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Thermodynamics at non-zero baryon number density: a comparison of lattice and hadron resonance gas model calculations

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Abstract

We compare recent lattice studies of QCD thermodynamics at non-zero quark chemical potential with the thermodynamics of a hadron resonance gas. We argue that for $T \leqslant T_c$ the equation of state derived from Monte Carlo simulations of two flavour QCD at non-zero chemical potential can be well described by a hadron resonance gas when using the same set of approximations as used in current lattice calculations. We estimate the importance of truncation errors arising from the use of a Taylor expansion in terms of the quark chemical potential and examine the influence of unphysically large quark masses on the equation of state and the critical conditions for deconfinement.

1. Introduction

While the thermodynamics of strongly interacting matter at vanishing baryon number density or chemical potential has been studied [1] in lattice calculations for quite some time, the first investigations of the equation of state at non-vanishing quark chemical potential (μ_q) have started only recently [2–4]. These studies of bulk thermodynamics have been performed with different lattice actions and also have used different methods (exact matrix inversion [2] or Taylor expansion [3,4]) to extend previous calculations performed at $\mu_q = 0$ into the region of $\mu_q > 0$. Nonetheless they led to qualitatively and even quantitatively similar results.

The basic pattern found for the temperature dependence of, e.g., the pressure at finite density, follows closely that already seen at $\mu_q = 0$; the pressure changes rapidly in a narrow temperature interval and comes close to the Stefan–Boltzmann value of an ideal gas of quarks and gluons at about twice the transition temperature. Consequently, the density dependence of the equation of state in the high temperature plasma phase has successfully been compared [5] with quasiparticle models that were also used at $\mu_q = 0$.

In this Letter we concentrate on a discussion of the thermodynamics of the hadronic phase of QCD in the regime of low baryon number density ($\mu_q/T \lesssim 1$) but

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high temperature ($T \sim T_c(\mu_q = 0)$). In a recent paper [6] we have shown that the partition function of a hadron resonance gas yields quite a satisfactory description of lattice results on bulk thermodynamic observables in the low temperature, hadronic phase of QCD at $\mu_q = 0$. We will extend here our previous study to finite chemical potential and compare the predictions of the resonance gas model calculations with lattice results obtained in 2-flavour QCD using a Taylor expansion for small μ_q/T [4]. The reference system for these calculations is a previous analysis [7] of the temperature dependence of the pressure in 2-flavour QCD performed at $\mu_q = 0$. Unlike the approach based on an exact inversion of the fermion determinant [2] the Taylor expansion, obviously, has the disadvantage of being approximate. There is, however, good reason to expect, that at least at high temperature, the contribution of terms that are beyond $\mathcal{O}((\mu_q/T)^4)$ order is small. Moreover, as will become clear from our discussion below, it turns out that the expansion coefficients themselves provide useful information on the relevant degrees of freedom controlling the density dependence of thermodynamic quantities. We argue that baryons and their resonances are these relevant degrees of freedom that govern thermodynamics in the confined phase at finite density. We show that for $T \leqslant T_c$ the equation of state at non-zero chemical potential which has been obtained in lattice calculations can be well described by a baryonic resonance gas when using the same set of approximations as used in current lattice studies. We examine the importance of truncation effects in the Taylor expansion and discuss the influence of unphysically large quark mass values on thermodynamic observables and the critical conditions for deconfinement.

2. Finite density QCD and Taylor expansion

The basic quantity that describes thermodynamics at non-vanishing chemical potential is the pressure. In the grand-canonical ensemble it is obtained[1] from the partition function as

$$p(T,\mu_q) = \lim_{V\to\infty} \frac{T}{V} \ln Z(T,\mu_q,V). \tag{1}$$

For small values of μ_q/T the pressure may be expanded in a power series,

$$\frac{p(T,\mu_q)}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_q}{T}\right)^{2n}. \tag{2}$$

In recent lattice calculations this series has been analyzed up to $\mathcal{O}((\mu_q/T)^4)$ and in addition to the density dependent change of the pressure (Δp) quantities like the quark number density (n_q) and quark number susceptibility (χ_q) have been calculated. The latter are obtained from Eq. (2) through appropriate derivatives with respect to the quark chemical potential,

$$\begin{aligned}
\frac{\Delta p}{T^4} &= \frac{p(T,\mu_q) - p(T,0)}{T^4} \\
&\simeq c_2(T)\left(\frac{\mu_q}{T}\right)^2 + c_4(T)\left(\frac{\mu_q}{T}\right)^4, \\
\frac{n_q}{T^3} &= \frac{\partial p(T,\mu_q)}{\partial \mu_q} \\
&\simeq 2c_2(T)\left(\frac{\mu_q}{T}\right) + 4c_4(T)\left(\frac{\mu_q}{T}\right)^3, \\
\frac{\chi_q}{T^2} &= \frac{\partial^2 p(T,\mu_q)}{\partial \mu_q^2} \\
&\simeq 2c_2(T) + 12c_4(T)\left(\frac{\mu_q}{T}\right)^2.
\end{aligned} \tag{3}$$

In the asymptotically high temperature limit this expansion terminates, in fact, at the order given in Eq. (3). The expansion coefficients are then given by $c_2(\infty) = n_f/2$ and $c_4(\infty)/c_2(\infty) = 1/2\pi^2$, respectively.[2] In this limit the ratio c_4/c_2 is small and the leading order term, consequently, dominates in the Taylor expansion for a wide range of values for μ_q/T. The lattice results for the expansion coefficients, ob-

[1] Although the volume dependence of thermodynamic quantities calculated on the lattice requires a careful analysis and has not yet been performed for most thermodynamic studies with dynamical quarks, we will for simplicity of notation suppress in the following any volume dependence in our formulas.

[2] Also at $O(g^2)$ the high temperature expansion terminates at $O((\mu_q/T)^4)$. This, however, changes in the resumed $O(g^3)$ contribution. The complete expansion up to $O(g^6 \ln g)$ has recently been presented in [8]. A discussion of the n_f dependence of the pressure at non-zero chemical potential and high temperature is given in [9].

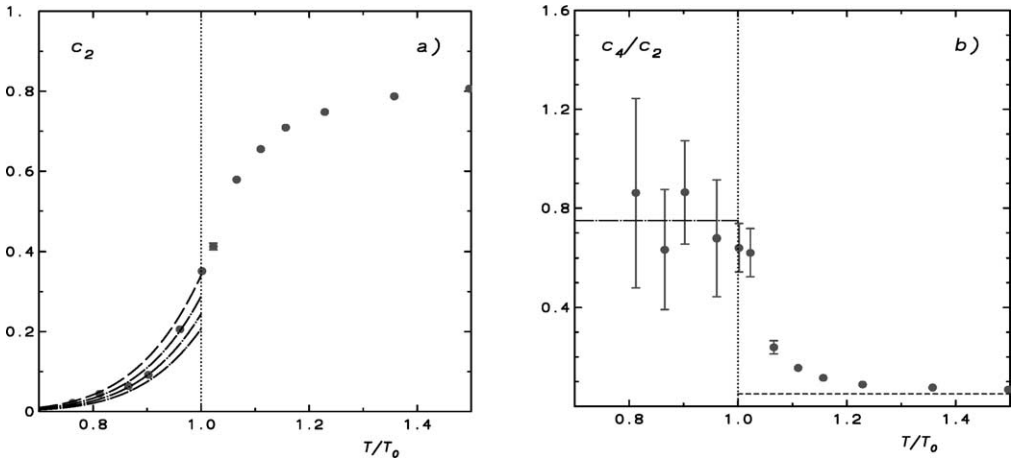

Fig. 1. Temperature dependence of the second order expansion coefficients c_2 and c_4 of the pressure in powers of μ_q/T obtained in 2-flavour QCD [4]. In (a) we show $c_2(T)$ and (b) shows the ratio $c_4(T)/c_2(T)$. The temperature scale is given in units of the transition temperature at $\mu_q = 0$ which for the quark masses used in the calculation of Ref. [7] is $T_0 \simeq 200$ MeV. For $T > T_0$ the dashed horizontal line shows the massless Fermi gas value of c_4/c_2. The dashed-dotted curves in (a) show results of a resonance gas model calculation for $A = 0.9, 1.0, 1.1, 1.2$ (from top to bottom) as discussed in Section 3. The dashed-dotted curve in (b) represents the resonance gas value $c_4/c_2 = 0.75$.

tained in 2-flavour QCD are shown in Fig. 1. It can be seen, that at $T \simeq 1.5T_0$ the numerical results for $c_2(T)$ still deviate by about 20% from the ideal gas value while the ratio c_4/c_2 is already close to the corresponding result expected in the infinite temperature limit.

3. Resonance gas and Boltzmann approximation

The analysis of experimental data on the production cross-sections of various hadrons in heavy-ion collisions shows astonishingly good agreement with corresponding thermal abundances calculated in a hadronic resonance gas model at appropriately chosen temperature and chemical potential [10]. Our recent analysis of the equation of state calculated on the lattice at $\mu_q = 0$ also has shown that a gas of non-interacting resonances can provide a good description of the low-temperature phase of QCD [6]. We want to extent here our analysis to the case of non-vanishing chemical potential.

The partition function of a resonance gas can be specified through the mass spectra for the mesonic and baryonic sectors of QCD, respectively. In a non-interacting resonance gas the partition function reads,

$$\ln Z(T,\mu_B,V) = \sum_{i\in\text{mesons}} \ln Z^B_{m_i}(T,V) + \sum_{i\in\text{baryons}} \ln Z^F_{m_i}(T,\mu_B,V), \quad (4)$$

where $Z^B_{m_i}$ ($Z^F_{m_i}$) denote single particle partition functions for bosons and fermions with mass m_i and $\mu_B = 3\mu_q$ is the baryon chemical potential. Here the fermion partition function contains the contribution from a particle and its anti-particle. The total pressure of the resonance gas is then obtained from Eq. (1) and builds up as a sum of contributions from particles of mass m_i. The dependence of the pressure on the chemical potential at a fixed temperature is thus entirely due to the baryonic sector. The contribution, p_m, of baryons of mass m to the total pressure is given by

$$\frac{p_m}{T} = \frac{d}{2\pi^2}\int_0^\infty dk\,k^2 \ln\left[\left(1+z\exp\left\{\frac{-\varepsilon(k)}{T}\right\}\right)\times\left(1+z^{-1}\exp\left\{\frac{-\varepsilon(k)}{T}\right\}\right)\right], \quad (5)$$

where $z \equiv \exp\{\mu_B/T\}$ is the baryonic fugacity with $\mu_B = 3\mu_q$, d is the spin–isospin degeneracy factor and $\varepsilon(k) = \sqrt{k^2+m^2}$ is the relativistic single particle en-

ergy. The pressure may be expressed in terms of a fugacity expansion as

$$\frac{p_m}{T^4} = \frac{d}{\pi^2}\left(\frac{m}{T}\right)^2 \sum_{\ell=1}^{\infty} (-1)^{\ell+1} \ell^{-2} K_2\left(\frac{\ell m}{T}\right) \times \cosh\left(\frac{\ell \mu_B}{T}\right), \tag{6}$$

where K_2 is the Bessel function.

In the hadronic phase of QCD the relevant temperatures will always be smaller than the transition temperature to the plasma phase determined in lattice calculations at $\mu_B = 0$, i.e., we are interested in the regime $T \lesssim 200$ MeV [7]. As the mass of the lightest baryon (m_N) is about five times larger than this value, the Bessel functions appearing in Eq. (6) can always be approximated by the asymptotic form valid for large arguments, i.e., $K_2(x) \simeq \sqrt{\pi/2x}\exp(-x)$. This shows that higher order terms in Eq. (6) are suppressed by factors $\exp(-\ell(m-\mu_B)/T)$. As long as $(m_N - \mu_B) \gtrsim T$ the contribution of baryons to the resonance gas partition function is thus well approximated by the leading term in Eq. (6), which constitutes the Boltzmann approximation. In this case each baryon/anti-baryon pair contributes to the pressure with

$$\frac{p_m}{T^4} \simeq \frac{d}{\pi^2}\left(\frac{m}{T}\right)^2 K_2\left(\frac{m}{T}\right)\cosh\left(\frac{\mu_B}{T}\right). \tag{7}$$

Thus, the total baryonic contribution to the pressure of a resonance gas reads

$$\frac{p_B}{T^4} = F(T)\cosh\left(\frac{\mu_B}{T}\right), \tag{8}$$

where $F(T)$ is defined by

$$F(T) \equiv \sum_i \frac{d_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 K_2\left(\frac{m_i}{T}\right), \tag{9}$$

and the sum is taken over all known baryons and their resonances.

Current lattice studies of the hadronic phase of QCD with non-vanishing chemical potential concentrate on a temperature regime $0.8T_0 \lesssim T \lesssim T_0$ with quark chemical potentials $\mu_q \lesssim T$. The baryon chemical potential $\mu_B = 3\mu_q$ thus stays considerably smaller than the nucleon mass. Under these conditions the Boltzmann approximation will be applicable and the dependence of the pressure on the baryon chemical potential will just be mediated through a multiplicative factor as given in Eq. (8). We stress that this simple relation is independent of details of the mass spectrum as long as all fermions are sufficiently heavy. The factorization of the part depending on the mass spectrum and that depending on the chemical potential, however, also relies on the assumption that interactions among the hadrons and resonances are negligible. The validity of this assumption can be verified through a direct comparison with lattice calculations for $\mu_q > 0$ and for $T < T_0$.

Following Eqs. (7), (8) we can specify the results for the change in pressure, the quark number density and quark number susceptibility. In order to compare the predictions of the resonance gas model with lattice results one still needs to perform the Taylor expansion up to the same order as given in Eq. (3). In the Boltzmann approximation we have,

$$\begin{aligned}
\frac{\Delta p}{T^4} &= F(T)\left[\cosh\frac{\mu_B}{T} - 1\right] \\
&\simeq F(T)\left(\tilde{c}_2\left(\frac{\mu_q}{T}\right)^2 + \tilde{c}_4\left(\frac{\mu_q}{T}\right)^4\right), \\
\frac{n_q}{T^3} &= 3F(T)\sinh\frac{\mu_B}{T} \\
&\simeq F(T)\left(2\tilde{c}_2\left(\frac{\mu_q}{T}\right) + 4\tilde{c}_4\left(\frac{\mu_q}{T}\right)^3\right), \\
\frac{\chi_q}{T^2} &= 9F(T)\cosh\frac{\mu_B}{T} \\
&\simeq F(T)\left(2\tilde{c}_2 + 12\tilde{c}_4\left(\frac{\mu_q}{T}\right)^2\right),
\end{aligned} \tag{10}$$

with $\tilde{c}_2 = 9/2$ and $\tilde{c}_4 = 27/8$. In the resonance gas model the expansion coefficients introduced in Eq. (3) are given by $c_{2n} = \tilde{c}_{2n}F(T)$. We note that ratios of these quantities indeed are independent of the resonance mass spectrum and only depend on the chemical potential. For fixed μ_q/T we thus expect to find that any ratio of two of the above quantities is temperature independent in the hadronic phase. Using the same order of the Taylor expansion as used in the lattice calculations such ratios only depend on $\tilde{c}_4/\tilde{c}_2 = 3/4$, i.e., the resonance gas model yields a temperature independent ratio c_4/c_2. As can be seen in the right-hand part of Fig. 1 this is indeed in good agreement with the

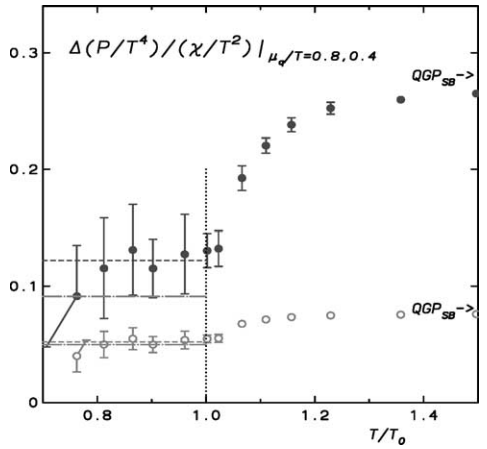

Fig. 2. The ratio of pressure and quark number susceptibility versus temperature for fixed values of μ_q/T. The horizontal lines are the results of hadron resonance gas model. The points are the lattice values from Ref. [4]. While the dashed-dotted curves represent the complete expression (Eq. (11)) the dashed curves give the result of a Taylor expansion to the same order as that used in the lattice calculations.

lattice results. We note that this result is independent of details of the hadron mass spectrum. It thus should also be insensitive to the change in the quark mass used in the lattice calculation. In Fig. 2 we show the ratio $\Delta p/T^2\chi_q$ for two values of the chemical potential. The good agreement between lattice calculations and the hadronic gas results merely reflects the agreement found already for the ratio c_4/c_2. In the resonance gas model we can, however, provide also the complete result for this ratio,

$$\frac{\Delta p}{T^2\chi_q} = \frac{1}{9}\left(1 - \cosh^{-1}\left(\frac{3\mu_q}{T}\right)\right). \tag{11}$$

This is shown as a dashed-dotted line in Fig. 2.

The agreement of the ratio c_4/c_2 calculated in the resonance gas model and on the lattice also implies that, for all values of μ_q/T, the temperature dependence of Δp and its derivatives like, e.g., the quark number susceptibility is to a large extent controlled by the same function, $c_2(T) = (9/2)F(T)$. To determine, however, the function $F(T)$ we have to specify the baryon mass spectrum explicitly. This is, in general, known experimentally. However, to facilitate a direct comparison with lattice calculations we have to take into account that the spectrum is distorted due to the still quite large quark masses used in calculations of the equation of state.[3] In addition we have to take into account that also the transition temperature is quark mass dependent. For the quark mass value used in the numerical study of the equation of state [4,7] the transition temperature has been determined as $T_0 \simeq 200$ MeV. While an extrapolation to the chiral limit yields the transition temperature of about 170 MeV. We use T_0 to express the hadron masses in units of the temperature, $m/T \equiv (m/T_0)(T_0/T)$ with T scaled in units of T_0.

The distortion of the hadron mass spectrum due to unphysically large quark masses (m_q) can in general be deduced from lattice calculations at zero temperature. A generic feature of such studies is that the deviation from the physical mass value due to unphysically large values of the quark mass becomes smaller for heavier hadronic states (see, e.g., [11]). Moreover, one finds [12,13] that the quark mass dependence is well parametrized through the relation, $(m_H a)^2 = (m_H a)^2_{\rm phys} + b(m_\pi a)^2$, where $(m_H a)_{\rm phys}$ denotes the physical mass value of a hadron expressed in lattice units and $(m_H a)$ is the value calculated on the lattice for a certain value of the quark mass or equivalently a certain value of the pion mass ($m_\pi^2 \sim m_q$). Until now, however, the masses of only a few baryonic states constructed from (u,d)-quarks have been studied in more detail on the lattice [11–13]. This is obviously not sufficient to fix the function $F(T)$ in Eq. (7) that requires the contributions from a large set of baryonic resonances.

The above quadratic parametrization of the quark mass dependence of baryon masses shows at least for nucleon, delta and their parity partners only a weak dependence on the hadron mass. We thus take this as a general ansatz for the parametrization of the dependence of baryon masses on the pion mass,

$$\frac{m(m_\pi)}{m_H} \simeq 1 + A\frac{m_\pi^2}{m_H^2}, \tag{12}$$

where $m(m_\pi)$ is the distorted hadron mass at fixed m_π and m_H is its corresponding physical value. This parametrization is consistent with our previous

[3] The phase transition temperature has been calculated at a large set of quark mass values, including rather small values which lead to "almost" physical hadron masses. So far the equation of state, however, has been studied with improved actions only for one set of quark masses corresponding to a pion mass of about 770 MeV [7].

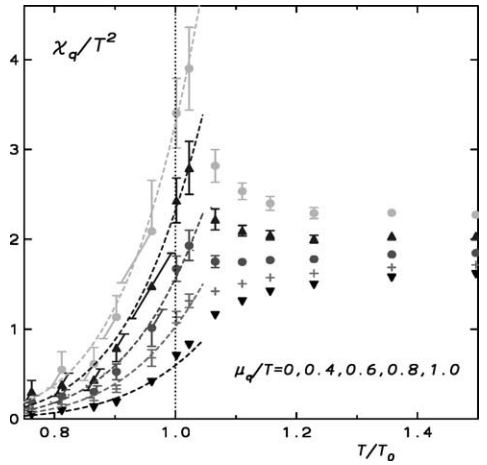

Fig. 3. Lattice results from Ref. [4] for the quark number susceptibility in 2-flavour QCD calculated in next-to-leading order Taylor expansion for different values of the quark chemical potential. The lines are results from the resonance gas model using a distorted baryon spectrum (Eq. (12) with $A = 1$) and treated within the same approximation as in the lattice study.

analysis [6] where we have used the MIT bag model to determine the m_π-dependence of hadron masses.

The lattice results [4] for QCD thermodynamics at finite μ_q were obtained in 2-flavour QCD. An immediate consequence of the restriction to only two quark flavour is that we have to suppress the contribution from strange baryons to the resonance gas in the low temperature phase. Moreover, since the lattice results were obtained with a quite large value of m_q, corresponding to $m_\pi/\sqrt{\sigma} = 1.84 \pm 0.04$, we account for modifications of the baryon mass spectrum using Eq. (12). From the pion mass dependence of the nucleon, delta and their parity partners [12,13] we estimate $0.9 \lesssim A \lesssim 1.2$ in Eq. (12). This range of values is also expected from the bag model study in Ref. [6].

The above discussion fixes our parametrization of the baryonic sector of the resonance gas model at unphysical values of the quark mass as they are used in current lattice calculations. The resulting temperature dependence of the expansion coefficient $c_2(T)$ is shown in Fig. 1(a). The corresponding result for the quark number susceptibility at different values of the quark chemical potential is shown in Fig. 3 for the choice $A = 1.0$. The agreement of the resonance gas model and results obtained from lattice calculations is indeed quite satisfactory. This indicates that the thermodynamics of the confined phase of QCD at finite density is to large extent governed by the baryonic resonances. As can be seen in Fig. 1(a) the 15% uncertainty in the parametrization of the baryonic mass spectrum (Eq. (12)) results in 20% error on the values of physical observables, i.e., $c_2(T)$, at $T = T_0$.

4. Relaxing the lattice constraints

We want to discuss here in somewhat more detail what the resonance gas model calculation suggests for the quark mass dependence of current thermodynamic studies on the lattice and what the influence of the truncation of the Taylor series expansion on the behavior of thermodynamic quantities in the hadronic phase could be. The latter clearly depends on the observable under consideration. As anticipated also in Ref. [4] the influence of a truncation of the Taylor expansion for the pressure at $\mathcal{O}((\mu_q/T)^4)$ becomes more severe in calculations of the quark number susceptibility as the expansion stops here already at $\mathcal{O}((\mu_q/T)^2)$. The resonance gas model suggests that for $\mu_q/T = 1$ the truncated result for the pressure differs only by 15% from the complete result. These truncation errors rise to about 80% in calculations of χ_q/T^2 at $\mu_q/T = 1$. The major part of this truncation error could be removed by calculating the $\mathcal{O}((\mu_q/T)^6)$ contribution to $\Delta p/T^4$. These properties are seen in Fig. 4 where the results of the resonance gas calculation performed with the complete (μ_q/T)-dependence and a Taylor expansion truncated at $O((\mu_q/T)^4)$ are shown for pressure and quark number susceptibility.

Already in the discussion of the $\Delta p/T^2\chi_q$ we indicated that within the Boltzmann approximation this and similar ratios are independent of the resonance mass spectrum and thus also on the quark masses. Changes in the quark mass thus will influence calculations of the pressure and its derivatives in a similar way. Replacing in the resonance gas calculation the modified baryon spectrum by the experimentally known spectrum will increase the value of the pressure as all baryons become lighter. This effect is somewhat reduced as also the relevant temperature scale is shifted to smaller values, i.e., the transition temperature will shift from $T_0 \simeq 200$ MeV to $T_0 \simeq 170$ MeV. As can be seen in Fig. 4 this increases the pressure and its derivatives. The extrapolation to the physical case depends, however, quite sensitively on the value

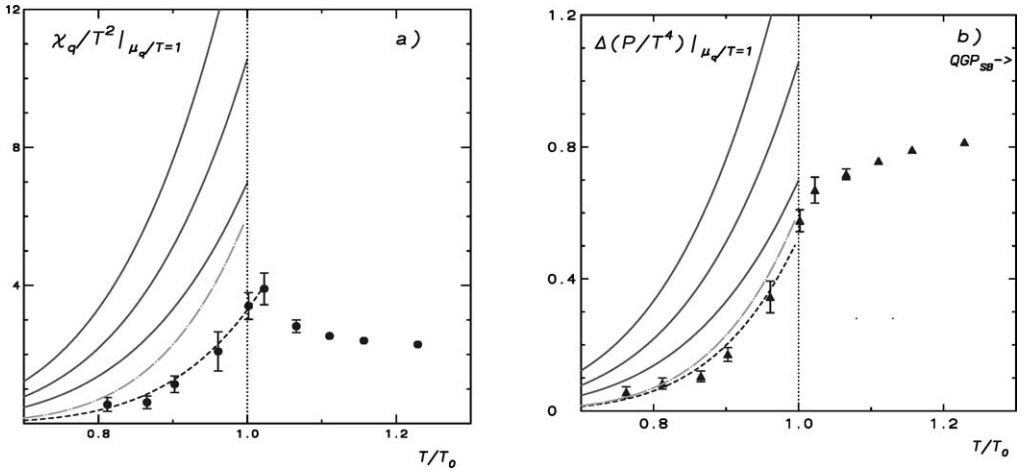

Fig. 4. Quark number susceptibility (a) and change in pressure (b) for fixed quark chemical potential $\mu_q/T = 1$ as a function of T/T_0. The points are lattice results from Ref. [4] and lines are the resonance gas model results. The dashed and dashed dotted lines are obtained with a baryon mass spectrum appropriate for the unphysical quark masses used in lattice calculations and with a Taylor expansion truncated at $\mathcal{O}((\mu_q/T)^4)$ and the full result, respectively. The full lines are resonance gas model results obtained with physical hadron masses, no expansion in μ_q/T and for three values of the transition temperature, $T_0 = 160$ MeV (lower), 170 MeV (middle) and 180 MeV (upper), which cover the range of current lattice estimates for the chiral limit extrapolation of T_c in 2-flavour QCD.

for the critical temperature. We note that the resonance gas model calculation favours a small critical temperature as it seems to be unlikely that $\Delta p/T^4$ will exceed the corresponding ideal gas value at T_0.

Finally, we want to comment on the convergence radius of the Taylor expansion in (μ_q/T). The resonance gas model as we have introduced it here, i.e., as a finite sum of non-interacting Fermi gases, of course does not lead to critical behaviour. Consequently also the dependence on (μ_q/T) is given by an analytic function. The resulting Taylor expansion has an infinite convergence radius, which in terms of the convergence criterium used in Ref. [4] is reflected in the fact that,

$$\lim_{n\to\infty} \sqrt{\frac{c_{2n}}{c_{2n+2}}} = \infty,$$

for all temperatures in the resonance gas model. In the case of QCD, we expect, however, that the convergence radius is bounded from above by the location of the phase boundary to the quark gluon plasma phase. In particular, we expect that the ratios c_{2n}/c_{2n+2} stay close to unity for temperatures close to T_0. In fact, this also is the case for the low-order expansion coefficients in the resonance gas model, i.e., $c_2/c_4 = 4/3$. This, however, changes already at the next order, $c_4/c_6 = 10/3$. We thus expect that differences between lattice results on the QCD thermodynamics and resonance gas model calculations should show up at $\mathcal{O}((\mu_q/T)^6)$.

5. Conclusion and outlook

We have shown that basic features of the density dependence of the QCD equation of state in the hadronic phase observed in recent lattice studies can be understood in terms of the thermodynamics of a baryonic resonance gas. A quite robust result, independent of the detailed structure of the hadron mass spectrum, is the dependence of thermodynamic quantities on μ_q/T at fixed temperature. This indicates that the thermodynamics at low temperatures is dominated by heavy degrees of freedom which justify a Boltzmann approximation for the partition function. Within the resonance gas model these degrees of freedom are non-interacting which leads to simple relations among different thermodynamic observables. In particular, we argued that the ratio $\Delta p/T^2\chi_q$ is independent of temperature at fixed μ_q/T. This is in agreement with current lattice calculations. It is, however, clear that this has to change, when the system undergoes a true phase transition at some temperature $T_c(\mu_q) < T_0$ for a sufficiently large value of μ_q/T. In particular, we expect

that the quark number susceptibility will diverge at the chiral critical point, i.e., at the second order endpoint of a line of first order phase transitions [14], which should lead to a dip in the ratio $\Delta p/T^2\chi_q$ at $T_c(\mu_q)$.

Finally we note that the relevance of the resonance gas model predictions for the description of the low temperature phase of QCD can also be tested in the imaginary chemical potential approach [15] which also is used in lattice calculations. The resonance gas model, for instance, suggests that the change in pressure is quite well described by a simple trigonometric function, $\cos(3\mu_q/T)$.

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