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Twistor space structure of the box coefficients of $N = 1$ one-loop amplitudes

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Abstract

We examine the coefficients of the box functions in $N = 1$ supersymmetric one-loop amplitudes. We present the box coefficients for all six point $N = 1$ amplitudes and certain all n example coefficients. We find for "next-to MHV" amplitudes that these box coefficients have coplanar support in twistor space.

1. Introduction

Recently a "weak–weak" duality has been proposed between $N = 4$ supersymmetric gauge theory and topological string theory [1]. This relationship becomes manifest by transforming amplitudes into twistor space where they are supported on simple curves. A consequence of this picture is that tree amplitudes, when expressed as functions of spinor variables $k_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$, are annihilated by various differential operators corresponding to the localization of points to lines and planes in twistor space. In particular the operator corresponding to collinearity of points i, j, k in twistor space is

$$[F_{ijk}, \eta] = \langle i\, j\rangle \left[\frac{\partial}{\partial \tilde{\lambda}_k}, \eta\right] + \langle j\, k\rangle \left[\frac{\partial}{\partial \tilde{\lambda}_i}, \eta\right] + \langle k\, i\rangle \left[\frac{\partial}{\partial \tilde{\lambda}_j}, \eta\right] \tag{1.1}$$

and similarly annihilation by

$$K_{ijkl} = \langle i\, j\rangle \epsilon^{\dot{a}\dot{b}} \frac{\partial}{\partial \tilde{\lambda}_k^{\dot{a}}} \frac{\partial}{\partial \tilde{\lambda}_l^{\dot{b}}} + \text{perms} \tag{1.2}$$

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indicates *co-planarity* of points i, j, k and l, i.e., these four points lie on a plane in twistor space.

As an example, the collinear operator annihilates the "maximally helicity violating" (MHV) n-gluon tree amplitudes,

$$[F_{ijk}, \eta] A_n^{\text{tree MHV}}(1, 2, \dots, n) = 0, \tag{1.3}$$

indicating that such amplitudes only have non-zero support on a line in twistor space. These MHV colour-ordered amplitudes, where exactly two of the gluons have negative helicity, have a remarkably simple form (see Eq. (2.3)), conjectured by Parke and Taylor [2] and proved by Berends and Giele [3]. Using "cut-constructibility" and collinear limits, the one-loop MHV amplitudes have also been constructed for $N = 4$ [4] and $N = 1$ supersymmetric theories [5]. "Cut-constructible" implies that the entire amplitude can be reconstructed from a knowledge of its four-dimensional cuts [4–6]. The MHV tree amplitudes appear to play a key role in gauge theories. Cachazo, Svrček and Witten have conjectured that Yang–Mills amplitudes could be calculated using off-shell MHV vertices [7]. This construction can be extended to other particle types [8] and has already had multiple applications [9].

An understanding of the twistor structure of loop amplitudes has proven more difficult. However, Brandhuber et al. [10] demonstrated by computation of the $N = 4$ MHV n-point amplitudes how the CSW construction can be extended to one-loop amplitudes. Furthermore, the twistor space structure has been shown to manifest itself in the coefficients of the integral functions defining one-loop amplitudes. For $N = 4$ one-loop amplitudes where only box integral functions appear, the coefficients of these box functions satisfy co-planarity and collinearity conditions in twistor space [11,12]. These twistor space conditions have been shown to be useful in determining the coefficients of $N = 4$ one-loop amplitudes [13–16].

For $N = 1$ supersymmetric one-loop amplitudes much less is known. In Refs. [17,18] it was shown that the CSW constructions [7,8] can be employed in computing one-loop $N = 1$ amplitudes by reproducing the MHV n-point amplitude and in Ref. [19] it was shown how the holomorphic anomaly applies to $N = 1$ amplitudes. $N = 1$ amplitudes have a more complicated structure than $N = 4$ amplitudes, containing box, triangle and bubble integral functions. In this Letter, we present the box coefficients of all six point one-loop $N = 1$ amplitudes and some specific box coefficients in n-point amplitudes. We find that, as in the case of $N = 4$, for amplitudes with three negative helicities ("next-to-MHV") the box coefficients have coplanar support in twistor space, while for $q > 3$ negative helicities this simple behaviour is no longer true.

2. Organization of one-loop gauge theory amplitudes

Tree-level amplitudes for $U(N_c)$ or $SU(N_c)$ gauge theories with n external gluons can be decomposed into colour-ordered partial amplitudes multiplied by an associated colour-trace [20–22]. Summing over all non-cyclic permutations reconstructs the full amplitude $\mathcal{A}_n^{\text{tree}}$ from the partial amplitudes $A_n^{\text{tree}}(\sigma)$,

$$\mathcal{A}_n^{\text{tree}}(\{k_i, \lambda_i, a_i\}) = g^{n-2} \sum_{\sigma \in S_n/Z_n} \operatorname{Tr}(T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(n)}}) A_n^{\text{tree}}(k_{\sigma(1)}^{\lambda_{\sigma(1)}}, \dots, k_{\sigma(n)}^{\lambda_{\sigma(n)}}), \tag{2.1}$$

where k_i, λ_i, and a_i are respectively the momentum, helicity ($\pm$) and colour-index of the ith external gluon, g is the coupling constant and S_n/Z_n is the set of non-cyclic permutations of $\{1, \dots, n\}$. The $U(N_c)$ ($SU(N_c)$) generators T^a are the set of Hermitian (traceless Hermitian) $N_c \times N_c$ matrices, normalized such that $\operatorname{Tr}(T^a T^b) = \delta^{ab}$. The colour decomposition (2.1) can be derived in conventional field theory simply by using $f^{abc} = -i \operatorname{Tr}([T^a, T^b]T^c)/\sqrt{2}$, where the T^a may be either $SU(N_c)$ matrices or $U(N_c)$ matrices.

In a supersymmetric theory amplitudes with all helicities identical, or with all but one helicity identical, vanish due to supersymmetric Ward identities [23]. Tree-level gluon amplitudes in super-Yang–Mills and in purely gluonic Yang–Mills are identical (fermions do not appear at this order), so that

$$A_n^{\text{tree}}(1^\pm, 2^+, \dots, n^+) = 0. \tag{2.2}$$

The non-vanishing Parke–Taylor formula [2] for the MHV partial amplitudes is,

$$A_{jk}^{\text{tree MHV}}(1,2,\ldots,n) \equiv A_n^{\text{tree}}\big(1^+,\ldots,j^-,\ldots,k^-,\ldots,n^+\big), = i\frac{\langle j\,k\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle n\,1\rangle}, \tag{2.3}$$

for a partial amplitude where j and k are the only legs with negative helicity. In our convention all legs are outgoing. The result (2.3) is written in terms of spinor inner-products, $\langle j\,l\rangle \equiv \langle j^-|l^+\rangle$, $[j\,l] \equiv \langle j^+|l^-\rangle$, where $|i^\pm\rangle$ is a massless Weyl spinor with momentum k_i and chirality $\pm$ [22,24]. In terms of the variables λ_m^i, $\tilde\lambda_{\dot m}^i$,

$$\langle i\,j\rangle = \epsilon^{mn}\lambda_m^i\lambda_n^j, \qquad [i\,j] = \epsilon^{\dot m\dot n}\tilde\lambda_{\dot m}^i\tilde\lambda_{\dot n}^j. \tag{2.4}$$

The spinor products are related to the momentum invariants by $\langle i\,j\rangle[j\,i] = 2k_i\cdot k_j \equiv s_{ij}$ with $(\langle i\,j\rangle)^* = [j\,i]$. The tree amplitudes contain many residual features of higher symmetries [25]. For one-loop amplitudes, one may perform a similar colour decomposition to the tree-level decomposition (2.1) [26]. In this case there are two traces over colour matrices and one must also sum over the different spins, J, of the internal particles circulating in the loop. When all particles transform as colour adjoints, the result takes the form,

$$\mathcal{A}_n\big(\{k_i,\lambda_i,a_i\}\big) = g^n\sum_J\sum_{c=1}^{\lfloor n/2\rfloor+1}\sum_{\sigma\in S_n/S_{n;c}}\mathrm{Gr}_{n;c}(\sigma)A_{n;c}^{[J]}(\sigma), \tag{2.5}$$

where $\lfloor x\rfloor$ is the largest integer less than or equal to x. The leading colour-structure factor,

$$\mathrm{Gr}_{n;1}(1) = N_c\,\mathrm{Tr}\big(T^{a_1}\cdots T^{a_n}\big), \tag{2.6}$$

is just N_c times the tree colour factor, and the subleading colour structures ($c>1$) are given by

$$\mathrm{Gr}_{n;c}(1) = \mathrm{Tr}\big(T^{a_1}\cdots T^{a_{c-1}}\big)\,\mathrm{Tr}\big(T^{a_c}\cdots T^{a_n}\big). \tag{2.7}$$

S_n is the set of all permutations of n objects, and $S_{n;c}$ is the subset leaving $\mathrm{Gr}_{n;c}$ invariant. Once again it is convenient to use $U(N_c)$ matrices; the extra $U(1)$ decouples [26]. (For internal particles in the fundamental ($N_c + \bar N_c$) representation, only the single-trace colour structure ($c=1$) would be present, and the corresponding colour factor would be smaller by a factor of N_c. In this case the $U(1)$ gauge boson will *not* decouple from the partial amplitude, so one should only sum over $SU(N_c)$ indices when colour-summing the cross-section.)

For one-loop amplitudes, the subleading in colour amplitudes $A_{n;c}c>1$ may be obtained by summations of permutations of the leading in colour amplitude [4],

$$A_{n;c}(1,2,\ldots,c-1;c,c+1,\ldots,n) = (-1)^{c-1}\sum_{\sigma\in COP\{\alpha\}\{\beta\}}A_{n;1}(\sigma), \tag{2.8}$$

where $\alpha_i\in\{\alpha\}\equiv\{c-1,c-2,\ldots,2,1\}$, $\beta_i\in\{\beta\}\equiv\{c,c+1,\ldots,n-1,n\}$, and $COP\{\alpha\}\{\beta\}$ is the set of all permutations of $\{1,2,\ldots,n\}$ with n held fixed that preserve the cyclic ordering of the α_i within $\{\alpha\}$ and of the β_i within $\{\beta\}$, while allowing for all possible relative orderings of the α_i with respect to the β_i. Hence, we need only focus on the leading in colour amplitude and use this relationship to generate the full amplitude if required.

For $N=1$ super-Yang–Mills with external gluons there are two possible supermultiplets contributing to the one-loop amplitude: the vector and the chiral matter multiplets, which can be decomposed into single particle contributions,

$$A_n^{N=1\text{ vector}} \equiv A_n^{[1]} + A_n^{[1/2]}, \qquad A_n^{N=1\text{ chiral}} \equiv A_n^{[1/2]} + A_n^{[0]}. \tag{2.9}$$

For spin-0 we always consider a complex scalar. Throughout we assume the use of a supersymmetry preserving regulator [27–29]. For $N=4$ super-Yang–Mills theory there is a single multiplet whose contribution is given by

$$A_n^{N=4} \equiv A_n^{[1]} + 4A_n^{[1/2]} + 3A_n^{[0]}. \tag{2.10}$$

The contributions from the three supersymmetric multiplets are not independent but satisfy

$$A_n^{N=1 \text{ vector}} \equiv A_n^{N=4} - 3A_n^{N=1 \text{ chiral}}. \tag{2.11}$$

Thus, provided the $N=4$ amplitude is known, one need only calculate one of the two possibilities for $N=1$. The $N=4$ six-point amplitudes are known [4,5] and their twistor space structure has been examined. In this Letter we focus on the $A_6^{N=1 \text{ chiral}}$ amplitudes.

3. Basis of functions

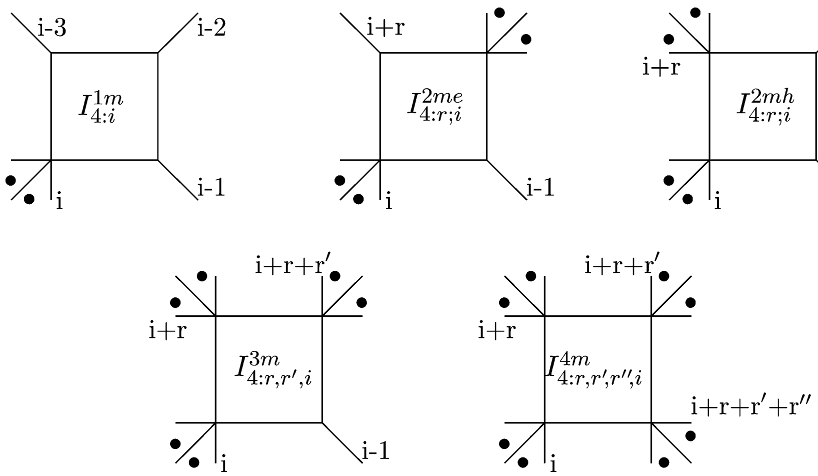
In general, one-loop amplitudes can be decomposed in terms of a set of basis functions, I_i, with coefficients, c_i, that are rational in terms of spinor products,

$$A = \sum_i c_i I_i. \tag{3.1}$$

In a Feynman diagram calculation the coefficients may, in principle, be obtained from a Passarino–Veltman reduction [30]. For supersymmetric amplitudes, the set is restricted due to cancellations within the loop-momentum integrals. For $N=1$ amplitudes the set can be taken to contain scalar boxes, I_4, scalar triangles, I_3, and scalar bubbles, I_2. In this Letter we focus on the behaviour of the box functions. In general, we can organize the box functions according to the number of legs with non-null input momenta and the relative labeling of legs. Specifically we have,

$$I_{4:i}^{\text{1m}}, \quad I_{4:r;i}^{\text{2me}}, \quad I_{4:r;i}^{\text{2mh}}, \quad I_{4:r,r',i}^{\text{3m}}, \quad I_{4:r,r',r'',i}^{\text{4m}} \tag{3.2}$$

with the labeling as indicated,



There is a choice as to which basis of functions to use, particularly with the bubble and triangle functions. For the boxes there is rather less freedom. Nevertheless we can consider three choices of basis, each of which has advantages in certain circumstances:

- $D=4$ scalar box integrals;
- $D=6$ scalar box integrals;
- $D=4$ scalar box F-functions.

The $D=4$ scalar box integrals are the natural choice, but $D=6$ scalar box integrals have several practical advantages. Firstly they are IR finite, which makes determining their collinear limits particularly simple. Secondly, for the $N=1$ chiral multiplet, the amplitude has a leading ϵ^{-1} singularity in dimensional regularization [31]. Scalar triangles have $1/\epsilon^2$ and $\ln(s)/\epsilon$ singularities. As the $D=6$ boxes are IR finite, there can be no cancellation of the ϵ^{-2} and $\ln(s)/\epsilon$ terms between them and the triangles. This implies the absence of the scalar triangle integrals in these amplitudes. The relationship between the $D=4$ boxes and $D=6$ boxes involves an overall factor and triangles functions, specifically, using the notation of Ref. [32],

$$I_4^{D=4} = \frac{1}{2N_4}\left[\sum_i \alpha_i \gamma_i I_3^{(i)} + (-1+2\epsilon)\hat{\Delta}_4 I_4^{D=6}\right]. \tag{3.3}$$

The $\hat{\Delta}_4$ are rational functions of the momentum invariants,

$$\begin{aligned}
\frac{\hat{\Delta}^{1\mathrm{m}}_{4:i}}{2N_4} &= -2\left(\frac{t_{i-3}^{[2]}+t_{i-2}^{[2]}-t_i^{[n-3]}}{t_{i-3}^{[2]}t_{i-2}^{[2]}}\right) = 2\frac{(k_{i-1}+k_{i-3})^2}{(k_{i-3}+k_{i-2})^2(k_{i-2}+k_{i-1})^2},\\
\frac{\hat{\Delta}^{2\mathrm{mh}}_{4:r;i}}{2N_4} &= -2\left(\frac{(t_{i-1}^{[r+1]}-t_i^{[r]})(t_{i-1}^{[r+1]}-t_{i+r}^{[n-r-2]})+t_{i-1}^{[r+1]}t_{i-2}^{[2]}}{t_{i-2}^{[2]}(t_{i-1}^{[r+1]})^2}\right) = \frac{\mathrm{tr}(\not{k}_{i-1}\not{P}_{i-1\ldots i+r-1}\not{k}_{i-2}\not{P}_{i-1\ldots i+r-1})}{(k_{i-2}+k_{i-1})^2(P^2_{i-1\ldots i+r-1})^2},\\
\frac{\hat{\Delta}^{2\mathrm{me}}_{4:r;i}}{2N_4} &= -2\left(\frac{t_{i-1}^{[r+1]}+t_i^{[r+1]}-t_i^{[r]}-t_{i+r+1}^{[n-r-2]}}{t_{i-1}^{[r+1]}t_i^{[r+1]}-t_i^{[r]}t_{i+r+1}^{[n-r-2]}}\right),
\end{aligned} \tag{3.4}$$

where $t_a^{[p]} \equiv (k_a+k_{a+1}+\cdots+k_{a+p-1})^2 = P^2_{a\ldots a+p-1}$ and $P_{i\ldots j} = k_i+k_{i+1}+\cdots+k_j$.

The four-dimensional boxes have dimension -2. It is convenient to define dimension zero F-functions by removing the momentum prefactors of the $D=4$ scalar boxes [5],

$$I_4^{D=4} = \frac{1}{K}F_4. \tag{3.5}$$

For the $N=4$ amplitudes, it is the coefficients of these F-functions which the collinearity and co-planarity operators annihilate [11,13,14]. Explicitly,

$$\begin{aligned}
&I^{1\mathrm{m}}_{4:i} = -2r_\Gamma \frac{F^{1\mathrm{m}}_{n:i}}{t_{i-3}^{[2]}t_{i-2}^{[2]}}, \qquad I^{2\mathrm{me}}_{4:r;i} = -2r_\Gamma \frac{F^{2\mathrm{me}}_{n:r;i}}{t_{i-1}^{[r+1]}t_i^{[r+1]}-t_i^{[r]}t_{i+r+1}^{[n-r-2]}}, \qquad I^{2\mathrm{mh}}_{4:r;i} = -2r_\Gamma \frac{F^{2\mathrm{mh}}_{n:r;i}}{t_{i-2}^{[2]}t_{i-1}^{[r+1]}},\\
&I^{3\mathrm{m}}_{4:r,r',i} = -2r_\Gamma \frac{F^{3\mathrm{m}}_{n:r,r';i}}{t_{i-1}^{[r+1]}t_i^{[r+r']}-t_i^{[r]}t_{i+r+r'}^{[n-r-r'-1]}}, \qquad I^{4\mathrm{m}}_{4:r,r',r'',i} = -2\frac{F^{4\mathrm{m}}_{n:r,r',r'';i}}{t_i^{[r+r']}t_{i+r}^{[r'+r'']}\rho}.
\end{aligned} \tag{3.6}$$

For the box functions it is easy to switch between bases since

$$A|_{\text{boxes}} = \sum_i c_i^{D=4} I_i^{D=4} = \sum_i c_i^{D=6} I_i^{D=6} = \sum_i c_i^F F_i, \tag{3.7}$$

thus the coefficients must satisfy

$$c_i^{D=4} = \frac{c_i^{D=6}}{(-\hat{\Delta}_4/2N_4)} = c_i^F K. \tag{3.8}$$

4. Box coefficients of the six point $N=1$ amplitudes

We can organise the six point amplitudes according to the number of negative helicities; amplitudes with zero, one, five or six vanish in any supersymmetric theory. The amplitudes with two negative helicities are the MHV amplitudes, which were computed previously [5], while those with four are the "googly" MHV amplitudes which are obtained by conjugation of the MHV amplitudes. Here we present the remaining box coefficients and examine the twistor structure of all the six point amplitudes.

The two independent types of six point amplitude have rather different box structures. The MHV amplitudes contain "two-mass easy" and single mass boxes, whereas the amplitudes with three negative helicities contain "two-mass hard" and single mass boxes. This feature does not extend to higher point functions.

4.1. MHV amplitudes

There are three independent MHV amplitudes. In terms of the $D=6$ boxes the box parts of these amplitudes are,

$$\begin{aligned}
&A\big(1^-,2^-,3^+,4^+,5^+,6^+\big)\big|_{\text{box}}=0,\\
&A\big(1^-,2^+,3^-,4^+,5^+,6^+\big)\big|_{\text{box}}=b_1^{D=6}I_{4:3}^{2\text{me}}+b_2^{D=6}I_{4:5}^{1\text{m}}+b_3^{D=6}I_{4:3}^{1\text{m}},\\
&A\big(1^-,2^+,3^+,4^-,5^+,6^+\big)\big|_{\text{box}}=c_1^{D=6}I_{4:1}^{2\text{me}}+c_2^{D=6}I_{4:3}^{2\text{me}}+c_3^{D=6}I_{4:6}^{1\text{m}}+c_4^{D=6}I_{4:3}^{1\text{m}},
\end{aligned}\tag{4.1}$$

where,

$$\begin{aligned}
&b_1^{D=6}=A_{13}^{\text{tree MHV}}\frac{\text{tr}_+(1325)\,\text{tr}_+(1352)}{s_{13}^2s_{25}}, \qquad b_2^{D=6}=A_{13}^{\text{tree MHV}}\frac{\text{tr}_+(1324)\,\text{tr}_+(1342)}{s_{13}^2s_{24}},\\
&b_3^{D=6}=A_{13}^{\text{tree MHV}}\frac{\text{tr}_+(1326)\,\text{tr}_+(1362)}{s_{13}^2s_{26}},
\end{aligned}\tag{4.2}$$

$$\begin{aligned}
&c_1^{D=6}=A_{14}^{\text{tree MHV}}\frac{\text{tr}_+(1436)\,\text{tr}_+(1463)}{s_{14}^2s_{36}}, \qquad c_2^{D=6}=A_{14}^{\text{tree MHV}}\frac{\text{tr}_+(1425)\,\text{tr}_+(1452)}{s_{14}^2s_{25}},\\
&c_3^{D=6}=A_{14}^{\text{tree MHV}}\frac{\text{tr}_+(1435)\,\text{tr}_+(1453)}{s_{14}^2s_{35}}, \qquad c_4^{D=6}=A_{14}^{\text{tree MHV}}\frac{\text{tr}_+(1426)\,\text{tr}_+(1462)}{s_{14}^2s_{26}},
\end{aligned}\tag{4.3}$$

where $\text{tr}_+(\text{abcd})=[\text{a b}]\langle\text{b c}\rangle[\text{c d}]\langle\text{d a}\rangle$. If we examine the coefficients of the F-functions we have, for example,

$$b_1^F=A_{13}^{\text{tree MHV}}\times\frac{\text{tr}_+(1325)\,\text{tr}_+(1352)}{s_{13}^2s_{25}^2}=A_{13}^{\text{tree MHV}}\times\frac{\langle 3\,2\rangle\langle 1\,5\rangle\langle 3\,5\rangle\langle 2\,1\rangle}{\langle 1\,3\rangle^2\langle 2\,5\rangle^2},\tag{4.4}$$

which is a holomorphic function (i.e., a function of λ alone). The amplitude has an overall factor in dimensional regularisation of r_Γ, where

$$r_\Gamma=\frac{(\mu^2)^\epsilon}{(4\pi)^{2-\epsilon}}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)},\tag{4.5}$$

which we will not write explicitly here or in following cases.

4.2. Amplitudes with three minus helicities

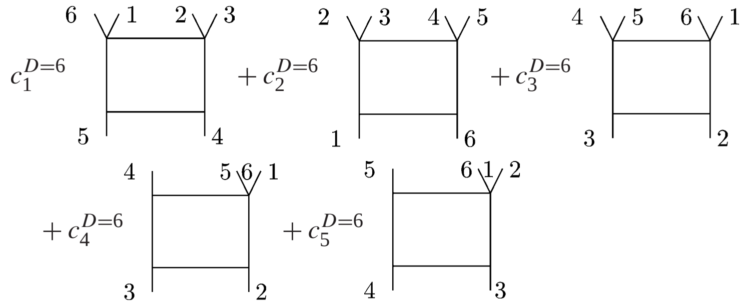
There are also three independent amplitudes with three minus helicities: $A(1^-,2^-,3^-,4^+,5^+,6^+)$, $A(1^-,2^-,3^+,4^-,5^+,6^+)$ and $A(1^-,2^+,3^-,4^+,5^-,6^+)$. Of these, the first consists only of triangle and bubble integrals [19] so we have a trivial box structure,

$$A\big(1^-,2^-,3^-,4^+,5^+,6^+\big)\big|_{\text{box}}=0.\tag{4.6}$$

The next amplitude, $A(1^-,2^-,3^+,4^-,5^+,6^+)$, does have a non-trivial box structure, which we express in terms of $D=6$ boxes as,

$$A\left(1^-,2^-,3^+,4^-,5^+,6^+\right)\big|_{\text{box}} = c_1^{D=6} I_{4:4}^{\text{2mh}} + c_2^{D=6} I_{4:6}^{\text{2mh}} + c_3^{D=6} I_{4:2}^{\text{2mh}} + c_4^{D=6} I_{4:2}^{\text{1m}} + c_5^{D=6} I_{4:3}^{\text{1m}}, \tag{4.7}$$

where the integral boxes are,



and the coefficients have been computed to be,

$$\begin{aligned}
c_1^{D=6} &= i\,\frac{(\langle 3|\not{P}|1\rangle)^2\langle 5|\not{P}|4\rangle\langle 3|\not{P}|5\rangle}{\langle 4|\not{P}|5\rangle\langle 2|\not{P}|5\rangle}\,\frac{\langle 5\,1\rangle}{[2\,3]\langle 5\,6\rangle\langle 6\,1\rangle P^2}, && P=P_{234},\\
c_2^{D=6} &= i\,\frac{(\langle 3|\not{P}|4\rangle)^2\langle 6|\not{P}|1\rangle}{\langle 1|\not{P}|6\rangle}\,\frac{[3\,1]\langle 6\,4\rangle}{[1\,2][2\,3]\langle 4\,5\rangle\langle 5\,6\rangle P^2}, && P=P_{123},\\
c_3^{D=6} &= i\,\frac{(\langle 6|\not{P}|4\rangle)^2\langle 2|\not{P}|4\rangle\langle 3|\not{P}|2\rangle}{\langle 2|\not{P}|3\rangle\langle 2|\not{P}|5\rangle}\,\frac{[6\,2]}{\langle 4\,5\rangle[6\,1][1\,2]P^2}, && P=P_{345},\\
c_4^{D=6} &= i\,\frac{(\langle 3|\not{P}|1\rangle)^2\langle 2|\not{P}|1\rangle}{\langle 2|\not{P}|5\rangle}\,\frac{\langle 2\,4\rangle}{\langle 5\,6\rangle\langle 6\,1\rangle P^2[2\,4]}, && P=P_{234},\\
c_5^{D=6} &= i\,\frac{(\langle 6|\not{P}|4\rangle)^2\langle 6|\not{P}|5\rangle}{\langle 2|\not{P}|5\rangle}\,\frac{[3\,5]}{[6\,1][1\,2]P^2\langle 3\,5\rangle}, && P=P_{345},
\end{aligned} \tag{4.8}$$

where $\langle a|\not{K}|c\rangle \equiv \langle a^+|\not{K}|c^+\rangle$.

The remaining amplitude, $A(1^-,2^+,3^-,4^+,5^-,6^+)$, contains all six one-mass and all six "two-mass-hard" boxes,

$$\begin{aligned}
A\left(1^-,2^+,3^-,4^+,5^-,6^+\right)_{\text{box}} = {}& a_1^{D=6} I_{4:1}^{\text{1m}} + a_2^{D=6} I_{4:2}^{\text{1m}} + a_3^{D=6} I_{4:3}^{\text{1m}} + a_4^{D=6} I_{4:4}^{\text{1m}} + a_5^{D=6} I_{4:5}^{\text{1m}} + a_6^{D=6} I_{4:6}^{\text{1m}}\\
&+ b_1^{D=6} I_{4:1}^{\text{2mh}} + b_2^{D=6} I_{4:2}^{\text{2mh}} + b_3^{D=6} I_{4:3}^{\text{2mh}} + b_4^{D=6} I_{4:4}^{\text{2mh}} + b_5^{D=6} I_{4:5}^{\text{2mh}}\\
&+ b_6^{D=6} I_{4:6}^{\text{2mh}}.
\end{aligned} \tag{4.9}$$

Fortunately these are not all independent and symmetry demands relationships amongst the $a_i^{D=6}$'s,

$$\begin{aligned}
a_3^{D=6}(123456) &= a_1^{D=6}(345612), & a_5^{D=6}(123456) &= a_1^{D=6}(561234),\\
a_4^{D=6}(123456) &= a_2^{D=6}(345612), & a_6^{D=6}(123456) &= a_2^{D=6}(561234),\\
a_2^{D=6}(123456) &= \bar{a}_1^{D=6}(234561), & a_1^{D=6}(123456) &= a_1^{D=6}(321654),
\end{aligned} \tag{4.10}$$

where $\bar{a}_1^{D=6}$ denotes $a_1^{D=6}$ with $\langle i\,j\rangle \leftrightarrow [i\,j]$. Thus there is a single independent $a_i^{D=6}$. Similarly we can use symmetry to generate all the $b_i^{D=6}$'s from $b_1^{D=6}$. The expressions for $a_1^{D=6}$ and $b_1^{D=6}$ are,

$$a_1^{D=6} = i\,\frac{\langle 2|\not{P}|5\rangle^2\langle 1|\not{P}|5\rangle\langle 3|\not{P}|5\rangle}{\langle 3|\not{P}|6\rangle\langle 1|\not{P}|4\rangle P^2}\,\frac{\langle 3\,1\rangle}{[1\,3]\langle 4\,5\rangle\langle 5\,6\rangle}, \qquad P=P_{123},$$

$$b_1^{D=6}=i\frac{\langle 2|\not{P}|5\rangle^2\langle 3|\not{P}|5\rangle\langle 2|\not{P}|4\rangle\langle 4|\not{P}|3\rangle}{\langle 3|\not{P}|6\rangle\langle 1|\not{P}|4\rangle\langle 3|\not{P}|4\rangle P^2}\frac{1}{[1\,2]\langle 5\,6\rangle},\quad P=P_{123}. \tag{4.11}$$

4.3. Googly MHV amplitudes

The googly MHV amplitudes can be obtained from the MHV amplitudes by conjugation. These amplitudes are useful for testing hypotheses regarding amplitudes containing four minus helicities. For example, we have,

$$A\left(1^+,2^-,3^+,4^-,5^-,6^-\right)\Big|_{\text{box}}=b_1^{D=6\prime}I_{4:3}^{2\text{me}}+b_2^{D=6\prime}I_{4:5}^{1\text{m}}+b_3^{D=6\prime}I_{4:3}^{1\text{m}}, \tag{4.12}$$

with $b_i^{D=6\prime}=\bar{b}_i^{D=6}$. The coefficients of the F-functions are anti-holomorphic functions, e.g.

$$b_1^{F\prime}=\bar{A}_{24}^{\text{tree}}\times\frac{[3\,2][1\,5][3\,5][2\,1]}{[1\,3]^2[2\,5]^2}. \tag{4.13}$$

5. Cut constructibility

In this section, we review and discuss the status of the box coefficients calculated by evaluating the cuts in one-loop amplitudes. In Ref. [4] the concept that an amplitude was "cut constructible" was introduced. At first sight the meaning of this term appears obvious: that one may calculate an amplitude from a knowledge of its cuts,

$$C_{i\ldots j}\equiv\frac{i}{2}\int d\text{LIPS}\left[A^{\text{tree}}(\ell_1,i,i+1,\ldots,j,\ell_2)\times A^{\text{tree}}(-\ell_2,j+1,j+2,\ldots,i-1,-\ell_1)\right] \tag{5.1}$$

in all channels $i\ldots j$.

Any amplitude involving massless particles can be reconstructed from a *full* knowledge of its cuts (see Ref. [33] for a modern review). This means that if we calculate the cuts precisely and regularize them in the same fashion as the amplitude, then we can determine any amplitude. Specifically, if we regularize the amplitude by dimensional regularization then, for consistency, in the cut $C_{i\ldots j}$ we should use tree amplitudes with external momenta in four dimensions, while the momenta crossing the cut should reside in $4-2\epsilon$ dimensions. These are not the normal tree amplitudes. In Ref. [34] this was explicitly realized and used to determine a specific non-supersymmetric amplitude. This method may also be used for amplitudes beyond one-loop [35,36].

Fortunately, for $N=4$ and $N=1$ supersymmetric gauge theory amplitudes it is not necessary to evaluate the cuts in this precise manner, instead one may calculate the cut using amplitudes where the cut legs lie in four-dimensions. This means that the cut can be evaluated using the conventional four-dimensional tree amplitudes. In principle this introduces errors in the trees at $O(\epsilon)$. It is non-trivial that these errors do not produce finite terms within the possibly divergent integrals. The proof of this lies in a detailed study of the possible integral functions which may occur within a one-loop calculation. For the restricted case of supersymmetric theories the cuts contain enough information to determine the coefficients of these functions unambiguously. This is a more precise definition of 'cut-constructibility'. A small number of supergravity amplitudes are also cut-constructible [37]. For $N=4$ amplitudes the integral functions are precisely scalar boxes. For $N=1$ amplitudes we have scalar boxes plus scalar triangles and bubbles. As presented in Refs. [4,5] the uniqueness of the coefficients hinges on the uniqueness of the classes of logarithms appearing in the cuts. Generically, the boxes are in a different class of functions from bubbles and triangles since the latter do not contain terms like

$$\ln\left(P_{i\ldots j}^2\right)\ln\left(P_{i'\ldots j'}^2\right), \tag{5.2}$$

which reside in boxes. By considering such terms, or specifically the coefficients of $\ln(P_{i'\ldots j'}^2)$ in the $P_{i\ldots j}$-channel cut, together with the limited number of boxes which may contain such a term in the $P_{i\ldots j}$-channel, one can show that the coefficients are uniquely defined. Generically this makes it unambiguous to extract the coefficients of boxes from a single cut. In performing a cut in, e.g., the $P_{i\ldots j}$-channel we can determine boxes with terms like $\ln(P^2_{i\ldots j})\ln(P^2_{i'\ldots j'})$ by determining the coefficients of $\ln(P^2_{i'\ldots j'})$ in this cut. In fact we tend not to evaluate the cut directly, but rather manipulate the cut into a form where it can be recognized as the cut of specific scalar boxes with coefficients.

We have carried out such a process in evaluating the coefficients of the box functions in the six point amplitudes. We shall illustrate this explicitly in the following section where we evaluate the coefficients of certain boxes in higher point functions.

6. Higher point box coefficients

In this section we evaluate some sample box coefficients for certain n-point amplitudes. This will enable us to examine whether the twistor space structure of the six-point amplitudes extends to higher point amplitudes.

For higher point amplitudes the number of helicity configurations grows quite rapidly with increasing numbers of legs. As our first example we will consider the specific amplitude,

$$A^{N=1,\ \text{chiral}}\big(1^-2^-\ldots j^+(j+1)^-5^+\ldots n^+\big). \tag{6.1}$$

We calculate the $123\ldots j$-cut of this amplitude, i.e.,

$$\begin{aligned} C_{123\ldots j} = \frac{i}{2}\int d\text{LIPS} \sum_{h\in\{-1/2,0,1/2\}} & A^{\text{tree}}\big(\ell_1^h, 1^-, 2^-, \ldots, j^+, \ell_2^{-h}\big) \\ \times\, & A^{\text{tree}}\big((-\ell_2)^h, (j+1)^-, \ldots, n^+, (-\ell_1)^{-h}\big). \end{aligned} \tag{6.2}$$

The sum is over the particles in the $N=1$ chiral multiplet. The two tree amplitudes are a MHV amplitude and a MHV-googly amplitude. For MHV amplitudes the different tree amplitudes for different particle types are related by simple cofactors determined by solving the supersymmetric Ward identities [22,23]. Using these to replace the tree amplitudes by the amplitudes for scalars with cofactors and summing the cofactors we obtain,

$$\begin{aligned} C_{123\ldots j} = \frac{i}{2}\int & d\text{LIPS}\, A^{\text{tree MHV}}\big(\ell_1^s, 1^-, 2^-, \ldots, j^+, \ell_2^s\big) \\ & \times A^{\text{tree MHV googly}}\big((-\ell_2)^s, (j+1)^-, \ldots, n^+, (-\ell_1)^s\big) \times \rho^{N=1}, \end{aligned} \tag{6.3}$$

where,

$$\rho^{N=1} = -x + 2 - \frac{1}{x} = -\frac{(x-1)^2}{x}, \quad \text{with } x = \frac{[j\,\ell_2]\langle j+1\,\ell_2\rangle}{[j\,\ell_1]\langle j+1\,\ell_1\rangle}$$

so that,

$$\rho^{N=1} = -\frac{[j\,\ell_1]\langle j+1\,\ell_1\rangle}{[j\,\ell_2]\langle j+1\,\ell_2\rangle}\left(\frac{[j\,\ell_2]\langle j+1\,\ell_2\rangle}{[j\,\ell_1]\langle j+1\,\ell_1\rangle} - 1\right)^2 = -\frac{\langle j|P_{123\ldots j}|j+1\rangle^2}{[j\,\ell_1]\langle j+1\,\ell_1\rangle[j\,\ell_2]\langle j+1\,\ell_2\rangle}. \tag{6.4}$$

This gives the integrand above as,

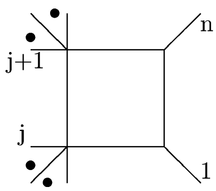
$$\begin{aligned} & \frac{[j\,\ell_1]^2[j\,\ell_2]^2}{[1\,2][2\,3]\cdots[j-1\,j][j\,\ell_2][\ell_2\,\ell_1][\ell_1\,1]} \times \frac{\langle j+1\,\ell_1\rangle^2\langle j+1\,\ell_2\rangle^2}{\langle j+1\,j+2\rangle\langle j+2\,j+3\rangle\cdots\langle n-1\,n\rangle\langle n\,\ell_1\rangle\langle\ell_1\,\ell_2\rangle\langle\ell_2\,j+1\rangle} \\ & \quad \times \frac{\langle j|P_{123\ldots j}|j+1\rangle^2}{[j\,\ell_1]\langle j+1\,\ell_1\rangle[j\,\ell_2]\langle j+1\,\ell_2\rangle} \\ & \quad = \frac{\langle j|P_{123\ldots j}|j+1\rangle^2}{[1\,2][2\,3]\cdots[j-1\,j]\langle j+1\,j+2\rangle\langle j+2\,j+3\rangle\cdots\langle n-1\,n\rangle P^2_{123\ldots j}} \times \frac{[j\,\ell_1]}{[\ell_1\,1]} \times \frac{\langle j+\rangle 1\ell_1}{\langle n\,\ell_1\rangle} \end{aligned}$$

$$= \frac{\langle j|P_{123\ldots j}|j+1\rangle^2}{[1\,2][2\,3]\cdots[j-1\,j]\langle j+1\,j+2\rangle\langle j+2\,j+3\rangle\cdots\langle n-1\,n\rangle P_{123}^2} \times \frac{[j\,\ell_1]\langle\ell_1\,1\rangle\langle j+1\,\ell_1\rangle[\ell_1\,n]}{(\ell_1-k_1)^2(\ell_1+k_n)^2}. \quad (6.5)$$

This corresponds to the cut of a box integral with integrand quadratic in the loop momentum, i.e.,

$$C_{123\ldots j} = \frac{\langle j|P_{123\ldots j}|j+1\rangle^2}{[1\,2]\cdots[j-1\,j]\langle j+1\,j+2\rangle\langle j+2\,j+3\rangle\cdots\langle n-1\,n\rangle P_{123\ldots j}^2} \times \left(I_2^{2\mathrm{mh}}\big[[j\,\ell_1]\langle\ell_1\,1\rangle\langle j+1\,\ell_1\rangle[\ell_1\,n]\big]\right)_{\mathrm{cut}}. \quad (6.6)$$

The specific box integral is the "two mass hard" depicted below,



with a non-trivial (quadratic in loop momenta) numerator.

Rewriting the numerator,

$$\langle j|\not{\ell}_1|1\rangle\langle n|\not{\ell}_1|j+1\rangle = \frac{\langle j^+|\not{\ell}_1|1^+\rangle\langle 1^+|P|n^+\rangle\langle n|\not{\ell}_1|j+1\rangle}{\langle 1^+|P|n^+\rangle} = \frac{\langle j|\not{\ell}_1\not{1}\not{P}\not{k}_n\not{\ell}_1|j+1\rangle}{\langle 1|\not{P}|n\rangle}, \quad (6.7)$$

and commuting the cut momenta toward $\not{P} = \not{\ell}_1 - \not{\ell}_2$,

$$\begin{aligned}\not{\ell}_1\not{1}\not{P}\not{k}_n\not{\ell}_1 &= (2\ell_1\cdot k_1)\not{P}\not{k}_n\not{\ell}_1 - \not{1}\not{\ell}_1\not{P}\not{k}_n\not{\ell}_1\\ &= (2\ell_1\cdot 1)\not{P}\not{k}_n\not{\ell}_1 - (2\ell_1\cdot k_n)\not{1}\not{\ell}_1\not{P} + \not{1}\not{\ell}_1\not{P}\not{\ell}_1\not{k}_n\\ &= -(\ell_1-k_1)^2\not{P}\not{k}_n\not{\ell}_1 - (\ell_1+k_n)^2\not{k}_1\not{\ell}_1\not{P} + (2\ell_1\cdot P)\not{k}_1\not{\ell}_1\not{k}_n. \end{aligned} \quad (6.8)$$

In this expression the first two terms cancel a propagator yielding triangle integrals—which we discard for the present purposes—and the third term can be rearranged as $(2\ell_1\cdot P) = -(\ell_1-P)^2+\ell_1^2+P^2 = -\ell_2^2+\ell_1^2+P^2 \equiv P^2$ discarding momenta null on the cut. The remaining expression is a box with linear integrand which can be evaluated and the result expressed as a $D=6$ scalar box function,

$$C_{123\ldots j} = \frac{\langle j|P_{123\ldots j}|(j+1)\rangle^2\langle n|\not{P}|1\rangle[1\,j]\langle j+1\,n\rangle}{\langle 1|\not{P}|n\rangle[1\,2][2\,3]\cdots[j-1\,j]\langle j+1\,j+2\rangle\langle j+2\,j+3\rangle\cdots\langle n-1\,n\rangle P_{123\ldots j}^2}\left(I_4^{2\mathrm{mh},D=6}\right)_{\mathrm{cut}} \quad (6.9)$$

so we deduce, using the arguments of the previous section, that the coefficient of the box is

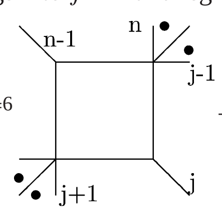
$$f_1^{D=6} = i\frac{\langle j|\not{P}|(j+1)\rangle^2\langle n|\not{P}|1\rangle[1\,j]\langle j+1\,n\rangle}{\langle 1|\not{P}|n\rangle[1\,2][2\,3]\cdots[j-1\,j]\langle j+1\,j+2\rangle\langle j+2\,j+3\rangle\cdots\langle n-1\,n\rangle P^2}, \quad P = P_{123\ldots j}, \quad (6.10)$$

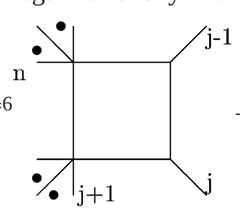
which is a generalisation of the coefficient c_2 within the six point amplitude $A(1^-,2^-,3^+,4^-,5^+,6^+)$.

As a further example, by looking at the $C_{n\ldots j-1}$ cut we can deduce that the amplitude,

$$A^{N=1,\mathrm{chiral}}\big(1^-2^-\ldots(j-1)^-j^+(j+1)^+\ldots k^-\ldots(n-1)^+n^+\big), \quad (6.11)$$

(where legs 1 to $j-1$ and leg k have negative helicity and the remainder have positive helicity) contains boxes,

$g_1^{D=6}$ 

$+\, g_2^{D=6}$ 

$+\cdots$

The first appearance of the two-mass easy box in non-MHV amplitudes occurs at seven point amplitudes. The coefficients are

$$g_1^{D=6} = -i\frac{\langle n|\not{P}|k\rangle^2\langle n|\not{P}|n-1\rangle\langle k\,n-1\rangle[n-1\,j]\langle j\,k\rangle}{[n\,1][1\,2]\cdots[j-2\,j-1]\langle j\,j+1\rangle\langle j+1\,j+2\rangle\cdots\langle n-2\,n-1\rangle\langle j-1|\not{P}|n-1\rangle\langle n-1\,j\rangle P^2},$$
$$g_2^{D=6} = i\frac{\langle n|\not{P}|k\rangle^2\langle j-1|\not{P}|k\rangle\langle j|\not{P}|j-1\rangle[n\,j-1]\langle j\,k\rangle}{[n\,1][1\,2]\cdots[j-2\,j-1]\langle j\,j+1\rangle\langle j+1\,j+2\rangle\cdots\langle n-2\,n-1\rangle\langle j-1|\not{P}|j\rangle\langle j-1|\not{P}|n-1\rangle P^2}. \tag{6.12}$$

Using symmetry arguments various other box coefficients can be obtained from these expressions by relabeling.

7. Twistor structure

It was observed by Witten [1] that the twistor space properties of amplitudes expressed in terms of the helicity states $(\lambda_i, \tilde\lambda_i)$ can be investigated using particular differential operators. Specifically, if a function has non-zero support when points i, j and k are *collinear* in twistor space, then it is annihilated by the operator

$$[F_{ijk},\eta] = \langle i\,j\rangle\left[\frac{\partial}{\partial\tilde\lambda_k},\eta\right] + \langle j\,k\rangle\left[\frac{\partial}{\partial\tilde\lambda_i},\eta\right] + \langle k\,i\rangle\left[\frac{\partial}{\partial\tilde\lambda_j},\eta\right], \tag{7.1}$$

where the square brackets indicate spinor products rather than commutators. Similarly, annihilation by the operator

$$\begin{aligned} K_{ijkl} = \frac{1}{4}\bigg[&\langle ij\rangle\epsilon^{\dot a\dot b}\frac{\partial}{\partial\tilde\lambda_k^{\dot a}}\frac{\partial}{\partial\tilde\lambda_l^{\dot b}} - \langle ik\rangle\epsilon^{\dot a\dot b}\frac{\partial}{\partial\tilde\lambda_j^{\dot a}}\frac{\partial}{\partial\tilde\lambda_l^{\dot b}} + \langle il\rangle\epsilon^{\dot a\dot b}\frac{\partial}{\partial\tilde\lambda_j^{\dot a}}\frac{\partial}{\partial\tilde\lambda_k^{\dot b}} + \langle jk\rangle\epsilon^{\dot a\dot b}\frac{\partial}{\partial\tilde\lambda_i^{\dot a}}\frac{\partial}{\partial\tilde\lambda_l^{\dot b}} \\ &+ \langle jl\rangle\epsilon^{\dot a\dot b}\frac{\partial}{\partial\tilde\lambda_k^{\dot a}}\frac{\partial}{\partial\tilde\lambda_i^{\dot b}} - \langle kl\rangle\epsilon^{\dot a\dot b}\frac{\partial}{\partial\tilde\lambda_j^{\dot a}}\frac{\partial}{\partial\tilde\lambda_i^{\dot b}}\bigg], \end{aligned} \tag{7.2}$$

indicates *co-planarity* of points i, j, k and l in twistor space.

Here we will explore the twistor space structure of the box coefficients of the $N=1$ amplitudes. At tree-level an important implication of the CSW-formalism is that the twistor space properties of amplitudes are completely determined by the number of minus legs. For this reason we organise the one-loop amplitudes according to the number of negative helicities. We have investigated the twistor space properties for all the possible 5-point box coefficients and all the 6-point box coefficients together with the n-point coefficients of the previous section. This was carried out by generating sets of on-shell kinematic points consisting of specific values of λ_i and $\tilde\lambda_i$ and determining the action of the operators at these points.

For the six point amplitudes there are three different classes of amplitudes organised by the number of negative helicities: MHV-amplitudes, next-to-MHV amplitudes and googly MHV-amplitudes. For the n-point amplitudes we have extended certain six point amplitudes by adding extra plus legs to the MHV side of the cut and extra minus legs to the googly side. This produces the following classes of n-point configurations: $(-\cdots-+\cdots+-+\cdots+)$ and $(-\cdots-+-+\cdots+)$.

For the MHV-amplitudes all helicity configurations for the box coefficients are holomorphic and are thus annihilated by any F_{ijk} and K_{ijkl} operator, as noted in [38]. The geometric picture of these configurations is simply a line in twistor space.

Now we consider next-to-MHV amplitudes with three minus helicities. By acting with the K_{ijkl} operators we find that the box coefficients are annihilated for any four points,

$$K_{ijkl}\left[c^F_{\text{next-to-MHV}}\right] = 0, \tag{7.3}$$

indicating a geometric picture where all points lie in a plane in twistor space.

The line structure of the box coefficients can be deduced by acting with the F_{ijk} operators. In the cuts we have used to determine these coefficients, there is a MHV tree amplitude on one side of the cut (the "mostly plus side") and a googly MHV tree amplitude on the other (the "mostly minus side"). The box coefficients calculated from each cut will be annihilated by F_{ijk} when i, j and k are any legs lying on the mostly plus side of that cut, indicating that these legs define points in twistor space that lie on a line. Similar behaviour was found for the box coefficients in $N=4$ amplitudes [11,12].

For the $q(>3)$ minus configurations, the box coefficients are only annihilated by F_{ijk} operators where all three of the points lie on the MHV, mostly plus, side of the cut used to calculate them. These points will lie on a line in twistor space. Hence the box coefficients are annihilated by any K_{ijkl} operator where three or more of these points lie on the line. For generic points in twistor space, we have confirmed explicitly that only these K_{ijkl} operators annihilate the box coefficients. The geometric interpretation is thus of $n-q$ points lying on a line with no restriction on the positions of the remaining q points. In general, if a box has a cut in the channel $C_{i\ldots j}$ and $A^{\text{tree}}(i\ldots j)$ is a MHV tree amplitude, then the box coefficient is supported on configurations in twistor space where points $i\ldots j$ are collinear. If there are two or more such cuts, this would imply a support of two or more lines with the remaining points unrestricted. When any pair of these cuts have a common leg, the corresponding lines intersect at the common point.

We have presented explicitly the results for the $N=1$ chiral multiplet. Since the $N=1$ vector multiplet is a linear combination of this and the $N=4$ multiplet, the box coefficients of the $N=1$ vector multiplet will also have planar support for next to MHV amplitudes.

8. Conclusions

In the twistor space realisation of gauge theory amplitudes many fascinating geometric features appear. These are of interest both formally and, possibly, practically in the determination of scattering amplitudes. One-loop amplitudes can be expressed as a sum of integral functions whose coefficients, in particular the coefficients of the box functions, contain interesting twistor space structure. For example, in $N=4$ gauge theory it has been shown that the box coefficients of next to MHV amplitudes have planar support in twistor space, analogous to the behaviour of the tree amplitudes. In this Letter we have investigated whether similar behaviour exists for $N<4$ by computing and examining the box coefficients for all six point $N=1$ amplitudes and certain classes of n point $N=1$ amplitudes. It would be interesting to extend this analysis to $N=0$ amplitudes, although in this case, the box coefficients represent a smaller fraction of the information contained in the amplitude. We find that for next to MHV amplitudes these coefficients have planar support in twistor space, explicitly confirming that the $N=4$ structure persists to $N=1$.

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