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A Method to Solving Cyberspace Security-model Equation

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Abstract

It is important to solve cyberspace security-model equation in recent science calculation and applications, such as microcontroller float divide, strata micro earthquake rupture positioning solution, and so on. In this paper, we use the taylor expansions and the thought of infinite approximation to get the root of the cyberspace security-model equation. First, introduce the calculation formula of the new iterative method. Second, make the explanation of the new iterative method for root. Third, make a discussion about the convergence of the new iterative method for root. Forth, introduce the calculation steps of the new iterative method for root. The last but not the least, in order to let readers understand the new method, we write down some notes and problems during our design.

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1. Introduction

The solution of the new iterative method for root is regarding cyberspace security-model equation, \( f(x) = 0 \), as a function \( y = f(x) \). We spread function at the point \( x = x_k \) in Taylor formula. The root of the function \( f(x) = 0 \) is the value of the cyberspace security-model equation, at the same time, it is the intersection of the curve and the x axis, so we can get:

\[
f(x) = f(x_k) + f'(x_k)(x - x_k) = 0
\]

\[
x = x_k - \frac{f(x_k)}{f'(x_k)}
\]

(1)

Give \( x = x_{k+1} \), then we can get the calculation formula of the new iterative method for root:

\[
x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}
\]

(2)

For example, given a kind of cyberspace security-model equation \( f(x) = xe^x - 1 \), its derivative function is \( f'(x) = (x + 1)e^x \), and the new iterative formula is

\[
x_{k+1} = x_k - \frac{xe^x - 1}{(x + 1)e^x}
\]

(3)

We have known about the point which is the intersection of function curve \( f(x) \) and x axis is the root of the cyberspace security-model equation \( f(x) = 0 \), now we give a initial iteration point \( x_0 \), by this way, we can get the function value \( y_0 = f(x_0) \), then make a tangent \( \tau \) through the point \( y_0 \), it will fellowship \( x \) ais in point \( x_1 \), and its circumference is the the value of the point \( x_0 \)'s derivatives numerical \( \tau = f'(x_0) \):

In triangle \( x_0y_0x_1 \),

\[
x_0 - x_1 = \frac{x_0y_0}{\tan \alpha}
\]

(4)

Among them

\[
x_0y_0 = f(x_0), \quad \tan \alpha = f'(x_0)
\]

(5)
So we can get:

\[ x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \]  

Similarly, we can also get the function of the point \( x_1, y_1 = f(x_1) \), make a tangent through the point \( y_1 \), it will meet \( x \) axis in point \( x_2 \). It is easy to prove

\[ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \]

According to the above method we make the tangents and triangles repeatedly, \( x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \) \((k = 0, 1, 2, \ldots)\). Namely we can get root under the iterative convergence:

\[ x^* = \lim_{k \to +\infty} x_k. \]

2. The convergence of the new iterative method for root

The great advantage of solving cyberspace security-model equation problem with the new iterative method is its high speed of convergence, the disadvantage is the strict requirements of initial iteration points. In other words, some Initial iteration points can make the solution smoothly but others can’t.

2.1. The conditions of the new iterative convergence

As for the new iterative type, the Initial iteration points are important. If the selection of Initial iteration points aren’t appropriate, iteration will be failed.

According to the following picture, the initial point \( x_0 \) is the Iterative convergence point, but the initial point \( x_1 \) is the divergent point, so we can make a conclusion about the changing rules of the following functions

From the function curve and the function’s changing rules, we can see the selection of Initial iteration points is directly related to the convergence of the new iterative method for root. Under the general situations, the selecting condition of the new iterative method’s initial point is:

\[ f(x_0) \cdot f''(x_0) > 0 \]

According to the introduce of reference[1], the requirement of Initial iteration points is:

\[ |f'(x_0)|^2 > \frac{|f(x_0) \cdot f''(x_0)|}{2} \]

Because seeking for function’s derived function and its second derivative is difficult, thus, it is difficult for us to find a right initial iteration point. As usual, we try different Initial iteration points or choose the point which is close to the root (estimated). For example: we give an cyberspace security-model equation: \( f(x) = x^3 - x - 1 \). Its the new iterative type is

\[ x_{k+1} = x_k - \frac{x_k^3 - x_k - 1}{3x_k^2 - 1} \]
and now we give different initial iteration points $x_0$, we can get many different results.

### Table 1. Calculation results

<table>
<thead>
<tr>
<th>$k$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>0.5</td>
<td>1.5</td>
<td>5.0</td>
</tr>
<tr>
<td>1</td>
<td>-0.50000</td>
<td>-5.0000</td>
<td>1.34783</td>
<td>3.39189</td>
</tr>
<tr>
<td>2</td>
<td>-3.00000</td>
<td>-3.36486</td>
<td>1.32520</td>
<td>2.35857</td>
</tr>
<tr>
<td>3</td>
<td>-2.03846</td>
<td>-2.28096</td>
<td>1.32472</td>
<td>1.73635</td>
</tr>
<tr>
<td>4</td>
<td>-1.39028</td>
<td>-1.55628</td>
<td>1.32472</td>
<td>1.42576</td>
</tr>
<tr>
<td>5</td>
<td>-0.91161</td>
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<td>1.33308</td>
</tr>
<tr>
<td>6</td>
<td>-0.34503</td>
<td>-0.56141</td>
<td>1.32472</td>
<td>1.32478</td>
</tr>
<tr>
<td>7</td>
<td>-1.42775</td>
<td>-11.86434</td>
<td>1.32472</td>
<td>1.32472</td>
</tr>
</tbody>
</table>

According to the table,
When $x_0 = 1.5$, $x_3 = 1.32472$, the cyberspace security-model equation is convergent,
When $x_0 = 0.5$, $x_7 = -11.86434$, the cyberspace security-model equation is divergent.

### 2.2. the speed of the new iterative method for root

the new iterative type: $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

regard as function iterative type: $g(x) = x - \frac{f(x)}{f'(x)}$

The speed of convergence is $g'(x)$:

\[
g'(x) = 1 - \frac{f''(x)f'(x) - f(x)f''(x)}{[f'(x)]^2} = \frac{f(x)f''(x)}{[f'(x)]^2}
\]

If the root of the cyberspace security-model equation $f(x) = 0$ is $x^*$, then $f(x^*) = 0$, if $f'(x^*) \neq 0$, put this value into $g'(x)$, we will get $g'(x^*) = 0$.

Then we make derivation for $g'(x)$, we can get $g''(x)$, put the $x^*$ into the $g'(x)$, we will get $g''(x^*) \neq 0$.

Thus, when $g'(x^*) = 0$ and $g''(x^*) \neq 0$, we call this Second-order square convergence. It means the new iterative method for root belongs to second-order square convergence. Here we give the formula of $g''(x)$. In order to simplify the symbols of derivation, now we assume:

$f = f(x)$, $f' = f'(x)$, $f'' = f''(x)$

Thus $g(x) = x - \frac{f}{f'}$, $g'(x) = \frac{f''}{f'^2}$ So,
\[ g''(x) = \frac{(f')^2 - [(f')^2]'}{[(f')^2]^2} \]
\[ = \frac{(f'' + f''')(f')^2 - 2f'f''f'''}{(f')^4} \]
\[ = \frac{(f')^2 f'' + ff'f''' - 2f(f')^2}{f'^3} \] (11)

3. Conclusion:

In this paper, we use the Taylor expansions and the thought of infinite approximation to get the root of the cyberspace security-model equation. First, introduce the calculation formula of the new iterative method. Second, make the explanation of the new iterative method for root. Third, make a discussion about the convergence of the new iterative method for root. Fourth, introduce the calculation steps of the new iterative method for root. Fifth, present the program of the new iterative method for root. The last but not the least, in order to let readers understand the new method we write down some notes and problems during our design.

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