ENGINEERING PHYSICS AND MATHEMATICS

Wavelet based decoupled method for the investigation of surface roughness effects in elastohydrodynamic lubrication problems using couple stress fluid

S.C. Shiralashetti *, M.H. Kantli

Department of Mathematics, Karnataka University, Dharwad 580003, India

Received 7 December 2015; revised 21 March 2016; accepted 12 April 2016

KEYWORDS
Wavelet based preconditioners; Elastohydrodynamic lubrication; Daubechies filters; Couple stress; Surface roughness

Abstract In this paper, we developed the wavelet based decoupled method for the numerical solution of elastohydrodynamic lubrication problems. The standard decoupled method with Newton-generalized minimum residual procedure performs poorly or may break down when it is used to solve such type of problems. Zargari et al. (2007) presented decoupled and coupled methods, in which for the limitations of decoupled method for some set of physical parameters and slight variation in these values the non-convergence solution was tabulated. The wavelet based decoupled technique is used to overcome these limitations. Residual errors are presented in comparison with the existing methods to demonstrate the versatility and applicability of the proposed method. And also investigations of the effects of couple stress fluids on elastohydrodynamic lubrication behavior in smooth and rough contacts at low-speed-high load and high-speed-low load conditions are discussed. The elastohydrodynamic lubrication characteristics computed for couple stress fluids are found to have strong dependence on couple stress parameter.

1. Introduction

The elastohydrodynamic lubrication (EHL) is one of the important topics in tribology. EHL problems are characterized by the significant elastic deformation of the contacting surfaces and the dramatic increase in the viscosity of the lubricant with increasing pressure. The most common types of bearings are rolling bearings, gears, contacting surfaces with low geometric conformity and human joints, etc. The relevant introduction and other developments are effectively presented by Dowson and Higginson [2]. The deformation of the bearing surface...
results in changing the geometry of the lubricating film and is coupled with changes in pressure developed. The EHL is modeled by two main groups of equations, first is concerned with the physical model of the lubricant and second one is governing the lubrication problem itself. The lubricant model specifies the dependence of the fluid viscosity ($\eta$) and density ($\rho$) on the pressure ($P$). These are empirical relations and are highly nonlinear since most lubricants are non-newtonian fluids. The governing equations consist of the following: the Reynolds equation, the film thickness equation and the force balance equation. The system is derived from a non-dimensional thin-film approximation of the Stokes equation coupled with a linear elastic model of the contacting surfaces. The constitutive relationship for couple stress fluids was presented by Stokes [3] in the analysis to derive the modified Reynolds equation in terms of a non-dimensional couple stress parameter which represents the molecular length of the additives. The surface roughness is assumed to be single sided, transverse, and sinusoidal.

The ill-conditioned matrices arising in EHL for the solution of system of nonlinear algebraic equations limit the application of the powerful iterative schemes. The dense and diagonal singular structure of the Jacobian demands more sophisticated iterative schemes and the methods are of choice from Krylov subspaces. The linear system is $Ax = b$, if $A$ is symmetric and positive definite then the conjugate gradient (CG) method works well, whereas for non-symmetric and other equations, generalized minimum residual method (GMRES) can be used [4-6]. As the coefficient matrix is dense in EHL problems, the convergence is not guaranteed and takes large number of iterations to converge. The suitable remedy is preconditioning for such matrices. Classical preconditioners are incomplete LU (ILU) factorization, approximate inverse preconditioner and others [7-9]. But there are large classes of matrices occurring in the modeling of problems, which are not amenable to these latest nonstationary iterative schemes with classical preconditioners for their solutions. Zargari et al. [1] used versatile code from MATLAB interface KINSOL for analyzing EHL line contact problems, in which they observed that decoupled scheme is one of the methods and does not give converging solution for some set of physical parameters. The Jacobian matrices in EHL are dense with non-smooth diagonal and sinusoidal.

2. Wavelets

A major problem in the development of wavelets since from four decades was the search for a multiresolution analysis, where the scaling function was compactly supported and continuous. As already we know, the Haar [12,13] multiresolution analysis is generated by a compactly supported scaling function but it is not continuous. The B-splines are continuous and compactly supported but fail to form an orthonormal basis. A family of multiresolution analyses is generated by scaling functions, which are both compactly supported and continuous. These multiresolution analyses were first constructed by Daubechies [14,15], that created great eagerness among mathematicians and scientists performance research in the area of wavelets.

2.1. Multi-resolution analysis

Wavelets are functions generated from one single function called the mother wavelet by the simple operations of dilation and translation. A mother wavelet gives rise to a decomposition of the Hilbert space $L^2(R)$, into a direct sum of closed subspaces $W_j$, $j \in Z$.

$$
W_j = \text{ clos}_{L^2(R)}[\psi_{jk} : k \in Z]
$$

Then every $f \in L^2(R)$ has a unique decomposition

$$f(x) = \ldots + s_{-1} + s_0 + s_1 + \ldots$$

where $s_j \in W_j$ for all $j \in Z$, it is

$$L^2(R) = \sum_{j \in Z} W_j = \ldots \oplus W_{-1} \oplus W_0 \oplus W_1 \oplus \ldots$$

Using this decomposition of $L^2(R)$, a nested sequence of closed subspaces $V_j$, $j \in Z$ of $L^2(R)$ can be obtained, defined by

$$V_j = \ldots \oplus W_{j-2} \oplus W_{j-1}$$

These closed subspaces $\{V_j : j \in Z\}$ of $L^2(R)$, form a “multiresolution analysis” with the following properties:

(i) $\ldots \subset V_{-1} \subset V_0 \subset V_1 \subset \ldots$

(ii) $\text{ clos}_{L^2(R)}(\cup V_j) = L^2(R)$

(iii) $\cap V_j = \{0\}$

(iv) $V_{j+1} = V_j \oplus W_j$

(v) $f(x) \in V_j \iff f(2x) \in V_{j+1}, j \in Z$

Let $\phi \in V_0$ the so-called scaling function that generates the multiresolution analysis $\{V_j\}_{j \in Z}$ of $L^2(R)$. Then

$$\{\phi(-k) : k \in Z\}$$

is a basis of $V_0$, and by setting

$$\phi_{jk}(x) = 2^{j/2}\phi(2^jx - k)$$

it follows that, for each $j \in Z$, the family...
\{ \phi_{jk} : k \in \mathbb{Z} \} \tag{2.7}

is also a basis of \( V_j \).

Then, since \( \phi \in V_0 \) is in \( V_1 \) and since \( \{ \phi_{jk} : k \in \mathbb{Z} \} \) is a basis of \( V_1 \), there exists a unique sequence \( a_k \) that describes the following “two-scale relation”:

\[ \phi(x) = \sum_{k=-\infty}^{\infty} a_k \phi(2x - k) \tag{2.8} \]

of the scaling function \( \phi \) [16].

2.2. Daubechies’s wavelets

Different choices for \( \phi \) may yield different multiresolution analyses and the most useful scaling functions are those that have compact support. As an example of multiresolution analysis, a family of orthogonal Daubechies wavelets with compact support has been constructed by Daubechies [15].

A wavelet basis is orthonormal if any two translated or dilated wavelets satisfy the condition:

\[ \int_{-\infty}^{\infty} \psi_{n,k}(x) \psi_{m,l}(x) \, dx = \delta_{n,m} \delta_{k,l} \tag{2.9} \]

where \( \delta \) is the Kronecker Delta function.

Each wavelet family is governed by a set of \( L \) (an even integer) coefficients, \( a_k : k = 0, 1, \ldots, L - 1 \) through the two-scale relation

\[ \phi_L(x) = \sum_{k=0}^{L-1} a_k \phi_L(2x - k) \tag{2.10} \]

Based on the scaling function \( \phi_L(x) \), the mother wavelet can be written as,

\[ \psi_L(x) = \sum_{k=2}^{L-2} b_k \phi_L(2x - k) \tag{2.11} \]

Since the wavelets are orthonormal to the scaling basis the coefficients of the scaling function and the mother wavelet for the two-scale equation are related by the following:

\[ b_k = (-1)^k a_{L-1-k} \tag{2.12} \]

Daubechies [14] introduced scaling functions satisfying the property and distinguished by having the shortest possible support. The scaling function \( \phi \) has support \([0, L - 1]\), while the corresponding wavelet \( \psi_L \) has support \([1 - L/2, L/2]\). Thus, according to Eq. (2.8) Daubechies scaling functions of order \( L \) can exactly represent any polynomial of order up to, but not greater than \( L/2 - 1 \) [13]. For example, Daubechies wavelet family of order \( L = 4 \), we have filter coefficients [16], \( h_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}, h_1 = \frac{1}{2}, h_2 = \frac{-1}{2}, h_3 = \frac{-1-\sqrt{3}}{4\sqrt{2}} \) are low pass filter coefficients and \( g_0 = \frac{1}{\sqrt{2}}, g_1 = \frac{-1}{\sqrt{2}}, g_2 = \frac{-1}{\sqrt{2}}, g_3 = \frac{-1-\sqrt{3}}{4\sqrt{2}} \) are the high pass filter coefficients.

2.3. Discrete wavelet transform (DWT)

A DWT is a linear transformation that transforms vectors from the standard basis to a wavelet basis. Certain classes of linear operators that correspond to dense matrices in the standard basis can be approximated by sparse matrices in a suitably chosen wavelet basis. We use this property to design preconditioners for linear systems based on representing the system in a wavelet basis and forming a sparse approximation to the transformed matrix [17]. The process of applying a DWT to a dense matrix in order to obtain a sparse approximation is known as wavelet compression. The orthogonal DWT matrices \( (W_1, W_2, \ldots, W_k) \) are as follows,

\[
W_1 = \begin{bmatrix}
    h_0 & h_1 & h_2 & h_3 & 0 & 0 & \ldots & 0 \\
    0 & 0 & h_0 & h_1 & 2h_3 & 0 & \ldots & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    \end{bmatrix}
\]

\[
W_2 = \begin{bmatrix}
    g_0 & g_1 & g_2 & g_3 & 0 & 0 & \ldots & 0 \\
    0 & 0 & g_0 & g_1 & 2g_3 & 0 & \ldots & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    \end{bmatrix}
\]

\[
W_k = \begin{bmatrix}
    h_0 & h_1 & h_2 & h_3 & 0 & 0 & \ldots & 0 \\
    0 & 0 & h_0 & h_1 & h_2 & h_3 & \ldots & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    \end{bmatrix}
\]

where \( N = 2^J, J \) is the level of resolution and \( k \) is the coarsest level.

2.4. Discrete wavelet transform with permutation (DWTPer)

The standard DWT is not idea for compressing matrices with non-smooth (NS) diagonal bands for preconditioning purposes because the pattern of nonzero elements in the compressed matrix tends to a large amount of fill-in during LU factorization. One way of avoiding this problem is to use NS-forms, which we briefly consider here. Then we present the non-standard DWTPer and look in detail at its effect on dense matrices with non-smooth features both on and off the diagonal. Ford [18] proposed the DWTPer as an alternative way of avoiding the creation of finger pattern matrices. This is implemented by performing a standard DWT followed by a permutation of the rows and columns of the matrix to center the fingers about the leading diagonal. The DWTPer matrix \( (WP_k) \) is,
Here each $I$ is an identity matrix of dimension $2^{k-1} - 1$ and the
$\Phi$'s are block zero matrices of the appropriate sizes. For $k = 1$, both $I$ and $\Phi$ are of dimension 0.

3. Wavelet based decoupled method for the solution of EHL problems

Multigrid method is commonly used to solve the Reynolds equation efficiently [19,20]. Since EHL problems are nonlinear
due to governing equation from the physical model, the wavelet based decoupled method is appropriate, because of convergence,
where the existing method is not converged. The algorithm of the proposed scheme is followed by, (i) we begin
with an initial guess for $P$, $H_0$ and the cavitation point $X_c$, (ii) evaluate $H$ from the film thickness equation, (iii) solve the Reynolds
equation for $P$, (iv) update $H_0$ by using the force balance equation, (v) satisfy $\frac{dP}{dx} = 0$ at the cavitation point, (vi) while not converged go to step (ii). (vii) The overall method is such that, the cavitation point is only updated on the finest grid and $H_0$ is only updated on the coarsest level so as to ensure the smooth convergence of the whole scheme. (viii) Numerical under-relaxation parameters for pressure ($C_1$) and film thickness ($C_2$) are applied in steps (iii) and (iv) respectively. A detailed procedure of the proposed scheme is as follows.

To initiate the wavelet based decoupled method for the solution of EHL problem with line contacts using couple stress fluid and surface roughness effects, an initial guess is made for pressure distribution and the offset film thickness, $H_0$. These values are used for the calculation of film thickness ($H$), density ($\rho$) [21] and viscosity ($\eta$) [22]. The Reynolds equation is discretized over a uniform mesh in different grids with finite difference approximation and then solved along with the force balance equation using wavelet based decoupled method as follows.

Consider the Reynolds equation of the form,

$$\frac{\partial^2 P}{\partial X^2} = F(X, P, \frac{\partial P}{\partial X}, H, \eta, \rho, X_c)$$

(3.1)

where $P$ – pressure, $H$ – film thickness, $\eta$ – viscosity, $\rho$ – density and $X_c$ – cavitation.

with $P(a) = P(b) = \frac{\partial P(X_c)}{\partial X} = 0$

(3.2)

Taking the initial values of Hertzian pressure

$$P(X) = \begin{cases}
\sqrt{1 - \frac{X^2}{X_0^2}} & \text{if } |X| < 1, X_c = 0 \text{ and } H_0 = \frac{X_0}{2} - \frac{1}{2} \log 2, \\
0 & \text{otherwise}
\end{cases}$$

then compute $H$ from the film thickness equation.

Finite difference discretization of Eq. (3.1) gives the system of nonlinear algebraic equations,

$$F_j(P_1, P_2, \ldots, P_N, H_1, H_2, \ldots, H_N) = 0, \quad 1 \leq i, j \leq N$$

(3.3)

where $N$ is the number of grid points.

Solve Eq. (3.3) using Newton’s method,

$$P_{i+1} = P_i - C_1 \frac{F(P_i)}{F'(P_i)} \Rightarrow P_{i+1} = P_i - C_1 S_i$$

(3.4)

where $C_1 = 0.4(0 < C_1 < 1)$ is the under-relaxation parameter.

Let,

$$S_i = \frac{F(P_i)}{F'(P_i)}$$

(3.5)

Then Eq. (3.5) reduces to the linear system of equations,

$$Ax = b$$

(3.6)

where $A = [F(P_i)]_{N \times N}, \quad x = [S_i]_{N \times 1}, \quad b = [F(P_i)]_{N \times 1}$.

Here we have two cases for the solution of above system using DWT and DWTPer matrices.

Case-I: Applying DWT based decoupled method (DWT-DM) up to $k$th level to Eq. (3.6) as,

First level: Eq. (3.6) reduces to

$$[W_1]_{N \times N}[A]_{N \times N}[W_1]^T_{N \times N}[x]_{N \times 1} = [W_1]_{N \times N}[b]_{N \times 1} \Rightarrow A_1x_1 = b_1$$

(3.7)

Second level: Eq. (3.7) reduces to

$$[W_2]_{N \times N}[W_1]_{N \times N}[A]_{N \times N}[W_1]^T_{N \times N}[W_2]^T_{N \times N}[x]_{N \times 1} = [W_1]_{N \times N}[W_1]^T_{N \times N}[b]_{N \times 1} \Rightarrow A_2x_2 = b_2$$

(3.8)

$k$th level: At the coarsest level gives (3.8).

$$[W_{k+1}]_{N \times N}[A]_{N \times N}[W_{k+1}]^T_{N \times N}[x]_{N \times 1} = [W_1]_{N \times N}[W_1]^T_{N \times N}[b]_{N \times 1} \Rightarrow A_kx_k = b_k$$

(3.9)

Consider the transformed linear system at the $k$th level (coarsest level) in (3.9) from which we can write $A_k = D_k + C_k$ [10] based on operator (matrix) splitting then we need to choose an efficient preconditioner matrix $M = D_k$. Multiplying $M^{-1}$ on both sides of Eq. (3.9), we get,

$$M^{-1}A_kx_k = M^{-1}b_k$$

(3.10)

By solving Eq. (3.10) iteratively, we get $x_k$. Now applying successively Inverse discrete wavelet transform (IDWT) to $x_k$ at the level $k$, which reduces to

$$x_{k-1} = \frac{[W_1^T_{N \times N}]_{N \times N}x_k}{[W_1^T_{N \times N}]_{N \times N}[W_1]_{N \times N}[W_1]^T_{N \times N}[W_2]^T_{N \times N}[x]_{N \times 1}}$$

(3.11)

Again, applying IDWT to $x_{k-1}$, at the level $k - 1$, we get

$$x_{k-2} = \frac{[W_1^T_{N \times N}]_{N \times N}x_k}{[W_1^T_{N \times N}]_{N \times N}[W_1]_{N \times N}[W_1]^T_{N \times N}[W_2]^T_{N \times N}[x]_{N \times 1}}$$

(3.12)

And so on up to finer level of IDWT to $x_1$, at the level first, finally, which gives

$$x = [W_1^T_{N \times N}]_{N \times N}[W_1]_{N \times N}[W_1]^T_{N \times N}[W_2]^T_{N \times N}[x]_{N \times 1}$$

(3.11)

Case-II: Similarly, applying DWTPer based decoupled method (DWTPer-DM), the above procedure follows.

Eq. (3.6) reduces to

$$[W_{P_1}]_{N \times N}[A]_{N \times N}[W_{P_1}]^T_{N \times N}[x]_{N \times 1} = [W_{P_1}]_{N \times N}[b]_{N \times 1} \Rightarrow A_{P_k}x_k = b_k$$

(3.12)

By varying $k$, we get different levels as in the Case-I. Finally we get $x = [W_{P_1}]_{N \times N}[W_{P_1}]^T_{N \times N}[x]_{N \times 1}$ as in Eq. (3.11).

Substitute $x$ in Eq. (3.4) we get the required pressure $P$ if it satisfies the following Eq. (3.13) then stop, otherwise continue the procedure till the desired accuracy.
Investigation of surface roughness effects in elastohydrodynamic lubrication problems

\[ \sum_{j=1}^{N_x} \left| \frac{P_{j+1} - P_j}{P_j} \right| \leq \text{tol} \]  
(3.13)

and obtained the residual \( R \) from the following force balance Eq. (3.14) using the \( P \) from (3.13),

\[ R = h \sum_{j=0}^{N_x-1} \left( \frac{P_j + P_{j+1}}{2} \right) - \frac{\pi}{2} = 0 \]  
(3.14)

and then update \( H_0 \) in the following Eq. (3.15),

\[ H_0 = H_0 + RC_2 \]  
(3.15)

where \( C_2 = 0.03 \) \((0 < C_2 < 1)\) is the under-relaxation parameter.

4. Numerical experiment

Consider the governing modified Reynolds equation [23] in non-dimensional form,

\[ \frac{\partial}{\partial X} \left( \frac{\rho H^2 \partial P}{\eta \sigma} \right) - K \frac{\partial}{\partial X} (\rho H) = 0 \]  
(4.1)

where \( K = \frac{\omega_{m}^{2} L_{m}}{a_{m}^{3}} \) and \( \xi = \left[ 1 - \frac{\pi}{7} + \frac{\eta_{c}}{\rho_{c}} \right] ^{-1} \) where, \( \Omega = \frac{\omega_{m}^{2} L_{m}}{a_{m}^{3}} \), \( L_{m} \) is couple stress parameter with boundary conditions, inlet boundary \( P = 0 \) at \( X = X_0 \) and outlet boundary \( P = P_{\infty} \) at \( X = X_e \). The film thickness equation is containing the sinusoidal roughness term in non-dimensional form is,

\[ H(X) = H_0 + \frac{X^2}{2} + \bar{v} + a \sin \left( \frac{2\pi X}{\lambda} \right) \]  
(4.2)

where \( \bar{v} = \frac{\omega_{m}^{2} L_{m}}{a_{m}^{3}} \), \( \alpha \) is the amplitude. The non-dimensional surface displacement is given by

\[ \bar{v} = \frac{1}{2\pi} \int_{X_0}^{X} P \log (X - S)^3 dS \]  
(4.3)

The non-dimensional force balance equation is as,

\[ \int_{X_0}^{X} P dX = \frac{\pi}{4} \]  
(4.4)

The non-dimensional form of viscosity \( \eta(P) \) [21] and density \( \rho(P) \) [22], are as follows

\[ \bar{\rho} \left( 1 + \frac{0.6E - 09P_{H}}{1 + 1.7E - 09P_{H}} \right) \]  
(4.5)

and

\[ \bar{\eta} = \exp \left[ \left( \log \eta_0 + 0.67 \right) \times \left( -1 + \left( 1 + 5.1E - 09P_{H} \right)^{-1} \right) \right] \]  
(4.6)

where \( \eta_0 = 0.67 \) is the viscosity index, \( \eta_0 = 1.98E + 08 \) is the ambient pressure and \( P_{H} \) is the maximum Hertzian pressure. The three non-dimensional physical parameters characterize the line contact problem velocity (\( U \)), load force (\( W \)) and elasticity (\( G \)) [1] respectively.

The spatial domain \( X \in [X_0, X_e] \) is discretized with a uniform grid of \( N \) points \( X_j \) \((1 \leq i \leq N)\). Consider the cavitation point \( X_e \) to be located at an unknown internal point \( X_j \) \((2 \leq j \leq N - 1)\). Eqs. (4.1)–(4.4) are discretized using second order finite differences with uniform grid of \( N \) points \( X_j \), \( 1 \leq i \leq N \), and the domain of interest is \([X_0, X_e] = [-2, 1.5], X_e \), the cavitation point to be determined in the solution process, is an internal point near exit. The discretized form of modified Reynolds equation [23],

\[ \frac{(\Delta X)}{2} \left( \frac{P_{j+1} - P_j}{\Delta X} \right) - K \frac{(\rho_{j+1} H_j - \rho_{j} H_{j-1})}{\Delta X} = 0 \]  
(4.7)

where \( \epsilon_i = \frac{\rho_{Hj}}{\rho_{j+1} H_j} \).

The film thickness equation approximated at \( X_j \) on the regular grid is given by

\[ H(X_j) = H_0 + \frac{X_j^2}{2} + \frac{1}{2\pi} \sum_{i=0}^{N_x-1} K_i P(X_i) + a \sin \left( \frac{2\pi X_j}{\lambda} \right) \]  
(4.8)

where

\[ K_j = \left( X_j - X_j + \frac{\Delta X}{2} \right) \left( \log \left| X_j - X_j - \frac{\Delta X}{2} \right| - 1 \right) \]  
(4.9)

for \( i = 0, 1, 2, \ldots, N \) and \( j = 0, 1, 2, \ldots, N \), and the force balance equation in discrete form is

\[ \Delta X \sum_{j=0}^{N_x-1} \left( \frac{P_j + P_{j+1}}{2} \right) - \frac{\pi}{2} = 0. \]  
(4.10)

using boundary conditions in Eqs. (4.7)-(4.10) and then by taking the initial values of \( P, X_e \) and \( H_0 \), solve Eq. (4.8) we get \( H \). Now we take Eq. (4.7) in the form of \( F(P) = 0 \) to evaluate pressure \( P \) as follows,

\[ P_{j+1} = P_j - C_1 \frac{F(P_j)}{F'(P_j)} \]  
(4.11)

where

\[ S_i = \frac{F(P_i)}{F'(P_i)} \]  
(4.12)

Eq. (4.12) can be rewritten in the linear form as,

\[ F'(P_i) S_i = F(P_i) \Rightarrow Ax = b \]  
(4.13)

where \( A = F'(P_i) \), \( x = S_i \), and \( b = F(P_i) \) for \( i = 1, 2, 3, \ldots, N \).

We get the results for \( N = 8 \) with \( U = 2E - 11, W = 4E - 05, G = 5E + 03 \), couple stress \( (Lm = 1E - 05) \) and surface roughness effects \((a = 0.2, \lambda = 0.12)\).

Apply Case-I (DWT-DM) to system \( Ax = b \)

First level: Eq. (4.13) gives,

\[ W_1 \text{AW} \text{W}^T x = W_1 b \Rightarrow A_1 x_1 = b_1 \]  
(4.14)

where

\[ \begin{bmatrix} h_0 & h_1 & h_2 & h_3 & 0 & 0 & 0 & 0 \\ g_0 & g_1 & g_2 & g_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0 & h_1 & h_2 & h_3 & 0 & 0 \\ 0 & 0 & g_0 & g_1 & g_2 & g_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_0 & h_1 & h_2 & h_3 \\ 0 & 0 & 0 & 0 & g_0 & g_1 & g_2 & g_3 \\ h_2 & h_3 & 0 & 0 & 0 & 0 & h_0 & h_1 \\ g_2 & g_3 & 0 & 0 & 0 & 0 & g_0 & g_1 \end{bmatrix} \]  
(8 x 8)
Second level: Eq. (4.14) becomes,

\[
\begin{bmatrix}
\mathbf{W}_3 & \mathbf{D}_{\Phi} & \mathbf{D}_F \\
\mathbf{D}_\Phi & \mathbf{A}_1 & \mathbf{D}_F \\
\mathbf{D}_F & \mathbf{D}_F & \mathbf{A}_2
\end{bmatrix}
\begin{bmatrix}
\mathbf{W}_3 \\
\mathbf{W}_2 \\
\mathbf{W}_1
\end{bmatrix}
= \begin{bmatrix}
\mathbf{f}_1 \\
\mathbf{f}_2 \\
\mathbf{f}_3
\end{bmatrix}
\]

where \( \mathbf{W}_2 \) = 

\[
\begin{bmatrix}
\mathbf{h}_0 & \mathbf{g}_0 & \mathbf{g}_1 \\
\mathbf{g}_0 & \mathbf{h}_1 & \mathbf{g}_2 \\
\mathbf{g}_1 & \mathbf{g}_2 & \mathbf{h}_3
\end{bmatrix},
\]

\( \mathbf{A}_i \) = 

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\( \mathbf{D}_i \) = 

\[
\begin{bmatrix}
-0.0028 & -0.0570 & 0.0802 \\
-0.0570 & -3.4363 & 1.6448 \\
0.0802 & 1.6448 & -2.3132
\end{bmatrix}
\]

\( \mathbf{C}_i \) = 

\[
\begin{bmatrix}
-3.60E-05 & 4.45E-05 \\
0 & -9.27E-08 \\
0 & 0
\end{bmatrix}
\]

We hope to select an efficient preconditioner \( M^{-1} \) to matrix \( A_3 \) based on operator (matrix) splitting, as \( A_3 = D_3 + C_3 \) and let \( M = D_3 \), where

The convergence of three schemes for the physical parameters with \( G = 5E + 03 \), couple stress \( (Lm = 1E - 05) \) and surface roughness effects \( (a = 0.2, \lambda = 0.12) \).

<table>
<thead>
<tr>
<th>( N )</th>
<th>( U )</th>
<th>( W )</th>
<th>Newton-GMRES</th>
<th>DWT-DM</th>
<th>DWTPer-DM</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>2E - 11</td>
<td>4E - 05</td>
<td>Yes (201)</td>
<td>Yes (41)</td>
<td>Yes (18)</td>
</tr>
<tr>
<td>2E - 11</td>
<td>4E - 04</td>
<td>Yes (201)</td>
<td>Yes (44)</td>
<td>Yes (19)</td>
<td></td>
</tr>
<tr>
<td>20E - 11</td>
<td>4E - 05</td>
<td>No</td>
<td>Yes (43)</td>
<td>Yes (19)</td>
<td></td>
</tr>
<tr>
<td>128</td>
<td>2E - 11</td>
<td>4E - 05</td>
<td>No</td>
<td>Yes (134)</td>
<td>Yes (30)</td>
</tr>
<tr>
<td>2E - 11</td>
<td>4E - 04</td>
<td>No</td>
<td>Yes (147)</td>
<td>Yes (35)</td>
<td></td>
</tr>
<tr>
<td>20E - 11</td>
<td>4E - 05</td>
<td>No</td>
<td>Yes (145)</td>
<td>Yes (32)</td>
<td></td>
</tr>
<tr>
<td>256</td>
<td>2E - 11</td>
<td>4E - 05</td>
<td>No</td>
<td>Yes (100)</td>
<td>Yes (39)</td>
</tr>
<tr>
<td>2E - 11</td>
<td>4E - 04</td>
<td>No</td>
<td>Yes (109)</td>
<td>Yes (45)</td>
<td></td>
</tr>
<tr>
<td>20E - 11</td>
<td>4E - 05</td>
<td>No</td>
<td>Yes (108)</td>
<td>Yes (41)</td>
<td></td>
</tr>
</tbody>
</table>
Multiplying $M^{-1}$ on both sides of Eq. (4.16),

$$M^{-1} Ax_3 = M^{-1} b_3 \quad (4.17)$$

By solving Eq. (4.17) iteratively, we get $x_3$.

Coarsest level to Second level: Now applying successively inverse discrete wavelet transform decoupled method (IDWT-DM) to $x_3$ at the level 3, which reduces to

$$x_2 = \left[ W_2^T \right]_{2 \times 2} \left[ \Phi \right]_{2 \times 6} \left[ I \right]_{6 \times 6} [x_3]_{8 \times 1}. $$

Second level to First level: Again apply IDWT-DM to $x_2$ at the level 2, we get

$$x_1 = \left[ W_1^T \right]_{4 \times 4} \left[ \Phi \right]_{4 \times 4} \left[ I \right]_{4 \times 4} [x_3]_{8 \times 1}. $$

First level to Finer level: Finally, apply IDWT-DM to $x_1$ in the level first, which gives

$$x = \left[ W_1^T \right]_{8 \times 8} [x_1]_{8 \times 1}. $$

i.e. $x = \left[ -0.1617, -2.73E -06, -0.2801, 0.0026, -0.6073, -0.7600, -0.0433, 5.94E -05 \right]$. Substitute $x$ in Eq. (4.11), we get the required pressure $P$. If it satisfies the Eq. (4.10) then stop, otherwise continue the procedure till the desired accuracy as explained in Section 3. Finally that gives pressure ($P$),

$$P = \left[ -0.0162, 2.73E -06, 0.02801, 0.2601, 1.3072, 1.7600, 0.0433, -5.94E -05 \right].$$

Similarly, apply Case-II (DWTPer-DM) to system $Ax = b$ and then follow the same procedure as explained in the above implementation to get required solution. Numerical results for higher values with different parameters and their effects are discussed in the next section.

5. Results and discussion

The EHL characteristics are presented mainly for two different combinations of operating speed and load conditions, i.e., low speed-high load ($U = 2E -11, W = 4E -04$), Maximum Hertzian pressure ($p_h = 5.8E +08$) and high speed-low load ($U = 20E -11, W = 4E -05$), Maximum Hertzian pressure

![Figure 1](image1.png)

Figure 1 Convergence of the EHL problem using different schemes with couple stress ($Lm = 1E -05$) and surface roughness parameters ($a = 0.2, \lambda = 0.12$), ($U = 2E -11, W = 4E -05$ and $G = 5E +03$).

![Figure 2](image2.png)

Figure 2 Numerical solution of the EHL problem with couple stress using DWTPer-DM ($N = 256, U = 2E -11, W = 4E -05$ and $G = 5E +03$).
Figure 3  Detailed numerical solution of the EHL problem with couple stress around the Petrusevich spike using DWTPer-DM $(N = 256, U = 2E - 11, W = 4E - 05$ and $G = 5E + 03)$.

Figure 4  Numerical solution of the EHL problem using DWTPer-DM with couple stress and surface roughness parameters $(N = 256, U = 2E - 11, W = 4E - 05$ and $G = 5E + 03)$.

Figure 5  Detailed numerical solution of the EHL problem with couple stress and surface roughness around the Petrusevich spike using DWTPer-DM $(N = 256, U = 2E - 11, W = 4E - 05$ and $G = 5E + 03)$.
is observed at high speed-low load through the results. On the other hand, the enhancement in film thickness caused by using couple stress fluids at high speed-low load is much lower than that at low speed-high load. The maximum value of couple stress parameter is used in this analysis to observe the validity of the scheme. Also, the reduction in minimum film thickness due to surface roughness may be compensated for film thickening caused by couple stress fluid and the desired film thickness is obtained by selecting an appropriate value of couple stress parameter.

Acknowledgments

We are thankful to the anonymous reviewers for their valuable suggestions.

References

Dr. S.C. Shiralashetti was born in 1976. He received M.Sc., M.Phil, PGDCA, and Ph.D. degree, in Mathematics from Karnataka University, Dharwad. He joined as a Lecturer in Mathematics in S. D. M. College of Engineering and Technology, Dharwad, in 2000 and worked up to 2009. He worked as Assistant professor in Mathematics in Karnataka College Dharwad from 2009 to 2013. From 2013 onward he is working as Associate Professor in the P.G. Department of studies in Mathematics, Karnataka University Dharwad. He has attended and presented more than 35 research articles in National and International conferences. He has published more than 36 research articles in National and International Journals and proceedings.

Area of Research: Numerical Analysis, Wavelet Analysis, CFD, Differential Equations, Integral Equations, Integro-Differential Eqns. H-index: 06; Citation index: 80; Ph.D. Working: 06; Research Projects Completed: 01; UGC-MRP (PI) (2013); Research Project Applied: UGC-MRP-01(2014); Life Member of the Association/Academy/Pari shat/Society: 03; Awards: 04; Special Lectures Delivered: 20; Conference/Workshop organized as an organizing secretary/coordinator/Member: 10; Chaired the session in the National and International Conferences: 10; Conference/Workshop/Orientation/Refresher course attended: 23; Administrative Assignments completed: 02; Service given to the University in different capacities: 07.

Mr. M.H. Kantli received his M.Sc., degree in Mathematics (2009) from Karnataka University Dharwad. He is pursuing his Ph.D degree in Dept of Mathematics, from Karnataka University Dharwad in the field of Wavelet based Numerical Methods to solve Elastohydrodynamic lubrication problems. His area of Interest includes Wavelets analysis, Numerical Methods and Differential equations.

Please cite this article in press as: Shiralashetti SC, Kantli MH, Wavelet based decoupled method for the investigation of surface roughness effects in elastohydrodynamic lubrication problems using couple stress fluid, Ain Shams Eng J (2016), http://dx.doi.org/10.1016/j.asej.2016.04.024