Spectroscopic factors of cluster decays in an algebraic cluster model

P.O. Hess \textsuperscript{a}, Ş. Mişicu \textsuperscript{b}

\textsuperscript{a} Instituto de Ciencias Nucleares, UNAM, Circuito Exterior, CU, A.P. 70-543, 04510 México, DF, Mexico
\textsuperscript{b} National Institute for Nuclear Physics, PO Box MG6, Bucharest, Romania

Received 12 December 2003; received in revised form 12 May 2004; accepted 19 May 2004
Available online 23 June 2004
Editor: W. Haxton

Abstract

We propose a new ansatz for the cluster spectroscopic factor in an algebraic cluster model. The parameters of the ansatz are fitted to the experimental and Blendowske–Walliser values of the spectroscopic factor for light emitted clusters and then applied to predict the spectroscopic factors for heavier clusters up to the spontaneous cold fission in two equal fragments. We show that due to the cluster and daughter shell structure, the calculated spectroscopic factors are strongly deviating from the empirical law for increasing cluster number. The observed gap between the alpha-like and cluster-like fission on one side and the spontaneous cold fission on the other side for actinides is explained by the moderate increase of spectroscopic factors, as computed in the present Letter, combined with the simultaneous decrease of the barriers when approaching symmetric mass division. This conclusion is inferred by computing the half-lives of cluster emission and spontaneous cold fission for $^{234}\text{U}$.

© 2004 Elsevier B.V. Open access under CC BY license.

PACS: 21.60.Fw; 21.60.Gx; 23.70.+j; 25.85.Ca

Keywords: Cluster radioactivity; Spontaneous fission; Spectroscopic factor; Algebraic models of clusterization

The cluster spectroscopic factor ($S$) gives the probability of finding a cluster preformed in the parent nucleus [1]. This quantity can be obtained microscopically by evaluating the squared product of the overlaps between the single-particle states in the many-body state of the cluster, $\phi_{A_1}$, and those in the many-body state of the parent $\phi_{A_1 + A_2}$. As explained in [1], disregarding the normalization effects and the specific structure of the nuclei involved, this product contains effectively $A_1 - 1$ single particle overlaps because $\phi_{A_1}$ depends on $A_1 - 1$ internal coordinates. This structure implies that many single particle states contribute to $S$, and that $S$ will approximately scale with the $(A_1 - 1)$st power of the square of a single-particle overlap. This point suggested the use of the bulk formula [2]

$$S = (S_\alpha)^{(A_1-1)/3},$$

where the $\alpha$ spectroscopic factor $S_\alpha$ is fitted to the empirically estimated spectroscopic factor and $A_1$ is the mass of the light cluster. Fitting the even and
the odd parents separately, Blendowske and Walliser obtained in the case of favored decays [2]:

\[(S_{\alpha})^{\text{even}} = 6.3 \times 10^{-3},\]
\[(S_{\alpha})^{\text{odd}} = 3.2 \times 10^{-3}.\]  
(2)

This empirical formula gave a quite reasonable fit to the experimental data existing at that time (up to the cluster \(^{28}\text{Mg}\)) and was in good concordance with the microscopical calculations. It was also mentioned that the unsuccessfully search for \(^{30}\text{Mg}\) and \(^{34}\text{Si}\) decays is explained by this exponential law. However in 1999 the decay of \(^{242}\text{Cm}\) by the emission of \(^{34}\text{Si}\) was reported [3] and deviations from the logarithmic Geiger–Nuttal systematics were observed. From the inspection of Geiger–Nuttal lines corresponding to the emission of different clusters as a function of their mass \(A_1\), for the cold decay of \(^{242}\text{Cm}\), it was concluded that the preformation probability of \(^{34}\text{Si}\) is much larger than what could be expected from the empirical law.

If one takes for example the cold splitting (the division into two fragments) \(^{234}\text{U} \rightarrow 106\text{Zr} + 134\text{Te}\), and if one employs the formula (1), then \(T_{1/2} \sim 10^{62}\) s, i.e., a huge number compared to the partial half-lives of \(^{234}\text{U}\) spontaneous fission, which is \(T_{1/2} \sim 4.73 \pm 0.63 \times 10^{23}\) s according to [4]. The authors of [5] attempted to extract the half-lives of the cold fission of \(^{238}\text{U}\) from the experimental mass spectrum of cold fission of \(^{233}\text{U}(n_{th}, f)\) [6] and arrived at the conclusion that the corresponding half-lives are \(T_{1/2} \sim 10^{30}–10^{33}\) s which is a number much smaller compared to the prediction based on the empirical ansatz (1), and thus closer to the experimental one.

Indications that the spectroscopic factors are deviating from the Blendowske–Walliser law are also found in the case of the cold fission of \(^{252}\text{Cf}\) (see [7] and references therein). In this case, except for the strong \(\alpha\)-decay component, the experiment confirms the existence of two regions of large fission yields at high cluster masses [8]. In the region of smaller clusters there is to the date no experimental evidence for emission channels. As has been shown in [9,10], considering the spectroscopic factors to be constant in the mass region \(A_1 = 96–114\), the occurrence of the observed splittings is due to the large barrier penetrabilities. The noticeable decrease of the barriers is in turn entailed by the large fragments quadrupole and hexadecupole deformations.

In [11] a series expansion was proposed to obtain the cluster spectroscopic factors in the frame of the vibron model [12] with \(U(3)\) dynamical symmetries such that it has a functional form similar to that of the energy (see also the original publications [13,14]):

\[S_{n_{\pi}L} = a_0 + a_1 n_{\pi} + a_2 n_{\pi}^2 + \beta_2 L(L + 1) + \cdots.\]  
(3)

The vibron quantum number \(n_{\pi}\) was determined from the Wildermuth condition [15] using a spherical harmonic oscillator shell-model scheme for the participating clusters. (The minimal number of relative oscillation quanta is given by the difference of the total number of quanta in the mother nucleus to the sum of total number of quanta in the clusters. Any number less implies that at least two nucleons are in the same state, i.e., it is Pauli forbidden.) The spectroscopic factors for some core-plus-alpha-particle systems and some cluster bands in \(^{16}\text{O}\) and \(^{20}\text{Ne}\) were discussed.

The advantage of an algebraic parametrization lies in the possibility to obtain the spectroscopic factors in a simple, economic way, including in regions where methods from first principles are bound to fail due to numerical and conceptual difficulties. The drawback is the introduction of parameters which have to be adjusted to a sufficient number of data points.

The purpose of the present Letter is to propose a new ansatz for the spectroscopic factor which applies to the cluster radioactivity (CR) [16,17] as well to the cold fission [7], i.e., our intention is to show that CR and cold fission can be described on the same footing. We will show that fitting the parameters to some systems of CR the fission probability can already be obtained in the right ball park.

For this purpose we consider instead of the simple vibron model, \(U(4)\), an extended version of the semi-microscopic algebraic cluster model (SACM) [13, 14], which also includes heavy nuclei [18]. In the SACM each cluster \(i = 1, 2\) is described by the \(U_{C_i}(3)\) shell model and the relative motion by the \(U_{R}(4)\) group, which introduces an upper cut off by including \(s\)-bosons. For most purposes one uses the dynamical symmetry \(U_{C_i}(3) \times U_{C_i}(3) \times U_{R}(4) \supset SU_{C_1}(3) \times SU_{C_2}(3) \times SU_{R}(3) \supset SU_{C}(3) \times SU_{R}(3) \supset SU_{C}(3)\), with the irreducible representations (irreps) \((\lambda, \mu, i)\) \((i = 1, 2), (n_{\pi}, 0), (\lambda_{C}, \mu_{C})\) and \((\lambda, \mu)\) for the \(SU_{C}, SU_{R}(3), SU_{C}(3)\) (which describes the coupling of the
two clusters) and SU(3) group, respectively. The Pauli exclusion principle is observed by matching the resulting SU(3) irreps with the shell model space. Only irreps are retained which are also contained in the shell model space. Note, that the model contains microscopic information and observes the Pauli exclusion principle. It is called semimicroscopic due to the appearance of parameters in the model operators like the Hamiltonian. A more detailed description, with examples, of how to get the microscopic model space is given in [13,14] and programs are available on request.

In the above group chain $U_R(4)$ describes the relative motion of the two clusters and its generators are composed by $p$-bosons and an auxiliary $s$-boson. The group $U_C(3)$ describes the shell structure of the $i$th cluster, as determined in [18]. The structure is obtained by filling the protons and neutrons into the Nilsson orbitals at a given deformation, using the values listed in [19]. In [20] it is shown that to it one can associate in the asymptotic limit an effective SU(3) irrep and in [18] the extension to lower deformation is given. The effective SU(3) contains all contributions like the spin–orbit and pairing interaction. The $(\lambda_C, \mu_C)$ is obtained by coupling in SU(3) the irreps of the two individual clusters. The size of the SU(3) irreps can be related to the deformation (extension) of a system [21], e.g., the second order Casimir operator is proportional to the deformation squared. The irrep $(\lambda_C, \mu_C)$ can be also geometrically related to the relative orientation of the clusters [22].

Though, we will introduce a parametrization of the spectroscopic factor, the size of the parameters will give valuable information about the microscopic influence on the spectroscopic factor, which is otherwise difficult, if not impossible, to obtain from first principles. The results may give in the future some insight how to select a microscopic approach.

Contrary to Ref. [11], where the spectroscopic factor has a linear or a quadratic form in the number of $\pi$ bosons $n_{\pi}$, we propose an exponential dependence which, for light emitted clusters, is close to the empirical Blendowske–Walliser law. The ansatz is similar to the one proposed in [1] which is also exponential, $S = \exp[ (A_1 - 1) \ln S_i / 3 ]$, but now the $A_1$ dependence is changed by the one of $n_{\pi}$. For small clusters with mass $A_1$ the two numbers $A_1$ and $n_{\pi}$ are approximately proportional because mainly all nucleons of the cluster are transferred to one shell of the united system. However, a significant deviation appears when $A_1$ is increased and the nucleons are distributed over different shells. This new ansatz can be motivated using the geometrical mapping of the SACM. Since $S$ is the probability of finding the two clusters at a distance $R$ with respect to each other, then $S \sim |F(R)|^2$ [15], where $F(R)$ is the relative-motion wave function. In the harmonic cluster model $F(R) \sim e^{-aR^2}$ [15]. On the other hand $R \sim \sqrt{n_{\pi}}$ when the SACM is mapped to the geometrical picture [24] and mutatis mutandis $S \sim e^{-a n_{\pi}}$.

If the cluster and the daughter nucleus, resulting from the cold splitting of the mother nucleus, are deformed, then a possible choice for the functional form of the spectroscopic factor is

$$S = \exp\left( A + B n_{\pi} + C \lambda_{C2}(\lambda_C, \mu_C) + D \mu_{C2}(\lambda, \mu) \right).$$

where the constants $A$, $B$, $C$ and $D$ are to be determined from fitting the available experimental points. The total SU(3) representation is given by $(\lambda, \mu)$ and the corresponding Casimir value is

$$C_2(\lambda, \mu) = \lambda^2 + \mu^2 + 3(\lambda + \mu).$$

For the cluster SU$C(3)$ representation $(\lambda_C, \mu_C)$ we take

$$\lambda_C = \lambda_1 + \lambda_2, \quad \mu_C = \mu_1 + \mu_2,$$

which in general is Pauli allowed, where $(\lambda_i, \mu_i) (i = 1, 2)$ is the SU(3) representation of the $i$th cluster. The above choice of $\lambda_C$ and $\mu_C$ corresponds to use the most aligned configuration of the clusters [22]. Note, that this definition automatically contains the symmetry $A_1 \leftrightarrow A_2$ and the microscopic information is represented by $n_{\pi}$, which is limited from below due to the Wildermuth condition [15], and by the SU(3) irreps. A possible heuristic explanation of the appearance of the second order Casimir operators in the exponent might be that they are related to the deformation, hence extension, of the system and, thus, have a similar effect as the operator of the total number of relative quanta.

To investigate the behavior of the spectroscopic factor with increasing cluster masses (decreasing mass asymmetry) we have to provide for each mass splitting $(A_1, A_2)$ a corresponding charge splitting $(Z_1, Z_2)$. Thus, for a mother nucleus with mass number $A = A_1 + A_2$ we compute the optimal charge cluster charge
Fig. 1. Spectroscopic factors of clusters emitted by various parent nuclei accompanied by the double-magic daughter nucleus $^{208}$Pb. The fit is performed with the data for the 7 clusters $\alpha$, $^{14}$C, $^{20}$O, $^{24}$Ne, $^{28}$, $^{30}$Mg, $^{34}$Si.

Z$_1$ by minimizing the driving potential [23]

$$\frac{\partial}{\partial Z_1} (V - Q) = 0,$$

where $V$ is the interaction potential and $Q$ the decay energy. Later on, when discussing the penetrability calculation, we give details on the potential.

In Fig. 1 we represented the dependence of the Blendowske–Walliser formula, the algebraic model and the experimental spectroscopic factors for various mother nuclei which are splitting into a cluster of mass $A_1$ and the daughter nucleus $^{208}$Pb. In this case ($\lambda_2$, $\mu_2$) is fixed and corresponds to the effective SU(3) representation of $^{208}$Pb (0, 16). As the best parameter fit we obtained $A = 7.764$, $B = -0.0324$, $C = -0.024$ and $D = -0.00024$. Taking into account the eigenvalues of the algebraic operators, one obtains a dominant contribution from the $Bn_{\pi}$ terms which varies (for different systems) from $-5.82$ to $-66.7$, while for the others it is less. The deformation associated to this representation is very small and the nonzero value of $\mu$, is due to the approximation made in [18]. Compilated values of the spectroscopic factor are taken from Fig. 9 of Ref. [25]. The last points on this figure are deviating from the empirical dependence. Our calculations are exhibiting some oscillations beyond that point, which are due to the SU(3) microscopic content in Eq. (4). Especially, the small peak at $A_1 = 40$ corresponds to $^{40}$Ar, which satisfies the minimum condition of (7).

Another example is to consider the mother nucleus $^{234}$U. We perform the fit on the clusters: $\alpha$, $^{24}$Ne, $^{28}$Mg. In this case the effective SU(3) representation ($\lambda$, $\mu$) of the mother nucleus is fixed and equal to (62, 16) and thus the fourth term in the exponential of Eq. (4) can be discarded (it is constant and can be joined with $A$).

In Fig. 2 the plotted solid line depicts the Blendowske–Walliser dependence of $-\log S$ and the dots show the dependence as given by Eq. (4). Our approach, which relies on the nuclear shell structure, predicts a deviation of the spectroscopic factor from the empirical law for increasing cluster mass number. When $A_1$ approaches the mass-symmetric splitting point ($A_1 = 116$), the spectroscopic factors tends to saturate. This behavior can be easily understood by inspecting Fig. 3 where we plotted the vibron number $n_{\pi}$ versus $A_1$. From this figure it can be inferred that the steady increase of the vibron number is saturating beyond $A_1 = 90$. Therefore, in our approach the spectroscopic factors are following the behavior of the vibron number and apparently the SU(3) Casimir invariants, which provide a link to the deformation of individual clusters, are influencing to a lesser extent the spectroscopic factor.

In Fig. 4 we show how the inclusion of the spectroscopic factor in the definition of the lifetime,
Fig. 3. Dependence of the vibron quantum number on the emitted cluster mass number for the cluster decay of 234U.

is influencing the importance of different mass-splittings regions starting from α-decay, passing through the CR region and ending with the spontaneous cold fission region. The penetrability $P$ was computed in the WKB-approximation for deformed clusters using the double folding potential with M3Y effective $N$–$N$ forces. Such an evaluation was used in [9] to explain the cold fission mass yields of 252Cf. The penetrability on the barrier was denoted by $\nu$.

The life-times of $\alpha$, 24Ne and 28Mg emission are fitted to the experimental values taken from [26].

From there we try to estimate the life-times of cold fission events. We cannot expect an excellent agreement because the fitting is done in the highly asymmetric region but the result shows that the ansatz of the parametrization of the spectroscopic factor is justified. Surely, a better agreement could be obtained using at least one experimental point in the near symmetric fission region. Also, we do not intend to give a thorough description of the penetration factor itself. The main emphasis in this Letter is the spectroscopic factor and the penetration factor we need to see if the life-time will fall into the right ball park. Our intention is to show that the algebraic ansatz of the spectroscopic factor leads to reasonable predictions of $S$ in the region of fission. The life-times for cold fission predicted by our approach, in the mass region around the large cluster $^{100}$Zr, are in average only a couple of orders of magnitude smaller (which gives a rough idea about the uncertainties involved from extrapolating the life-time at fission from the far end of CR decay) than the one inferred from experiment as quoted above. Besides the uncertainty mentioned above in the spectroscopic factor, the difference is due to the above mentioned approximations used in the calculation of the penetrability $P$ in Eq. (8). Nevertheless, the increase in the fission probability, as obtained by us, points in the correct direction. Further investigation is needed.

Two very important conclusions should be drawn from the inspection of Fig. 4:

1. If the empirical law of Blendowske–Walliser would work out from the point of highest asymmetry ($A_1 = 4, A_2 = A - 4$) to the point of total symmetry ($A_1 = A_2 = A/2$), then the spontaneous fission life-times would increase dramatically, and we would not see this decay phenomenon in 234U, a fact which is in contradiction to the experiment [4], where the ratio between the spontaneous fission and $\alpha$-decay half-lives is $(T_{1/2})_{sf}/(T_{1/2})_{\alpha} < 10^{11}$.

2. If the spectroscopic factors would be considered constant for the entire range of mass splittings, then the spontaneous fission region would provide the smallest life-times/highest mass-yields. In this case the CR would be several order of magnitude less probable than the spontaneous fission, a fact which is again in contradiction with the above mentioned experimental observations.

Actually in 234U, CR and spontaneous fission are occurring on an approximately equal footing. It is known that besides the emission of the clusters 24,25Ne
and $^{28}\text{Mg}$ observed experimentally, a broad group of splittings at $A_1 = 80–110$ is present in the fission mass-distribution with comparable intensity. The presence of nuclear shell structure in our approach, determines a steady increase of the spectroscopic factor with cluster mass, but much more tempered than the empirical one, until reaching the cold spontaneous fission region. Beyond $A_1 = 90$, the spectroscopic factor is saturating and the fission mass-distribution is determined mostly by the large penetrabilities for fragmentations in which the heavier fragment is close to $Z_2 = 50$ and $N_2 = 82$. We therefore conclude that the hiatus in experimental observations of mass splittings between the CR and the spontaneous fission regions is due to the decrease of the spectroscopic factor which is not compensated enough by the increase of the barrier penetrabilities.

The algebraic-based formula (4) proposed in this Letter and which depends on the shell structure of the nuclei, is therefore able to predict qualitatively the half-lives of cold spontaneous fission and give information about the dependence on the microscopic structure by fixing the half-lives of CR. It also shows how the spectroscopic factor depends on the quantum numbers, which are related to the underlying microscopic structure. In this picture the former process (weak asymmetric–symmetric fission) is an extension of the last one (highly asymmetric fission).

Acknowledgements

The authors acknowledge financial support from DGAPA (IN119002). S.M. is grateful to the European Community for financial support through a Marie Curie fellowship.

References