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Everyone knows that the four days of Galileo's Discorsi mark the highest achievement of his mathematical physics. However, the existence of uncompleted "additional days," the fifth and the sixth, is less well known. One of these, the "fifth day," dictated to Torricelli during the last years of Galileo's life (1641–1642), consists of two arguments about the theory of proportion: an alternative to the definition of proportion in Book V of the Elements, and an uncompleted short explanation of compounded ratio. This fragment, however, has attracted little attention even among Galileo scholars because Galileo's argument is too rash and apparently erroneous. Thus, Galileo's "fifth day" has been abandoned as the stuttering of an old man, if not condemned as a scandal perpetrated by the great scientist. In the present book, Giusti successfully tries to rescue the aged Galileo from the judgment of incompetence imposed on him by posterity.

This study begins with a survey of the status of the Euclidean theory of proportion in the 16th century (Chap. 1). Humanist–mathematicians restored the text and the sense of the Elements, thus clearing up the typical medieval misunderstanding about the definition of proportionality in Campanus' edition. Though Oronce Finé still defended Campanus' interpretation in his commentary to the Elements (1536), Tartaglia clearly criticized it in his Italian translation of the Elements (1543), followed by Clavius, Commandino, and Guidobaldo.

The disappearance of the medieval interpretation did not, however, mean that all questions concerning the interpretation of Book V of the Elements had been resolved. Giusti points out two problems which constitute the key for understanding the later 'reform' of the theory of proportion by the Galilean school.

First, the definition of proportionality remained at the center of the scene partly because of the existence of a definition which Heiberg in his modern edition omitted as spurious. The placement and the wording of this definition are not the same in Greek codices and, consequently, in printed editions of the time. In Clavius' version (1574), it was placed after Definition 3, and goes "Proportio vero est rationum similitudo," while Commandino (1572) places it after Definition 7. Though each version involves difficulties of interpretation, Clavius' edition inspired Galileo's 'reform.' In that edition the existence of the spurious definition (Def. 4, in Clavius) makes the genuine definition (Def. 5, or Def. 6 in Clavius) look like a repetition or paraphrase of the former. This apparent duplication of definitions, together with the operative complexity of the latter (Def. 5), gave Galileo an incentive to propose an alternative.

The other difficulty with the theory of proportion was the notion of compounded ratio. Its definition in VI-Def. 5, which we now know to be spurious, is not consistent with its use in Proposition VI-23.
Guidobaldo dal Monte's two articles, "In Quintum Euclidis Elementorum librum Commentarius" and "De proportione composita," published here for the first time, represent an attitude typical of the humanist–mathematicians of the 16th century in respecting and excessively adhering to the text. Guidobaldo's attitude is well illustrated by adequate examples including his interpretation of compounded ratio. As a faithful successor of Commandino, he tried to justify every phrase, even every word of Euclid, and thus he was surely not the precursor of Galileo's program of the mathematization of nature.

The first tentative reform of the theory of proportion is found in Benedetti's Diversarum Speculationum Mathematicarum et Physicarum Liber (1585). Of Benedetti's audacious arguments, Giusti emphasizes his systematic semialgebraic use of compounded ratio, though Galileo did not follow this way of algebraization.

Giusti then turns to Galileo and discusses the effects of his famous program of the mathematization of nature (Chap. 2). Giusti establishes the following: Galileo's application of mathematics to physics (or natural philosophy) not only meant a revolution in the latter but also inevitably influenced the former. In developing his mathematical physics, Galileo had no other mathematics at his disposal except the Euclidean theory of proportion. This forced choice largely determined the style and the limitations of his physics. At the same time, the application of the theory of proportion to the description of the physical world could not but modify this ancient mathematics, revealing its weakness and inadequacy as a tool for physics. For example, compounded ratio, a marginal notion in antiquity, had to play a central role (this explains Galileo's interest in compounded ratio in the second argument of the "fifth day"); the existence of the fourth proportional also had to be required as an axiom because it is indispensable in handling physical magnitudes such as time or velocity. Besides technical necessities, the mathematics used as a basis of physics had to fulfill the requirements of simplicity and intuitive clarity, which Book V of the Elements lacks altogether. In short, Galileo's application of the theory of proportion to physics necessitated a reform of the former. Reasonably enough, Giusti does not mention Drake [1], because the relationship between the theory of proportion of Galilean physics has turned out to be not so simple and happy as Drake expected. Throughout this chapter, accurate studies in [3] make Giusti's clear arguments even more convincing.

Having seen the necessity of a reform of the theory of proportion on Galileo's part, we now come to Galileo's alternative definition in the "fifth day" (Chap. 3). According to Giusti, the negative view of this work is due to its first publication (1574) by Viviani, Galileo's last disciple. It was accompanied by Viviani's elaboration of the same topic, which, unfortunately, served to propagate the misunderstandings of this unworthy disciple as the thoughts of the master.

To correct these misunderstandings, Giusti carefully examines Galileo's arguments and their tacit assumptions. He also embarks on textual studies, reexamining Torricelli's manuscript of the "fifth day" (Ms. Gal. 75), which, elaborated by Torricelli after dictation from Galileo, contains numerous cancellations and additions. Later Viviani, too, added at least marginal notes to it. Giusti judges that
the revisions on this manuscript, though partly due to Torricelli himself and not to Galileo's dictation, are nonetheless useful in revealing Galileo's intentions because his disciple must have attempted to clarify his master's thoughts. Giusti's new edition of this manuscript, which reproduces the process of changes in as far as possible, is included in this volume. Through this reexamination, Giusti arrives at the conclusion that Galileo's argument is not so erroneous as commonly believed, and that though the text of the "fifth day" is far from perfect, its arguments are remediable because they are rooted in a sound idea. For the details, and Giusti's interpretation of Galileo's alleged errors, I would invite the reader to refer directly to Giusti's arguments.

Torriceilli and Borelli, in fact, developed Galileo's ideas and tried to overcome their logical weakness (Chaps. 4 and 5, respectively). Torricelli's De Proportionibus Liber (1647), written in the year of his death though not published until [4], enjoyed circulation among his colleagues. Borelli, in his Euclides Restitutus (1658), expressed a position different from that of Galileo and Torricelli. Giusti, who examined the manuscripts of Torricelli's work, restores the trace of alterations from the first draft and establishes a critical edition which enables us to witness Torricelli's process of revision. As is expected, the phrases which suffered more cancellations or alterations involve logical difficulties and weaknesses in the theory.

Torriceilli, following Galileo, rejected the operative Definition V-5, and started from an intuitive definition of proportionality as similarity of ratios. He was well aware that this necessitated further axioms, and, in fact, introduced several concerning the operation of ratios. Borelli, on the other hand, insisted on the necessity of some operative definition. In the demonstrations, both develop Galileo's idea of proceeding from commensurable magnitudes to incommensurable ones in a more rigorous manner.

In connection with Torricelli's theory, Giusti very interestingly points out that it is so similar to Archimedes' arguments in Proposition 7 of On the Equilibrium of Planes that the latter may possibly have been the source of Galileo's reform. This possibility is all the more intriguing because, as Giusti also indicates, [2] shows that a theory of proportion can be reconstructed from this proposition of Archimedes. Works of minor epigones of the Galilean school such as Viviani, Marchetti, and Noferi, however, turn out to be of less interest (Chap. 6).

In the appendix, Giusti proposes a modern reconstruction of Galileo's theory of proportion. This is a small but neat work by an author who is no less famous as a reputed mathematician (in the calculus of variations) than as an historian of mathematics.

In the edition of the manuscripts, which occupies nearly half of this volume (pp. 175–340), cancellations and additions are included in the text between different styles of parentheses so that the first draft as well as the final version can be traced without consulting the critical apparatus.

As a whole, this work supplies precious source texts and sets them in their proper context with accurate and reasonable arguments. We thus have a much clearer vision of Galileo's mathematics and its limitation as a tool for his physics.
A serious scholar has therefore no less reason to learn Italian today than at the time of Galileo.

REFERENCES


