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# Improved Threshold Denoising Method Based on Wavelet Transform

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**Abstract**

Signals are easily polluted by noises in their transmission process and then they can't be received in the receiver correctly. So the polluted signals should be processed to reduce noises and improve the quality of received signals. After analyzing the theory of wavelet transform and the characteristics of traditional soft and hard wavelet threshold denoising methods, a modified threshold denoising method based on wavelet transform is adopted to improve the quality of a signal which has been polluted by noises. The method overcomes the discontinuous in hard threshold denoising method and reduces the permanent bias in soft threshold denoising method. At last soft threshold denoising, hard threshold denoising and modified threshold denoising are used to reduce noises in the same signal by simulation. The results show that the modified threshold denoising method is superior to the traditional soft and hard wavelet threshold denoising methods in improving SNR and decreasing RMSE.



Keywords- wavelet transform; threshold deniosing; SNR; multi-resolution analysis

## Introduction

In traditional denoising method, Fourier transform is used to separate the signal into high and low frequency components and the noises are removed by removing the high-frequency part. Because the high-frequency useful information and the noises are both eliminated at the same time in this method, the signal is distorted in high-frequency part. Wavelet transform is a time-frequency localization analysis method. The size of its window is fixed and its shape can be changed. It overcomes the weakness of Fourier transform that the instantaneous changes in time domain can't be reflected in frequency domain. Wavelet

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transform is adapable to signal. It has higher frequency resolution and lower time resolution in low-frequency part and has higher time resolution and lower frequency resolution in high-frequency part.

Currently there are threshold denoising method, coefficient correlation denoising method and modulus maxima denoising method in wavelet transform domain. In threshold denoising method the signal polluted by noises is decomposed by wavelet transform. and then the wavelet coefficients of useful signal are retained while the most wavelet coefficients of noises are set zero according to the selected appropriate thresholds. In coefficient correlation denoising method, wavelet coefficients in finer scale are directly multiplied by wavelet coefficients in its neighboring scale to enhance signal and reduce the noises accord to the characteristics that the wavelet coefficients of signal in neighboring scale are correlative and the coefficients of noises in neighboring scale are independent. This method is suitable for analyzing the characteristics of a signal at the edge[1]. Noises and useful signal have different variational trends in wavelet transform domain. The modulus maxima of singularities in useful signal increases in form of positive Lipchitz exponential with scale increasing and the modulus maxima of singularities in noises decreases in form of negative Lipchitz exponential with scale increasing. In modulus maxima denoising method, the modulus maximas of useful signal are found to reconstitute the signal to realize denoising. When the signal contains a lot of singularities, this method can reduce noises better. But this method spends a lot of time.

In these methods above, the wavelet threshold denoising method is the simplest in realization and it spends the least computational time. Thus it has been used widely. In this paper the theory of wavelet transform and the process of threshold denoising is analyzed. After the classic soft and hard threshold denoising method are studied, a modified method is proposed. To analyzing the denoising effects, three methods (soft threshold denoising, hard threshold denoising and modified threshold denoising) are simulated in Matlab7.0.

## Wavelet Transform

In wavelet transform, an analyzing function called the mother wavelet $\psi(t) \in L^2(R)$ is used. It meets the allowable condition $C_\psi = \int_R |\Psi(w)|^2 |w|^{-1} dw < +\infty$, where $\Psi(w)$ is the Fourier transform of $\psi(t)$. For a continuous wavelet transform (CWT), the translation and dilation parameters: $b$ and $a$ respectively, vary continuously. In other words, CWT uses shifted and scaled copies of $\psi(t)$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right), \qquad (1)$$

where $a, b \in R; a \neq 0$. For any function $f(t) \in L^2(R)$, its CWT is

$$W_f(a,b) = \frac{1}{\sqrt{|a|}} \int f(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt. \qquad (2)$$

In order to facilitate computer analysis and processing, signal $f(t)$ is usually discretized into discrete sequence, and its wavelat transform should be also discretized. If the function $f(t)$ is discretized into $f(n)$ ($n = 0,1,2,\cdots,N-1$) and $a$ is set $a = 2^j$, $b$ is set $b = k2^j$, the discrete dyadic wavelet transform is

$$Wf(j,k) = 2^{\frac{-j}{2}} \sum_{n=0}^{N-1} f(n) \psi(2^{-j} n - k). \qquad (3)$$

**Wavelet Threshold Denoising**

*A. Multi-resolution Analysis*

In practical application the fast algorithm based on multi-resolution analysis: Mallat, is often adopted to improve the calculation speed of discrete dyadic wavelet transform and Mallat algorithm is

$$\begin{cases} Sf(j+1,k) = Sf(j,k) * h(j,k) \\ Wf(j+1,k) = Sf(j,k) * g(j,k) \end{cases}, \tag{4}$$

where $h$ and $g$ are respectively corresponding with the low-pass filter of the scaling function $\phi(x)$ and the high-pass filter of the wavelet function filter $\psi(x)$, $Sf(0,k)$ is the original signal, $Sf(j,k)$ is the the scale coefficients (the coefficients of signal's approximate part), $Wf(j,k)$ is the wavelet coefficients (the coefficients of signal's details)[2]. The reconstruction of discrete dyadic wavelet transform is as follows

$$Sf(j-1,k) = Sf(j,k)\tilde{h}(j,k) + Wf(j,k)\tilde{g}(j,k) . \tag{5}$$

In the equation above, $\tilde{h}$ is the conjugate of $h$ and $\tilde{g}$ is the conjugate of $g$ .

*B. Threshold Denoising*

The essence of multi-resolution analysis is that the signal is decomposed in different space which has different frequencies. The information in different scales can show the characteristics of a signal in different frequencies. The signal is decomposed into two parts in each decomposition. One is the details which contain the high frequency information of the signal and the other is the approximate part which contains the low-frequency information of the signal. The approximate part can be further decomposed to obtain the details and approximations in bigger scale. The diagram of three layers wavelet decomposition is shown in Fig. 1. In the diagram A1, A2, A3 are respectively corresponding to the sinal's approximate parts in different scales and D1, D2, D3 are respectively corresponding to the sinal's details in different scales. The noises existing in useful signal are usually contained in D1, D2, D3. The wavelet coefficients in D1, D2, D3 are processed by threshold denoising method to reduce noises and the effective signal is reconstructed by these processed wavelet coefficients through inverse wavelet transform. Generally the denoising process of one-dimensional signal includes three steps[3]:

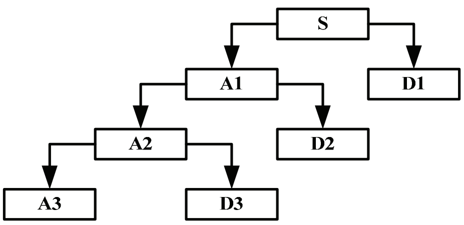


Figure 1. The diagram of three layers wavelet

- The signal polluted by noises is decomposed by orthogonal wavelet transform to obtain the corresponding wavelet coefficients after the appropriate wavelet and wavelet decomposition layer M are selected.

- In each layer, a suitable threshold is selected and the high-frequency coefficients are processed in the form of threshold quantization.
- The signal is reconstructed by the low-frequency coefficients from No. M layer and the high-frequency coefficients from layer 1 to layer M via inverse wavelet transform.

In the three steps, the key is how to select the threshold and how to quantify the wavelet coefficients according to the threshold and it relates to the denoising effect.

Using Mallat algorithm, the signal $S$ polluted by noises in time domain is transformed into wavelet coefficients $w_{j,k}$ which is $Wf(j,k)$ in (3). λ is set as the threshold and $w'_{j,k}$ is set as the new wavelet coefficients. The function of traditional hard threshold denoising function is

$$w'_{j,k} = \begin{cases} w_{j,k} & |w_{j,k}| \geq \lambda \\ 0 & |w_{j,k}| < \lambda \end{cases}. \qquad (6)$$

The function of traditional soft threshold denoising function is

$$w'_{j,k} = \begin{cases} sign(w_{j,k}) \times (|w_{j,k}| - \lambda) & |w_{j,k}| \geq \lambda \\ 0 & |w_{j,k}| < \lambda \end{cases}. \qquad (7)$$

The two methods have been widely used in practice and have achieved good results in denoising. But they have some potential weakness. For example, the hard threshold function is discontinuous in the whole wavelet domain and there are interrupted points in the place of $\pm\lambda$. So the signal $S'$ which has been reconstructed by $w'_{j,k}$ may appear some oscillation. In the soft thresholding function, there is a constant deviation between $w'_{j,k}$ and $w_{j,k}$ when $|w_{j,k}| \geq \lambda$ . This will causes a deviation between the reconstructed signal $S'$ and the real signal.

In order to overcome the shortcomings of the two methods above, an improved method based on them is proposed. Its function is as follows

$$w'_{j.k} = \begin{cases} w_{j,k} - \dfrac{2\lambda}{1+\exp(\lambda - w_{j,k})} & w_{j,k} \geq \lambda \\ 0 & |w_{j,k}| < \lambda \\ w_{j,k} + \dfrac{2\lambda}{1+\exp(w_{j,k} + \lambda)} & w_{j,k} \leq -\lambda \end{cases}. \qquad (8) \qquad \alpha + \beta = \chi. \qquad (1) \qquad (1)$$

In the improved function above, $w'_{j,k} = w_{j,k}$, when $w_{j,k}$ tends to $\infty$ and the deviation between $w'_{j,k}$ and $w_{j,k}$ is reduced when it is compared with soft threshold denoising function. It meets the trend that the coefficients of noises decreases while the coefficients of signal increases in wavelet domain. When $w_{j,k}$ closes to the threshold $\pm\lambda$, $w'_{j,k}$ closes to zero gradually and this makes the function continuous.

**Experimental Simulation**

In order to analyze and compare three denoising methods, Matlab 7.0 is selected as the simulation software. Signal-to-noise ratio (SNR) and root mean square error (RMSE) are used to measure denoising effects. SNR is defined as

$$SNR = 10\log\frac{\sum_i f^2(i)}{\sum_i (S(i)-f(i))^2}(dB), \qquad (9) \qquad \alpha + \beta = \chi. \qquad (1) \qquad (1)$$

RMSE is defined as

$$RMSE = (N^{-1}\sum_i (S'(i)-f(i))^2)^{1/2}. \qquad (10) \qquad \alpha + \beta = \chi. \qquad (1) \qquad (1)$$

Considering that CoifN wavelet is the best, SymN wavelet is better and DbN wavelet is poorer than the previous two in denoising effect at the same filter length. To CoifN wavelet, the denoising effect is best when the filter length is 4. To SymN wavelet, the de-noising effect is good when the filter length is between 4 and 11 [4]. In the experimental simulation, Coif4 wavelet is used and the wavelet decomposition level M is 4. When SNR is smaller than 15, the denoising effect is the worst[5]. Minimax rule is adopted to select the threshold in ev... ment. One is heavysine shown in Fig. 2(a) whic ... is shown in Fig. 2(b) whose SNR is 9.7474. T ... signals with mutations and its polluted signal is ... ıd data after denoising process by three methoc ... 3(a),(d) the signals are processed by hard thresh ... essed by hard threshold denoising method. In Fig ... denoising method. The data from experimental si

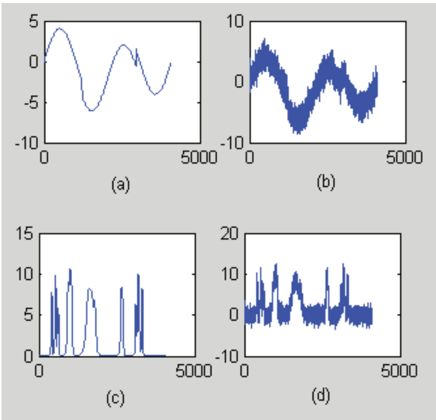


Figure 2. The useful signal and polluted signa

## Conclusion

The characteristics of ... thods are analyzed and based on them, an impro ... e performance of three methods are compared tl ... how that the improved threshold overcomes the ... ntinuous in the whole wavelet domain and ther ... signal. It is superior to the traditional soft and ha ... and decreasing RMSE.

Table 1 Comparison of Three Methods

| **Performance** | **Wavelet threshold deniosing method** | | |
|---|---|---|---|
| | ***Hard*** | ***Soft threshold*** | ***Improved threshold*** |
| | | 21.041 | 21.41 |
| | | 21.929 | 22.267 |
| | | 0.27651 | 0.2657 |
| | | 0.28095 | 0.27022 |

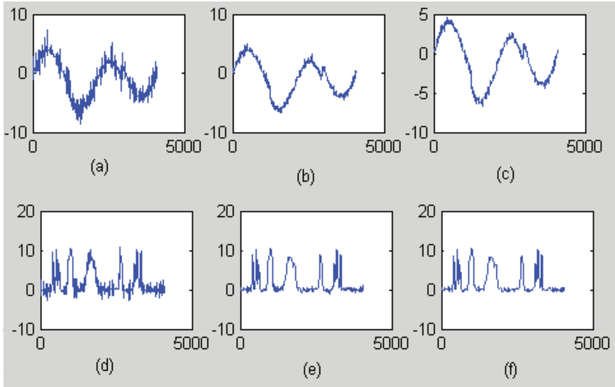


Figure 3. The denoised signals

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