cases when the curvaton decays before or after it dominates the Universe. Suppose, at first, that the curvaton decays before domination ($\Omega_{\mathrm{dec}} \ll 1$). In this case, during the radiation epoch and after the onset of the oscillations, for the curvaton density fraction we have $\rho_\sigma/\rho \propto a(t) \propto H^{-1/2}$. Hence, we find

$$\Omega_{\mathrm{dec}} \sim \left(\frac{\min\{m_\sigma, \Gamma_{\mathrm{inf}}\}}{\Gamma_\sigma}\right)^{1/2} \left(\frac{\sigma_{\mathrm{osc}}}{m_{\mathrm{P}}}\right)^2, \tag{51}$$

where we have used Eq. (13) and

$$\left.\frac{\rho_\sigma}{\rho}\right|_{\mathrm{osc}} \sim \left(\frac{\sigma_{\mathrm{osc}}}{m_{\mathrm{P}}}\right)^2, \tag{52}$$

with $(\rho_\sigma)_{\mathrm{osc}} \simeq \frac{1}{2} m_\sigma^2 \sigma_{\mathrm{osc}}^2$ and $\rho_{\mathrm{osc}} \sim m_\sigma^2 m_{\mathrm{P}}^2$. Using Eq. (49) into Eq. (51) and also Eqs. (13) and (38) we obtain

$$\varepsilon \sim \frac{\sqrt{g\,\Omega_{\mathrm{dec}}}}{\zeta} \left(\frac{m_{3/2}}{m_{\mathrm{P}}}\right)^{\frac{1}{2}\left(\frac{n+2}{n+1}\right)}, \tag{53}$$

where we have also used that $\Gamma_{\mathrm{inf}} < H_* \sim m_{3/2} \sim m_\sigma$ and

$$\Gamma_{\mathrm{inf}} \sim g^2 m_{3/2}, \tag{54}$$

with the mass of the inflaton field s taken to be $m_s \lesssim H_* \sim m_{3/2}$ and g being the coupling of the inflaton to its decay products. In principle, g can be as low as m_s/m_{P} if the inflaton decays gravitationally. However, since reheating has to occur before BBN, g has to lie in the range:

$$10^{-14} \sim 10\frac{m_{3/2}}{m_{\mathrm{P}}} < g < 1. \tag{55}$$

Combining Eqs. (44) and (53) we find the relation

$$\frac{g}{\Omega_{\mathrm{dec}}} \sim \frac{1}{\theta^2} \left(\frac{m_{3/2}}{m_{\mathrm{P}}}\right)^{\frac{n-2}{n+1}}. \tag{56}$$

Let us now consider the case when the curvaton decays after domination ($\Omega_{\text{dec}} \approx 1$). In this case, the curvaton dominates the energy density of the Universe when $H = H_{\text{dom}}$, where H_{dom} is given by

$$H_{\text{dom}} \sim \left(\frac{\sigma_{\text{osc}}}{m_P} \right)^4 \min\{m_\sigma, \Gamma_{\text{inf}}\}. \tag{57}$$

Now, using Eqs. (43), (49) and (54) it can be shown that the requirement $\Gamma_\sigma < H_{\text{dom}}$ results in the bound

$$g > \frac{1}{\theta^2} \left(\frac{m_{3/2}}{m_P} \right)^{\frac{n-2}{n+1}}. \tag{58}$$

The fact that the case of curvaton domination requires a larger value of g [compare the above with Eq. (56)] is to be expected because, this means that the inflaton decays earlier and, therefore, the density fraction ρ_σ / ρ grows substantially, allowing the latter to dominate the Universe before its decay. The higher g is the more dominant the curvaton will be.

Note also, that, when the curvaton decays after it dominates the Universe, the hot big bang begins after curvaton decay, which suggests the reheating temperature

$$T_{\text{reh}} \sim \sqrt{\Gamma_\sigma m_P} \sim m_{3/2} \left(\frac{m_{3/2}}{m_P} \right)^{\frac{1}{2}\left(\frac{n-1}{n+1}\right)}. \tag{59}$$

It can be easily checked that the above is higher that T_{BBN} when $n \leqslant 9$, in agreement with the bound from Eq. (50).

From Eqs. (56) and (58) we see that, in general,

$$g \gtrsim \frac{\Omega_{\text{dec}}}{\theta^2} \left(\frac{m_{3/2}}{m_P} \right)^{\frac{n-2}{n+1}}. \tag{60}$$

For $\theta \sim 1$ and in view of Eqs. (5) and (55) the above bound suggests

$$n \geqslant 2, \tag{61}$$

which is tighter than the bound in Eq. (47).

We now concentrate of the evolution of the radial field $|\phi|$, which has to be such, as to achieve the required value for ε. Let us assume, at first, that the radial field follows the growth of the temporal minimum given by Eq. (37). In this case we can calculate the amplification factor as

$$\varepsilon \equiv \frac{(|\phi|_{\text{min}})_*}{v_0} = \left[1 - C_\phi \left(\frac{H_*}{m_\phi} \right)^2 \right]^{\frac{1}{2(n+1)}}. \tag{62}$$

Using Eqs. (37) and (39) one finds that, in this case, the curvaton's mass is given by

$$\tilde{m}_\sigma^2 \approx (C_A H + A) \sqrt{m_\phi^2 - C_\phi H^2}. \tag{63}$$

The above and Eq. (62) suggest that

$$\left(\frac{\tilde{m}_\sigma}{H_*} \right)^2 \simeq (A + C_A H_*) \frac{m_\phi}{H_*^2} \varepsilon^{n+1}, \tag{64}$$

which agrees with Eq. (42), given that $C_A \sim 1$ and $H_* \sim A \sim m_\phi \sim m_\sigma \sim m_{3/2}$. From Eq. (37) it is easy to show that the rate

of growth of the order parameter is

$$\frac{\dot{v}}{v} = \frac{|\dot{\phi}|_{\text{min}}}{|\phi|_{\text{min}}} = \frac{\epsilon}{n+1} \left(\frac{m_\phi^2}{C_\phi H^2} - 1 \right)^{-1} H, \tag{65}$$

where ϵ is defined in Eq. (29). From Eqs. (62) and (65) we obtain

$$(\dot{v}/v)_* \sim \epsilon_* \varepsilon^{-2(n+1)} H_*. \tag{66}$$

Comparing this with Eq. (22), we find that, for the scale invariance of the spectrum to be preserved, we require

$$\epsilon_* \ll \varepsilon^{2(n+1)}, \tag{67}$$

where $\epsilon_* = \epsilon(s_*)$.

Now, if the growth of $|\phi|_{\text{min}}$ is so rapid that the radial field cannot follow it, then we expect $|\phi|$ to roll, instead, down the potential hill. In this case the order parameter is determined by the rolling $|\phi|$.

When the cosmological scales exit the horizon the radial field has to be slowly rolling because we need the order parameter to vary slowly enough, not to destabilise the approximate scale invariance of the perturbation spectrum [cf. Eq. (22)]. Therefore, the Klein–Gordon equation for $|\phi|$ is:

$$3H_* |\dot{\phi}| - \bar{m}_\phi^2 |\phi| \simeq 0, \tag{68}$$

where

$$\bar{m}_\phi^2 \equiv m_\phi^2 - C_\phi H^2. \tag{69}$$

Using the above, the rate of growth of the order parameter, in this case, can be easily found to be

$$\frac{\dot{v}}{v} = \frac{|\dot{\phi}|}{|\phi|} = \frac{1}{3} C_\phi \left(\frac{m_\phi^2}{C_\phi H^2} - 1 \right) H. \tag{70}$$

The variation of the order parameter is expected to follow the less rapidly changing rate of growth. Hence, by comparing the two rates in Eqs. (65) and (70), we see that the order parameter follows the variation of $|\phi|_{\text{min}}$ only if

$$\varepsilon^{4(n+1)} > \epsilon_*, \tag{71}$$

where we used again Eq. (62) and also $C_\phi \sim 1$. It is evident that, if the above constraint is satisfied, then so is the requirement in Eq. (67). Note, however, that if the above constraint is violated then the order parameter v is determined by the rolling $|\phi|$ and not by the varying $|\phi|_{\text{min}}$, in which case the requirement in Eq. (67) is not valid, while also the amplification factor is not the one shown in Eq. (62).[5]

In this latter case, we find the amplification factor as follows. Using Eq. (28) we can write $|\phi|$ as a function of the number N of the remaining e-foldings of inflation. Starting from Eq. (68) and after a little algebra we obtain

$$\frac{3}{C_\phi} \frac{d \ln |\phi|}{dN} = \frac{e^{-2F_s N_x} - e^{-2F_s N}}{1 - e^{-2F_s N}}, \tag{72}$$

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[5] If v follows the growth of $|\phi|$ instead of $|\phi|_{\text{min}}$ then ε is expected to be smaller that the one in Eq. (62) because, at any given time, $v(t) < |\phi|_{\text{min}}(t)$.

where N_x corresponds to the phase transition which changes the sign of $\bar{m}_\phi^2$. By definition

$$m_\phi^2 \equiv C_\phi H_x^2 \simeq C_\phi H_{\mathrm{inf}}^2 \left(1 - e^{-2F_s N_x}\right), \tag{73}$$

where $H_x \equiv H(N_x)$. Integrating Eq. (72) we get

$$\frac{6}{C_\phi} \ln\left(\frac{|\phi|_*}{|\phi|_x}\right)$$
$$= \left(1 - e^{-2F_s N_x}\right) F_s^{-1} \ln\left|\frac{e^{2F_s N_x} - 1}{e^{2F_s N_*} - 1}\right| - 2(N_x - N_*), \tag{74}$$

where $|\phi|_x \equiv |\phi|(N_x)$.

The displacement of the field from the origin at the phase transition is determined by its quantum fluctuations. This means that

$$|\phi|_x \simeq H_x/2\pi. \tag{75}$$

We also have

$$\varepsilon = \frac{|\phi|_*}{v_0} = \frac{|\phi|_*}{H_*} \frac{H_*}{v_0} \quad \Rightarrow \quad |\phi|_* \simeq \frac{\varepsilon}{\varepsilon_{\mathrm{min}}} \frac{H_*}{2\pi}, \tag{76}$$

where we used Eq. (14).

In view of Eq. (70), the requirement in Eq. (22) becomes

$$\frac{3}{C_\phi} e^{-2F_s N_*} \left(\frac{1 - e^{-2F_s(N_x - N_*)}}{1 - e^{-2F_s N_x}}\right) \ll 1, \tag{77}$$

where we took into account Eq. (73).

Finally, another issue to be addressed concerns the requirement that the radial field *does* slow roll at the time when the cosmological scales exit the horizon. In order for this to occur, its quantum fluctuations should not dominate its motion, i.e. $|\phi|$ has to be outside the quantum diffusion zone. The condition for this to occur is $H_*/2\pi < (\dot{\phi}/H)_*$ or equivalently

$$\left.\frac{\partial V}{\partial |\phi|}\right|_* \simeq 2\bar{m}_\phi^2(H_*)|\phi|_* > H_*^3. \tag{78}$$

Using Eqs. (69) and (73) and working as before, the above constraint is recast as

$$\ln\left(\frac{|\phi|_*}{|\phi|_x}\right) > 2F_s N_* - \ln(C_\phi/\pi) + \ln\left(\frac{1 - e^{-2F_s N_*}}{1 - e^{-2F_s(N_x - N_*)}}\right)$$
$$+ \frac{1}{2} \ln\left(\frac{1 - e^{-2F_s N_*}}{1 - e^{-2F_s N_x}}\right). \tag{79}$$

To illustrate the above we present an example, taking

$$n = 2 \quad \text{and} \quad \lambda, \theta \sim 1. \tag{80}$$

The bound in Eq. (50) suggests that this is acceptable provided $m_\sigma \gtrsim 5$ GeV. Using Eq. (44) we obtain the value of the amplification factor, necessary for the model to work:

$$\varepsilon \sim 10^{-5} \Omega_{\mathrm{dec}}. \tag{81}$$

If the curvaton decays after domination then Eq. (58) demands $g > 1$, which is not compatible with the range in Eq. (55). Therefore, we have to assume that the curvaton decays before domination, in which case $\Omega_{\mathrm{dec}} \lesssim 1$, with the bound saturated when the curvaton decays approximately when it is about

to dominate the Universe. In this case, Eq. (56) suggests

$$g \sim \Omega_{\mathrm{dec}} \lesssim 1. \tag{82}$$

Such a large coupling can be understood only if the VEV of the inflaton modulus is an enhanced symmetry point. As a result of the above, the reheating temperature after the end of inflation is found to be

$$T_{\mathrm{reh}} \sim g \sqrt{m_{3/2} m_{\mathrm{P}}} \sim g \times 10^{10.5} \text{ GeV}. \tag{83}$$

From the above we see that, in order not to challenge the gravitino constraint, we have to choose the lowest possible value of g, which, according to Eqs. (5) and (82) corresponds to

$$\Omega_{\mathrm{dec}} \sim 10^{-2}. \tag{84}$$

Hence, from Eqs. (81) and (82) we obtain the values

$$\varepsilon \sim 10^{-7} \quad \text{and} \quad g \sim 10^{-2}. \tag{85}$$

From Eq. (31) and (26) and also, using Eq. (29), it is easy to see that

$$\epsilon_* > 10^{-30}, \tag{86}$$

where we considered that $s_* > s_{\mathrm{in}}$. Hence, from Eqs. (85) and (86) it is straightforward to see that the constraint in Eq. (71) is badly violated, which means that the order parameter v follows the slow roll of the $|\phi|$ field and not the variation of the minimum of the potential $|\phi|_{\mathrm{min}}$. Consequently, the amplification factor ε is *not* given by the expression in Eq. (62) in this case. Instead, we can estimate the amplification factor with the use of Eq. (76).

Using Eq. (45) with $n = 2$ we find

$$\varepsilon_{\mathrm{min}} \sim 10^{-10}. \tag{87}$$

Comparing this with Eq. (85) we see that the value of ε saturates the bound in Eq. (15). This is expected since Eq. (84) also saturates the non-Gaussianity bound in Eq. (5).

Eqs. (45), (76), (85) and (87) suggest

$$|\phi|_* \sim 10^3 \frac{H_*}{2\pi}. \tag{88}$$

The above can, in principle, be used in Eqs. (74) and (79) to constrain the parameters of the underlying model.

A useful quantity to calculate in order to evaluate Eqs. (74) and (79) is the number of e-foldings, which corresponds to the cosmological scales N_*. The cosmological scales range from a few times the size of the horizon today $\sim H_0^{-1}$ down to scales $\sim 10^{-6} H_0^{-1}$ corresponding to masses of order $10^6 M_\odot$ [16]. Typically this spans about 13 e-foldings of inflation. For the estimate of N_* we will chose a scale roughly in the middle of this range; the scale that re-enters the horizon at the time when structure formation begins, i.e. at the time t_{eq} of matter–radiation equality. Then, in the case when the curvaton decays before domination it is straightforward to obtain

$$\exp(N_*) \sim H_*^{1/3} \Gamma_{\mathrm{inf}}^{1/6} \sqrt{t_{\mathrm{eq}}} \sim g^{1/6} \sqrt{m_{3/2} t_{\mathrm{eq}}}, \tag{89}$$

where we have used Eq. (54) and that $H_* \sim m_\sigma \sim m_{3/2}$. Using Eq. (85), we obtain

$$N_* \simeq 43. \tag{90}$$

The number of e-folds that corresponds to decoupling (when the CMBR is emitted) is roughly $N_* + 1.5$, while the one which corresponds to the present horizon is $\sim N_* + 9$.

In the attempt to obtain the allowed parameter space for our model it soon becomes clear that, while the requirement in Eq. (79) is relatively easy to satisfy, the major difficulty is reconciling Eq. (74) with the bound in Eq. (77) coming from the spectral index requirements. This is especially true in view of the recent WMAP results [17], which correspond to spectral index $n_s = 0.96 \pm 0.02$, i.e. $n_s \geqslant 0.92$ at 95% c.l. This means that the left-hand side of Eq. (77) should not exceed 0.04.[6] By careful investigation of Eqs. (74) and (77) it is found that the above difficulty is more alleviated the larger the value of N_x is, i.e. the earlier the phase transition occurs. In fact, a solution is only possible if

$$2 F_s N_x \gg 1. \tag{91}$$

In view of the above Eqs. (74) and (77) can be respectively approximated as

$$\ln\left(\frac{|\phi|_*}{|\phi|_x}\right) \simeq -\frac{C_\phi}{6 F_s} \ln\left(1 - e^{-2 F_s N_*}\right), \tag{92}$$

$$C_\phi \leqslant 0.12\left(e^{2 F_s N_*} - 1\right). \tag{93}$$

Now, using Eq. (75) we can write:

$$\frac{|\phi|_*}{|\phi|_x} \simeq \frac{2\pi |\phi|_*}{H_*}\left(\frac{H_*}{H_x}\right)$$

$$\Rightarrow \quad \ln\left(\frac{|\phi|_*}{|\phi|_x}\right) \simeq \ln\left(\frac{2\pi |\phi|_*}{H_*}\right) + \frac{1}{2}\ln\left(\frac{1 - e^{-2 F_s N_*}}{1 - e^{-2 F_s N_x}}\right), \tag{94}$$

where we have considered also Eq. (28), using that $H^2(N) \simeq V(N)/3 m_P$. In view of the above and according to the approximation in Eq. (91) we can recast Eq. (92) as

$$\ln\left(\frac{2\pi |\phi|_*}{H_*}\right) \simeq -\left(\frac{1}{2} + \frac{C_\phi}{6 F_s}\right) \ln\left(1 - e^{-2 F_s N_*}\right). \tag{95}$$

Under the same approximation, Eq. (79) becomes

$$\ln\left(\frac{2\pi |\phi|_*}{H_*}\right) > 2 F_s N_* - \ln(C_\phi/\pi) + \ln\left(1 - e^{-2 F_s N_*}\right), \tag{96}$$

where we have also used Eq. (94).

Solving Eq. (95) in terms of C_ϕ and using Eq. (93) we obtain

$$-\frac{2 F_s N_*}{86}\left[1 + \frac{6 \ln 10}{\ln(1 - e^{-2 F_s N_*})}\right] - 0.04\left(e^{2 F_s N_*} - 1\right) \leqslant 0, \tag{97}$$

where we have also employed Eqs. (88) and (90). Solving numerically we obtain the bound

$$F_s \lesssim \frac{1}{1780} \simeq 5.6 \times 10^{-4}. \tag{98}$$

This bound, in view of Eq. (27) results in

$$m_s \leqslant 0.041 H_{\text{inf}}, \tag{99}$$

which is somewhat tight and implies that inflation is not really of the fast-roll type, but the inflaton is light enough to roll slowly down its potential hill. From Eqs. (32) and (98) one obtains

$$N_{\text{tot}} \geqslant 6.1 \times 10^4. \tag{100}$$

Thus, if the phase transition, which releases $|\phi|$ from the origin, occurs not much later than the onset of inflation, then the approximation in Eq. (91) can be well justified. Similarly, using Eqs. (90) and (98), Eq. (93) gives the bound

$$C_\phi \leqslant 5.9 \times 10^{-3}. \tag{101}$$

In view of Eqs. (73) and (91) the above bound suggests

$$m_\phi \leqslant 0.077 H_{\text{inf}}. \tag{102}$$

The values of m_s and m_ϕ can approach H_{inf} if one decreases N_x but then the constraint in Eq. (77) becomes seriously challenged. It can be easily checked that, with the above values the requirement in Eq. (96) is satisfied as well.

The above results suggest that, for the $n = 2$ case and when $\lambda, \theta \sim 1$, the model can work for masses of the order

$$m_\phi \lesssim 0.1 m_{3/2} \quad \text{and} \quad m_s \lesssim 0.01 m_{3/2}, \tag{103}$$

where $m_{3/2} \sim 1$ TeV. Such values imply only a mild tuning on the masses; predominantly on the mass of the inflaton modulus. This is necessary because the variation of H should be kept small, since only then can the tachyonic effective mass of the radial field $\bar{m}_\phi$ remain small enough for $|\phi|$ to be slow-rolling and the constraint in Eq. (22) to be satisfied. Note that, a tuning of the inflaton mass is quite plausible, since the latter is a string axion.

One may wonder why, since both the inflaton field s and the radial field $|\phi|$ turn-out to be light when the cosmological scales exit the horizon during inflation, we cannot use those fields to generate the observed curvature perturbations. The reason is that, in contrast to the PNGB curvaton, the perturbations of those fields are not amplified. Hence their contribution to the overall curvature perturbation is insignificant. Indeed, for the inflaton we have $\zeta_s \sim (m_s/s_*) \sim 10^{-17}$, which is much smaller than the observed value $\zeta \simeq 2 \times 10^{-5}$. Similarly, for $|\phi|$ it is easy to show that $\zeta_\phi \sim \varepsilon \zeta_\sigma \sim 10^{-12}$, where we used that $\zeta_\sigma \approx \zeta$.

In conclusion, we have seen that our mechanism can work with natural values of the parameters with only a mild tuning on the inflaton mass. Another important requirement is that the phase transition, which releases the radial field from the origin, occurs much earlier than the time when the cosmological scales exit the horizon, in order not to destabilise the flatness of the curvature perturbation spectrum.

Our PNGB curvaton is such that can be easily accommodated in simple extensions of the standard model. Indeed, in Ref. [19] we present in detail such a realisation, using as curvaton an angular degree of freedom orthogonal to the QCD axion

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[6] Note that, in our model, all other contributions to the deviation of the spectral index from unity [18] are negligible.

in a class of supersymmetric constructions of the Peccei–Quinn symmetry. Presumably, other PNGB curvatons, such as the ones in Ref. [7], can also be utilised.

We should note here that, although the modular inflation model, which we considered, is highly motivated, it is by no means the only possibility. Other inflationary models with Hubble-scale of order 1 TeV may also be applied [20]. Needless to say that designing inflationary models at such energy scale can allow direct contact with particle physics.

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