Attitude Control Considering Variable Input Saturation Limit for a Spacecraft Equipped with Flywheels

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Abstract
A new attitude controller is proposed for spacecraft whose actuator has variable input saturation limit. There are three identical flywheels orthogonally mounted on board. Each rotor is driven by a brushless DC motor (BLDCM). Models of spacecraft attitude dynamics and flywheel rotor driving motor electromechanics are discussed in detail. The controller design is similar to saturation limit linear assignment. An auxiliary parameter and a boundary coefficient are imported into the controller to guarantee system stability and improve control performance. A time-varying and state-dependent flywheel output torque saturation limit model is established. Stability of the closed-loop control system and asymptotic convergence of system states are proved via Lyapunov methods and LaSalle invariance principle. Boundedness of the auxiliary parameter ensures that the control objective can be achieved, while the boundary parameter’s value makes a balance between system control performance and flywheel utilization efficiency. Compared with existing controllers, the newly developed controller with variable torque saturation limit can bring smoother control and faster system response. Numerical simulations validate the effectiveness of the controller.

Keywords: spacecraft attitude control; variable input saturation limit; Lyapunov methods; flywheels; brushless DC motor

1. Introduction
Flywheel is widely applied spacecraft attitude control actuator for its simple structure and no consumption of nearly nonrenewable fuel[1]. Flywheel is usually driven by brushless DC motor (BLDCM), which has trapezoidal back electromotive force (EMF) and keeps outstanding control performance [2-3]. Due to weight miniaturization of the driving motor and limited power supply voltage of the spacecraft, the flywheel’s torque output ability is limited and this causes input saturation problem in spacecraft attitude control. Actual characteristics of attitude control actuators including saturation nonlinearity of flywheel were introduced in Ref. [4], but no control methods were given. If torque command keeps exceeding its saturation limit, error between it and the actual output torque may cause non-smooth control, even divergence of system.

A lot of reported work has focused on the input saturation problem in spacecraft attitude control. Controller design methods in literatures could be mainly sorted into two kinds according to design process: 1) establish torque saturation limit model then develop controller; 2) develop controller then prove its boundedness. In the first kind, researchers established torque saturation limit model first, then formed controller with the limit acting as a constant parameter [5-8]. The second kind could be viewed as an inverse process in which the controller was proposed based on system model then proved to be bounded [9-13]. Derivation of the controller boundedness was always involved with bounded system states and several adaptive parameters. Recently, some researchers approximated the saturation nonlinearity through neural network or wavelet transformation [14], in which the saturation nonlinearity was considered as a compensation term in controller. Bošković,
et al. [5-6] developed several controllers which included the constant torque saturation limit directly. Wallsgrove and Akella [19] proposed a classical form of controller, which was similar to linear assignment of torque saturation limit, and feedback information of attitude position and angular velocity entered the controller with different weights. Cruz, et al. [9] proved that torque command produced by the given controller was within its limit, and they dealt with the torque input saturation problem by cutting-off or scaling methods.

Consideration of actuator saturation possesses much importance especially in attitude control of spacecraft with flexible parts, such as solar arrays and extendable antennas. Attitude controller ignoring actuator’s saturation may cause undesired dither. This is harmful to the convergence of vibration modes of flexible parts and attitude stabilization of whole spacecraft. Dr. Hu and his team have made great progress in the area. Attitude control methods applied by them included back-stepping control [14] and variable structure control [15-17]. Besides saturation nonlinearity, dead-zone nonlinearity was also considered and coped with [15]. Among existing studies, adaptive parameters such as spacecraft moment of inertia and system state feedback gains were frequently utilized. The boundedness and convergence of these parameters were crucial to the achievement of control objective. State feedback gains were always discussed in detail [6-7,11,15]. External disturbance torque was usually considered to maintain the robustness of control system [7,11-12,15]. Furthermore, actuator’s rate saturation was also mentioned for cautious control [18-19].

Nearly all of the controllers mentioned above were designed and discussed under the assumption that the saturation limit of every actuator unit was the same for simplicity. The saturation limit was simply modeled as a constant parameter and directly utilized. Only in Ref. [15] the actual dynamics of flywheel was brought into the system model for controller development, but there lacked further discussion of control torque saturation limit. Variability of input saturation limit was not embodied apparently and detailedly. Actually, the torque saturation limit of actuator such as BLDCM is variable in a wide range. Saturation limit model is based on deep analysis of BLDCM electromechanical dynamics and pulse width modulation (PWM) control method. Inspired by the form of controllers proposed in Ref. [7] and Ref. [16], we apply the first kind of controller design method and propose a new attitude controller considering flywheel output torque saturation nonlinearity. The key parameters contained in the controller are designed and analyzed in detail to make the control process stable and fluent.

2. Control System and Variable Input Saturation Limit Modeling

The spacecraft studied here is a small satellite with three identical flywheels orthogonally mounted on board, as shown in Fig. 1. $O_x X_b Y_b Z_b$ is spacecraft body frame. Rotating axes of flywheel rotors are all in parallel with three axes of the body frame.

![Fig. 1 Sketch of a spacecraft equipped with three orthogonally mounted flywheels.](image)

Attitude dynamics equation of a rigid spacecraft is given as follows:

$$J_s \frac{d \omega_b}{dt} + \omega_b \times (J_s \omega_b + C_w J_w \Omega) = T_w + T_{dbh} \quad (1)$$

where $J_s$ is the total moment of inertia matrix of the spacecraft with flywheels viewed as mass points. $J_w$ represents the diagonal inertia matrix of flywheel group, i.e. $J_w = \text{diag}(J_{w1}, J_{w2}, J_{w3})$, and $J_w$ is the united inertia of flywheel rotor and motor rotor. $\omega_b$ is the spacecraft inertial angular velocity expressed in the body frame. $C_w$ is the constant mounting matrix of flywheel group and it turns to be a three-order identity matrix $I_3$ here. $\Omega = [\Omega_1 \quad \Omega_2 \quad \Omega_3]^T$ is the angular velocity vector of flywheel group. $T_{dbh}$ is the external distur-
bance torque while \( T_w = [T_{w1}, T_{w2}, T_{w3}] \) the active control torque on three axes produced by flywheels. \( T_{db} \) is usually much less than \( T_w \), so ignoring \( T_{db} \) here does not affect the design of attitude controller. \( T_w \) equals the flywheels’ reaction torque:

\[
T_w = -C_w I_{ws} \frac{d\Omega}{dt} \quad (2)
\]

Situations of attitude stabilization and maneuver are both considered in this paper, so kinematics equation is expressed in quaternion form for singularity avoidance:

\[
\frac{dQ_e}{dt} = -\frac{1}{2}Q_e^T \omega_e, \quad \frac{d\omega_e}{dt} = \frac{1}{2}(Q_e I + I^T_q) \omega_e \quad (3)
\]

where \( Q_e = [q_{e0}, q_{e1}, q_{e2}, q_{e3}]^T \) is the attitude quaternion error of spacecraft with respect to the orbit frame and there exists a restriction:

\[
q_{e0}^2 + q_{e1}^2 + q_{e2}^2 + q_{e3}^2 = 1. \quad q_i \quad \text{is skew-symmetric matrix of the vector} q_i, \quad \omega_e = [\omega_{e1}, \omega_{e2}, \omega_{e3}]^T \text{is the attitude velocity error between the orbit frame and the orbit frame and expressed in the body frame:}
\]

\[
\omega_e = \omega_b - C_{bo} \omega_o \quad (4)
\]

where \( C_{bo} \) is the attitude transformation matrix from the orbit frame to the current body frame, and \( \omega_o \) the orbit angular velocity vector expressed in the orbit frame, i.e. \( \omega_o = [0 \quad -\omega_o \quad 0]^T \), so for a standard circle orbit, \( \omega_o \) is a constant vector and

\[
\frac{d\omega_o}{dt} = \frac{d\omega_o}{dt} - \frac{dC_{bo}}{dt} \omega_o = \frac{d\omega_o}{dt} + \omega_o^T C_{bo} \omega_o \quad (5)
\]

The objective of attitude control is to make the attitude errors between the body frame and the orbit frame vanish with time, i.e.

\[
\lim_{t \to \infty} \omega_e = 0, \quad \lim_{t \to \infty} \omega_o = 0 \quad (6)
\]

The PWM controlled three-phase-six-state BLDCM (see Fig. 2) is selected as the driving motor of flywheel. There are always two conducting windings in each state.

![PWM controlled three-phase-six-state BLDCM](image)

Fig. 2  PWM controlled three-phase-six-state BLDCM.

In Fig. 2, IGBT 1 to IGBT 6 are all power electronics, and \( A, B, C \) represent three phases of the motor. The current of two conducted windings can be solved from the voltage balance equation

\[
I_{wd} = \frac{U_{dc} - 2U_f - 2N_p K_{emf} \Omega}{2(R_{wd} + R_{in})} \left[1 - \exp\left(-\frac{\Gamma_{on}}{\Gamma_{ec}}\right)\right] \quad (7)
\]

where \( U_{dc} \) is the limited supply voltage which causes the input saturation problem in spacecraft attitude control, and \( U_f \) the forward voltage drop of power electronics used in PWM control, such as insulated gate bipolar transistor (IGBT) or metal-oxide-semiconductor field-effect transistor (MOSFET). \( \Omega \) is the motor speed equals that of flywheel rotor, \( T_w \) the windings’ conducting time during one PWM period \( T_{PWM} \), and \( \Gamma_{ec} \) the windings’ electromagnetic time constant which is usually much less than \( T_{PWM} \). \( N_p \) and \( K_{emf} \) are respectively the number of pole pairs and motor back EMF coefficient. \( R_{wd} \) is the single winding’s resistance, while \( R_{in} \) the internal resistance of one power electronics unit. According to the power equivalence principle, the equivalent current during one \( T_{PWM} \) is

\[
I_{eq} = \frac{I_{max}}{T_{PWM}} \Gamma_{on} \left[1 - \exp\left(-\frac{\Gamma_{on}}{\Gamma_{ec}}\right)\right] \quad (8)
\]

\[
I_{max} = \frac{U_{dc} - 2U_f - 2N_p K_{emf} \Omega}{2(R_{wd} + R_{in})} \quad (9)
\]

\( I_{eq} \) can reach its maximum value under the conditions of \( \Gamma_{on} = T_{PWM} \), which means the two windings keep conducting during the whole \( T_{PWM} \) period. Considering \( \Gamma_{ec} \ll \Gamma_{PWM} \),

\[
\max I_{eq} = \frac{I_{max}}{T_{PWM}} \Gamma_{on} \left[1 - \exp\left(-\frac{\Gamma_{ PWM}}{\Gamma_{ec}}\right)\right] \approx I_{max} \quad (10)
\]

The equivalent electromagnetic torque \( T_{eq} \) of BLDCM is proportional to the equivalent current. Its counteractive part is the flywheel control torque when friction torque is neglected:

\[
T_{eq} = J_w \frac{d\Omega}{dt} = K_{tor} I_{eq} = -T_w \quad (11)
\]

where \( K_{tor} \) is the motor torque coefficient, \( T_w \) is the output torque of the flywheel driven by the BLDCM and can be anyone of \( T_{wi} (i = 1, 2, 3) \). So there exists

\[
T_{wmax} = \max T_w = \max T_{eq} = K_{tor} \max I_{eq} \quad (12)
\]

Combining Eqs. (8)-(12) and considering the rotor speed may be positive or negative yield

\[
T_{wmax} = T_{wmax0} - K_{com} \left[\int_{0}^{T_{w}} T_w dt\right] \quad (13)
\]

\[
T_{wmax0} = \frac{K_{tor}(U_{dc} - 2U_f)}{2(R_{wd} + R_{in})} \quad (14)
\]

\[
K_{com} = \frac{N_p K_{tor} K_{emf}}{J_w (R_{wd} + R_{in})} \quad (15)
\]

where \( | \cdot | \) is the absolute value of a scale, \( T_{wmax0} \) the maximum of torque saturation limit when \( \Omega = 0 \), and \( K_{com} \) a combined constant parameter. Equation (13)
establishes the torque saturation limit model which is obviously time-varying and state-dependent. The comparison of former linear saturation model with constant torque limit and saturation model established here with variable limit is shown in Fig. 3.

![Torque Saturation Models](image)

Fig. 3 Linear torque saturation model mostly applied in former studies and variable torque saturation model applied in this paper.

According to spacecraft attitude dynamics, the total angular momentum of the spacecraft here can be expressed as

\[ H = J_0 \omega_0 + C_w J_{w,\Omega} \]  \hspace{1cm} (16)

According to the conservation characteristic of \( H \), the angular velocities of flywheels at the end of the control process would be (here, \( C_w = I_3 \))

\[ \Omega(t) = J_{w,1}^{-1} C_{bb}(t_0, t_0) (J_0 \omega_0(t_0) + J_{w,2} \Omega(t_0)) - J_{w,3} \omega_0(t_0) \]  \hspace{1cm} (17)

where \( t_0 \) means the initial moment, while \( t \) the moment that control process ends. \( C_{bb}(t_0, t_0) \) is the attitude transforming matrix from the initial body frame to the final one, and

\[ C_{bb}(t_0, t_0) = C_{bb}(t_0, t_0) C_{bb}^T(t_0) \]  \hspace{1cm} (18)

The initial speed of flywheels \( \Omega(t_0) \) is usually set to be zero. \( \omega_0(t_0) \) converges to the orbit angular velocity \( \omega_b \) which is very small. The attitude transforming matrix \( C_{bb}(t_0, t_0) \) from the orbit frame to the body frame almost equals \( I_3 \) according to the control objective, so

\[ \Omega(t) \approx J_{w,1} C_{bb}(t_0) J_0 \omega_0(t_0) \]  \hspace{1cm} (19)

Elements of \( C_{bb}(t_0) \) are all sine or cosine functions. So

\[ \| \Omega(t) \| = \frac{1}{J_{w,1}} \| \omega_b(t_0) \| \]  \hspace{1cm} (20)

where \( \| \cdot \|_2 \) is the Euclidean norm of a vector, while \( \| \cdot \|_\infty \) the induced infinity norm of a matrix.

Since the initial angular velocity \( \omega_0(t_0) \) is within some range in normal working stage, we know from Eq. (20) that each flywheel rotor speed cannot exceed its maximum value when a proper type of BLDCM with higher maximum speed is chosen. Combining Eq. (9), Eq. (11), Eq. (13) and Eq. (20) yields

\[ T_{w,\max} \geq T_{w,\max} > 0 \]  \hspace{1cm} (21)

3. Attitude Control Considering Variable Input Saturation Limit

Rewrite Eq. (13) in vector and matrix form:

\[ T_{w,\max} = \begin{bmatrix} T_{w,\max 1} \\ T_{w,\max 2} \\ T_{w,\max 3} \end{bmatrix} = \begin{bmatrix} T_{w,\max 0} - K_{\text{con}} \left[ \int_0^t T_{w,1} \, dt \right] \\ T_{w,\max 0} - K_{\text{con}} \left[ \int_0^t T_{w,2} \, dt \right] \\ T_{w,\max 0} - K_{\text{con}} \left[ \int_0^t T_{w,3} \, dt \right] \end{bmatrix} \]  \hspace{1cm} (22)

\[ D(T_{w,\max}) = \text{diag}(T_{w,\max 1}, T_{w,\max 2}, T_{w,\max 3}) = \begin{bmatrix} D(T_{w,\max 0}) - K_{\text{con}} D \left[ \int_0^t T_{w} \, dt \right] \end{bmatrix} \]  \hspace{1cm} (23)

where \( D(\cdot) \) is an operator which transforms a vector to a diagonal matrix whose non-diagonal elements are all zero. \( T_{w,\max 0} = [T_{w,\max 0} \ T_{w,\max 0} \ T_{w,\max 0}]^T \). Obviously, the matrices \( D(T_{w,\max}) \) and \( D(T_{w,\max 0}) \) are all positive definite based on Eq. (21). \( T_{w,\max}(t) \) (\( i = 1, 2, 3 \)) are the torque commands produced by attitude controller, and \( T_{w,\max}(t) \) (\( i = 1, 2, 3 \)) are actual output torque saturation limits of three BLDCMs. \( T_{w} \) can also be viewed as the torque commands for three BLDCMs and \( T_{w,\max} \) also represents the torque commands saturation limits because of the special configuration of flywheel group. \( T_{w,\max} \) is obviously time-varying and dependent on the rotor angular velocity \( \Omega \).

With the modeling and analysis above, the input saturation problem here can be described as

\[ T_{w,\max}(t) = T_{w,\max 0} - K_{\text{con}} \left[ \int_0^t T_{w}(t) \, dt \right] \hspace{1cm} (i = 1, 2, 3) \]  \hspace{1cm} (24)

An attitude controller is proposed as follows:

\[ T_w = -\alpha D(T_{w,\max}) \Psi_s - (1-\alpha) D(T_{w,\max}) \Psi(s) \]  \hspace{1cm} (25)

where \( \alpha \) is a positive scalar coefficient, and \( 0 < \alpha < 1 \).

\[ \Psi(s) = [\rho(s_1) \ \rho(s_2) \ \rho(s_3)]^T \] is a vector function, and
its definition will be given later. \( s = \omega_x + k \hat{q}_e = [s_1, s_2, s_3] \) is a compound state including feedback information of attitude quaternion and velocity. The positive scalar \( \kappa \) in state \( s \) acts as an auxiliary parameter and its first-order time derivative is given as

\[
\frac{d\kappa}{dt} = -\frac{1}{\lambda} [1 - (1 - \alpha) q_e^T D(T_{\text{max}}) \rho(s) + \omega_e^T D(T_{\text{max}}) \omega_e + \omega_e^T (D(T_{\text{max}}) - D(T_{\text{max}})) q_e]
\]

where \( \lambda \) is a positive constant parameter and usually set to be larger than \( T_{\text{max}} \), i.e., \( \lambda > T_{\text{max}} \). \( \kappa \) is crucial to the convergence of system states and there exists a constraint for its initial value \( \kappa_i \):

\[
\kappa_i - \kappa_e \geq \int_0^\infty \| q_e(t) \|_2 dt + \int_0^\infty \| \omega_e(t) \|_2 dt + \int_0^\infty \| \omega_e(t) \|^2_2 dt
\]

where \( \kappa_e > 0 \) is a constant parameter, and \( \| \cdot \|_2 \) the 1-norm of a vector.

In Eq. (25), components of the function \( \rho(s) \) must satisfy \(-1 \leq \rho(s_i) \leq 1\), and represent the magnitude of \( s_i \). There are many functions available, such as \( \text{tanh}(\cdot) \).

A Lyapunov function \( V \) containing system states is chosen as

\[
V = \kappa q_e^T D(T_{\text{max}}) q_e + \kappa T_{\text{max}} (1 - q_e^T q_e) + \frac{1}{2} \omega_e^T J_s \omega_e + \frac{1}{2} \lambda \kappa^2
\]

It is obvious that \( V \) is positive definite and radially unbounded. The first-order differential of \( V \) with respect to time is

\[
\frac{dV}{dt} = 2 \kappa q_e^T D(T_{\text{max}}) \frac{dq_e}{dt} - 2 \kappa T_{\text{max}} (1 - q_e^T q_e) \frac{dq_e}{dt} + \omega_e^T \frac{d\omega_e}{dt} + \lambda \kappa \frac{d\kappa}{dt} + \kappa T_{\text{max}} (1 - q_e^T q_e) \omega_e + \omega_e^T J_s \omega_e + \kappa T_{\text{max}} q_e^T \frac{dq_e}{dt} + \omega_e^T (T_{\text{w}} - \omega_e^T J_s \omega_e + J_{\text{w}} \Omega) \frac{d\omega_e}{dt} + \omega_e^T C_{\text{w}} \omega_e + \frac{\lambda \kappa}{dt} \frac{d\kappa}{dt}
\]

Substituting Eq. (1), Eq. (25) and Eq. (26) into Eq. (30) yields

\[
\frac{dV}{dt} = -(1 - \alpha) s^T D(T_{\text{max}}) \rho(s) - \kappa \omega_e^T D(T_{\text{max}}) \omega_e
\]

where \( q_e^T q_e = 0 \) and \( \omega_e^T C_{\text{w}} \omega_e = 0 \) are used.

Definition of function \( \rho(s) \) means: \( s_i, \rho(s_i) \geq 0 \) (\( i = 1, 2, 3 \)). The second term on the right side of Eq. (31) has quadratic form, so \( dV/dt \leq 0 \) can be guaranteed with the positive definiteness of \( D(T_{\text{max}}) \). According to the LaSalle invariance principle, all of the system states will converge to the invariant set \( M = \{ s, \omega_e \}: dV/dt = 0 \). So there exists

\[
\lim_{t \to \infty} q_e = 0, \quad \lim_{t \to \infty} (\omega_e + \kappa q_e) = 0
\]

Equation (32) cannot guarantee that the control objective expressed in Eq. (6) can be achieved for sure, because the time-varying parameter \( \kappa \) contained in the state \( s \) may converge faster than \( q_e \). In some special case, \( \kappa \) may converge to zero and the convergence property of \( q_e \) will be unknown in this situation. Analyzing \( d\kappa/dt \) given in Eq. (26) yields

\[
\frac{d\kappa}{dt} = -\left( 1 - \alpha \right) \sum_{i=1}^3 \frac{T_{\text{max}}}{\lambda} q_e \rho(s_i) + \sum_{i=1}^3 \frac{T_{\text{max}}}{\lambda} q_e \omega_e \frac{d\omega_e}{dt} + \sum_{i=1}^3 \frac{T_{\text{max}}}{\lambda} \omega_e \frac{d\omega_e}{dt} + \sum_{i=1}^3 \frac{T_{\text{max}}}{\lambda} \omega_e \frac{d\omega_e}{dt}
\]

where the following inequalities are applied: \( T_{\text{max}}/\lambda \leq 1, (T_{\text{max}} - T_{\text{max}}/\lambda) \leq 1, 0 < \alpha < 1, |q_e| \leq 1, |\rho(s)| \leq 1, i = 1, 2, 3 \), integrate Eq. (33) with time and submit Eq. (27) into it, then we obtain

\[
\kappa_e - \kappa_0 \geq -\int_0^\infty \| q_e(t) \|_2 dt - \int_0^\infty \| \omega_e(t) \|_2 dt - \int_0^\infty \| \omega_e(t) \|^2_2 dt \geq \kappa_e - \kappa_0
\]

Therefore, the auxiliary parameter \( \kappa \) varies with time but will not converge to zero due to \( \kappa_e \geq \kappa > 0 \). With this conclusion, \( \lim_{t \to \infty} q_e = 0 \) can be derived from Eq. (32). Furthermore, \( 1/\lambda \) can be selected with small value to make \( \kappa \) converge more slowly than \( q_e \).
4. Numerical Simulation and Analysis

Spacecraft studied here has the inertia matrix

\[
\mathbf{J} = \begin{bmatrix}
32 & 1.85 & 3.5 \\
1.85 & 25 & 2 \\
3.5 & 2 & 32
\end{bmatrix}
\text{kg m}^2.
\]

The orbit angular velocity is \(\omega_0 = 1.1 \times 10^{-3}\) rad/s. Other parameters are listed in Table 1.

Table 1 Parameters of attitude control and BLDCM

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<th>Parameter</th>
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| \(a_0(t_0)/\left(\text{°} \cdot \text{s}^{-1}\right)\) | \(1 \ 1 \ 1\) | \(e_0(t_0)/\left(\text{°} \cdot \text{s}^{-1}\right)\) | \(0 \ 0 \ 0\)
| \(\Omega(t_0)\) | \(0.940 \ 2\) | \(\Omega(t_0)\) | \(0.0286\)
| \(U_a/V\) | 28 | \(U_f/V\) | 0.6
| \(R_{\omega}/\Omega\) | 0.15 | \(R_{\omega}/\Omega\) | 0.001
| \(N_p\) | 4 | \(J_0/\text{kg m}^2\) | 0.0042
| \(K_{\text{sat}}/(\text{V} \cdot \text{rad s}^{-1})^{-1}\) | 0.006 | \(T_{\text{sat}}/\text{rad s}^{-1}\) | \(8 \times 10^{-6}\)
| \(K_{\text{sat}}/(\text{N m} \cdot \text{rad s}^{-1})^{-1}\) | 0.024 | \(T_{\text{max}}/\text{rad s}\) | \(1 \times 10^{-4}\)
| \(K_{\text{com}}/\text{s}^{-1}\) | 0.96 | \(T_{\text{max}}/\text{N m}\) | 2.13
| \(k_u\) | 4 | \(\lambda\) | 5

Simulations are constituted of three parts:
1) Value determination of the boundary coefficient \(\xi\);
2) Comparison between controller in literatures with constant torque saturation limit and controller (25);
3) Comparison between controllers in literatures designed via the second method and controller (25).

Torque saturation limit model applied are all variable. \(\rho(s)\) affects the control performance much, and its shape can be adjusted by \(\xi\). Relatively larger \(\xi\) would lead to too cautious control that wastes control ability of flywheels. However, relatively smaller \(\xi\) would make torque command approach its saturation limit too closely and cause unwanted dither. Simulation applying the controller (25) and variable torque saturation limit expressed in Eq. (24) are conducted. The relationship between \(\rho(s)\) and \(\xi\) is shown in Fig. 4. Figure 5 gives torque commands (solid lines) and their limits (dotted lines) on \(Z_b\) axis with \(\xi\) of three different values. There appear several dithers (one of them is marked by a solid line rectangle) in Fig. 5(a), while the torque command is far away from its respective limit in Fig. 5(c). We finally determine \(\xi = 1 \times 10^{-3}\) as a tradeoff between improving control performance and exerting flywheels control capability.

Former studies always applied the constant torque-saturation limit model. Attitude controllers had the following form \([7,14]\):

\[
T_w = -\alpha T_{\text{max}} q_e - (1 - \alpha) T_{\text{max}} \rho(s)
\]  

(35)

where \(T_{\text{max}}\) is the constant torque saturation limit and equals \(T_{\text{max0}}\) in Eq. (14). Numerical simulation with controller (35) is conducted for comparison. Here the torque command from controller (35) must pass through a saturation limit loop (24) and then transform to be torque command that BLDCMs execute. Figures 6-7 show the simulation results.

In Fig. 7, the dotted lines are the actual output torque saturation upper and lower limits (\(T_{\text{max}}\) and \(-T_{\text{max}}\)) of three BLDCMs, while the solid lines are the three axes torque commands \(T_{\text{eq}}\) from controller (35). Errors between torque commands and actuator’s output torque limits lead to control failure described by the divergent attitude quaternion and angular velocities in Fig. 6. This situation occurs more frequently when the spacecraft conducts large angle maneuver.

Besides comparison of torque saturation limit models applied, comparison of controller design methods is also made. We choose one controller designed by the method of controller to control boundedness in Ref. [13];
There are three adjustable parameters in controller (36): $\mu_\alpha$, $\xi$ and $\kappa$ in $s_\alpha$. $\xi$ can also be viewed as boundary coefficient and set as $1\times10^{-3}$. $\kappa$ is always required to converge slowly. Its first-order time derivative was given in Ref. [13]:

$$\frac{d \kappa}{dt} = \frac{1}{\lambda \kappa} \sum_{i=1}^{3} \left\{ \mu_i \left[ - \frac{|\omega_i|}{2\sqrt{\xi} |s_i|} - |\kappa| |q_{i\alpha}| \right] \right\}$$

To meet the variable torque saturation limit, there exists a restriction for $\mu_i$:

$$\frac{\mu_i}{2\sqrt{\xi}} \leq T_{\text{wmax}} \quad (i = 1, 2, 3)$$

$T_{\text{wmax}} \quad (i = 1, 2, 3)$ are variable torque saturation limits given in Eq. (24). For simplicity, we determine $\mu_i$ as $2\sqrt{\xi} T_{\text{wmax}} \quad (i = 1, 2, 3)$. Simulation results with controller (36) and variable torque saturation limit (24) are shown in Figs. 8-9. Other simulations are all undertaken with the controller given in Eq. (25) and variable torque saturation limit (24). Results are shown in Figs. 10-11. Change of $\xi$ with different controllers is shown in Fig. 12.
Figure 8 and Fig. 10 show that both of the controller (25) and controller (36) are effective. Errors of attitude quaternion and angular velocities all converge near zero after a while. As to torque saturation, all of the torque commands from the two controllers are within their respective saturation limits, as shown in Fig. 9 and Fig. 11. But controller (36) is too cautious. System states in Fig. 9 take more time to converge near zero than those in Fig. 11. It is apparent that in Fig. 9 the torque commands are far away from their saturation limits, while in Fig. 11, there exists appropriate space between torque commands and their limits due to proper value of $\kappa$ and reasonable structure of the controller. Controller (25) leads to higher utilization efficiency of output torque ability of flywheels, so the control transition time is shorter than that of controller (36). Furthermore, control process in Fig. 10 is smoother than that in Fig. 8, and this is more obvious as to angular velocities. Comparing with controller (25), torque commands from controller (25) can adjust automatically according to states of BLDCMs, and this is helpful for control stability. Figure 12 describes the change of the important auxiliary parameter $\kappa$ in controllers (25) and (36). In both situations, $\kappa$ decreases slowly as expected and converges to a positive constant. It is proved that $\kappa$ gains enough margin relative to the initial states of attitude quaternion and angular velocities.

5. Conclusions

1) The torque saturation model with variable limit in attitude control is established via detailed analysis of driving motor electromechanical dynamics of flywheel and PWM controlled method. Torque saturation limit in matrix form is guaranteed to be positive definite, and this is important for the development of attitude controller.

2) The new attitude controller is similar to linear assignment of torque limits. Torque commands are all within their respective variable limits. Stability of controlled system and asymptotic convergence of system states are proved. Proper values for the auxiliary parameter and the boundary coefficient are also determined.

3) Compared with the existing controllers, the new controller can not only guarantee the stability of controlled system, but also bring more smooth control process and higher flywheels’ utilization efficiency.

References


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