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Analysis of the Co-routing Problem in Agent-based Carpooling Simulation[☆]

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Abstract

Carpooling can cut costs and help to solve congestion problems but does not seem to be popular. Behavioral models allow to study the incentives and inhibitors for carpooling and the aggregated effect on the transportation system. In *activity based modeling* used for travel forecasting, *cooperation* between actors is important both for schedule planning and revision. Carpooling requires cooperation while commuting which in turn involves *co-scheduling* and *co-routing*. The latter requires combinatorial optimization. Agent-based systems used for activity based modeling, contain large amounts of agents. The agent model requires helper algorithms that deliver high quality solutions to embedded optimisation problems using a small amount of resources. Those algorithms are invoked thousands of times during agent society evolution and schedule execution simulation. Solution quality shall be sufficient in order to guarantee realistic agent behavior. This paper focuses on the *co-routing* problem.

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agent based modeling, activity based modeling, routing, optimisation

1. Introduction and Context

Carpoolers need to solve two essential problems while negotiating for shared rides: finding a suitable route (*coRouting*) and finding a new timing for their agenda for the day (*reScheduling*).

Activity based modeling (ActBM) is used to predict daily schedules for each individual in a synthetic population based on data mining and statistical methods application to census and survey data on one hand and stated preference evidence on the other. A *schedule* (daily agenda) is a sequence of *episodes* each one consisting of a *trip* to a specific location and an *activity* executed at that location. *ActBM* integrates behavioral rules stating the individual's sensitivity to external factors. The result is a set of almost mutually

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independent agents whose joint behavior reproduces the statistic distributions found in the actual society. Generated schedules specify activities, their duration and location : those are used for traffic demand prediction under several scenarios.

Agent based modeling (AgnBM) simulates interactions between individuals in order to assess the effect on the society as a whole. We use *AgnBM* to investigate interaction between carpooling people. Agent interaction influences travel *timing*, *mode choice* and *routing*. The carpooling case has been selected as a first study domain because the problem is well-defined, because the characteristics of the participating population probably make the problem tractable and because it contains both *coRouting* (i.e. determining a route that suits all carpoolers) and *reScheduling* (i.e. schedule adaptation) problems.

Symbol	Meaning
\bar{C}	Set of carpool parkings
$c_t(a, b)$	Cost to travel from a to b
\bar{D}	Set of destinations
$\bar{d}(p)$	Destination for participant p
M_{PT}	Function mapping participants to feasible transferia sets
M_{TP}	Function mapping transferia to supported participants sets
$\bar{o}(p)$	Origin for participant p
\bar{O}	Set of origins
P	Set of participants
$\mathcal{P}(S)$	Set of all partitions of set S
$ps(t)$	Participant set that can be supported by transferium T
$ts(p)$	Transferia set suitable for use by participant p
\bar{T}	Set of all transferia
$u_c(p)$	Upper limit for the cost accepted by participant p to travel from $\bar{o}(p)$ to $\bar{d}(p)$

2. AgentBased model for carpooling

The *agentBased* model simulates between 1000 and 5000 individuals belonging to the synthetic population generated for Flanders (Belgium). This amount of agents is sufficient to investigate the carpooling phenomenon and is expected to be small enough to keep the problem computationally tractable. A social network joining the agents is built and evolves as described in [1], [2]. Small sets of agents (typically 2...5) negotiate *route choice* and *travel time* in order to carpool e.g. for commuting on a specific day of the week. Schedule execution is simulated and introduces stochastic deviations between the actual and planned schedule versions. Behaviorally relevant factors such as VOT (value of time) and time use flexibility are involved. The model is used to evaluate both the effect of (a) travel-parking costs and carpool parks availability on the overall travel demand and (b) the complexity of the drivers cooperation process itself as an inhibiting factor (due to required schedule adaptation).

Carpooling candidates explore their social networks in order to detect possible fellow travelers and negotiate a route (*coRouting*) which requires schedule adaptation (*reScheduling*). Key components are exploration, negotiation (requiring *coRouting* and *reScheduling*) and schedule execution. Those are coordinated by the agentBased model. *Rescheduling* involves shifting activities (and hence travel) in space-time using limited activity reordering and making use of VOT, disutility functions and lists of feasible locations for activity execution. *CoRouting* includes *route choice* and *mode selection* (walk, bike, car, public transportation) and affects *route duration* but not absolute time (trip start time). *CoRouting* and *reScheduling* thus are orthogonal concepts: they can be studied independently. By negotiating, each agent tries to minimize their total cost which is the sum of travel cost and schedule adaptation disutility cost. Each passenger pays a weighted part of the drivers *original* trip distance cost plus a weighted part of the *excess generalized cost* for the driver caused by trip distance and duration increase. Both *coRouting* and *reScheduling* involve frequent solution of moderately sized optimisation problems. The *coRouting* subproblem is covered in this paper.

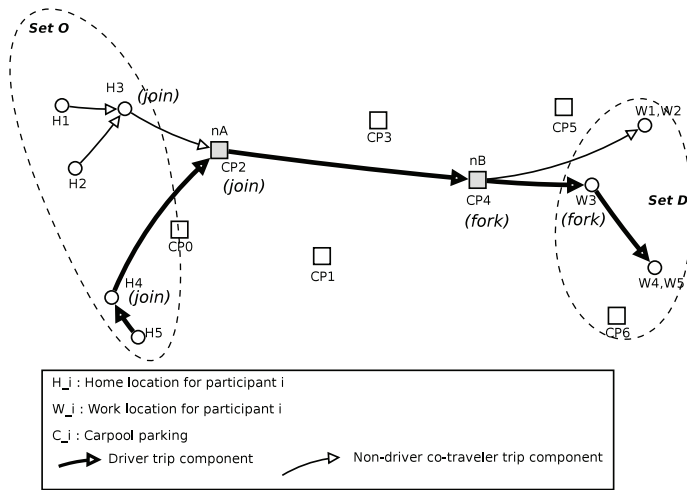


Fig. 1. Trips driven by carpooling people P_i numbered from 1 to 5. H_i are home locations (set O of origin locations on the left). W_i are work locations (set D of destination locations on the right). CP_i are carpool parkings. Person P_5 is the driver. P_1 and P_2 leave the participants set at N_B where they continue to their work location using a different mode (e.g. subway). P_3 is dropped at its destination. P_4 and P_5 work at the trip endpoint. n_A is the head of the *join* (backward) hyperArc, n_B is the tail of the *fork* (forward) hyperArc.[3]

3. CoRouting in the carpooling context

A set P of identified participants $p_i \in P$ and for each participant the origin $o(p_i) \in \bar{O}$ and destination $d(p_i) \in \bar{D}$ locations, the upper limit $u_c(p_i)$ for the generalized route cost (duration, distance) acceptable by p_i to travel from $o(p_i)$ to $d(p_i)$ are considered to be given (i.e. supplied by the agentBased model). Furthermore, a set \bar{C} of carpool parks on the road network is given. $\bar{O} \cup \bar{C} \cup \bar{D}$ is the set of *transferia* i.e. the set of locations where joint rides can start or end. We assume that on each shared ride, all participants are on board on at least one link in the network (from n_A to n_B). As a consequence, the shared ride route consists of a *join* subtree (where participants come on board) and a *fork* subtree (where participants alight from the car) as shown in Fig. 1. The problem is to find the route that brings all participants from their origin to their destination via a set of transferia while minimizing the overall cost. Mode selection is not covered by this study. The root nodes (n_A and n_B in the figure) for the *join* and *fork* trees respectively, are determined by the agentBased model and thus considered to be given here. This paper analyses the *join* subtree.

4. Calculations a priori

Before tackling the *coRouting* problem, some supporting concepts will be explained.

4.1. Reduced network

The generalized cost $c_i(t_0, t_1)$ to travel between transferia t_0 and t_1 is calculated a priori for all pairs in $(\bar{O} \times \bar{O}) \cup (\bar{O} \times \bar{C}) \cup (\bar{C} \times \bar{C}) \cup (\bar{C} \times \bar{D}) \cup (\bar{D} \times \bar{D})$ (hence for a graph that contains some complete subgraphs).

4.2. Limited detour network

Let N_{RN} denote the set of nodes and L_{RN} denote the set of links in the road network represented by the digraph $RN = \langle N_{RN}, L_{RN} \rangle$ with $L_{RN} \subseteq N_{RN} \times N_{RN}$. For each candidate participant p the *Limited Detour Network* $LDN(p)$ (space-time prism) is calculated a priori. $LDN(p)$ is a subgraph of the road network RN .

$$LDN(p) = \langle N_{LDN(p)}, L_{LDN(p)} \rangle, N_{LDN(p)} \subseteq N_{RN} \wedge L_{LDN(p)} \subseteq L_{RN} \wedge o(p) \in N_{LDN(p)} \wedge d(p) \in N_{LDN(p)} \quad (1)$$

$$\forall n \in N_{LDN(p)} : (\exists q = path(o(p), n, d(p)) | cost(q) \leq u_c(p)) \quad (2)$$

$path(a, b, c)$ is a path joining a to c and containing b . Hence $d(p)$ can be reached from $o(p)$ via each node in $LDN(p)$ at a cost acceptable to participant p . If a transferium $t \in N_{LDN(p)}$ then t is said to be contained in $LDN(p)$ (see Fig. 2).

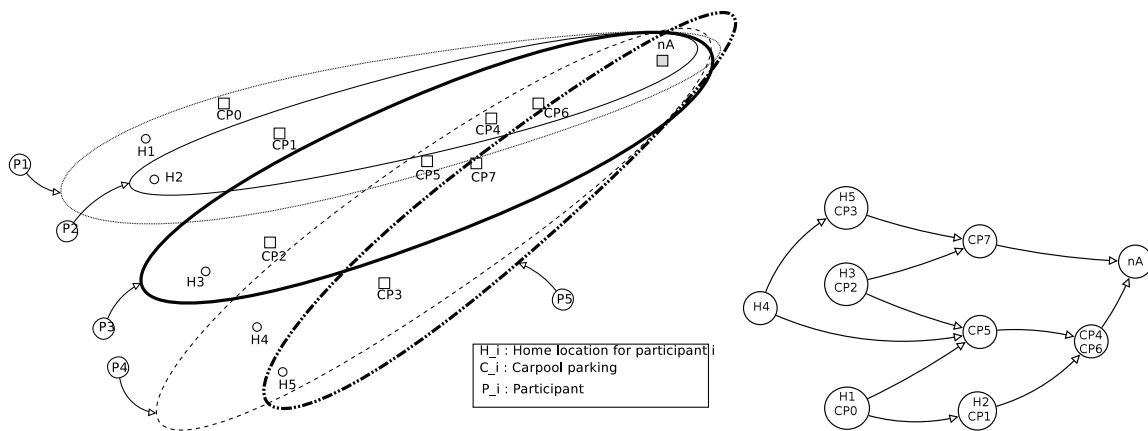


Fig. 2. **Left:** Participant specific *Limited Detour Network* perimeter encloses its home location (H_i), the target (n_A) and zero or more transferia (home locations H_j and carPoolParks (CP_k)). The *containment* relation defined over the participant sets associated with the carPoolParks, defines a partial order over the carPoolParks. See also table 1, fig. 2/**Right**. **Right:** Transitive reduction of the partial order on the set of transferia induced by the containment relation between the corresponding participants sets ($ps(H4) \subset ps(CP3) \subset ps(CP7) \subset ps(n_A) \wedge ps(H4) \subset ps(CP5) \subset ps(CP6) \subset ps(n_A) \wedge ps(CP2) \subset ps(CP7) \wedge ps(CP2) \subset ps(CP5) \wedge ps(CP0) \subset ps(CP5) \wedge ps(CP0) \subset ps(CP1) \subset ps(CP6)$). See also table 1, fig. 2

4.3. Transferium usability partial order relations

Transferia not contained in the *LDN* of any participant are ignored. For each participant, the set of usable transferia $t \in N_{LDN(p)}$ is determined: this maps each participant to a set of transferia $M_{PT} : P \Rightarrow 2^{\bar{T}} : p \mapsto ts(p)$. From this, the *reverse* mapping $M_{TP} : \bar{T} \Rightarrow 2^P : t \mapsto ps(t)$ follows. Examples corresponding to Fig. 2/**Right** are shown in tables 1. Transferium t_0 is said to be more specific than t_1 if and only if the participant set for t_0 is a subset of the one for t_1 and thus can also be serviced by t_1 . Since set containment induces a partial order over 2^P the *ISM* (*isMoreSpecificThan*) relation is a partial order. Refer to Fig. 2/**Right** for the *ISM* relation derived from Fig. 2.

$$(t_0 < t_1 \Leftrightarrow ps(t_0) \subset ps(t_1)) \wedge (t_0 \leq t_1 \Leftrightarrow ps(t_0) \subseteq ps(t_1)) \tag{3}$$

5. Problem model

The problem model consists of a mathematical structure one part of which representing the ways people can combine to cooperate and the other one representing the carpool parking selection.

5.1. Combining participants

While establishing the *join* tree we need to decide who will join at a specific transferium. A priori all possible combinations of participants need to be evaluated. Therefore, every partition of the participants set P is considered. The number of partitions is given by the Bell number $B_{|P|}$ and grows rapidly with the set size (see [4] and table 2).

Consider all partitions having the same number of *cells*. The relation *hasSameNumberOfCells* induces a partition on $\mathcal{P}(P)$ whose equivalence classes are called *layers*. Layers are numbered by the cardinality of the elements they contain. Low numbered layers are at the top of the graph in Fig. 3/**Right**. Figure 3/**Left** shows a Hasse diagram (see [5] for more info) for $\mathcal{P}(P)$ for $|P| = 4$. Each rectangle represents a partition and edges represent the *refinement* relation. In this representation, each arrow shows the target is derived from the source by combining exactly two *cells*. Each arrow corresponds to a *join* operation in the carpooling problem. This diagram is called the *joinGraph*.

$P \Rightarrow T$		
	Transferia	
Participant	Homes	CarPoolParks
P1	H1, H2	CP0, CP1, CP4, CP5, CP6
P2	H2	CP1, CP4, CP6
P3	H3	CP2, CP4, CP5, CP6, CP7
P4	H4, H5	CP3, CP4, CP5, CP6, CP7
P5	H5	CP3, CP7

$T \Rightarrow P$	
Transferium	Participants
H1	P1
H2	P1, P2
H3	P3
H4	P4
H5	P4, P5
CP0	P1
CP1	P1, P2
CP2	P3
CP3	P4, P5
CP4	P1, P2, P3, P4
CP5	P1, P3, P4
CP6	P1, P2, P3, P4
CP7	P3, P4, P5

Table 1. **Left:** mapping of participants to transferia that *can* be feasible (are not infeasible). See also fig. 2, fig. 2/**RightRight:** mapping of transferia to sets of participants for whom use of the transferia *can* be feasible (is not infeasible). See also table 1, fig. 2, fig. 2/**Right**

$B_1 = 1$	$B_7 = 877$
$B_2 = 2$	$B_8 = 4140$
$B_3 = 5$	$B_9 = 21147$
$B_4 = 15$	$B_{10} = 115975$
$B_5 = 52$	$B_{11} = 678570$
$B_6 = 203$	$B_{12} = 4213597$

Table 2. Bell numbers (taken from <http://oeis.org/A000110> (*The On-Line Encyclopedia of Integer Sequences*))

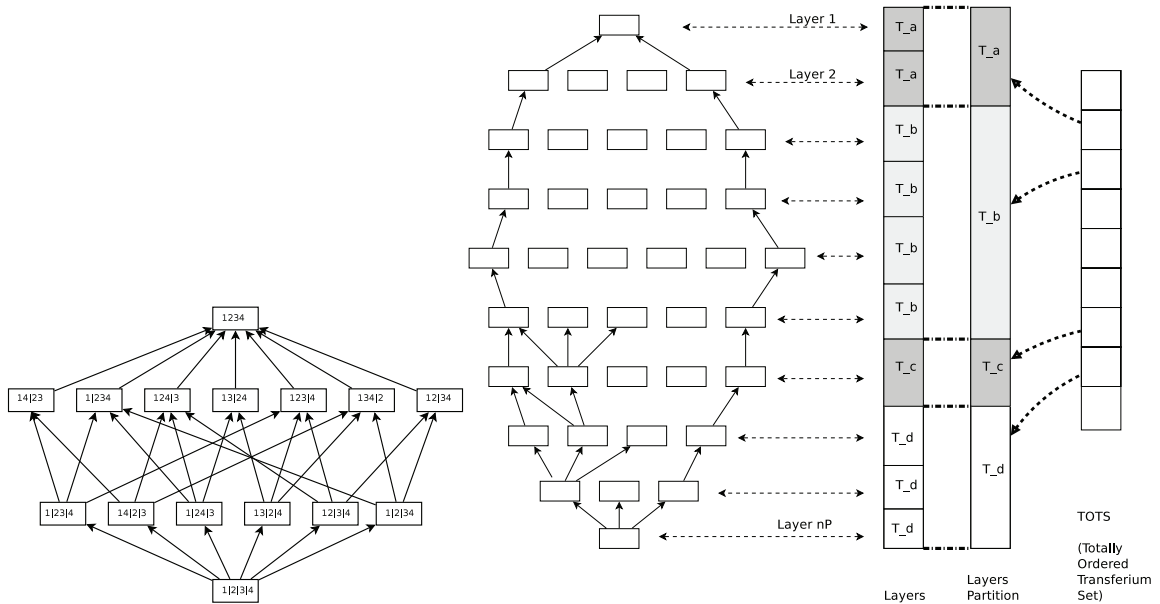


Fig. 3. **Left:** Hasse diagram for the transitive reduction of the refinement relation for a 4-element set partitioning. **Right:** the graph represents the Hasse diagram (most edges not drawn). The *layer* number gives the number of *parts* in each participants set partition. The leftmost of the vertical bars shows the ordered set of *layer-transferium* assignments. The central vertical bar represents a partition of the ordered set: different transferia have been assigned to each *part* (cell). The rightmost vertical bar represents the ordered set of transferia (see section 4.3).

5.2. Transferium assignment

Fig. 3/Right combines the concepts developed before in sections 4.3 and 5.1. The idea is to assign a transferium to each *layer* in the *joinGraph*. Each level transition in the *joinGraph* denotes exactly one *join* operation. Joining more subgroups of participants at a single transferium t_0 corresponds to successive *layers* to get assigned t_0 . In order to realize this, the set of *layers* L in turn is partitioned in all possible ways $\mathcal{P}(L)$. A different transferium is assigned to each *layer* partition. Each *layer* inherits the transferium assigned to the *cell* it belongs to.

The order in which participant groups $g_0 = ps(t_0)$ and $g_1 = ps(t_1)$ are joined is irrelevant in case g_0 and g_1 are unrelated with respect to *ISM* (i.e. $(g_0, g_1) \notin ISM$). Hence, it is not relevant in which order t_0 and t_1 get considered for join operations. On the other hand, if the optimal solution contains t_0 and t_1 where $t_0 < t_1$ then t_0 is assigned to a higher level *layer* since t_0 can server less people.

Consider a totally ordered set of transferia *TOTS* using an order relation R so that $ISM \subset R$. The element order in every subset of *TOTS* complies with *ISM* and thus can be used to assign transferia to *layers*. For efficiency reasons it is mportant that only one transferium order is to be investigated.

6. Algorithm

The smallest number of feasible transferium assignments is generated so that all possible different solutions are enumerated. Each of those assignments is used to prune the *joinGraph* by deleting links that join participants at transferia infeasible for them. After assigning a transferium to each *layer*, $ps(t)$ is used to label nodes in the *joinGraph* as (in)feasible. Finally the least cost path from the *infimum* to the *supremum* in the *joinGraph* is determined by a traversal algorithm and the cheapest one over all assignments is kept.

Observe that it does not make sense to assign transferium t_0 to a transferium cell τ in case $|ps(t_0)| < |\tau|$ because $|\tau|$ is the number of *layers* spanned and at least one participant joins another one at each *layer*. Other pruning techniques have not been commented due to lack of space.

Finally, the algorithm for transferium assignment (before *joinGraph* pruning) is

```

for all  $q \in \mathcal{P}(L)$  do                                ▶ for each layerSet partition
  for all  $\bar{\tau} \subseteq TOTS \ ||\bar{\tau}| = |q|$  do                ▶ foreach TOTS subset with size equal to the layerSet partition size
    if  $\forall i < |q| : |q[i]| \leq |ps(\bar{\tau}[i])|$  then          ▶ no oversized layerSet partition cells : partition usable
      for all  $i < |q|$  do                                ▶ foreach layerSet partition cell
         $\forall l \in q[i] : t_l \leftarrow \bar{\tau}[i]$                 ▶ assign transferium to all layers in cell
      end for
    end if
  end for
end for

```

7. Conclusion

The problem structure for *coRouting* in the carpooling context has been analysed in order to find an algorithm suitable in the agentBased modeling context. The idea is to constrain the search space as much as possible. After this analysis, algorithm implementation should not pose a problem. Experiments still are required to estimate the performance.

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