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Some q-Generating Functions Associated with Basic Multiple Hypergeometric Series

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Abstract—The authors derive basic (or q-) extensions of certain generalized hypergeometric generating functions of H. M. Srivastava. Some remarkable consequences of these q-generating functions are also considered.

1. INTRODUCTION AND DEFINITIONS

Motivated by some interesting generating functions for the classical Jacobi polynomials, Srivastava [1] established the following hypergeometric generating function:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n(\mu)_n}{(\rho)_n} {}_{A+1}F_B \left[\begin{matrix} -n, (a); \\ \\ (b); \end{matrix} \; x \right] \frac{t^n}{n!}$$
$$= (1-t)^{-\lambda} F^{1:1;A+1}_{1:0;B} \left[\begin{matrix} \lambda\colon \rho-\mu; \mu, (a); \\ \\ \rho\colon \quad -; \quad (b); \end{matrix} \; \tfrac{t}{t-1}, \tfrac{xt}{t-1} \right] \quad (|t|<1), \qquad (1.1)$$

where $(\lambda)_n = \Gamma(\lambda+n)/\Gamma(\lambda)$, (a) abbreviates the array of A parameters $a_1, \dots, a_A$, and $F^{A:B;B'}_{C:D;D'}$ is a generalized Kampé de Fériet function (see, e.g., [2]; see also Srivastava and Karlsson [3, p. 27]). For $\rho = \mu$, (1.1) would reduce immediately to Chaundy's result [4, p. 62, equation (25)]:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{n!} {}_{A+1}F_B \left[\begin{matrix} -n, (a); \\ \\ (b); \end{matrix} \; x \right] t^n = (1-t)^{-\lambda} {}_{A+1}F_B \left[\begin{matrix} \lambda, (a); \\ \\ (b); \end{matrix} \; \tfrac{xt}{t-1} \right] \quad (|t|<1), \qquad (1.2)$$

which indeed contains, as special cases, several familiar generating functions for Jacobi and Laguerre polynomials.

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Srivastava [5] gave a further generalization of (1.1) in the following form:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n(\mu)_n}{(\rho)_n} F_{D:\quad B;B'+1}^{C:A+1;A'+2} \left[\begin{matrix}(c)\colon -n,\, (a);\, \lambda+n,\, \mu+n,\, (a'); \\ (d)\colon \quad (b);\, \rho+n, \quad (b');\end{matrix}\ x, y\right] \frac{t^n}{n!}$$
$$= (1-t)^{-\lambda} F^{(3)} \left[\begin{matrix}\lambda\colon\colon\ -;\ \mu,\ (c);\ -\colon\ \rho-\mu;\ (a);\ (a'); \\ \rho\colon\colon\ -;\quad (d);\ -\colon\quad -;\ (b);\ (b');\end{matrix}\ \tfrac{t}{t-1},\ \tfrac{xt}{t-1},\ \tfrac{y}{1-t}\right] \quad (|t|<1), \tag{1.3}$$

where $F^{(3)}[x,y,z]$ denotes a generalized triple hypergeometric series (introduced by Srivastava [6, p. 428]). The main object of this paper is to establish the basic (or $q-$) extensions of Srivastava's results involving hypergeometric generating functions. We also consider several remarkable consequences of these hypergeometric $q-$generating functions.

For real or complex $q(|q|<1)$, put

$$(\lambda;q)_\infty = \prod_{j=0}^{\infty}(1-\lambda q^j), \tag{1.4}$$

and let $(\lambda;q)_\mu$ be defined by

$$(\lambda;q)_\mu = \frac{(\lambda;q)_\infty}{(\lambda q^\mu;q)_\infty} \tag{1.5}$$

for arbitrary parameters λ and μ, so that

$$(\lambda;q)_n = \begin{cases} 1, & \text{if } n=0, \\ (1-\lambda)(1-\lambda q)\dots(1-\lambda q^{n-1}), & \forall n \in \mathbb{N} = \{1,2,3,\dots\}. \end{cases} \tag{1.6}$$

Then a generalized basic (or $q-$) hypergeometric function is defined by (cf., e.g., Slater [7, Chapter 3] and Exton [8]; see also Srivastava and Karlsson [3, p. 347])

$${}_A\Phi_B \left[\begin{matrix}(a); & q;z \\ (b); & i\end{matrix}\right] = \sum_{n=0}^{\infty} q^{in(n-1)/2} \frac{\prod_{j=1}^{A}(a_j;q)_n}{\prod_{j=1}^{B}(b_j;q)_n} \frac{z^n}{(q;q)_n}, \tag{1.7}$$

where, for convergence,

$$|q|<1 \text{ and } |z|<\infty, \quad \text{when } i \in \mathbb{N},$$

or

$$\max\{|q|,|z|\}<1, \quad \text{when } i=0,$$

provided that no zeros appear in the denominator.

For $i,j,k \in \mathbb{N}_0 = \mathbb{N}\cup\{0\}$, a generalized basic (or $q-$) hypergeometric function of two variables is defined by (cf. Srivastava and Karlsson [3, p. 349])

$$\Phi_{C:D;D'}^{A:B;B'} \left[\begin{matrix}(a)\colon (b);\ (b'); & q;x,y \\ (c)\colon (d);\ (d'); & i,j,k\end{matrix}\right] = \sum_{m,n=0}^{\infty} q^{kmn+\{im(m-1)+jn(n-1)\}/2}$$
$$\cdot \frac{\prod_{\tau=1}^{A}(a_\tau;q)_{m+n} \prod_{\tau=1}^{B}(b_\tau;q)_m \prod_{\tau=1}^{B'}(b'_\tau;q)_n}{\prod_{\tau=1}^{C}(c_\tau;q)_{m+n} \prod_{\tau=1}^{D}(d_\tau;q)_m \prod_{\tau=1}^{D'}(d'_\tau;q)_n} \frac{x^m}{(q;q)_m} \frac{y^n}{(q;q)_n}, \tag{1.8}$$

provided that the series converges or terminates.

Finally, a basic (or $q-$) extension of the triple hypergeometric series $F^{(3)}[x,y,z]$ of Srivastava [6, p. 428] is defined by (cf. Denis [9])

$$\Phi^{A::B;B';B'':C;C';C''}_{E::G;G';G'':H;H';H''}\left[\begin{matrix}(a)::(b);(b');(b''):(c);(c');(c''); & q;x,y,z\\ (e)::(g);(g');(g''):(h);(h');(h''); & i,j,k,u,v,w\end{matrix}\right]$$

$$= \sum_{m,n,p=0}^{\infty} q^{umn+vnp+wpm+\{im(m-1)+jn(n-1)+kp(p-1)\}/2}$$

$$\cdot \frac{\prod_{\tau=1}^{A}(a_\tau;q)_{m+n+p}\prod_{\tau=1}^{B}(b_\tau;q)_{m+n}\prod_{\tau=1}^{B'}(b'_\tau;q)_{n+p}\prod_{\tau=1}^{B''}(b''_\tau;q)_{p+m}}{\prod_{\tau=1}^{E}(e_\tau;q)_{m+n+p}\prod_{\tau=1}^{G}(g_\tau;q)_{m+n}\prod_{\tau=1}^{G'}(g'_\tau;q)_{n+p}\prod_{\tau=1}^{G''}(g''_\tau;q)_{p+m}} \tag{1.9}$$

$$\cdot \frac{\prod_{\tau=1}^{C}(c_\tau;q)_m\prod_{\tau=1}^{C'}(c'_\tau;q)_n\prod_{\tau=1}^{C''}(c''_\tau;q)_p}{\prod_{\tau=1}^{H}(h_\tau;q)_m\prod_{\tau=1}^{H'}(h'_\tau;q)_n\prod_{\tau=1}^{H''}(h''_\tau;q)_p}\,\frac{x^m}{(q;q)_m}\,\frac{y^n}{(q;q)_n}\,\frac{z^p}{(q;q)_p},$$

provided that the series converges or terminates.

In the special case, when $i=j=k=0$, the first member of (1.8) will be written simply as

$$\Phi^{A:B;B'}_{C:D;D'}\left[\begin{matrix}(a):(b);(b'); & \\ & q;x,y\\ (c):(d);(d'); & \end{matrix}\right],$$

and a similar notational simplification will also be made for writting the first member of (1.7) when $i=0$.

It should be remarked in passing that, in the definition (1.8), the double series converges absolutely for all bounded values of the complex arguments x and y when $i,j,k\in\mathbb{N}$ and $|q|<1$, and also when $i=j=k=0$, provided further that

$$\max\{|q|,|x|,|y|\}<1.$$

The conditions of convergence of the triple series (1.9) can be stated in an analogous manner.

The following results will be required in our analysis (see, e.g., Slater [7, pp. 247–248]).

I. The q-Binomial Theorem.

$${}_1\Phi_0\left[\begin{matrix}\lambda; & \\ & q;z\\ —; & \end{matrix}\right] = \frac{(\lambda z;q)_\infty}{(z;q)_\infty} \quad (\max\{|q|,|z|\}<1). \tag{1.10}$$

II. The q-Gauss Theorem.

$${}_2\Phi_1\left[\begin{matrix}\alpha,\beta; & \\ & q;\frac{\gamma}{\alpha\beta}\\ \gamma; & \end{matrix}\right] = \frac{(\gamma/\alpha;q)_\infty(\gamma/\beta;q)_\infty}{(\gamma;q)_\infty(\gamma/\alpha\beta;q)_\infty}, \tag{1.11}$$

which, for $\beta=q^{-n}$ ($n\in\mathbb{N}_0$), yields the terminating version:

$${}_2\Phi_1\left[\begin{matrix}\alpha,q^{-n}; & \\ & q;\frac{\gamma}{\alpha}q^n\\ \gamma; & \end{matrix}\right] = \frac{(\gamma/\alpha;q)_n}{(\gamma;q)_n} \quad (n\in\mathbb{N}_0). \tag{1.12}$$

It should be mentioned here that, upon reversal of the order of its terms, this last result (1.12) would lead us to

III. q-Chu-Vandermonde Theorem.

$${}_2\Phi_1\left[\begin{matrix}\alpha,q^{-n}; & \\ & q;q\\ \gamma; & \end{matrix}\right] = \frac{(\gamma/\alpha;q)_n}{(\gamma;q)_n}\alpha^n \quad (n\in\mathbb{N}_0), \tag{1.13}$$

which incidentally is a widely useful result.

2. HYPERGEOMETRIC q-GENERATING FUNCTIONS

Such hypergeometric generating functions as (1.1), (1.2), and (1.3), and their various generalizations and q-extensions, were considered by Srivastava [10]. (See also a recent book on the subject of generating functions by Srivastava and Manocha [11].) In this section, we give a direct proof of the following q-extension of Srivastava's generating function (1.3):

$$\sum_{n=0}^{\infty} \frac{(\lambda;q)_n(\mu;q)_n}{(\rho;q)_n} \Phi^{C:A+1;A'+2}_{D:\ B;B'+1} \left[\begin{matrix} (c)\colon q^{-n},\ (a);\ \lambda q^n,\ \mu q^n,\ (a'); \\ (d)\colon \qquad (b); \qquad \rho q^n,\ (b'); \end{matrix} \ q; xq^n, y \right] \frac{t^n}{(q;q)_n}$$
$$= \frac{(\lambda t;q)_\infty}{(t;q)_\infty} \Phi^{1::0;C+1;0:1;A;A'}_{2::0;\ D;0:0;B;B'} \left[\begin{matrix} \lambda::\ —;\ \mu,\ (c);\ —\colon \rho/\mu;\ (a);\ (a'); & q; -\mu t, -xt, y \\ \rho,\ \lambda t::\ —;\quad (d);\ —\colon\ —;\ (b);\ (b'); & 1,1,0,1,0,1 \end{matrix} \right] \tag{2.1}$$
$$(\max\{|q|,|t|\} < 1).$$

PROOF. Denote, for convenience, the right-hand side of the q-generating function (2.1) by $\Omega(x,y;t)$. Then, by appealing to the definition (1.9), we have

$$\Omega(x,y;t) = \frac{(\lambda t;q)_\infty}{(t;q)_\infty} \sum_{\ell,m,p=0}^{\infty} \frac{(\lambda;q)_{\ell+m+p}(\mu;q)_{m+p}(\rho/\mu;q)_\ell}{(\rho;q)_{\ell+m+p}(\lambda t;q)_{\ell+m+p}}$$
$$\cdot \Delta(m,p) q^{\ell(m+p)+\{\ell(\ell-1)+m(m-1)\}/2} \frac{(-\mu t)^\ell}{(q;q)_\ell} \frac{(-xt)^m}{(q;q)_m} \frac{y^p}{(q;q)_p}$$

or, equivalently,

$$\Omega(x,y;t) = \sum_{\ell,m,p=0}^{\infty} \frac{(\mu;q)_{m+p}(\rho/\mu;q)_\ell}{(\rho;q)_{\ell+m+p}} \Delta(m,p) q^{\ell(m+p)+\{\ell(\ell-1)+m(m-1)\}/2}$$
$$\cdot \frac{(-\mu)^\ell}{(q;q)_\ell} \frac{(-x)^m}{(q;q)_m} \frac{y^p}{(q;q)_p} \sum_{n=0}^{\infty} \frac{(\lambda;q)_{\ell+m+n+p}}{(q;q)_n} t^{\ell+m+n} \tag{2.2}$$
$$(\max\{|q|,|t|\} < 1)$$

where we have also used the q-binomial theorem (1.10), and the definitions (1.5) and (1.7), $\Delta(m,p)$ being given by

$$\Delta(m,p) = \frac{\prod_{\tau=1}^{C} (c_\tau;q)_{m+p} \prod_{\tau=1}^{A} (a_\tau;q)_m \prod_{\tau=1}^{A'} (a'_\tau;q)_p}{\prod_{\tau=1}^{D} (d_\tau;q)_{m+p} \prod_{\tau=1}^{B} (b_\tau;q)_m \prod_{\tau=1}^{B'} (b'_\tau;q)_p}. \tag{2.3}$$

Upon writing $n-\ell-m$ for n in (2.2), and noting the elementary identity:

$$(q;q)_{n-k} = (-1)^k q^{k(k-2n-1)/2} \frac{(q;q)_n}{(q^{-n};q)_k} \qquad (0 \le k \le n), \tag{2.4}$$

we find from (2.2) that

$$\Omega(x,y;t) = \sum_{n,p=0}^{\infty} \sum_{\ell,m=0}^{\ell+m\le n} \frac{(\lambda;q)_{n+p}(\mu;q)_{m+p}(\rho/\mu;q)_\ell(q^{-n};q)_{\ell+m}}{(\rho;q)_{\ell+m+p}}$$
$$\cdot \Delta(m,p) q^{mn+\ell(n+p)} \frac{\mu^\ell}{(q;q)_\ell} \frac{x^m}{(q;q)_m} \frac{t^n}{(q;q)_n} \frac{y^p}{(q;q)_p}, \tag{2.5}$$

which readily yields

$$\Omega(x,y;t) = \sum_{n,p=0}^{\infty}\sum_{m=0}^{\infty} \frac{(\lambda;q)_{n+p}(\mu;q)_{m+p}(q^{-n};q)_m}{(\rho;q)_{m+p}} \Delta(m,p) \cdot {}_2\Phi_1\left[\begin{matrix}\rho/\mu, & q^{-n+m}; & \\ & & q;\mu q^{n+p} \\ & \rho q^{m+p}; & \end{matrix}\right] \frac{(xq^n)^m}{(q;q)_m}\frac{t^n}{(q;q)_n}\frac{y^p}{(q;q)_p}. \tag{2.6}$$

Finally, we sum the hypergeometric ${}_2\Phi_1$ series in (2.6) by means of the known result (1.12) with, of course,

$$\alpha = \frac{\rho}{\mu}, \qquad \gamma = \rho q^{m+p},$$

and n replaced by $n-m$, and we thus obtain the left-hand side of (2.1). The derivation of the q-generating function (2.1) is evidently completed.

3. APPLICATIONS

For $c_j = 0$ $(j = 1,\dots,C)$ and $d_j = 0$ $(j=1,\dots,D)$, (2.1) reduces immediately to the bilinear generating relation:

$$\sum_{n=0}^{\infty} \frac{(\lambda;q)_n(\mu;q)_n}{(\rho;q)_n} {}_{A+1}\Phi_B\left[\begin{matrix} q^{-n},(a); & \\ & q;xq^n \\ (b); & \end{matrix}\right] {}_{A'+2}\Phi_{B'+1}\left[\begin{matrix} \lambda q^n, \mu q^n, (a'); & \\ & q;y \\ \rho q^n, (b'); & \end{matrix}\right] \frac{t^n}{(q;q)_n}$$
$$= \frac{(\lambda t;q)_\infty}{(t;q)_\infty} \Phi^{1::0;1;0:1;A;A'}_{2::0;0;0:0;B;B'}\left[\begin{matrix} \lambda::\text{—};\ \mu;\text{—}:\ \rho/\mu;\ (a);\ (a'); & q;-\mu t,-xt,y \\ \rho,\ \lambda t::\text{—};\text{—};\text{—}:\ \text{—};\ (b);\ (b'); & 1,1,0,1,0,1 \end{matrix}\right]$$
$$(\max\{|q|,|t|\}<1). \tag{3.1}$$

Upon setting $\rho = \mu$ in (3.1), if we simplify the right-hand side by applying the definition (1.9), we obtain the formula:

$$\sum_{n=0}^{\infty} \frac{(\lambda;q)_n}{(q;q)_n} {}_{A+1}\Phi_B\left[\begin{matrix} q^{-n},(a); & \\ & q;xq^n \\ (b); & \end{matrix}\right] {}_{A'+1}\Phi_{B'}\left[\begin{matrix} \lambda q^n,(a'); & \\ & q;y \\ (b'); & \end{matrix}\right] t^n$$
$$= \frac{(\lambda t;q)_\infty}{(t;q)_\infty} \Phi^{1:A;A'}_{1:B;B'}\left[\begin{matrix} \lambda:\ (a);\ (a'); & q;-xt,y \\ \lambda t:\ (b);\ (b'); & 1,0,0 \end{matrix}\right] \quad (\max\{|q|,|t|\}<1), \tag{3.2}$$

which provides a q-extension of a known bilinear generating function (cf., e.g., [11, p. 229, equation 4.1(35)]). More generally, a q-extension of another known bilinear generating function [11, p. 231, equation 4.1(42)] is provided by the following special case of (2.1) when $\rho = \mu$:

$$\sum_{n=0}^{\infty} \frac{(\lambda;q)_n}{(q;q)_n} \Phi^{C:A+1;A'+1}_{D:\ B;\ B'}\left[\begin{matrix} (c):\ q^{-n},(a);\ \lambda q^n,(a'); & \\ & q;xq^n,y \\ (d):\ \ (b);\ \ (b') & \end{matrix}\right] t^n$$
$$= \frac{(\lambda t;q)_\infty}{(t;q)_\infty} \Phi^{C+1:A;A'}_{D+1:B;B'}\left[\begin{matrix} \lambda,\ (c):\ (a);\ (a'); & q;-xt,y \\ \lambda t,\ (d):\ (b);\ (b'); & 1,0,0 \end{matrix}\right] \quad (\max\{|q|,|t|\}<1). \tag{3.3}$$

A further special case of (3.1) when $y=0$ yields

$$\sum_{n=0}^{\infty} \frac{(\lambda;q)_n(\mu;q)_n}{(\rho;q)_n} {}_{A+1}\Phi_B\left[\begin{matrix} q^{-n},(a); & \\ & q;xq^n \\ (b); & \end{matrix}\right] \frac{t^n}{(q;q)_n}$$
$$= \frac{(\lambda t;q)_\infty}{(t;q)_\infty} \Phi^{1:1;A+1}_{2:0;\ B}\left[\begin{matrix} \lambda:\ \rho/\mu;\ \mu,(a); & q;-\mu t,-xt \\ \rho,\lambda t:\ \text{—};\ \ (b); & 1,1,1 \end{matrix}\right] \quad (\max\{|q|,|t|\}<1), \tag{3.4}$$

which is a q-extension of the hypergeometric generating function (1.1).

For $\rho = \mu$, (3.4) assumes the form:

$$\sum_{n=0}^{\infty} \frac{(\lambda;q)_n}{(q;q)_n} {}_{A+1}\Phi_B \begin{bmatrix} q^{-n}, (a); & \\ & q; xq^n \\ (b); & \end{bmatrix} t^n = \frac{(\lambda t;q)_\infty}{(t;q)_\infty} {}_{A+1}\Phi_{B+1} \begin{bmatrix} \lambda, (a); \, q; \, -xt \\ \lambda t, (b); \, 1 \end{bmatrix} \quad (\max\{|q|, |t|\} < 1), \tag{3.5}$$

which follows also from (3.2) and (3.3) when $y = 0$. Formula (3.5) provides a q-extension of Chaundy's result (1.2).

Next, setting

$$a_j = 0 \quad (j = 1, \dots, A) \quad \text{and} \quad b_j = 0 \quad (j = 1, \dots, B)$$

in (3.4), and applying the q-binomial theorem (1.10) with $\lambda = q^{-n}$ and $z = xq^n$, we get

$${}_3\Phi_1 \begin{bmatrix} \lambda, \mu, x; & \\ & q; t \\ \rho; & \end{bmatrix} = \frac{(\lambda t;q)_\infty}{(t;q)_\infty} \Phi^{1:1;1}_{2:0;0} \begin{bmatrix} \lambda\colon \rho/\mu; \ \mu; \ q; -\mu t, -xt \\ \rho, \lambda t\colon \ -; -; \ 1,1,1 \end{bmatrix} \quad (\max\{|q|, |t|\} < 1), \tag{3.6}$$

which, for $x = 0$, yields Jackson's transformation (cf. [12, p. 145, equation (4)] and [3, p. 348, equation 9.4 (279)]):

$${}_2\Phi_1 \begin{bmatrix} \lambda, \mu; & \\ & q; t \\ \rho; & \end{bmatrix} = \frac{(\lambda t;q)_\infty}{(t;q)_\infty} {}_2\Phi_2 \begin{bmatrix} \lambda, \rho/\mu; & q; -\mu t \\ \rho, \ \lambda t; & 1 \end{bmatrix} \quad (\max\{|q|, |t|\} < 1). \tag{3.7}$$

Many of the hypergeometic q-generating functions, considered in this paper, can be applied also to the various families of q-orthogonal polynomials including, for example, the little q-Jacobi polynomials defined by (cf. [13, p. 29] and [14, p. 48, equation (3.34)])

$$p_n^{(\alpha,\beta)}(x;q) = \frac{(\alpha q;q)_n}{(q;q)_n} {}_2\Phi_1 \begin{bmatrix} q^{-n}, \alpha\beta q^{n+1}; & \\ & q; qx \\ \alpha q; & \end{bmatrix} \tag{3.8}$$

and the q-Laguerre polynomials defined by (cf. [13, p. 29], [8, p. 188], and [15, p. 21, equation (2.3)])

$$L_n^{(\alpha)}(x;q) = \frac{(\alpha q;q)_n}{(q;q)_n} {}_1\Phi_1 \begin{bmatrix} q^{-n}; \, q; \, xq^n \\ \alpha q; \, 1 \end{bmatrix} = \lim_{\beta \to 0} \left\{ p_n^{(\alpha,\beta)} \left(-\frac{x}{\alpha\beta q^2}; q \right) \right\}. \tag{3.9}$$

The details may be omitted.

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