Research on Supply Chain Finance Pricing Problem under Random Demand and Permissible Delay in Payment

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Abstract

The research is carried on supply chain finance pricing under conditions of random demand and supplier’s permissible delay in payments, and the model considers both backorders and backlogging as well as discount episode. The objective of the bank is to maximize its own profit, and also it is the retailer under bank’s financing interest rate.

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1. Introduction

There often occurs cash constraint in a supply chain when upstream and downstream enterprises do business of purchases and sales. Facing these questions, traditionally, the supplier provides financing service, that is; the supplier permits a conditional delay in payment as a credit for retailer, and no interest is charged during this period. It can relieve retailer’s shortness of money to a certain extent, but a fact that supplier is not expertise in financing, it often results in high risk, not reasonable profit, not abundant cash demand and so forth other problems. Bank acts as a leader in the business of supply chain finance therefore, it can fill this gap. When banks and 3PLs cooperate tightly with each other, they can greatly reduce financing risk due to their synergistic advantages, and small and medium enterprises (SMEs) can get their dormant assets active through this business.

Comparing to traditional supplier finance, supply chain finance is much more complex for the sake of the attendance of the bank and 3PL. Studies of supplier finance is quite mature, but the theoretical research on supply chain finance is rare in both home and abroad.


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with partial backorders. Ouyang, Teng and Chen (2006)\textsuperscript{[7]} studied the partial backlogging model for deteriorating items. Dada, Hu (2008)\textsuperscript{[8]} extended the classical newsvendor model to capital constraint retailer, and the retailer and the lender are in a Stackelberg game. Geetha and Uthayakumar (2010)\textsuperscript{[9]} studied the inventory policy for non-instantaneous deteriorating items, in the model shortages were allowed and partially backlogged.

On the research of supply chain finance, Zhu, Liu and Xu (2007)\textsuperscript{[10]} firstly put up with an impawn financing pricing model under conditions of permissible delay in payments. They studied retailer’s decision-making of its optimal order period, as well as 3PL company’s optimal financing price. Jiang, Yang and Ye (2008)\textsuperscript{[11]} studied the pricing model for inventory impawn financing under conditions of dynamic permissible delay in payments and dynamic discount rate. Yuan and Wang (2010)\textsuperscript{[12]} researched 3PL financing pricing model for random demand to determine retailer’s optimal order quantity and reorder point as well as the 3PL company’s financing interest rate. In the same year they\textsuperscript{[13]} set up a 3PL financing pricing model for the exponentially deteriorating products. Chen and Cai (2011)\textsuperscript{[14]} studied the value of 3PL firms as credit providers in budget constraint supply chains, and the 3PL firms provide both transportation and trade credit to the retailer. They found that all players can be better off under 3PL financing than under bank financing, and 3PL financing conditionally outperforms supplier financing. He, Jiang and Wang, et al. (2012)\textsuperscript{[15]} proposed the way of setting the dynamic impawn rate by dividing the impawn periods into different risk windows. At each risk window, the return behaves quite differently. Therefore, the key to setting the impawn rate is to predict the long-term risk.

On the basis of predecessors’ research, the author will continue to explore supply chain finance pricing model under conditions of supplier’s permissible delay in payments and stochastic demand. Differs to predecessors’, the objective of this article is to maximize bank and supplier’s profit, while predecessors like Zhu, Yuan, et, al. are to maximize bank’s profit and minimize retailer’s cost. Backorders and backlogging are allowed in this paper, and supplier provides a discount when retailer repays within a certain credit period. This paper is organized as follows; After define the research boundaries in section 2, the symbols used in this paper will be defined in section 3. Section 4 to section 6 is the body of this work. Section 4 discusses retailer’s optimal order cycle under each potential reorder policy. Section 5 is about retailer’s choice of optimal order cycle. Bank’s pricing strategy is given in section 6. At last, we analyzed two examples of different cases.

2. Assumption

(1) Supplier permits retailer repaying before the end of a credit time and provides retailer with a discount episode within a certain time;
(2) The products are of single kind and same qualification;
(3) Retailer has no fund before the order, and he can financing through supply chain finance, and the sales income deposits immediately into a bank account which could earn interest after the transaction is completed every day, the retailer refunds with sales income and bank interest at the end of each cycle;
(4) Bank cooperates tightly with 3PL, and bank provides financing service, 3PL provides freight and warehousing services, because the charge of 3PL can be converted into supplier’s price, we don’t consider it in the model;
(5) Customer’s annual demand is stochastic, and follows normal distribution, the mean value is $\mu$, and the standard deviation is $\sigma$;
(6) Backorders and backlogging are allowed, to facilitate easy calculation, assume that they are of the same loss;
(7) Take multiple episodes into account, research the equilibrium state when the order process is balanced;
(8) No lead time, and don’t considerate safety stock;
(9) Bank and retailer are in a Stackelberg game, bank is leader, deciding its financing interest rate, retailer is a follower deciding his optimal order cycle and order quantity, all of their objectives are to maximize their own profit.

3. Symbols definition

$\mu$ – The mean value of customers’ annual demand of the product;
$\sigma$ – The standard deviation of customers’ annual demand of the product;
$S$ – Unit ordering cost;
$T$ – Order cycle;
$M$ – Supplier’s credit time;
$n$ – The discount episode provided by supplier, we assume $T \geq n$;
$r$ – Discount rate in the discount episode;
$Q$ – Retailer’s order quantity;
$h$ – Annual holding cost for unit product excluding capital opportunity cost;
$I_b$ – Bank’s interest, we assume it’s simple interest;
$I_p$ – Bank’s financing interest rate, assuming $I_p \geq I_b$;
$p_1$ – The price of the supplier;
$p_2$ – The price of the retailer, we assume $p_2 > p_1$;
$a$ – Unit cost for backorder or backlogging;
$f(x)$ – The function of probability density of standard normal distribution;
$F(x)$ – The function of probability distribution of standard normal distribution;
$r_{ipi}$ – Retailer’s annual profit for policy $i$, $i = 1, \cdots, 3$;
$b_{ipi}$ – Bank’s annual profit for retailer’s policy $i$, $i = 1, 3$.

4. Analysis of retailer’s optimal order cycle

As is shown in the following graph, retailer has three refund policies which can make his profit reach a extreme maximum. The first one is that retailer repay in time $n$; second, retailer repay in time $M$, and $M < T$; last, retailer repay in time $M$, and $M \geq T$.

Retailer’s objective is to maximize his own profit, and retailer’s annual profit equals to annual income subtracts annual cost.

Let’s analysis retailer’s annual cost first.
Retailer’s cost incorporates ordering cost($oc$), stock cost($sc$), backorder cost($boc$), backlogging cost($blc$), purchase cost($pc$), financing cost($fc$).
Retailer’s ordering cost($oc$) depends only on ordering cycle, so

$$oc = \frac{S}{T}. \tag{1}$$

Retailer’s stock cost($sc$) can be expressed as

$$sc = \frac{h \int_0^T I(t)dt}{T} = \frac{h \mu T}{2}. \tag{2}$$

Retailer’s backorders cost($boc$) is

$$boc = \frac{a}{T} \int_Q^{\infty} (x - Q)g(x)dx.$$

$g(x)$ is normal distributed in the equation, and follows $N \sim (uT, \sigma \sqrt{T})$, it’s the demand distribution function of a single order cycle.

Backorders and backlogging are of the same loss, so retailer’s order quantity is only for demand, and we have
\( Q - \mu T = 0 \), let \( x - \mu T = v \), so the value range for \( v \) is \((0, \infty)\).

The original equation equals to

\[
\text{boc} = \frac{a\sigma}{2\pi T} \int_{0}^{\infty} e^{-\frac{v^2}{2\sigma^2 T}} d(-\frac{v^2}{2\sigma^2 T}).
\]

For the sake of poisson integral \( \int_{0}^{\infty} e^{-v^2} dv = \frac{\sqrt{\pi}}{2} \), we get

\[
\text{boc} = \frac{a\sigma}{\sqrt{2\pi T}}. \tag{3}
\]

Retailer’s backlogging cost(\( \text{blc} \)) is

\[
\text{blc} = \frac{a}{T} \int_{0}^{Q} (Q - x)g(x)dx.
\] \tag{4}

The above four costs are independent of repay policies, but purchase cost(\( \text{pc} \)), and financing cost(\( \text{fc} \)) are not.

1. When retailer selects policy one, that is, repay in the end of discount episode, and \( T \geq n \).

Retailer’s purchase cost (\( \text{pc} \)) is

\[
\text{pc} = \mu(1 - r)p_1. \tag{5}
\]

Retailer’s financing cost(\( \text{fc} \)) is

\[
\text{fc} = \frac{[Q(1 - r)p_1 - \mu p_2(1 + \frac{n}{T}b)]I_b}{T}. \tag{6}
\]

When financing cost(\( \text{pc} \)) is less than zero, that is, \( T < \frac{np_2(1 + \frac{n}{T}b)}{(1 - r)p_1} \), then retailer don’t need financing, so the computation should be transferred to no need for financing case.

Retailer’s annual total income \( r_{ti} \) falls into two parts, the product income(\( p_{in} \)), and the interest income(\( r_{in} \)).

\[
p_{in} = \frac{p_2}{T} \int_{0}^{Q} xg(x)dx = \frac{p_2}{T} \int_{-\infty}^{Q} xg(x)dx = (\frac{\sqrt{\pi} \mu}{2} - \frac{\sigma}{\sqrt{2\pi T}})p_2. \tag{7}
\]

\[
r_{in} = \frac{1}{T} [\frac{1}{2} \mu p_2 I_b n + \frac{1}{2} \mu (T - n)p_2 I_b (T - n)] = \frac{1}{2} \mu p_2 I_b (\frac{2n^2}{T} + T - 2n). \tag{8}
\]

Retailer’s annual total income is

\[
r_{ti} = (\frac{\sqrt{\pi} \mu}{2} - \frac{\sigma}{\sqrt{2\pi T}})p_2 + \frac{1}{2} \mu p_2 I_b (\frac{2n^2}{T} + T - 2n). \tag{9}
\]

Now we can get the expression of annual profit(\( r_{tp1} \)) for policy one;

\[
r_{tp1} = \left( \frac{\sqrt{\pi} \mu}{2} - \frac{\sigma}{\sqrt{2\pi T}} \right) p_2 + \frac{1}{2} \mu p_2 I_b (\frac{2n^2}{T} + T - 2n) - \frac{\mu(p_2 I_b)}{2} - \frac{2\sigma p_2}{\sqrt{2\pi T}} - \mu(1 - r)p_1 \]
\[
T > \frac{np_2(1 + \frac{n}{T}b)}{(1 - r)p_1}. \tag{10}
\]

To solve the extreme value with constraint, we solve that with no constraint first, and then check whether the extreme point satisfy the constraint, if not, we select a best value according to the graphic of retailer’s profit.
function, and then we substitute the values we got to the objective function, at last we compare the results and take a larger one.

Take the first and second derivative of (10);
\[
\frac{dr_p}{dT} = \frac{1}{T^2} \left[ (p_2 + 2a)\sigma \sqrt{T} + \frac{1}{2} \mu (p_2 I_b - h) T^2 + (S - \mu p_2 n (1 + \frac{n}{2} I_b I_p - I_b n)) \right].
\]
\[
\frac{d^2 r_p}{dT^2} = -\frac{3\sigma (p_2 + 2a)}{4\sqrt{2\pi} T^3} + \frac{2\mu p_2 n ((1 + \frac{n}{2} I_b I_p - I_b n) - S)}{T^2}. \quad \text{Let } \frac{d^2 r_p}{dT^2} = 0, \text{ we get}
\]
\[
T_0 = \left( \frac{8 \sqrt{2\pi} \mu p_2 n ((1 + \frac{n}{2} I_b I_p - I_b n) - S)}{3\sigma (p_2 + 2a)} \right)^2.
\]
So when the root of the first order derivative is not \(T_0\), then this root is a extremum.

Let \(\frac{dr_p}{dT} = 0\), we get
\[
\frac{\sigma (p_2 + 2a)}{2 \sqrt{2\pi} T^3} = \frac{1}{2} \mu (p_2 I_b - h) T^2 + (S - \mu p_2 n (1 + \frac{n}{2} I_b I_p - I_b n)) = 0.
\]
If \(p_2 I_b - h = 0\)
\[
T^* = \left( \frac{\sqrt{2\pi} \mu p_2 n (2 + n I_b I_p - 2 I_b n) - 2 S}{(p_2 + 2a)\sigma} \right)^2 = \frac{9}{16} T_0.
\]
For \(T^* \neq T_0\), \(T^*\) is a extreme point.

If \(p_2 I_b - h \neq 0\), the optimal order cycle can be solved as follow (the solving process refers to the appendix).
If \(p_2 I_b - h < 0\)
\[
T^* = \begin{cases} 
\frac{j}{2\sqrt{3}} + \sqrt{\frac{1}{3} \sqrt{3} j - \frac{1}{3} y^2}, & k > 0 \\
\frac{j}{2\sqrt{3}} \pm \sqrt{\frac{1}{3} \sqrt{3} j - \frac{1}{3} y^2}, & -\frac{3y^2}{16} < k \leq 0 \\
y, & k = \frac{3y^2}{16} \\
n p_2 (1 + \frac{n}{2} I_b I_p) \pm \frac{y}{(1 - \frac{n}{2} I_p)}, & k < \frac{3y^2}{16}.
\end{cases}
\]

When \(-\frac{3y^2}{16} < k \leq 0\), one of the two roots is a extreme minimum, you can compute the second derivative with the corresponding \(T^*\), if the result is less than zero, then it's the right value.
If \(p_2 I_b - h > 0\)
\[
T^* = \begin{cases} 
\frac{-j}{2\sqrt{3}} - \sqrt{-\frac{1}{3} \sqrt{3} j - \frac{1}{3} y^2}, & k > 0 \\
\infty, & k \leq 0.
\end{cases}
\]

When retailer expects his cycle time as long as possible, he can assume the infinity to be an acceptable arbitrary large number \(N\).
When \(k \geq -\frac{3y^2}{16}\) and \(T^* \neq T_0\), \(T^*\) is an extreme value, and if \(r_{tp}'(T^*) > 0\), \(T^*\) is an extreme maximum point, if \(r_{tp}''(T^*) < 0\), \(T^*\) is an extreme minimum point. If \(T^* = T_0\), then substitute all the values that make the first order derivation zero and the constraint boundaries to the objective function and compare the results and select a bigger one.

Several variables are involved in the process of solving \(T^*\), now we express them as follow;
When \(j^4 + 4(\frac{4}{3} k)^3 \geq 0\)
\[
\begin{align*}
\begin{cases} 
y = s + t \\
s = (\frac{\hat{r} + \sqrt{\hat{r}^2 + 4(\frac{4}{3} k)^3}}{2})^\frac{1}{3} \\
t = (\frac{\hat{r} - \sqrt{\hat{r}^2 + 4(\frac{4}{3} k)^3}}{2})^\frac{1}{3}.
\end{cases}
\end{align*}
\]
When \( j^3 + 4\left(\frac{4}{3}k\right)^3 < 0 \), select anyone of
\[
y_1 = 2\sqrt{r\cos \frac{\theta}{3}}, \quad y_2 = 2\sqrt{r\cos \frac{\theta + 2\pi}{3}}, \quad y_3 = 2\sqrt{r\cos \frac{\theta + 4\pi}{3}}
\]
that is greater than zero, among which
\[
\begin{align*}
r &= \sqrt{u^2 + v^2}, \quad \theta = \arctan \frac{v}{u} \\
u &= \frac{j}{2}, \quad v = \frac{\sqrt{-j^3 + 4\left(\frac{4}{3}k\right)^3}}{2} \\
j &= \frac{\sigma(p_1 + 2a)}{\sqrt{2\pi(h - p_2)\mu}} \\
k &= \frac{2S - \mu p_2 n (2 + n) h - 2I_p}{\mu(h - p_2)p_1}.
\end{align*}
\]
No need for financing case \( f(c) \leq 0 \);
\[
r_{tp1}' = \left(\sqrt{\frac{n\mu}{2}} - \frac{\sigma}{\sqrt{2\pi}}\right)p_2 + \frac{1}{2}\mu p_2 I_b \left(\frac{m^2}{2} + T - 2n\right) - \frac{hyT}{2} - \frac{2avn}{\sqrt{2\pi}} - \mu(1 - r)p_1 - \frac{s}{T}
\]
\[
n \leq T \leq \frac{np_1(1 + \frac{4}{3}h_b)}{(1 - r)p_1}.
\]  
\( \frac{dr_{tp1}'}{dT} \) is an alteration of replacing \( S - \mu p_2 n (1 + \frac{n}{2}I_p - I_p n) \) with \( S - \mu p_2 n h_n \) in equation (11) here, so we can derive the optimal order cycle directly now. Different from \( fc > 0 \) case, here;
\[
T_0 = \left(\frac{2\sqrt{2\pi(h - p_2)p_1} \cdot 3}{(p_1 + 2a)\sigma}\right)^2.
\]
When \( p_2 I_b - h = 0 \), \( T^* = \left(\frac{2\sqrt{2\pi(h - p_2)p_1} \cdot 3}{(p_1 + 2a)\sigma}\right)^2 \);
When \( p_2 I_b - h \neq 0 \), let \( k = \frac{2(S - \mu p_2 n h_n)}{\mu(h - p_2) p_1} \).
If the resolution of \( T^* \) is greater than \( \frac{np_1(1 + \frac{4}{3}h_b)}{(1 - r)p_1} \), then assign \( T^* = \frac{np_1(1 + \frac{4}{3}h_b)}{(1 - r)p_1} \);
If the resolution is smaller than \( n \), then assign \( T^* = n \).

2. When retailer selects policy two, which means retailer refunds in the end of credit time \( M \), and \( T \leq M \), now retailer doesn’t have discount.
Retailer’s purchase cost is
\[
pc = \mu p_1.
\]  
No financing is needed, so financing cost is zero now.
Retailer’s interest income \( (r_{in}) \) is up to
\[
r_{in} = \frac{\frac{1}{2}\mu I p_2 I_b T}{T} = \frac{1}{2}\mu T p_2 I_b.
\]  
(14)
Now retailer’s annual income \( r_{tp2} \) becomes
\[
r_{tp2} = \left(\sqrt{\frac{n\mu}{2}} - \frac{\sigma}{\sqrt{2\pi}}\right)p_2 + \frac{1}{2}\mu T p_2 I_b - \frac{hyT}{2} - \frac{2avn}{\sqrt{2\pi}} - \mu p_1 - \frac{s}{T}
\]
\[
n \leq T \leq M.
\]  
(15)
In order to solve this problem with \( K - T \) condition, change the programming as follow;
\[
\begin{align*}
min r_{tp2} &= -r_{tp2} \\
g_1(T) &= M - T \geq 0 \\
g_2(T) &= T - n \geq 0.
\end{align*}
\]
Take the first and second derivative of $\tilde{r}_{tp2}$:

$$\frac{d\tilde{r}_{tp2}}{dT} = -\frac{(p_2 + 2a)\sigma}{2\sqrt{2\pi T^3}} - \frac{S}{T^2} - \frac{1}{2} \mu(p_2 I_b - h).$$

$$\frac{d^2\tilde{r}_{tp2}}{dT^2} = \frac{3\sigma(p_2 + 2a)}{4\sqrt{2\pi T^5}} + \frac{2S}{T^3} > 0.$$  

So $\tilde{r}_{tp}$ is a convex function of $T$, and $g_1(x)$ and $g_2(x)$ are concave functions, now we know that all the $K - T$ points are extreme values.

Introduce lagrangian multipliers $\tilde{\lambda}_1, \tilde{\lambda}_2$, we obtain

$$\left\{ \begin{align*}
-\frac{(p_2 + 2a)\sigma}{2\sqrt{2\pi T^3}} - \frac{S}{T^2} - \frac{1}{2} \mu(p_2 I_b - h) + \tilde{r} &= 0 \\
\tilde{\lambda}_1(M - T) &= 0 \\
\tilde{\lambda}_1(T - n) &= 0 \\
\tilde{\lambda}_1 &\geq 0, \tilde{\lambda}_2 &\geq 0.
\end{align*} \right.$$  

If $\tilde{\lambda}_1 > 0, \tilde{\lambda}_2 > 0$, there is no value for $T$.

If $\tilde{\lambda}_1 = 0, \tilde{\lambda}_2 > 0$, then $T^* = M$.

If $\tilde{\lambda}_1 > 0, \tilde{\lambda}_2 = 0$, then $T^* = n$.

If $\tilde{\lambda}_1 = 0, \tilde{\lambda}_2 = 0$, then

$$\frac{(p_2 + 2a)\sigma}{2\sqrt{2\pi}} + \frac{1}{2} \mu(p_2 I_b - h)T^2 + S = 0.$$

If $p_2 I_b - h = 0$, then $\frac{d\tilde{r}_{tp}}{dT} = -\frac{(p_2 + 2a)\sigma}{2\sqrt{2\pi T^3}} + \frac{S}{T^2} < 0$, so $r_{tp2}$ is an increasing function, now we receive $T^* = M$.

If $p_2 I_b - h \neq 0$, let $\sqrt{T} = x$, so we get

$$x^4 - \frac{(p_2 + 2a)\sigma}{\sqrt{2\pi \mu(h - p_2 I_b)}} x - \frac{2S}{\mu(h - p_2 I_b)} = 0.$$  

Here we keep $k = \frac{2S}{\mu(h - p_2 I_b)}$.

When $k \geq -\frac{3\sqrt{2\pi}}{16}$, the expression of $T^*$ is similar to policy one. If the result is $T^* > M$, then assign $T^* = M$; if the result is $T^* < n$, then assign $T^* = n$.

When $p_2 I_b - h > 0$, $k < -\frac{3\sqrt{2\pi}}{16}$, the original function is decreasing, now let $T^* = n$; When $p_2 I_b - h < 0$, $k > 0$, the original function is increasing, now let $T^* = M$.

3 When retailer select policy three, which implies the retailer will repay in the end of credit time, and $T > M$, and retailer can’t get discount now.

Retailer’s purchase cost ($pc$) is

$$pc = \mu p_1.$$  

(16)

Retailer’s financing cost ($fc$) arrives at

$$fc = \frac{[Qp_1 - \mu M p_2 (1 + \frac{M}{2} I_b)]p_1}{T}.$$  

(17)

When $fc < 0$, that is, $T < \frac{Mp_2 (1 + \frac{M}{2} I_b)}{p_1}$, retailer doesn’t need financing, so the computation should be transferred to no need for financing case.

Retailer’s interest income ($r_{in}$) becomes

$$r_{in} = \frac{1}{T} \left[ \frac{1}{2} \mu M p_2 I_b M + \frac{1}{2} \mu (T - M) p_2 I_b (T - M) \right]$$

$$= \frac{1}{2} \mu p_2 I_b (\frac{2M^2}{T} + T - 2M).$$  

(18)
Retailer’s annual profit ($r_{ip3}$) can be expressed as

$$r_{ip3} = \left( \frac{\sqrt{\mu}}{2} - \frac{\sigma}{\sqrt{2\pi\tau}} \right)p_2 + \frac{1}{2} \mu p_2 I_b((2M)^2 + T - 2M) - \frac{h p_T}{2} - \frac{2 \sigma \sqrt{\mu}}{\sqrt{2\pi\tau}}$$

$$T > \frac{M p_1 + \frac{\mu}{2} I_b}{p_1}.$$  

(19)

$$\frac{d r_{ip3}}{d T}$$ is an alteration of replacing $n$ in $\frac{d r_{ip1}}{d T}$ with $M$, so we can directly write down the optimal order cycle ($T^*$).

When $p_2 I_b - h = 0$

$$T^* = \left( \frac{\sqrt{2\pi} \mu p_2 M((2 + M I_b)I_p - 2I_b M) - 2S}{(p_2 + 2a)\sigma} \right)^2.$$  

(20)

What is different from policy one is that if $p_2 I_b - h < 0, k < -\frac{3\sqrt{2}}{16}$, then the original function is decreasing, now we assign $T^* = \frac{M p_1 + \frac{\mu}{2} I_b}{p_1}$.

And now we keep $k = \frac{2S - \mu p_2 M((2 + M I_b)I_p - 2I_b M)}{\mu (p_2 I_b)}$, $T_0 = (\frac{8 \sqrt{2\pi} \mu p_2 M((1 + \frac{\mu}{2} I_b)I_p - I_b M) - S)}{3 \sigma (p_2 + 2a)})^2$.

No need for financing case;  

$$r'_{ip3} = \left( \frac{\sqrt{\mu}}{2} - \frac{\sigma}{\sqrt{2\pi\tau}} \right)p_2 + \frac{1}{2} \mu p_2 I_b((2M)^2 + T - 2M) - \frac{h p_T}{2} - \frac{2 \sigma \sqrt{\mu}}{\sqrt{2\pi\tau}} - \frac{\mu}{\sqrt{2\pi\tau}} - \frac{S}{\sqrt{2\pi\tau}}$$

$$M < T \leq \frac{M p_1 + \frac{\mu}{2} I_b}{p_1}.$$  

(21)

Because $\frac{d r'_{ip3}}{d T}$ is replacing $n$ in $\frac{d r'_{ip1}}{d T}$ with $M$, we can acquire the optimal order cycle immediately according to no need for financing case in policy one.

5. Retailer’s choice of optimal order cycle

For a set of given parameters, substitute them into the optimal order cycle formulas under three policies, and we can get the optimal order cycle ($T^*_i, i = 1, \cdots, 3$) for each policy, and then substitute $T^*_i$ into retailer’s profit function $r_{ip}(T^*_i), i = 1, \cdots, 3$, from $\max\{r_{ip}(T^*_i), i = 1, \cdots, 3\}$ the retailer could get the optimal order policy.

6. Supply chain finance pricing strategy for bank

For a given set of parameters (among which $I_p$ is indeterminate), the objective of bank is to set a proper interest rate and maximize its profit, and bank should also take retailer’s reaction to the price into consideration as well. So bank should make prices according to retailer’s potential policies.

Firstly, the price of the bank, in other words, the financing interest rate $I_p$ should be within collection of $B_1 = \{ I_p \geq I_b \}$; and then, the financing interest rate should make the business profitable for retailer, so $I_p$ should be in the set $B_2 = \{ I_p | \text{max}\{r_{ip}\} > 0, i = 1, \cdots, 3 \}$; thirdly, bank should prevent retailer from choosing policy two, because retailer will not finance at this moment, so $I_p$ should satisfy $B_3 = \{ I_p | \text{max}\{r_{ip1}, r_{ip3}\} > \text{max}\{r_{ip2}\} \}$. Lastly, if retailer selects policy one, and $p_2 I_b - h > 0$, $I_p$ should be in $B_4 = \{ I_p > \frac{2S - \mu (p_2 I_b) + 2\mu p_2 I_b M^2}{\mu p_2 M(2 + n I_b)} \}$; if retailer selects policy three, and $p_2 I_b - h > 0$, $I_p$ should satisfy $B_5 = \{ I_p > \frac{2S - \mu (h - p_2 I_b) + 2\mu p_2 I_b M^2}{\mu p_2 M(2 + n I_b)} \}$, that’s because if $I_p$ is not in these collections, retailer’s optimal order cycle will become infinity, which means he won’t repay any more.

And now, retailer could only choose policy one and policy three. If $r_{ip1}(T^*_1) > r_{ip3}(T^*_3)$, then retailer chooses policy one, now $I_p$ must be in $B_6 = \{ I_p | r_{ip1}(T^*_1) > r_{ip3}(T^*_3) \}$; if $r_{ip1}(T^*_1) < r_{ip3}(T^*_3)$, then retailer chooses policy three, now let the collection of $I_p$ to be $B_7 = \{ I_p | r_{ip1}(T^*_1) < r_{ip3}(T^*_3) \}$. If $I_p \in B_6$, denote bank’s profit to be $b_{ip1}$, if $I_p \in B_7$, denote it to be $b_{ip3}$. Consequently bank’s largest profit can be expressed as $\max\{b_{ip} \} = \max\{\text{max}\{b_{ip1}\}, \text{max}\{b_{ip3}\}\}$, among which
It can be seen from the above expressions, $b_{tp1}$ and $b_{tp3}$ are functions of $I_p$, so are $T_1^*$ and $T_3^*$. It’s hard for us to express the extremum of $b_{tp}$ directly. We can apply trial value method to approach the optimal value gradually. It’s simple to use, and the approximation ability is strong enough to draw near to an arbitrary precision. When apply this method, we make the variation of $I_p$ wider first to find the variation feature between $b_{tp}$ and $I_p$, and then fix the variation in a selected range, when this range reach a satisfactory precision, then stop this process.

Find the optimal $I_p1$ and $I_p3$ that maximize $b_{tp1}$ and $b_{tp3}$ respectively, at last compare $b_{tp1}^*$ and $b_{tp3}^*$, and then take a larger one of which the corresponding $I_p$ is bank’s optimal price.

7. Example analysis

Assume $\mu = 6400$, $\sigma = 200$, $S = 420$, $p_1 = 5$, $p_2 = 7$, $r = 0.08$, $a = 3$, $h = 1$, $n = 0.1$, $I_b = 0.03$, $M = 0.3$. Retailer should find the optimal order cycle $T^*$, and bank should determine the optimal financing interest rate $I_p$.

$2I_b - h = -0.79 < 0$

If retailer chooses policy two, the annual profit is independent of $I_p$, when substitute the parameters into the formulas, we get $r_{tp2}^* = 4378.25$.

If retailer chooses policy one, and the computing will be influenced by $I_p$, now we solve it with trial value method, and the results are shown as follow:

Table 1. Policy one.

<table>
<thead>
<tr>
<th>$I_p$</th>
<th>$T_1^*$</th>
<th>$r_{tp1}$</th>
<th>$b_{tp1}$</th>
<th>$I_p$</th>
<th>$T_1^*$</th>
<th>$r_{tp1}$</th>
<th>$b_{tp1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0300</td>
<td>0.5153</td>
<td>6538.31</td>
<td>622.01264</td>
<td>0.1022</td>
<td>0.328</td>
<td>5377.06</td>
<td>1610.5880</td>
</tr>
<tr>
<td>0.0500</td>
<td>0.4731</td>
<td>6170.93</td>
<td>997.82948</td>
<td>0.1030</td>
<td>0.325</td>
<td>5368.69</td>
<td>1610.1891</td>
</tr>
<tr>
<td>0.0750</td>
<td>0.4277</td>
<td>5749.72</td>
<td>1392.6331</td>
<td>0.1100</td>
<td>0.2966</td>
<td>5306.85</td>
<td>1574.1751</td>
</tr>
<tr>
<td>0.0800</td>
<td>0.3988</td>
<td>5672.47</td>
<td>1455.2342</td>
<td>0.1200</td>
<td>0.245</td>
<td>5278.21</td>
<td>1335.1810</td>
</tr>
<tr>
<td>0.0900</td>
<td>0.3693</td>
<td>5527.42</td>
<td>1556.2265</td>
<td>0.1250</td>
<td>0.2075</td>
<td>5329.23</td>
<td>977.12173</td>
</tr>
<tr>
<td>0.0950</td>
<td>0.3532</td>
<td>5461.36</td>
<td>1590.1490</td>
<td>0.1280</td>
<td>0.1696</td>
<td>5448.29</td>
<td>382.71843</td>
</tr>
<tr>
<td>0.1000</td>
<td>0.336</td>
<td>5401.21</td>
<td>1608.6365</td>
<td>0.1285</td>
<td>0.1576</td>
<td>5502.01</td>
<td>123.93648</td>
</tr>
<tr>
<td>0.1015</td>
<td>0.3305</td>
<td>5384.57</td>
<td>1610.4356</td>
<td>0.1290</td>
<td>0.1524</td>
<td>4418.64</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.1020</td>
<td>0.3287</td>
<td>5379.19</td>
<td>1610.5911</td>
<td>0.1300</td>
<td>0.1524</td>
<td>4418.64</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>0.1021</strong></td>
<td><strong>0.3283</strong></td>
<td><strong>5378.13</strong></td>
<td><strong>1610.5943</strong></td>
<td><strong>0.135</strong></td>
<td><strong>0.1524</strong></td>
<td><strong>4418.64</strong></td>
<td><strong>0.0000</strong></td>
</tr>
</tbody>
</table>

If retailer selects policy three, we get the results here:

Table 2. Policy three

<table>
<thead>
<tr>
<th>$I_p$</th>
<th>$T_1^*$</th>
<th>$r_{tp4}$</th>
<th>$b_{tp4}$</th>
<th>$I_p$</th>
<th>$T_1^*$</th>
<th>$r_{tp4}$</th>
<th>$b_{tp4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>0.0300</strong></td>
<td><strong>0.4346</strong></td>
<td><strong>4548.89</strong></td>
<td><strong>28.01523</strong></td>
<td>0.1000</td>
<td>0.4219</td>
<td>4555.62</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.0316</td>
<td>0.4227</td>
<td>4574.34</td>
<td>1.8856</td>
<td>0.1100</td>
<td>0.4219</td>
<td>4555.62</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.0317</td>
<td>0.4219</td>
<td>4555.62</td>
<td>0.0000</td>
<td>0.1300</td>
<td>0.4219</td>
<td>4555.62</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.0500</td>
<td>0.4219</td>
<td>4555.62</td>
<td>0.0000</td>
<td>0.1500</td>
<td>0.4219</td>
<td>4555.62</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.0750</td>
<td>0.4219</td>
<td>4555.62</td>
<td>0.0000</td>
<td>0.2000</td>
<td>0.4219</td>
<td>4555.62</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Obviously, the optimal financing interest rate of bank falls to $I_p^* = 0.1021$, the largest profit is $b_{tp}^* = 1610.5943$; retailer’s optimal order cycle comes to $T^* = 0.3283$, annual profit $r_{tp}^* = 5378.13$, it can be seen that $I_p \in \{B_1 \cap B_2 \cap B_3 \cap B_6\}$.

8. Conclusions

Based on the researches of predecessors, the author continues to study how retailer determines his order cycle and refund strategies to maximize his annual profit. Because of the consideration of backorders, backlogging, discount episode, and retailer’s income, the model is much more practical, at the same time the model becomes more complex, even though, when compared to reality, it’s very simplified.

The research can be extended into multiple aspects. For example, (1) in order to simplify the model, the author assumes the losses of backorders and backlogging are the same, for the sake of reality, you can expand it into the different case; (2) in this paper, the author assume demand is stochastic and is normally distributed, and it can be extended to a more practical distribution; (3) the author assume that there is no lead time and no safety stock, you can assume that lead time is fixed, and safety stock is considered; (4) research the case that demand is a elastic function of price, etc.

Acknowledgements

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References

Appendix

If \( p_2 I_b - h \neq 0 \), let \( \sqrt{T} = x \), and because \( T > 0 \), we get

\[
x^4 - \frac{\sigma(p_2 + 2a)}{\sqrt{2\pi(h - p_2 I_b)}} x - \frac{2S - \mu p_2 n (2 + n I_b) I_p - 2 I_b n}{\mu(h - p_2 I_b)} = 0.
\]

Keep \( \frac{\sigma(p_2 + 2a)}{\sqrt{2\pi(h - p_2 I_b)}} = j, \frac{2S - \mu p_2 n (2 + n I_b) I_p - 2 I_b n}{\mu(h - p_2 I_b)} = k \), the equation becomes

\[
x^4 = jx + k. \quad (1)
\]

Add \( (x^2 y + \frac{1}{4} y^2) \) to both side of the equation, of which \( y > 0 \), and now we can access the following equations;

\[
x^4 + x^2 y + \frac{1}{4} y^2 = x^2 y + \frac{1}{4} y^2 + jx + k.
\]

\[
(x^2 + \frac{1}{2} y)^2 = yx^2 + jx + \frac{1}{4} y^2 + k.
\]

In order to form a perfect square of the right side, let \( \Delta = 0 \), so the following equation appears;

\[
y^3 + 4ky - j^2 = 0. \quad (2)
\]

Let \( y = s + t \), and make cube of both side, we get

\[
y^3 = s^3 + t^3 + 3st(s + t) = s^3 + t^3 + 3sty.
\]

Transpose all the terms to the left, we have

\[
y^3 - 3sty - (s^3 + t^3) = 0. \quad (3)
\]

Compare(2) and (3), we get

\[
\begin{cases} 
  s^3 t^3 = -(\frac{4}{3}k)^3 \\
  s^3 + t^3 = j^2.
\end{cases}
\]

It can be derived that \( s^3 \) and \( t^3 \) are roots of equation:

\[
w^2 - j^2w - (\frac{4}{3}k)^3 = 0. \quad (4)
\]

Now let’s discuss the root existence of equation (4);

1. If \( j^4 - 4(\frac{4}{3}k)^3 \geq 0 \), then

\[
\begin{cases} 
  y = s + t \\
  s = (\frac{j^2 + \sqrt{j^4 + 4(\frac{4}{3}k)^3}}{2})^{\frac{1}{3}} \\
  t = (\frac{j^2 - \sqrt{j^4 + 4(\frac{4}{3}k)^3}}{2})^{\frac{1}{3}}.
\end{cases}
\]

2. If \( j^4 - 4(\frac{4}{3}k)^3 < 0 \), choose anyone of

\[
y_1 = 2 \sqrt{T} \cos \frac{\theta}{3}, \quad y_2 = 2 \sqrt{T} \cos \frac{\theta + 2\pi}{3}, \quad y_3 = 2 \sqrt{T} \cos \frac{\theta + 4\pi}{3}.
\]
that is larger than zero, among which

\[
\begin{align*}
  r &= \sqrt{u^2 + v^2}, \quad \theta = \arctan \frac{v}{u} \\
  u &= \frac{\beta}{2}, \quad v = \frac{-\beta + \sqrt{\beta^2 + 4\lambda_1}}{2}.
\end{align*}
\]

If \( p_2 I_b - h < 0 \), we have

\[
2 \sqrt{\left(\frac{1}{4} y^2 + k\right)y} = j.
\]

The perfect square comes to

\[
(x^2 + \frac{1}{2} y)^2 = (\sqrt{y} x + \sqrt{\frac{1}{4} y^2 + k})^2.
\]

From \( x = \sqrt{T} > \sqrt{\mu} \), we know both sides of the equation are positive, so we obtain

\[
x^2 + \frac{1}{2} y = \sqrt{y} x + \sqrt{\frac{1}{4} y^2 + k}. \quad (A)
\]

It’s a quadratic equation of \( x \), the root discriminant is

\[
\Delta = 4 \sqrt{\frac{1}{4} y^2 + k - y}.
\]

Because the two terms of the right of the equation are positive, we can square them and make subtraction, and the result is

\[
\Delta' = 3y^2 + 16k.
\]

So, if \( k > -\frac{3y^2}{16} \), there are two roots; if \( k = -\frac{3y^2}{16} \), only one root; if \( k < -\frac{3y^2}{16} \), there’s no root.

If \( k < -\frac{3y^2}{16} \), the original function has no extremum, and because \( p_2 I_b - h < 0 \), the original function is decreasing, now we get \( x = \sqrt{\frac{np_1(1 + \frac{2}{z} I_b)}{(1 - r)p_1}} \).

Apply the root solution formula, we get

\[
x = \frac{1}{2} (\sqrt{y} \pm \sqrt{4 \sqrt{\frac{1}{4} y^2 + k - y}}, \quad 4 \sqrt{\frac{1}{4} y^2 + k - y} \geq 0.
\]

Square and make subtraction of the two terms in the bracket of the right of the equation, we get

\[
2y - 4 \sqrt{\frac{1}{4} y^2 + k}.
\]

Repeat once again, we get

\[
(2y)^2 - (4 \sqrt{\frac{1}{4} y^2 + k})^2 = -16k.
\]

Because \( x > 0 \), we obtain that there is one root when \( k > 0 \), and two roots when \( k \leq 0 \).

From above all, the roots of equation (1) can be expressed as follow;

\[
x^* = \begin{cases} 
\frac{1}{2} (\sqrt{y} + \sqrt{4 \sqrt{\frac{1}{4} y^2 + k - y}}, & k > 0 \\
\frac{1}{2} (\sqrt{y} \pm \sqrt{4 \sqrt{\frac{1}{4} y^2 + k - y}}, & -\frac{3y^2}{16} < k \leq 0 \\
\frac{\sqrt{y}}{2}, & k = -\frac{3y^2}{16} \\
\sqrt{\frac{np_1(1 + \frac{2}{z} I_b)}{(1 - r)p_1}}, & k < -\frac{3y^2}{16}.
\end{cases}
\]
If \( p_2I_b - h > 0 \), we have
\[
2 \sqrt{\left(\frac{1}{4}y^2 + k\right)}y = -j.
\]
And the perfect square becomes
\[
(x^2 + \frac{1}{2}y)^2 = (\sqrt{y}x - \sqrt{\frac{1}{4}y^2 + k})^2.
\]
So, if \( \sqrt{y}x - \sqrt{\frac{1}{4}y^2 + k} > 0 \), we get equation (B), if \( \sqrt{y}x - \sqrt{\frac{1}{4}y^2 + k} < 0 \), we get equation (C);
\[
\begin{align*}
 x^2 + \frac{1}{2}y &= \sqrt{y}x - \sqrt{\frac{1}{4}y^2 + k}. \quad (B) \\
 x^2 + \frac{1}{2}y &= \sqrt{\frac{1}{4}y^2 + k} - \sqrt{y}x. \quad (C)
\end{align*}
\]
The solving process is similar to that of the above, so get it omitted here.
Now we get the solutions of the original function.
If \( p_2I_b - h = 0 \)
\[
T^* = \left(\frac{\sqrt{2\pi}((2 + nl_b)I_p - 2I_p n) - 2\Sigma}{(p_2 + 2a)\sigma}\right)^2.
\]
If \( p_2I_b - h < 0 \), we can derive \( \sqrt{\frac{1}{4}y^2 + k} = \frac{j}{2\sqrt{y}} \) from \( 2 \sqrt{\left(\frac{1}{4}y^2 + k\right)}y = j \), so we get
\[
T^* = \begin{cases} 
\frac{j}{2\sqrt{y}} + \sqrt{\frac{1}{2} \sqrt{y}j - \frac{1}{4}y^2}, & k > 0 \\
\frac{j}{2\sqrt{y}} \pm \sqrt{\frac{1}{2} \sqrt{y}j - \frac{1}{4}y^2}, & -\frac{3y^2}{16} < k \leq 0 \\
\frac{y}{4}, & k = -\frac{3y^2}{16} \\
\frac{np_1(1 + 2nl_b)}{(1 - r)(p_1)}, & k < -\frac{3y^2}{16}.
\end{cases}
\]
If \( p_2I_b - h > 0 \), we can derive \( \sqrt{\frac{1}{4}y^2 + k} = -\frac{j}{2\sqrt{y}} \) from \( 2 \sqrt{\left(\frac{1}{4}y^2 + k\right)}y = -j \), so we get
\[
T^* = \begin{cases} 
-\frac{j}{2\sqrt{y}} - \sqrt{-\frac{1}{2} \sqrt{y}j - \frac{1}{4}y^2}, & k > 0 \\
\infty, & k \leq 0.
\end{cases}
\]