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THE SYMMETRY OF M. C. ESCHER'S "IMPOSSIBLE" IMAGES

CAROLINE H. MACGILLAVRY
Vijverhoef 40, NL-1081 AX Amsterdam, Netherlands

Abstract—Escher's preoccupation with symmetry is well known. His periodic plane-filling patterns, his Circle limits, have been fully analyzed by several authors. Less attention has been given to the symmetry—much less evident—in his graphic work. In some of his prints, it is directly visible: then it is two-dimensional symmetry. In other cases one has to visualize or construct the three-dimensional image that is evoked by the print. Examples of both cases are given.

1. WHAT IS IMPOSSIBLE?

This is the first question. Not Escher's prints and drawings: They exist! He not only thought them out but also executed them. However, among the images that they evoke in the spectator's mind, and were meant by the author to do so, there are many that we feel to be impossible, inconceivable to exist in our "objective" three-dimensional reality.

This is the case with the imagery of many modern artists. What distinguishes Escher's work is, first, that every print or drawing is composed of items (animals, plants, fragments of buildings, etc.) which are reproduced with academic, "life-like", precision, so that the spectator can immediately identify each separate object as something known from everyday experience. These details tend to distract the mind from Escher's intentions with the print as a whole, when he combines the conceivable fragments into an "impossible" whole. These intentions which he wants to convey to the onlooker, are mostly quite abstract: the properties of space itself, such as symmetry; the geometric relations between an object in three-dimensional space with its representation on the flat plane of the drawing-board; the effect of visual illusions; approaches to infinity within the contours of the print or drawing. Thus, and this is the second point to make, Escher's prints appeal to our intellect rather than to our aesthetic or emotional receptiveness. Emotions, although composed of different and conflicting tendencies, overwhelm us instantaneously, whereas the intellect works step by step. Picasso's "Guernica", also composed of many individually recognizable details, evokes an immediate image of the horrors of war. Escher's "Concave and convex" (Fig. 1), full of details like "Guernica", at first sight only evokes bewilderment. Something does not fit, but what? The worst thing happens in the lower middle of the print: a floor on which a man sits dozing (left side) changes towards the right into a ceiling, from which a lamp hangs. At this point a spectator either turns away in disgust, or tries to understand what has been in Escher's mind.

2. DIFFERENT VIEWPOINTS COMBINED IN ONE PRINT

Let us follow the second course, step by step. First, strip the building from all its movable objects, such as human figures, lizards, flower pots. Except the two little pavilions, left and right, the building then appears to have the bi-lateral symmetry that most monumental buildings in all cultures have. What then goes wrong when the animals, ladders, etc. are put back in their places? The key to the picture is the flag, hanging from the wall and the balcony on the right. The pattern embroidered on it shows a well-known visual trick: a set of three adjoining parallelograms, repeated in two directions, is interpreted by the eye and the brain as a stack of blocks in two different ways: Seen from below, the small grey face is the bottom of each block, the left edges of the white faces seem to stick out of the drawing. But one can also "see" the small grey faces as the tops of blocks; then the right edges of the white faces are protruding. It is impossible to see the two representations simultaneously. However, Escher has separated them in the print. In nearly the whole right half we are forced by lamp, flagpole, and especially

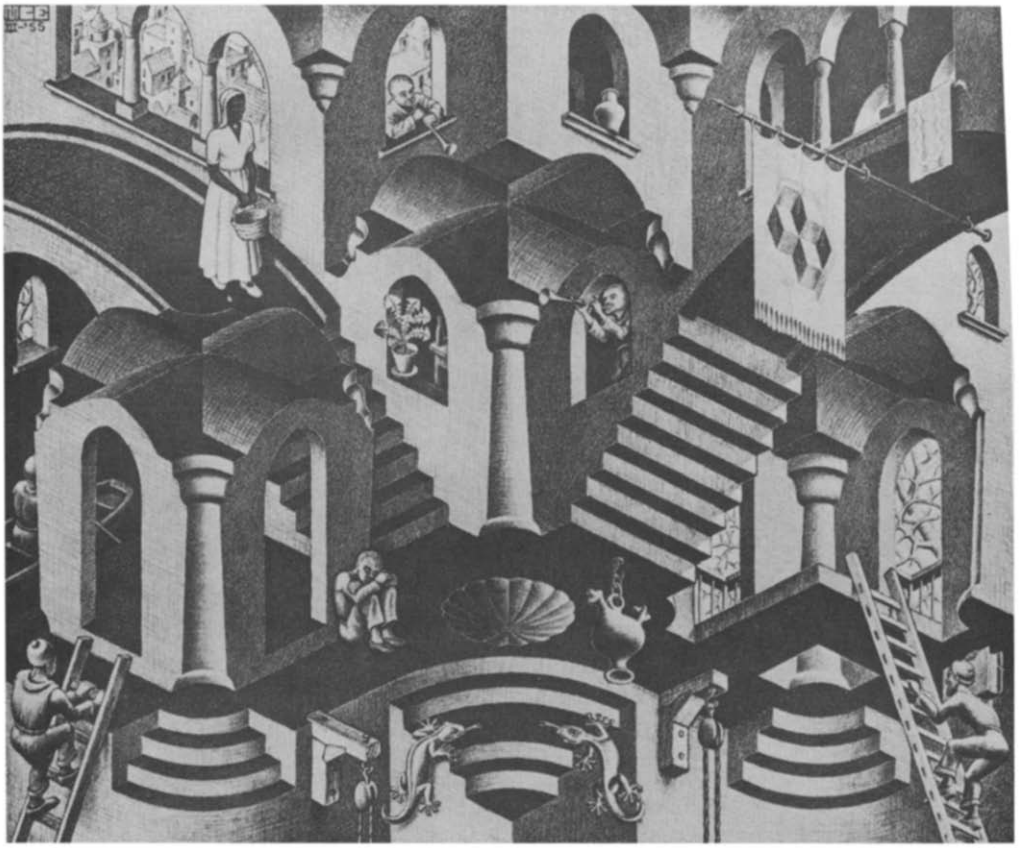


Fig. 1. "Concave and Convex". (Copyright M. C. Escher heirs c/a Cordon Art, Baarn, Holland. Used by permission.)

by the bottom of the little pavilion, to view this part of the print from below upwards ("grey is bottom"). In the left part of the picture our view is forced downwards ("grey face is top"): the woman walks on a bridge which by symmetry corresponds to a vault on the right. The roof of the pavilion on the left becomes the ceiling of the one on the right, etc. What about the central strip? It is truly ambidextrous. The lizards below do not seem to mind, but the man sitting above them, slightly towards the left, will fall off the ceiling when he ventures across the border line between left and right.

In other prints where different directions of view are combined, this may lead to different kinds of symmetry. Figure 2, "Another world", shows the interior of a cube-shaped building. In the faces of this cube there are large "windows", separated by pillars. The axes of all these pillars converge towards one point, exactly in the center of the print. Consequently, the building is clearly drawn in perspective, but how? Divide the print in three segments: a) upper wall with upper half of the right-hand side wall, (b) left side with central square, and (c) lower wall with lower half of right-hand side wall. Make three masks, each covering two of the segments, leaving a view on the third one. Covering (b) and (c) one sees a moon-like landscape through the two windows of segment (a); a bird is sitting on the sill of the upper window and a horn hangs from the arched top of the right one. The direction of view is downward, practically in the plane of the print. The seemingly parallel pillars converge towards the center of the print, which is the so-called *nadir* of segment (a). If only segment (b) is left uncovered, the direction of view is perpendicular to the plane of the print. One sees again the bird, now from the side, the horn on the left, the moon landscape viewed horizontally, and the sky above it. The *vanishing point* is again in the center of the picture and now lies on the horizon of the central panel. The third view (c) is from below, directly opposite to the first view, converging towards the *zenith*, showing the sky. These three directions of view, at angles of about 90°, induce an apparent fourfold rotational symmetry about an axis across the middle of the print, parallel to its lower and upper edges. This symmetry is seen in the three representations of the same bird in the

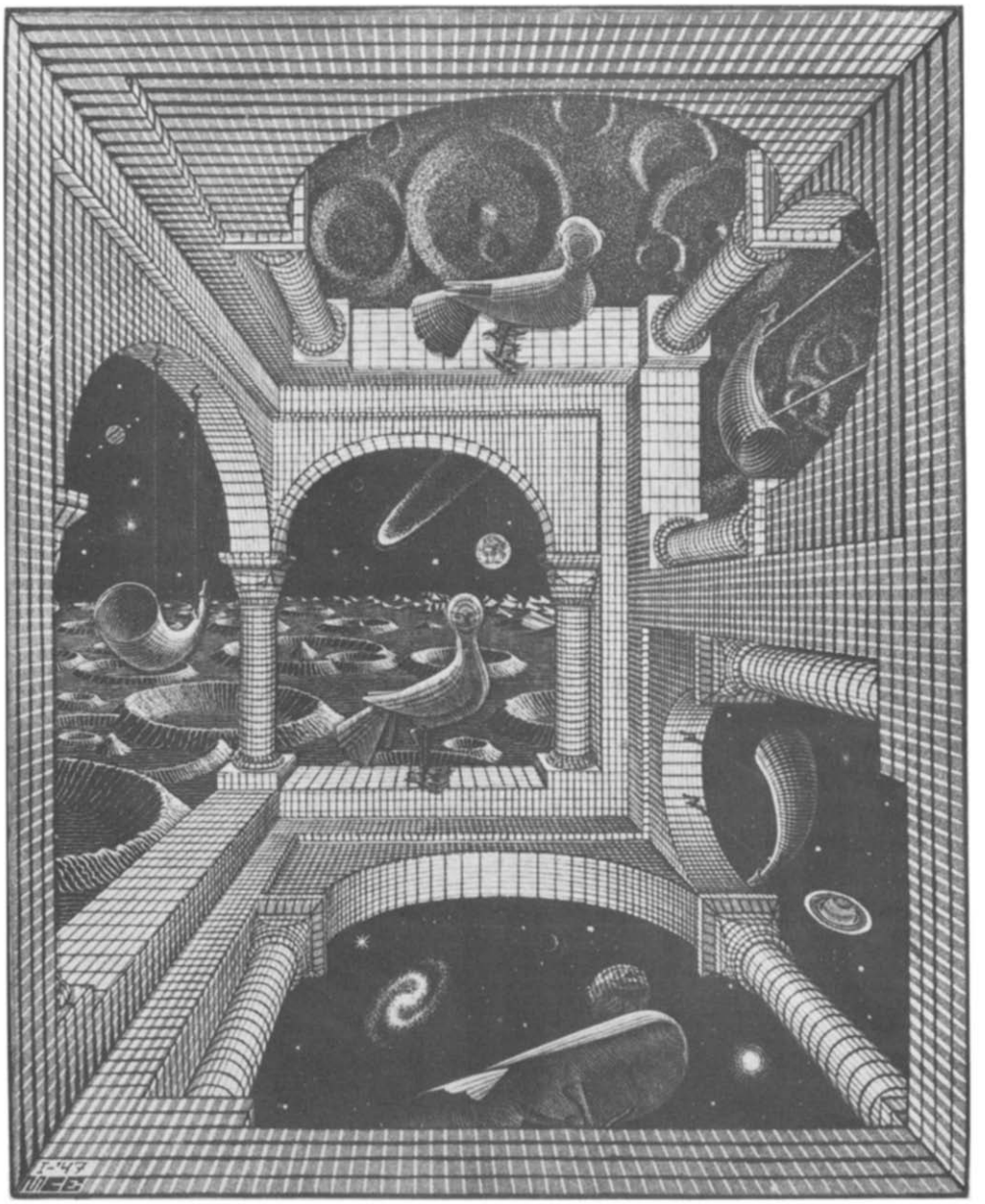


Fig. 2. "Another World". (Copyright M. C. Escher heirs c/a Cordon Arts, Baarn, Holland. Used by permission.)

three central panels. As to the side walls: the right one follows the rotation over 180° from view (a) to view (c). During view (b) this wall is covered. The left wall is only visible in view (b) so it is not bound by the symmetry.

Another print revealing three directions of view is "Relativity" (Fig. 3(a)). By eliminating all the niches, balconies, gardens and doors, we obtain the schematic Fig. 3(b). It shows that basically there are three walls, which appear to be perpendicular to each other, intersecting along the lines marked OA, OB and OC. By following in Fig. 3(a) the sets of lines supposedly parallel to these three lines, we find that they converge to three different vanishing points. Thus there are, as in "Another world", three directions of view, which in space would be perpendicular to the three walls. However, in this case the three corresponding vanishing points lie far outside the print, as indicated by the three arrows in Fig. 3(b). Next, take a look at the people who live in this strange building. The figures which I have colored blue are clearly subjected to a downward force, like we ourselves are attracted by gravity towards the center of the earth. These people are seen from below, in the direction AO towards the *zenith* A, as

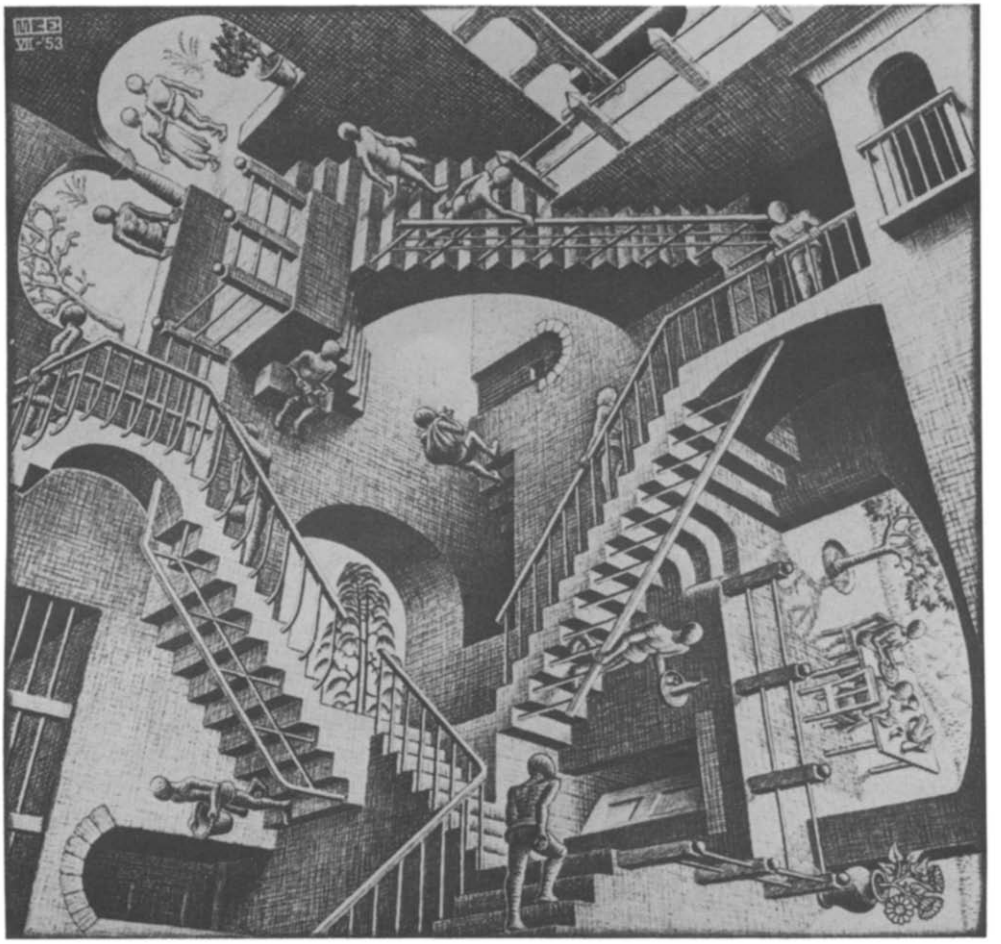


Fig. 3(a). "Relativity". (Copyright M. C. Escher heirs c/a Cordon Art, Baarn, Holland. Used by permission.)

indicated in Fig. 3(b). A second group of people, the red ones, seem to be attracted by a force in the direction *BO* towards the left side wall, marked red. This wall is their floor; they do not feel the gravity that pulls the blue persons down. Neither do the green people who walk with their feet towards the right wall. The red and the green people are viewed from head to feet, so the vanishing points of their worlds are towards the *nadirs B* and *C* respectively.

Finally, look at the set of staircases in Fig. 3(a). There are three major ones which appear to be arranged as an equilateral triangle. In fact, if the print represents a view into a cubic room, then the direction of view on the print as a whole would be close to that of a space diagonal of the cube, which does have trigonal symmetry. These three stairs are fixed sideways to the three "walls", *BOC*, *COA*, and *AOB* respectively, as shown in Fig. 3(b). The one perpendicular to the blue people's ceiling—and of course also to their floor, not visible in the print—has steps which are parallel to the red and the green wall respectively, as shown in Fig. 3(b). So the red and the green people can walk on these stairs. When they both go from left to right, one is going down the stairs, the other is going up, as shown in Fig. 3(a). Why this difference? The red man climbs up from his floor, the green one descends towards his. The banister of this staircase is marked blue in Fig. 3(b), because it is parallel to the blue ceiling. The banisters of the other two big staircases are marked red and green respectively because their stairs are fixed sideways against the red (left) and green (right) wall. Again, on these two staircases the two sets of "other"-colored people can walk: the blue and green ones on the "red" staircase; the blue and red people on the green. Remarkably, in these latter two cases the people of one color walk on one side of the stairs, the other group on what would appear to the first group to be the reverse of the stairs. Why this difference with the top staircase *A*? That one goes between the *floor B* and the *floor C*. The crystallographic denomination of the

plane of that staircase would be (011). The other big staircases, marked B and C run between the *floor* C (resp. B) and the *ceiling* A. Crystallographic planes $(\bar{1}01)$ and $(\bar{1}10)$. Thus, although these three staircases seem to be equivalent, in "reality" they are not. The same holds for the three vanishing points: zenith A, but nadirs B and C, as shown in Fig. 3(b). There are several other stairs in Fig. 3(a); some of them are indicated in Fig. 3(b). It is left to the reader to find out to which of the two types of stairs these belong. Crystallographers can also denominate the threefold axis, mentioned above, running perpendicular to the triangle of the three big staircases.

3. PLANE AND SPACE

Another quite frequent approach in Escher's work is the following. He starts from one of his truly periodical plane-filling patterns[1], constructed from adjoining, more or less schematically drawn animals (including angels and devils). By giving some of them an appropriate sort of relief in the drawing, these become more "life-like", they give a spatial suggestion and Escher then makes them walk or fly in seemingly three-dimensional space. Of course, they are still drawn on his plane drawing board! (See for example Fig. 4 "Reptiles".) His notebook is lying open, showing a trigonal pattern of reptiles in three different colors (in the litho: three different shades). The motifs are closely interlocked, and although recognizable as reptiles, they function simply as motifs in the pattern. However, two of them, in the lower left hand corner of the drawing, do come to life, one stretching a paw over the lower edge of the notebook and raising its head from the paper. The other has freed its whole right side, the left paws are still integrated in the repeating pattern. The print then shows the subsequent adventures of the creature: it climbs on a zoology book, and along a drawing-triangle on to the top of a regular polyhedron. From there it descends over an ashtray down into the pattern again, where it loses its individual identity.

Fig. 4. "Reptiles". (Copyright M. C. Escher heirs c/a Cordon Art, Baarn, Holland. Used by permission.)

It is the polyhedron to which I wish to draw attention. It represents Plato's regular pentagondodecahedron, a very symmetrical body, having *inter alia* five-fold rotation axes normal to each of its twelve faces and three-fold symmetry about each of its twenty corners. It seems that Escher wanted to say: "Look, in the plane pattern the creatures are arranged in close-packed hexagons. Three hexagons meet at each corner. But as long as you stay in the plane of the drawing, you cannot fill it in this way with regular pentagons. However, when you allow your paper to fold in three-dimensional space, and you fit the edges of adjoining pentagons together, you get the object on which the little alligator blows out its blast of triumph."

A possible sequel to this story is the following. Imagine that suddenly each face of the dodecahedron starts expanding in the five directions beyond its edges until it meets the extensions from the five adjoining pentagons. On the top of each face there is now formed a pentagonal pyramid. The originally convex body has become star-shaped with twelve fivefold spikes sticking out in the directions of the fivefold axes. If a little monster now tries to climb this stellated dodecahedron, it is lucky to find that the five side-faces of each spike have openings through which it can stick out its head and its four feet. (Its tail, if it has one, has to be tucked inside the pyramid.) Fig. 5 shows six of these monsters: one sitting on a central face and the others on its five adjoining faces; each under its own pyramid. Since they do not seem to fall off in any direction, they are apparently attracted by a gravitational force directed towards the center of the whole polyhedron. This is why Escher called the print "Gravitation". Note that the pyramids which should be at the back of the figure are also inhabited by monsters: some of their heads and feet are partly visible. The symmetry of the stellated polyhedron is still that of the original pentagondodecahedron. The creatures are not symmetrically arranged: Escher left them some freedom and individual rights within their prisons.

Among Escher's prints there are several more examples of creatures in his repeating patterns "coming to life" and seemingly going off into three-dimensional space, but in fact always staying in the plane of the picture. This is what Escher means to show: "Drawing is Fake!" In this group there is one print which from the point of view of symmetry needs further analysis. This is the well-known wood-cut "Day and Night" (Fig. 6). The landscape, seen in bird's eye view, appears quite symmetrical. Practically all the elements in the left-hand, "day" side of the print are found in mirror image on the right-hand side, as seen during the night: little town, river, etc. Remarkably, this most famous print is based on a typically Dutch landscape, in contrast with the many Italian ones of Escher's earlier period.

How do the birds come into the picture? Concentrate on the lower middle of the print, near the edge. Forgetting the rest of the picture, this part looks like the corner of a chess board. Going upwards and fanning out towards left and right, two changes occur. The fields of the chess board become fields in the landscape, receding in perspective towards the horizon. But at the same time, the plane of the chess board appears to fold upwards and to change gradually into a pattern of interlocking white and black birds. In the middle this pattern extends to the top of the print, blocking out the view on the landscape. Going to the right, the bird pattern resolves into a flock of white birds flying into the night, while the black birds fade away so the landscape is again visible. Towards the left, the complementary change occurs.

Why do I call this an "impossible" image? Because it is the superposition of two images: the vertical plane in which the birds move and the horizontal plane of the landscape. These two planes are welded together at the bottom of the print. By projecting the landscape on the plane of the birds, the complete picture is regenerated! Note how the contours of a vanishing bird cover more and more fields as you move up. The last point to make is about symmetry. The black–white mirror symmetry of the landscape seems at first sight also to be present in the bird pattern, although here coupled with a shift upwards over half the distance between two birds of the same color. However, the black birds are not the mirror images of the white ones: look at their tails!

4. CHANGING THE SCALE

Escher was much preoccupied by the problem of how to suggest infinity. Drawing in perspective is one solution, as we saw in "Day and night". Escher explored other, less conventional ways of the principle of perspective. His remarkable results have been fully described

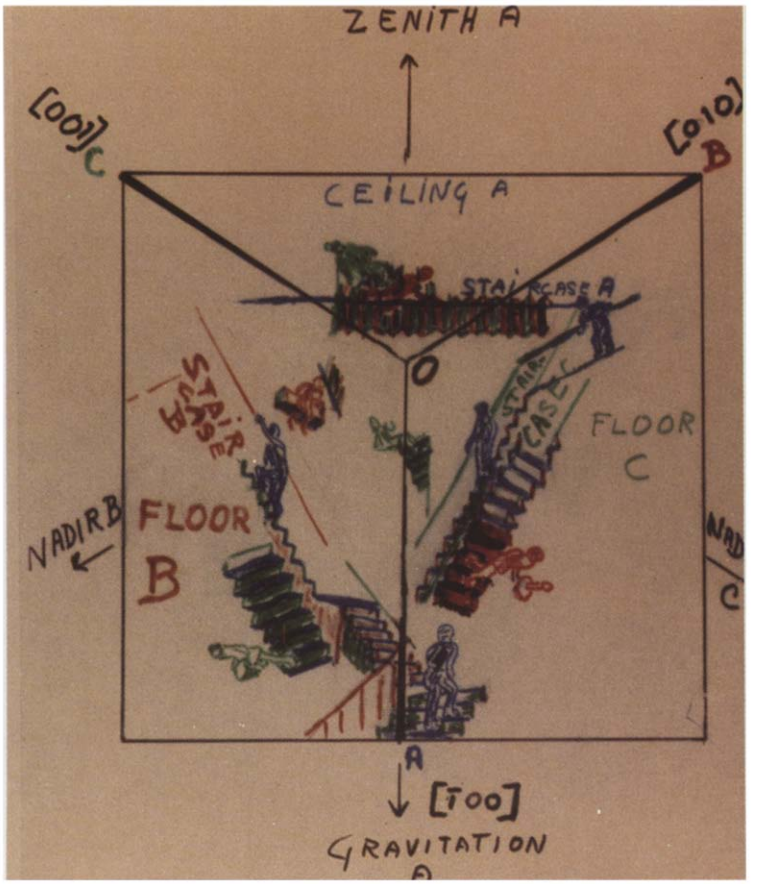


Fig. 3(b). Schematic drawing of "Relativity".

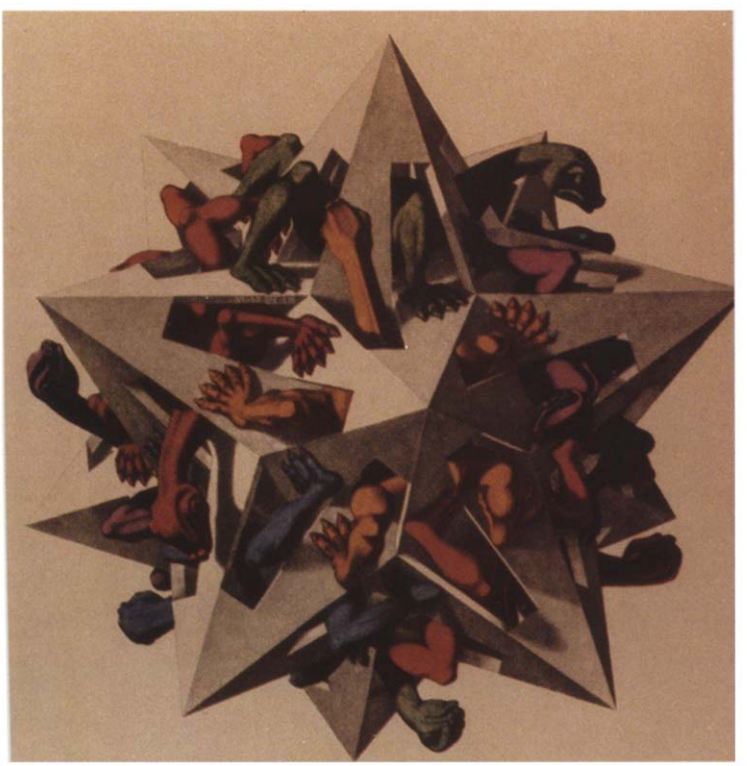


Fig. 5. "Gravitation". (Copyright M. C. Escher heirs c/a Cordon Art, Baarn, Holland. Used by permission.)

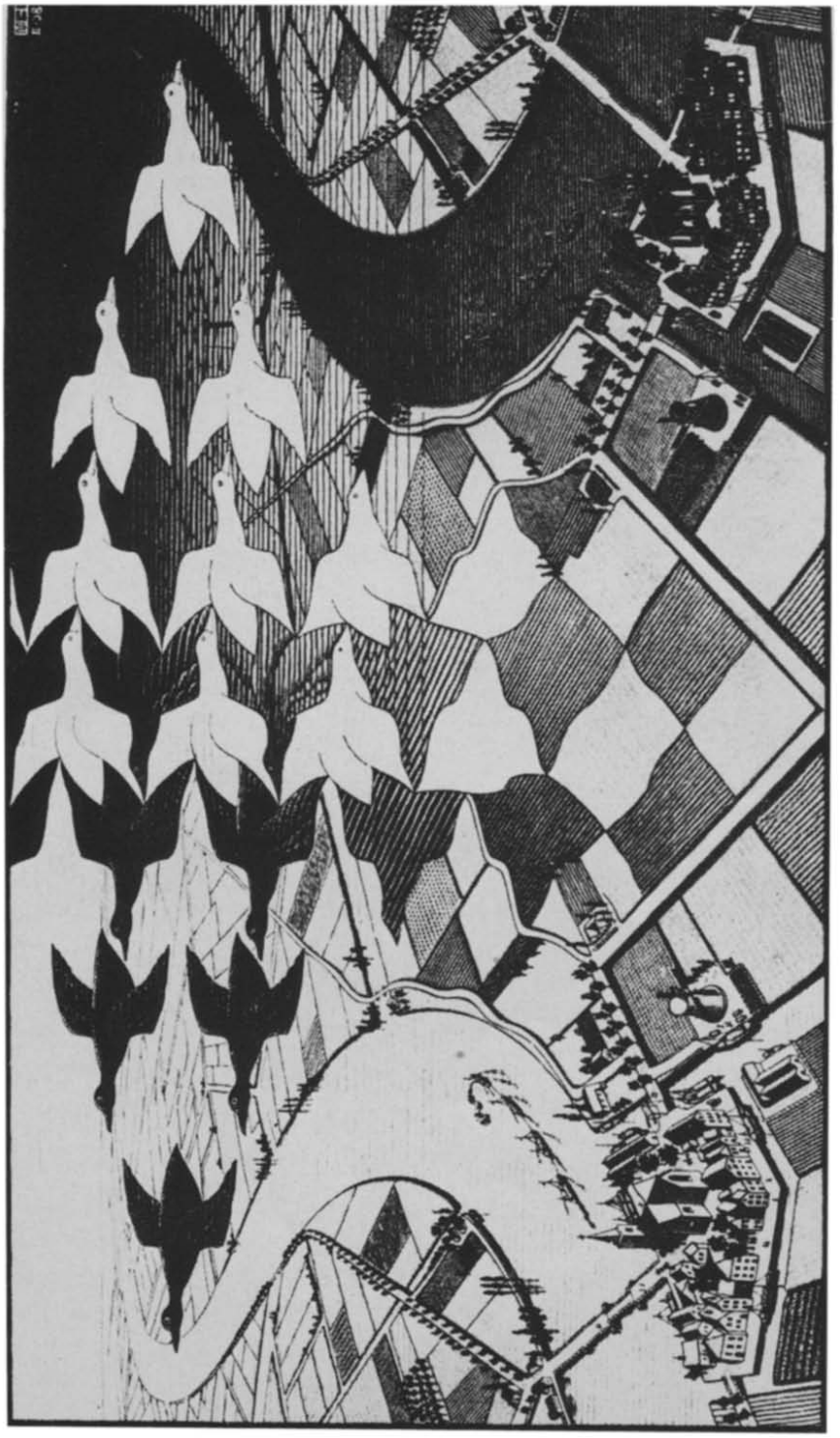


Fig. 6. "Day and Night". (Copyright M. C. Escher heirs c/a Cordon Art, Baarn, Holland. Used by permission.)

by Bruno Ernst in his book *The Magic Mirror of M. C. Escher*[2]. Another approach can be seen in Escher's numerous plane-filling repeating patterns: in principle these could be extended, like wall-paper or block printed material, to any desired length. But this did not satisfy him. He wanted to construct repeating patterns from motifs with ever smaller size, so that he could confine the design within the contours of his printing block or drawing paper. His first attempts had very small motifs in the middle of the picture, which grew larger and larger towards the edges. Again he did not find this satisfactory, because also here ever larger motifs could be added, so enlarging the size of the drawing. Inspired by a construction devised by the mathematician H. S. M. Coxeter, he used this frame of curves to construct his later "Circle-Limits I to IV", in which the largest motifs are in the center, becoming ever smaller towards the circumference of the circle. In principle, the construction would indeed allow a convergence towards infinitely small items. The very complicated, non-Euclidean symmetry of these circle limits has been fully analyzed by Coxeter[3], so I will not go into this. Instead I wish to draw attention to Fig. 7, "Fishes and scales". As always with Escher's work, it is essential to explore the print step by step. Below we see alternating rows of black and white fishes interlocking, but seeming to swim in opposite directions. Whereas the black fishes increase in size as they swim to the right, the white ones become smaller in their swimming direction. In the fourth and fifth rows from below, this effect gets enhanced. While the white fish in the lower right-hand side still stays in its row, the black one in the next row grows so large, both sideways and upwards, that it tends to occupy nearly the whole upper half of the figure. Something very odd happens: with the change of scale, its scales change into fishes! From here on the whole

Fig. 7. "Fishes and Scales". (Copyright M. C. Escher heirs c/a Cordon Art, Baarn, Holland. Used by permission.)

story repeats itself from right to left and then from top to bottom *ad infinitum*. What happens to the white fishes in the sixth row from below? As you follow them counter to their swimming direction, they get larger, as before, until they reach the center of the print. There they curve towards the right, changing their swimming direction and changing into scales of the big black fish. They become smaller and smaller until they reach the head of this fish. The only fixed point in this merry-go-round is the center of the print. You can check that it is a twofold rotation point: draw the contours of the fishes in the lower half of the print on a transparent sheet, rotate this sheet one half turn about the center of the print and you will see that it fits exactly the contours of the upper half. The only deviation from this twofold symmetry is the position of mouths and eyes: all the fishes look upward. Could this arrangement be extended beyond the limits of the print? One feels it could, but it would have required the geometric genius of Escher to do it. On the other hand, his sense of aesthetics may well have restrained him.

In Fig. 8, "Print gallery", the change of scale is an even more intriguing feature. You enter this gallery in the right lower corner of the figure and admire the prints (Escher's prints!). You pass a visitor and then you see a boy with somewhat abnormal body proportions. Comparing his head to his hand you see that he becomes larger from foot to crown. Also the framework of the gallery has extended abnormally. What at first, on the lower right, was a rather low arch, is now growing out of the top of the picture, just like the big fish in Fig. 7. The young man is looking at a print which undergoes the same transformation as he himself and as the gallery. Exploring it further, upwards and then towards the right this print expands more and more until its frame disappears out of the picture. Then, going down on the right, you find to your surprise that the gallery in which you started your exploration is part of the print. Going further around

Fig. 8. "Print Gallery". (Copyright M. C. Escher heirs c/a Cordon Art, Baarn, Holland. Used by permission.)

clockwise you find again the boy. So he is looking at a print in which he himself must be present somewhere. But then, where *is* he in the print, apart from being outside it? Start again from him, and this time go around withershins as witches do (after all, this *is* magic!). Now you find yourself rotating towards the white spot in the middle. Since, during this counter-clockwise turn, everything gets smaller, then boy must be inside the white spot, somewhere to the right, under the slanting roof of the gallery, of which you see the left end. But pursuing this gallery towards the right, is it not the same gallery from which you started? The riddle is solved when we try to imagine what is going on inside the white spot. Still moving counter-clockwise the print should become smaller and smaller until it dwindles in the center. Then, now turning again clockwise, the infinitesimally small print starts expanding spiralwise until it becomes the print as pictured on the left. Expanding further, the next layer of the spirally extending sheet begins to overlap the foregoing turn. The line where this happens starts just beyond the right of Escher's signature. It runs along the second rib of the slanting roof. Towards the right and down is the new layer of the spiral plane, towards the left, slightly upwards, the old one, as is seen by the two small arches. The second one, the top of which is just visible beyond the white spot, corresponds to the missing top of the large arch under which the boy in the left foreground stands! The whole litho is a sort of analogy in two dimensions of the Birth of the Universe. The "Big Bang" would take place in the center of the white spot; the overlapping sheets of the expansion spiral represent the time-scale of the expanding Universe.

Symmetry in this print is only a hint of a repetition along a spiral on an ever larger scale.

5. IMPOSSIBLE BUILDINGS

Under this heading I will discuss those buildings to which Escher himself gave this name. Of course, the structures in Figs. 1, 2, and perhaps 3(a) could have been included in this group.

The three prints to be considered now are all based on optical illusions. In Fig. 9(a), "Belvedere", the essential clues are the piece of paper lying on the floor and the object that the man sitting on a bench nearby holds in his hands. The sketch on the paper shows the conventional way of drawing a cube in elementary stereometry lessons. It can be interpreted either as a three-dimensional transparent cube seen from above, or alternately, the same cube seen from below (see the flag in Fig. 1!). In either case, the two pairs of lines that in the drawing intersect at points marked by a small circle, in space do not intersect but cross. One of the vertical edges of the cube is in front, the other is in the back of the cube. The corresponding horizontal lines represent edges in the back and in the front respectively. Just as in the flag of Fig. 1, when one changes the interpretation "seen from above" into "seen from below", those edges that first seemed to be in front, now appear to be in the back of the cube, *vice versa*. What happens if one starts with the two horizontal squares, the top and bottom faces of the cube, and makes the wrong connections between the two squares, so that the corners in front of the upper square are connected to the ones in the back of the lower square, and *vice versa*? When one does this, and it can be done with some distortion, one gets the object that the man on the bench holds in his hand. It combines the two views of the drawing on the paper: from above and from below. The man seems amazed to see that the bottom square appears to be rotated with respect to the top one. Above his head, this is actually the case: the man on the floor of the loggia in the Belvedere looks away to the right, whereas the lady upstairs turns half-face towards the onlooker. Evidently the distortion becomes less when the vertical edges, the pillars, are much longer than the edges of the base. If one follows the course of the pillars carefully, one sees that the four in the front row of the loggia floor are fixed to the back row of the ceiling, and *vice versa*. Escher has very cleverly camouflaged this crossing of the pillars by two tricks: firstly, the balustrade of the loggia floor is viewed nearly horizontally so that it is not immediately evident which pillars have their base in the front row, and which in the back. Secondly, the crossing over of the pillars is seen against a background of rugged mountain ridges, which distracts the onlooker's attention. Fig. 9(b) shows a very simple model of the relevant middle section of the building, the "loggia". In the view direction of Fig. 9(b) the pillars appear to be parallel, just as in Escher's print. Seen from the shorter side, Fig. 9(c), the crossing-over is evident. The symmetry of the loggia is that of the first stage of a fourfold screw-axis: rotation of 90° coupled with a shift from bottom to ceiling. This screw movement

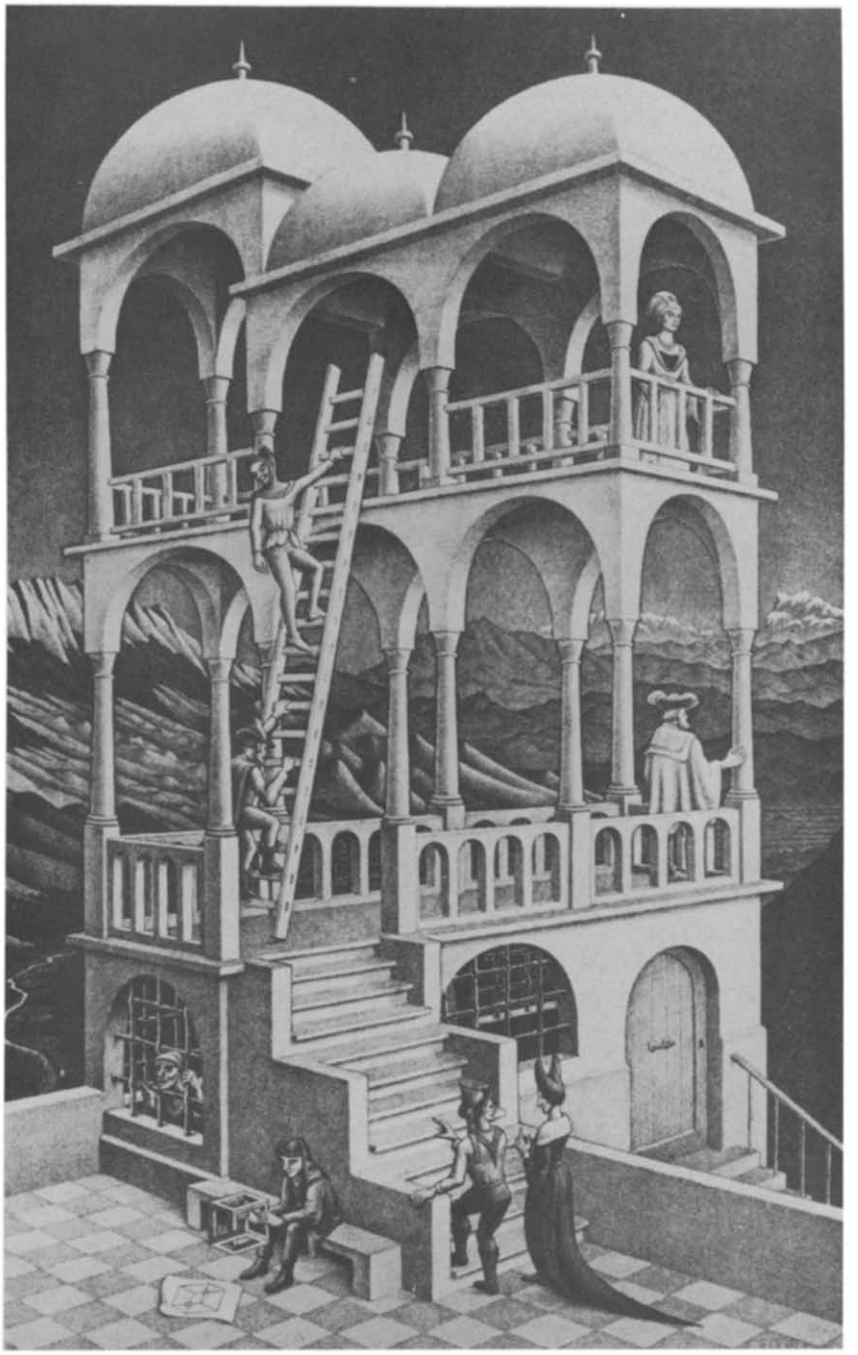
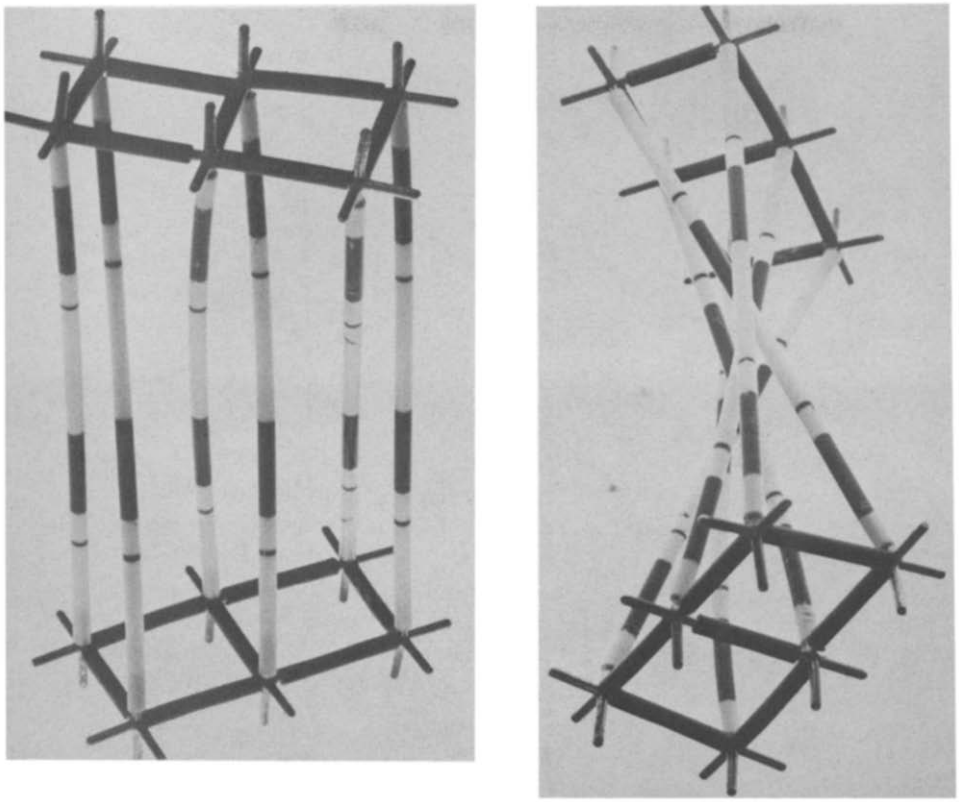


Fig. 9(a). "Belvedere". (Copyright M. C. Escher heirs c/a Cordon Art, Baarn, Holland. Used by permission.)

is evidently caused by the torsion induced by alternately bending the pillars forwards and backwards. Because the rectangular floor and the ceiling of the loggia are askew, it is possible for a ladder standing on the floor to lean against the outside of the balustrade of the top story of the Belvedere. This story is completely "normal". The same holds for the ground floor of the Belvedere: inside is inside, so there is no escape possible for the raving prisoner.

Figure 10, "Ascending and descending", has been fully discussed by Bruno Ernst[2]. This complicated building has a courtyard; the roof of the central part of the building surrounding it seems to be a staircase on which two groups of monks walk. Those on the inner side seem to climb down the stairs, while those on the outer side climb up. However, after a full turn around the courtyard, every monk ends up in exactly the same place and at exactly the same height as before! How is this possible? The answer must be: if the stairs go neither up nor



(b) (c)

Fig. 9(b). Simplified model of the loggia (middle story) in "Belvedere", viewed as in Fig. 9(a); (c). Same model seen at an angle of about 90° from the view in (b).

down, then the plane of the stairs must be horizontal: but then, what about the floors of the building? The ledge of the floor of the top story is seen to be at an angle with the plane of the roof; the floors on the sides of the building that are visible in the print appear to be parallel to the top floor, so all are at the same angle towards the roof. The problem is solved when you imagine the floors at the back of the building to go up from right to left. This is actually seen to be the case in the inner side of the courtyard! In other words: the building is a "spiral ramp", just like a modern car garage. You get in on street level, drive round and round until you find a parking place. Then on leaving your car, you discover that you have to take an elevator which brings you from the n-th floor down to street level again. How does Escher trick you into a false sense of what is horizontal? Note that this is the only Escher building which stands as it were in an empty world: no foreground, no background, no horizon! Only a vague shadow at the bottom of the building. The illusion is further enhanced, as pointed out by Bruno Ernst, by a false perspective suggested by the shape of the courtyard. Escher[4] describes it as "rectangular". However, by counting the sections of the banisters, you see that this is not the case. The four sides of the courtyard are all of different length! In this print Escher does not give you any clue, but fools you by false information. Thus, the apparent orthogonal symmetry of the helicoidal central part is broken by this deviation from orthogonality. It is further impaired by changes in the pitch of the helix: the stories are of different height.

Thus, whereas neither the Belvedere nor the Monastery are really impossible buildings, they are certainly uncomfortable to live in, just like the house in Fig. 3, "Relativity". On the other hand, the building of Fig. 1 "Concave and convex", with its pavillions added, is truly impossible. Equally impossible is the building in the last print I will discuss, "Waterfall" (Fig. 11). The water tumbling down from the left-hand tower ("Tower I") turns the wheel of a mill. Then the water runs down a duct zigzagging between the two towers until it reaches the *high* level of the left tower, from where it falls down again. The basic error in this apparent *perpetuum mobile* is again an optical illustion, the so-called Penrose triangle[5]. This is a set of three

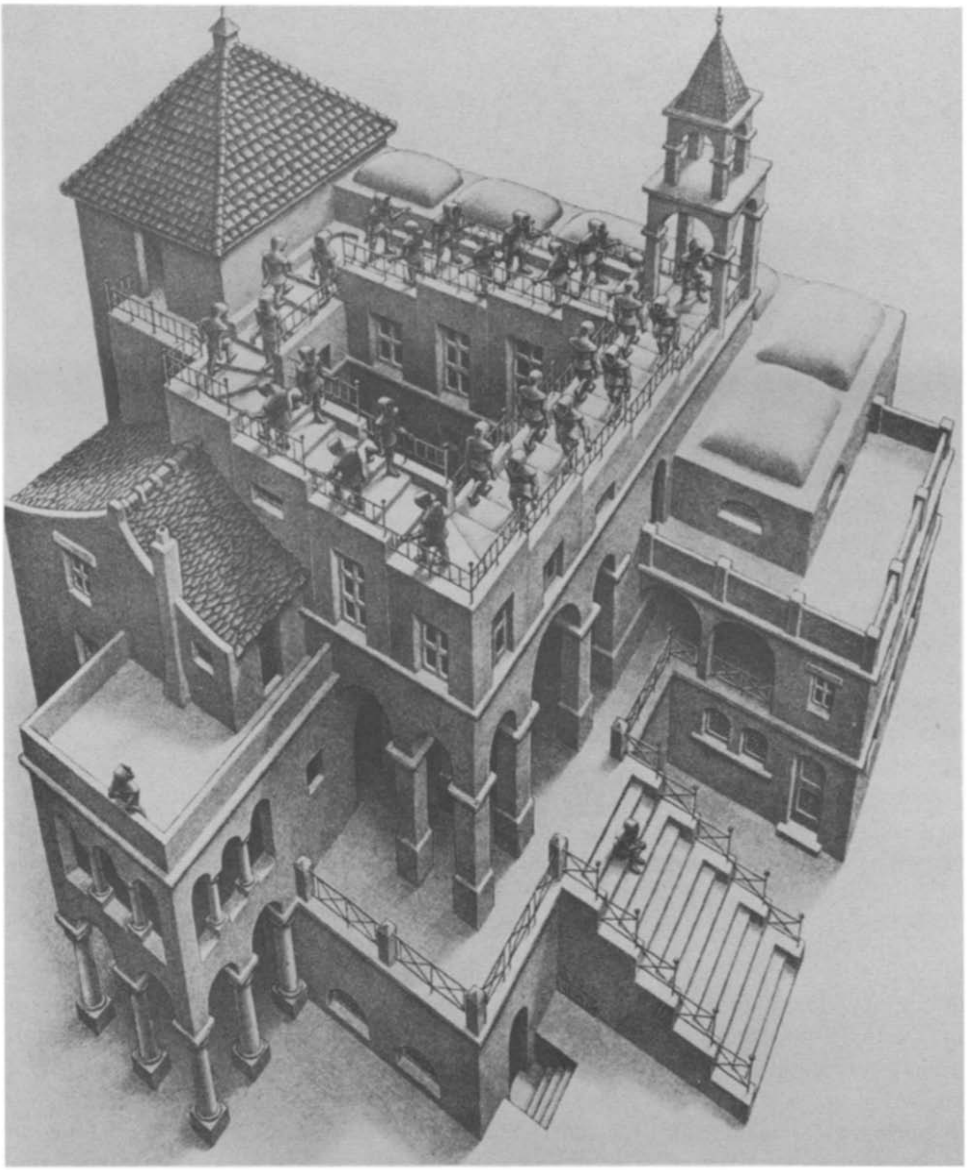


Fig. 10. "Ascending and Descending". (Copyright M. C. Escher heirs c/a Cordon Art, Baarn, Holland. Used by permission.)

beams, two fixed together at an angle of 90°, the third vertically linked to the end of one of the first two horizontal beams, so that the three are arranged like three consecutive edges of a rectangular parallelepiped. Viewed, with one eye closed, from the top of the vertical beam towards the end of the farthest horizontal beam, you are tricked into the illusion of "seeing" a triangle with three angles of 90°. This trick has been applied three times in Fig. 11. From the bottom of the left-side Tower I, the water-duct appears to run down towards the right, and *away* from you. Then, meeting the pillars of Tower II, it makes a right turn, still supposedly downward and away from you, until it appears to meet the second level pillars of Tower I: the first Penrose triangle! From there, running further again at right angles, it seems to meet Tower II at the bottom of the second set of pillars: the second Penrose triangle is formed. Then, along the last zigzag back in the direction of Tower I suggests the third Penrose triangle. What tricks you? First, the apparently very solid attachment of the ends of the three mutually orthogonal sides of the Penrose triangle. In reality these ends would be at a distance of the body-diagonal of the orthogonal parallelepiped with edges a and b of the zigzag of the duct and c, the height of a pillar. If the duct would really run away towards the horizon, then the zigzags should become shorter and narrower as seen in perspective. Moreover, the low parapets of the duct

suggest a downward course. But seen against the background of a terraced hillside, they seem to run slightly up. Thus the whole representation of the duct is full of inconsistencies. They puzzle you, because—as Escher knew very well—it is impossible for the human brain to register momentaneously such a complicated print as a whole. The only really symmetrical objects in this print are the two blocks on top of the towers: on Tower I three interlocking cubes, on Tower II a stellated regular rhombo-dodecahedron. Both blocks still have full cubic symmetry!

6. FINAL REMARKS

In this article I have discussed only a limited number of Escher's "impossible" prints which show more or less symmetry, in total or in details. An analysis of all of them would fill a book which to a large extent would overlap Bruno Ernst's *Magic Mirror*[2].

Since my point of view was to be "symmetry", my analysis covers only the "cerebral"

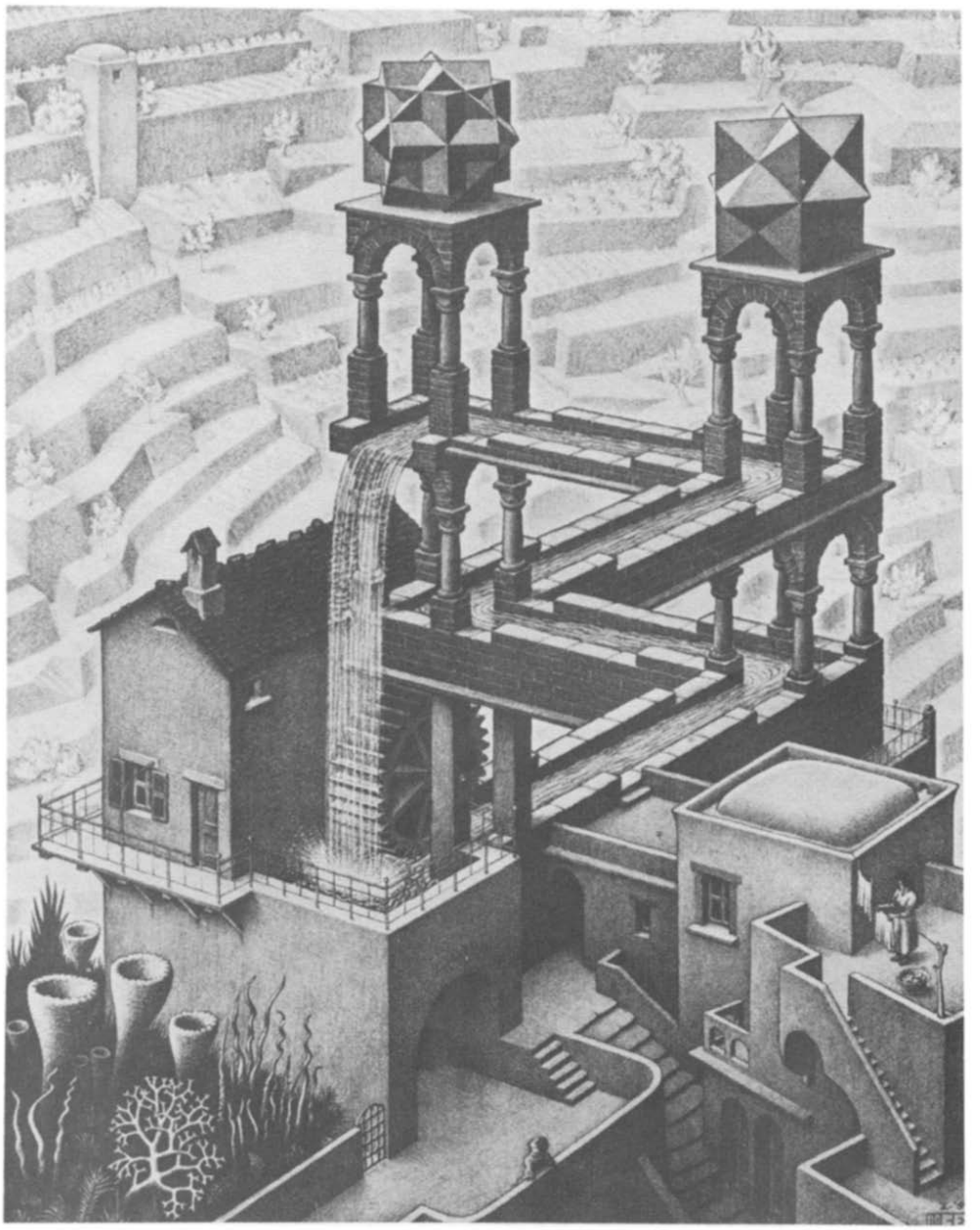


Fig. 11. "Waterfall". (Copyright M. C. Escher heirs c/a Cordon Art, Baarn, Holland. Used by permission.)

side of Escher's work. This does not imply that his work only impresses me from this point of view. In the introduction I mentioned other reactions to works of art: aesthetic and emotional. These are of course highly individualistic. Some prints which I know to be much appreciated by many people, do not appeal to me personally. Some others I admire and love. Perhaps Escher's own opinion may be quoted (Bruno Ernst, l.c. Chapter 3).: "I consider my own work as the most beautiful, and also as the ugliest." I would not go so far either way in my appreciation. But I always admire the profoundness of Escher's thoughts, the intensity of his concentration, and the excellency of his craftmanship, in which he put his pride.

Acknowledgements—I am indebted first and foremost to Escher himself: to what I learned from his writings, his lectures, but above all to what he himself told me about his work. I gained much insight into the mathematical background by studying Bruno Ernst's *Magic Mirror* and several articles by H. S. M. Coxeter. I hope there is still enough personal interpretation in this paper to make it a worthy tribute to the memory of a remarkable man and valued friend, Maurits Cornelis Escher.

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