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Proposal for measure of degressive proportionality

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Abstract

Degressive proportionality is an intermediary solution between equality and proportionality. Taking this fact into account, the article proposes a measure of degression of the degressively proportional division. The defined measure was used, among other instances, in allocation based on classical proposals of seat distribution in European Parliament by Pukelsheim and Ramirez as well as the distribution of seats during the Parliamentary term of 2014-2019. The outcome is confronted with other measures functioning in the literature.

Keyword: degressive proportionality; fair division; measure; European Parliament.

1. Introduction

The concept of degressive proportionality has been widely disseminated as a result of problems with its interpretation in the course of work on the construction of the composition of the European Parliament. The principle of degressive proportionality is included in the Lisbon Treaty, as a method of determining the number of representations of the Member States of the European Union. The Lisbon Treaty has introduced major changes in this regard to the Treaty on the European Union. Submitted Article 9a, the second point is the following:

The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be ***degressively proportional****,*

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with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats.

Work on the interpretation of the principle of degressive proportionality has been entrusted to the Committee on Constitutional Affairs, which in the draft resolution of the Parliament (Lamassoure & Severin, 2007) concluded that:

Considers that the principle of degressive proportionality means that the ratio between the population and the number of seats of each Member State must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State.

This provision was included in the resolution of the European Parliament (INI/2007 / 2169). Article 1 of the resolution reads as follows:

The principle of degressive proportionality provided for in Article [9a] of the Treaty on European Union shall be applied as follows:

- the minimum and maximum numbers set by the Treaty must be fully utilised to ensure that the allocation of seats in the European Parliament reflects as closely as possible the range of populations of the Member States;
- the larger the population of a country, the greater its entitlement to a large number of seats;
- the larger the population of a country, the more inhabitants are represented by each of its Members of the European Parliament.

In many studies regarding the distribution of seats in the European Parliament among Member States there are different terms used relating to the degree of degression of the given allocation or the method by which the division is carried out. For example, in the report of the Committee on Constitutional Affairs on the composition of the European Parliament with a view to the elections in 2014 (Gualtieri & Trzaskowski, 2013), which initiated the distribution of seats in the Parliament's eighth term of office appears to say that *among the various possible mathematical formulae for implementing the principle of degressive proportionality* [1]*, the 'parabolic' method is one of the most degressive.* The authors of the report do not present arguments that could justify such an opinion, they referred only to the attachment of the document which presents the method of parabolic distribution.

The thesis skating that the parabolic method is the most degressive can be validated on the basis of strictly mathematical argumentation. Degressive proportionality is a deviation from proportion in the direction of equality. Proportional division is based on a linear allocation function, whereas equal division uses a constant function for the same purpose. One may regard as the most degressive an allocation, which allocation function diminishes at a steady pace. The only function with a derivative that is constant and different than zero is a square function.

This paper proposes a method for measuring the degree of degression of a given allocation. The values assigned to the challengers to the shared good will be marked with symbols $p_1, p_2, \ldots, p_n$, and numbers of goods attributed to them $s_1, s_2, \ldots, s_n$. Although, the discussion will be depicted with the case of distribution of seats in the European Parliament, they are general in nature.

2. Measures of degressive proportionality

Geometric interpretation of degressive proportionality is shown in Figure 1. Sections with the start in the point (0,0) and ends in the points (p_i, s_i) must have a decreasing inclination angle to the x-axis . It is known that, assuming $p_i > 0$, $s_i > 0$ the sufficient condition of degressive proportionality is concavity of the polygonal chain with vertices (p_i, s_i) (Dniestrzański, 2011a). It is not, however, as shown in Figure 1, a necessary condition. Determination of degressive proportionality cannot contain the requirement of concavity of polygonal chain with vertices (p_i, s_i) as the same number of seats for two countries with different populations would force the same number of seats for all subsequent Member States with population of at least the same quantity.

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[1] For an analysis and a description of the various mathematical formulae, see the Special Issue of 'Mathematical Social Sciences', 63 (2012), pp. 65–191, especially Table 2, on p. 100.

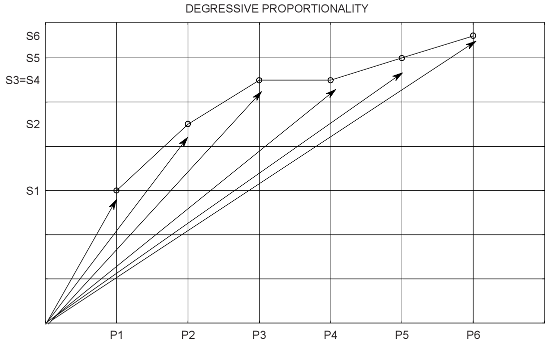

Fig.1. Degressive proportionality

Determining methods of measuring the degression of a given division or an algorithm to obtain it can be of great importance for precise usage of this allocation. In his work Łyko (Łyko, 2013) shows two different ways of measuring the degression of the division. The first, called the **boundary measure**, is based solely on boundary conditions [2] of a given allocation. Łyko proposed as a measure of degression the value

$$U_B = \frac{(p_n - p_1) - U^*}{(p_n - p_1) + U^*}, \text{ where } U^* = \frac{\sum_{i=1}^{n} p_i}{\sum_{i=1}^{n} s_i}(s_n - s_1) \tag{1}$$

The value U_B belongs to the range [–1,1] and equality $U_B = 0$ holds if and only if the considered distribution is proportional. Therefore, this measure is a numerical representation of the deviation from proportionality.

Second way of measuring the degression of the distribution proposed by the Łyko is based on the value of the expression

$$U_I = \max_{i \in N_0} \{|u_i|\}, \text{ where } u_i = \frac{z_i - u^*}{z_i + u^*},\ u^* = \frac{p_n - p_1}{s_n - s_1} \tag{2}$$

and
$$z_i = \begin{cases} \dfrac{p_{i+1} - p_i}{s_{i+1} - s_i} & \text{for } s_i \neq s_{i+1} \\ \dfrac{p_{i+k} - p_i}{s_{i+k} - s_i} & \text{for } s_{i-1} \neq s_i = s_{i+1} = \ldots = s_{i+k-1} \neq s_{i+k} \end{cases},\ i = 1, 2, \ldots, n-1 \tag{3}$$

N_0 is a set of those i for which the value u_i is defined.

The value U_I is called **internal measure** of degressive proportionality. He also proposes calling the value $U_G = \frac{1}{\mathrm{card} N_0} \sum_{i \in N_0} |u_i|$ **global measure** of degressive proportionality.

Another possibility of construction of the measure of degressive proportionality is presented by the result shown in the work by (Florek, 2012). Florek demonstrated the equivalence of the two following definitions of degressively proportional sequences.

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2 The boundary conditions mean minimum and maximum number of seats that must be granted and the total number of seats. Influence of conditions boundary on the possible distribution of seats is further analyzed in the work (Łyko, 2012).

Definition 1 (Florek, 2012)

The sequence $S = (s_1, s_2,\ldots, s_n)$, where $s_i \in N$ is degressively proportional with respect to the sequence $P = (p_1, p_2,\ldots, p_n)$, if and only if: $s_1 \geq s_2 \geq \ldots \geq s_n$ and $\frac{p_1}{s_1} \geq \frac{p_2}{s_2} \geq \ldots \geq \frac{p_n}{s_n}$.

Definition 2 (Florek, 2012)

Let (p_i), $1 \leq i \leq n$ be a fixed non-increasing sequence of positive real numbers ($p_1 \geq p_2 \geq \ldots \geq p_n > 0$) and $\lceil x \rceil$ is rounding the number x up to the nearest integer. Sequence of natural numbers (S_i), $1 \leq i \leq n$ is degressively proportional with respect to the sequence (p_i), if and only if

$$\begin{cases} s_1 = M \\ s_i = \min\left(\left\lceil \frac{s_{i-1} p_i}{p_{i-1}} + a_i \right\rceil, s_{i-1}\right) \quad \text{for} \quad 2 \leq i \leq n \end{cases} \tag{4}$$

for a certain sequence $a_i \geq 0$; $2 \leq i \leq n$ and certain $M \in N$ constant.

Definition 1 is passive in nature – it allows checking whether the sequence S is degressively proportional with relation to the sequence P, but does not specify the algorithm for constructing such sequences. Definition 2 has a constructive character, but does not allow direct determination whether a sequence S is degressively proportional with respect to the sequence P. An additional sequence (a_i) introduced to definition 2 can be the basis for constructing the measure of degression of distribution. For example, for $a_2 = a_3 = \ldots = a_n = 0$ we get a proportional distribution with the obtained values rounded up to the nearest integer, whereas for sufficiently large values of $a_2, a_3,\ldots a_n$ we obtain equal distribution. In order to obtain a sequence (s_i) of constant degression[3] one should , while determining the values $a_2, a_3,\ldots a_n$, take into account the values of the sequence (p_i).

3. Overview of selected methods of allocation of seats in European Parliament

Since the onset of the European Parliament the allocation of seats among Member States was not proportional. Detailed analysis of the composition of the consecutive term shows that, although it is not required by any legal act, distribution of seats in a number of parliamentary terms was in line with the principle of degressive proportionality, that is, increasing the ratio of population and seats with the increase in the population of each country. This principle was apparently intuitively natural alternative in the absence of the possibility of applying proportional distribution. Precise adjustment became necessary when there was rapid development of the Union[4]. The principle of degressive proportionality with its wide range of possible interpretations had to be clarified. Among number of proposals of algorithms for allocation of seats that have been proposed by mathematicians, economists and politicians, the most commonly mentioned are: a parabolic method by Ramirez and the method of shifted proportionality (Fix + prop apportionment) by Pukelsheim. Ramírez González (2007) proposed the usage of quadratic function in the process of distribution of parliamentary seats. He showed that for the population in 2007 and 27 Member States each function $f(x) = 6 + \frac{90}{M-m}(x-m) - cx^2$, for $c \in [0.00143257, 0.0015003]$, used as an allocation function gives the same distribution of seats. The parabolic method can be used in case of fluctuating populations and number of Member States.

This method is also mentioned by (Martinez-Aroza & Ramirez, 2008) and (Ramirez, Polomares, & Marquez, 2006).

A different approach was proposed by Pukelsheim (2010). He presented a very simple and natural design, which he called **a shifted proportionality** (Fix + prop apportionment). In a nutshell, the idea of shifted proportionality is as follows:

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[3] The constant degression can be understood as the decline in the growth rate of the sequence $s_1, s_2,\ldots s_n$ proportional to the increase in the value of the sequence $p_1, p_2,\ldots p_n$.

[4] In 2004 as a result of the accession of 10 new members, the biggest enlargement of EU in the history took place. The number of Member States increased from 15 to 25.

Each Member state receives 6 seats and the remaining ones are distributed proportionally.

In case of a Community comprising 28 member states, after dividing $28 \times 6 = 168$ seats, the rest of the available 583 seats is distributed proportionally taking into consideration the requirement of the Lisbon Treaty that none of the member states will be given more than 96 seats.

Pukelsheim's shifted proportionality became the basis for the most serious proposals to solve the problem of the allocation of seats called the **Cambridge Compromise** (Grimmett et al. 2011). It has been presented as a result of a symposium of mathematicians whose meeting was held at the Centre for Mathematical Sciences of University of Cambridge, on 28–29 January 2011. One of the differences with respect to the adoption of Pukelsheim's methods was that each Member State shall be given 5 seats at the start, which is called **the basis,** and the rest of the pot is divided proportionally rounding up the resulting values to the nearest integer. Additionally, the further details were devised so as the adopted solution could be used for an extended period of time. The symposium agreed on the manner of determining the base number of mandates in the future, depending on the number of Member States. The presented paper included a proposal of distribution of seats during the eighth term of the European Parliament. The distribution algorithm contained in the report from the meeting has been called Base+prop method. A detailed analysis and discussion of findings from the Cambridge Compromise can be found in the articles (Dniestrzański, 2011b) and (Grimmett, 2012). Cambridge Compromise has been rejected as a way to structure the composition of the European Parliament in the eighth term, but it still appears in many official documents of the Community as a valuable concept. An in-depth mathematical analysis of degressive proportionality and examples of other structural divisions can be found in the article (Słomczyński & Życzkowski, 2012). A precise algorithm for setting up degressively proportional distribution with any boundary conditions is provided in the article (Łyko & Rudek, 2013).

4. Newly proposed measure for degressive proportionality

Degressively proportional division is an intermediary solution between the equal and proportional divisions. It has been proposed that the design of the measure will be based on this fact.

Let contenders for the goods are characterized by numbers $p_1, p_2, \ldots, p_n$, where $p_1 \geq p_2 \geq \ldots \geq p_n > 0$. Values p_i are the basis for the allocation of the said goods s_i, the quantity of which is limited and equals H. Let us introduce the following designations:

$$\sum_{i=1}^{n} p_i = V\ ,\ \sum_{i=1}^{n} s_i = H \tag{5}$$

If we apply the equal division, the contender with the number i will receive $\frac{1}{n}H$ of the goods. If we apply the proportional distribution, the contender with the number i will be given the amount the shared goods equal to $\frac{H}{V}p_i$. The value $\left|s_i - \frac{H}{V}p_i\right|$ can be considered as a deviation of a given distribution from the proportional distribution. The value $\sum_{i=1}^{n}\left|s_i - \frac{H}{V}p_i\right|$ is the sum of these deviations for all contenders. Moreover, the number $\sum_{i=1}^{n}\left|\frac{1}{n}H - \frac{H}{V}p_i\right|$ can be interpreted as a variety of equal and proportional distributions. It has been therefore proposed that for the measure of degression of allocation $S = (s_1, s_2, \ldots, s_n)$ respect to the sequence $P = (p_1, p_2, \ldots, p_n)$ use the following value

$$\mathrm{MD(S)} = \frac{\sum_{i=1}^{n}\left|s_i - \frac{H}{V}p_i\right|}{\sum_{i=1}^{n}\left|\frac{1}{n}H - \frac{H}{V}p_i\right|} = \frac{\sum_{i=1}^{n}\left|s_i - \frac{p_i}{p_1 + p_2 + \ldots + p_n}\left(s_1 + s_2 + \ldots + s_n\right)\right|}{\sum_{i=1}^{n}\left|\frac{1}{n}\left(s_1 + s_2 + \ldots + s_n\right) - \frac{p_i}{p_1 + p_2 + \ldots + p_n}\left(s_1 + s_2 + \ldots + s_n\right)\right|} \tag{6}$$

The MD(S) ratio assumes values in the range of [0,1]. The closer the value of this coefficient is to 0, the closer it is to proportional distribution. Accordingly, shifting the value MD(S) in the direction 1 shows a greater equality of distribution. The values MD(S) = 0 and MD(S) = 1 are considered extreme cases which can be obtained in the case of proportional and equal distributions, respectively. It is also believed that the greater the value of MD(S) is equivalent to a larger degression of distribution S. Thus, **degression will be understood as a deviation from proportionality in the direction of equality**.

The defined MD factor will now be used to assess the degree of degression of the selected divisions of seats in the European Parliament among the Member States of the European Union. The analysis will cover five allocations. They are, in order:

1. Approved by the European Parliament distribution of seats in the Parliament during the term between 2014-2019 (column F, Table 1.).
2. Distribution obtained with the use of Base + prop method proposed in Cambridge Compromise, based on the shifted proportionality (Fix + prop) by Pukelsheim (column C).
3. Distribution obtained using the parabolic method by Ramirez (column D).
4. Distribution obtained using the method of shifted root (Fix + root) (Dniestrzański, 2013) (column E).
5. Purely proportional distribution (column G).

The figures for the distributions 1–3 were taken from the resolution of the European Parliament (Gualtieri & Trzaskowski, 2013). Distribution 4 was obtained using the allocation functions $f(x) = 5 + 0,000005719 \cdot x^{0,91}$ and rounding up the obtained values to the nearest integer. The method called shifted root (Fix+root), was developed by the author – a lecture titled: *The proposal of allocation of seats in the European Parliament - the shifted root* was presented at the *12th International Symposium in Management: Challenges and Innovation in Management and Leadership*, and an article which will serve as written record of the lecture is in print.

The figures regarding the population of the Member States as of 1 January 2012 were taken from the study (Gualtieri & Trzaskowski, 2013.)

Table 1 shows the values for the MD ratio for the above allocations. Values of the first four distributions are in the range $[0.2095, 0.2837]$. The highest value of the coefficient is obtained for the distribution of 2014-2019, and the lowest for the division Base+prop. The obtained results allow the classification of the concerned distributions as per the degree of degression understood as a location between equal and proportional distribution. Columns of the Table 1 containing analyzed allocations were ordered based on the value of the measure MD[5]. As one would expect most proportionate distribution (with the lowest MD value) turns out to be the Base + prop division. Though this is not a strictly proportional distribution, its design seems to be the most natural and closest to the idea of proportional distributions, taking into account the boundary conditions. The most distant from proportionality (MD = 0.2867) is the distribution of 2014–2019. It can be regarded at the same time as a distribution which is the closest (among the analyzed) to an equal distribution and is the most degressive. [6]

[5] With the exclusion of proportional distribution which despite the lowest value of the SD measure was placed in the last column. This distribution is only a point of reference for the other analyzed distributions.

[6] Taking into consideration an assumption that the bigger the deviation from proportion, the bigger the degression of the distribution.

Table 1. SD measures of the chosen allocations of seats in the European Parliament

A	B	C	D	E	F	G
Member State	Population 2012	Base+prop	Ramirez	Fix+root	2014-2019	Proportional
Germany	81843743	96	96	96	96	121
France	65397912	83	80	79	74	97
United Kingdom	62989550	80	78	77	73	93
Italy	60820764	78	75	74	73	90
Spain	46196276	61	60	59	54	68
Poland	38538447	51	51	50	51	57
Romania	21355849	31	32	31	32	32
Netherlands	16730348	25	26	26	26	25
Greece	11290935	19	20	20	21	17
Belgium	11041266	18	19	20	21	16
Portugal	10541840	18	19	19	21	16
Czech Republic	10505445	18	19	19	21	16
Hungary	9957731	17	18	18	21	15
Sweden	9482855	17	17	18	19	14
Austria	8443018	16	16	16	19	12
Bulgaria	7327224	15	15	15	17	11
Denmark	5580516	12	13	13	13	8
Slovakia	5404322	12	12	13	13	8
Finland	5401267	12	12	13	13	8
Ireland	4582769	11	11	12	11	7
Croatia	4398150	11	11	11	11	7
Lithuania	3007758	9	9	10	11	4
Slovenia	2055496	8	8	8	8	3
Latvia	2041763	8	8	8	8	3
Estonia	1339662	7	7	7	6	2
Cyprus	862011	6	7	7	6	1
Luxembourg	524853	6	6	6	6	1
Malta	416110	6	6	6	6	1
Total	508077880	751	751	751	751	751
MD		0.2095	0.2323	0.2474	0.2837	0.0087

Source. Own calculations

For comparison, column G of the table shows the proportional distribution concerning the roundings to the nearest integer. This comparison gives a more complete picture of the deviation of the distribution from proportion, due to the fact that the necessity to allocate total values excludes (except very improbable situations) the possibility of obtaining a pure proportion. Although, the factor for the distribution in column G is close to zero. Of course, because of the failure to fulfill boundary conditions presented in the Treaty of Lisbon, this distribution cannot be taken into account in the design of the composition the European Parliament.

5. Summary

Degressive proportionality has been legally approved as an idea distributing seats in the European Parliament. This was mainly because due to large diversity of populations of Member States both proportional allocation method and equal division proved useless. So the idea that somehow was a motto of proportional degression, saying “let's give the small more at the expense of the large, but not too much” must be refined so that it can actually be used in the subsequent terms of office of the European Parliament. One of the factors that may have an impact on reducing political bargaining accompanying each new Parliamentary elections can be the possibility to assign the degree of proportionality (and also the degree of equality) of the distributions. Then it will be possible to have a closer discussion on the proposed solutions from the mathematical perspective. The proposed measure of degression has a simple and clear interpretation. As its design is of general character, it can be used in any case where the allocation of goods was carried out on the basis of the degressive proportionality. Neither is it relevant whether there is a goods in question are divisible or indivisible. Of course, in case of indivisible goods achieving a strictly proportional division, ie, with the measure MD = 0, is often impossible due to necessary roundings. The MD measure show that the composition of the European Parliament during the term of 2014-2019 is the least proportional from the analyzed alternative distributions. For the proposed measure to be fully usable in constructing degressively proportional distributions all is characteristics must be thoroughly examined, which requires further research.

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