Analysis of a piezoelectric screw dislocation inside an elliptical inhomogeneity with confocal rigid line in piezoelectric material

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(Received 25 July 2011; accepted 25 September 2011; published online 10 November 2011)

Abstract This paper deals with the electro-elastic coupling interaction between a piezoelectric screw dislocation which is located inside the elliptical inhomogeneity and an electrically conductive confocal rigid line under remote anti-plane shear stresses and in-plane electrical loads in piezoelectric composite material. The analytical-functions of the complex potentials, stress fields and the image force acting on the piezoelectric screw dislocation are obtained based on the principle of conformal mapping, the method of series expansion, the technical of analytic continuation and the analysis of singularity of complex potentials. The rigid line and the piezoelectric material property combinations upon the image force and the equilibrium position of the dislocation are discussed in detail by the numerical computation.


Keywords complex variable method, rigid line, piezoelectric material, piezoelectric screw dislocation, elliptical inhomogeneity

The interaction of dislocations with inclusions or cracks plays an important role in analyzing the strengthening and hardening mechanisms of many materials, which is attributed to the fact that the mobility of dislocations can be significantly influenced by the presence of inclusions. Therefore, a large amount of research achievements1–6 have been brought out on this topic in non-piezoelectric materials till now. Piezoelectric composites have become an important branch of modern engineering materials with fast development of the intelligent materials and structures due to their intrinsic electromechanical coupling phenomenon. A great deal of work7–10 has been conducted on electro-elastic coupling characteristics of piezoelectric composite materials. Hard inclusions may be formed easily in piezoelectric composite materials in the course of processing and using. Liu et al.11 studied the electro-elastic interaction between a piezoelectric screw dislocation located either outside or inside inhomogeneity and circular interfacial rigid lines under anti-plane mechanical and in-plane electrical loads in linear piezoelectric materials. Chen et al.12 carried out the electro-elastic stress investigation on the interaction between a piezoelectric screw dislocation and collinear rigid lines under anti-plane mechanical and in-plane electrical load.

The elliptical inhomogeneity containing a confocal rigid line is a typical model, which is often used by some scholars because analytical solutions about related problems can be obtained, moreover, it can describe the influence of the schistose rigid inclusion in reinforcement. Wu et al.13 discussed the elastic field and electric field of a rigid line in a confocal elliptic piezoelectric inhomogeneity embedded in an infinite piezoelectric medium. In the present work, we discuss the problem of the electroelastic interaction between a piezoelectric screw dislocation and an electrically conductive confocal rigid line under anti-plane shear and in-plane electrical field in piezoelectric material.

Let us consider an infinite piezoelectric matrix (with electro-elasticity modulus $M_2$ occupied the region $S^-$) containing an elliptical cylindrical inhomogeneity (with electro-elasticity modulus $M_1$ occupied the region $S^+$) and a confocal rigid line, which is infinitely long in the $z$-direction and free of force, subjected to remote longitudinal shear, in-plane electrical field. Both the matrix and the inhomogeneity are assumed to be transversely isotropic with an isotropic $xoy$-plane. A piezoelectric screw dislocation $b = [b_x, b_y]^T$ is located at arbitrary point $z_0$ in the inhomogeneity. Longitudinal shear stresses $\tau_{xz}$ and $\tau_{yz}$, as well as electrical displacements $D_x^\infty$ and $D_y^\infty$, are applied at infinite.

Referring to the work by Liu et al.11 we introduce the vector of generalized displacement $U = [u, \varphi]^T$, generalized stresses $\Sigma_x = [\tau_{xz}, D_x]^T$, $\Sigma_y = [\tau_{yz}, D_y]^T$ and generalized strain $Y_x = [\gamma_{xx}, -E_x]^T$, $Y_y = [\gamma_{yz}, -E_y]^T$. All of these vectors can be expressed by a generalized analytical function vector $f(z) = [f_w(z), f_\varphi(z)]^T$, where $z = x + iy$ is the complex variable,

$$U = \text{Re}[f(z)], \quad Y_x - iY_y = f'(z), \quad \Sigma_x - i\Sigma_y = Mf'(z),$$

where

$$M = \left[ \begin{array}{cc} C_{44} & e_{15} \\ e_{15} & -d_{11} \end{array} \right], \quad C_{44} \text{ is the longitudinal}$$

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shear modulus at a constant electric field, $\varepsilon_{15}$ is the piezoelectric modulus, $d_{11}$ is the dielectric modulus at a constant stress field.

Using Eq. (1), we can represent the resultant force and the sum of the normal component of the electric displacement along any arc $AB$ as

$$T = \int_A^B (\Sigma_x \, dy - \Sigma_y \, dx) = M \text{Im} [f(z)]^B_A,$$

(2)

Introduce the following mapping function

$$z = \omega(\zeta) = \frac{c}{2} \left( R\zeta + \frac{1}{R\zeta} \right),$$

(3)

$$R\zeta = \frac{z}{c} \left[ 1 + \sqrt{1 - \left( \frac{c}{z} \right)^2} \right],$$

where $\zeta = \xi + i\eta$, $c = \sqrt{a^2 - b^2}$ and $R = \sqrt{(a + b)/(a - b)}$, $2a$ and $2b$ are the major and minor diameters of the elliptical inhomogeneity. Using the mapping function, we map the elliptical curve and the rigid line in the $z$-plane onto the concentric circles $L_1$, $L_2$ in the $\zeta$-plane with radius 1, $1/R$ respectively, see Figs. 1, 2.

Using the mapping function (3), we can rewrite Eqs. (1) and (2) in the $\zeta$-plane.

The boundary conditions for the above problem can be expressed as follows

$$U_1 = U_2, \quad T_1 = T_2, \quad |t| = 1.$$

(4)

Applying Riemann-Schwarz’s symmetry principle, we introduce the following new analytical function vectors in the corresponding region

$$f_1(\zeta) = -\bar{f}_1 \left( \frac{1}{\zeta} \right), \quad 1 < |\zeta| < R,$$

$$f_2(\zeta) = -\bar{f}_2 \left( \frac{1}{\zeta} \right), \quad |\zeta| < 1,$$

(5a)

(5b)

and can express the boundary conditions as

$$(f_1(t) + f_2(t))^+ = [f_2(t) + f_1(t)]^-,$$

$$|t| = 1, \quad t = e^{i\theta},$$

(6a)

$$(M_1 f_1(t) - M_2 f_2(t))^+ = [M_2 f_2(t) - M_1 f_1(t)]^-,$$

$$|t| = 1,$$

(6b)

where the subscripts 1 and 2 represent the regions of inhomogeneity $S^+$ and matrix $S^-$. The superscripts $+$ and $-$ denote the boundary values of the physical quantity as $z$ approaches the interface from $S^+$ and $S^-$, respectively.

The generalized analytical function vectors in region $S^+$ and $S^-$ can be written as

$$f_1(z) = B \ln(z - z_0) + f_{10}(z),$$

$$f_2(z) = G z + B \ln z + f_{20}(z),$$

(7a)

(7b)

where $B = b/2\pi i$, $G = M_2^{-1} \left\{ \frac{\tau_{zz}^\infty - i\tau_{yz}^\infty}{D_x^\infty - iD_y^\infty} \right\}$ is determined from the far-field loads, $f_{10}(z)$ and $f_{20}(z)$ are holomorphic in region $S^+$ and $S^-$ respectively.

Transforming the analytical function vectors into $\zeta$-plane, considering the boundary $L_0$, $U_1$ is a constant, and then using Cauchy integrals, we can obtain the following analytical function vectors.

$$f_1(\zeta) = B \left[ \ln(\zeta - \zeta_0) + \ln \left( 1 - \frac{1}{R^2 \zeta_0 \zeta} \right) \right] + \sum_{k=0}^{\infty} \frac{A^{-1}(M_1 - M_2) B}{(k + 1)} \left[ \frac{\zeta - k - 1}{\zeta_0 - k - 1} \right] \zeta^{k+1} +$$
\[
\sum_{k=0}^{\infty} \frac{A^{-1}}{k+1} \left[ (M_1+M_2) B \left( \zeta_0^{k-1} - \zeta_0^{-k-1} \right) + (M_1-M_2) B \left( \zeta_0^{k+1} + R^{-2k-2} \zeta_0^{-k-1} \right) \right] \zeta^{-k-1} + \\
\Omega^{-1} M_{2cR} \left( R^2 \Gamma \zeta - \mathcal{T} \zeta^{-1} \right),
\]
\[
f_2(\zeta) = B \left[ \ln(\zeta - \zeta_0) + \ln \left( 1 - \frac{1}{R^2 \zeta_0 \zeta} \right) \right] + \sum_{k=0}^{\infty} \frac{B}{(k+1)} \left( \zeta_0^{k-1} - \zeta_0^{-k-1} \right) \zeta^{-k-1} + \\
\sum_{k=0}^{\infty} \frac{A^{-1}}{k+1} \left[ (M_1+M_2) B \left( \zeta_0^{k-1} - \zeta_0^{-k-1} \right) + (M_1-M_2) B \left( \zeta_0^{k+1} + R^{-2k-2} \zeta_0^{-k-1} \right) \right] \left( 1 - R^{2k+2} \right) \zeta^{-k-1} + \\
c \frac{R \mathcal{T}}{2} \zeta - \frac{c R \mathcal{T}}{2} \zeta^{-1} - \Omega^{-1} M_{2cR} \mathcal{T} \left( 1 - R^2 \right) \zeta^{-1},
\]

where \( \Omega = \left[ (M_1+M_2) R^2 + (M_1-M_2) \right], \quad A = \left[ (M_1+M_2) R^{2k+2} + (M_1-M_2) \right]. \)

The image force acting on the dislocation is a significant physical parameter, which can be calculated by using the generalized Peach-Koehler formula by Pak

\[ F_x - i F_y = i b^T \left( \Sigma_x^0 - i \Sigma_y^0 \right), \]  

where \( \Sigma_x^0 - i \Sigma_y^0 \) denotes the perturbation generalized stress at the dislocation point, and they can be written as

\[ F_x - i F_y = \frac{1}{\pi} \left[ b^T M_1 b + \sum_{k=0}^{\infty} b^T M_1 A^{-1} (M_1-M_2) b \left( \zeta_0^{k+1} + R^{-2k-2} \zeta_0^{-k-1} \right) + \right. \]

\[ \sum_{k=0}^{\infty} b^T M_1 A^{-1} \left[ (M_1+M_2) b \left( \zeta_0^{k+1} + R^{-2k-2} \zeta_0^{-k-1} \right) - (M_1-M_2) b \left( \zeta_0^{k+1} + R^{-2k-2} \zeta_0^{-k-1} \right) \right] \frac{2 R \zeta_0^2}{c \rho^2 \zeta_0^2 - c}. \]

We take the piezoelectric screw dislocation vector \( b_1, b_2 \)^T = [1.0 nm, 0 m]^T and introduce \( u = C_{44}^{(2)}/C_{44}^{(1)}, v = c_{15}^{(2)}/c_{15}^{(1)}. \) In this section, we aim at discussing how the electroelastic parameters ratio affect the image force. Assume that the piezoelectric material is PZT-4 with the electroelastic properties: \( C_{44}^{(2)} = 2.56 \times 10^{10} \text{ N/m}^2, \quad c_{15}^{(2)} = 12.7 \text{ C/m}^2, \quad d_{11}^{(2)} = 6.46 \times 10^{-9} \text{ C/Nm}^2. \) The inhomogeneity is another piezoelectric material and \( d_{11}^{(2)}/d_{11}^{(1)} = 1. \) We consider the typical case that piezoelectric screw dislocation lines at the point \( x_0 \) on the positive \( x \)-axis define \( F_x = 2 \pi x_0 f_2 / C_{44}^{(1)} b_2^2. \) The variation of \( F_{x0} \) with respect to the relative position of the dislocation \( (x_0/a) \) is depicted in Fig. 3 with different values of \( u \) when \( v = 1. \)

Figure 4 shows \( F_{x0} \) versus \( x_0/a \) with different values of \( v \) when \( u = 1. \)

It is observed that the image force is positive first, and then becomes negative with the dislocation moving away from the rigid line to the interface when \( u > 1 \) (the inhomogeneity is softer than the matrix) and \( v = 1. \) There is a stable equilibrium position in \( x \)-axis. When \( u = 1 \) and \( v = 1, \) the dislocation is in the homogeneous piezoelectric material, and is repelled by the rigid line, therefore moves towards the interface of the inhomogeneity. When \( u < 1, \) the image force is always positive, and reaches a large value when the dislocation is tending to the interface of the two materials. This is because that the rigid line repels the dislocation while the soft matrix attracts the dislocation. It is shown that the effect of electroelastic properties on the image force
is significant only when the dislocation is very close to the interface. From Fig. 4, we observe that there is a stable equilibrium position where the image force equals zero when $v \neq 1$ and $u = 1$.

This work was supported by the Science Fund of State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body (60870005) and the National Natural Science Foundation of China (10872065).