Lu Advances in Difference Equations (2016) 2016:282
DOI 10.1186/s13662-016-0979-z
Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Periodicity and stability of an impulsive nonlinear competition model with infinitely distributed delays and feedback controls

Hongying Lu*

*Correspondence:
hongyinglu543@163.com
School of Mathematics, Dongbei University of Finance and Economics, Dalian, Liaoning 116025, P.R. China

Abstract

This paper is concerned with a periodic nonlinear competition model governed by impulsive differential equation with infinitely distributed delays and feedback controls. By means of coincidence degree theory and Lyapunov functional, a set of sufficient criteria are obtained to guarantee the existence and globally asymptotic stability of a unique positive periodic solution of the model. Furthermore, applying our main results to some important competition models which have been well studied in the literature, we establish some new criteria to supplement and generalize some well-known results.

Keywords: positive periodic solution; globally asymptotic stability; impulse; feedback control; delay; nonlinear competition model; coincidence degree; Lyapunov functional

1 Introduction

Lotka [1] and Volterra [2] proposed the following famous two-species model:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\bigl[r_1 - a_1x_1(t) - b_1x_2(t)\bigr], \\ \dot{x}_2(t) &= x_2(t)\bigl[r_2 - a_2x_1(t) - b_2x_2(t)\bigr]. \end{aligned} \tag{1.1}$$

It is a classical Lotka-Volterra competition model when $b_1 > 0$, $a_2 > 0$. Here, $x_1(t)$, $x_2(t)$ denote the population density of two competing species. r_1, r_2 represent the intrinsic growth rate of the two competing species; a_1, b_2 are the rate of intra-specific competition, b_1, a_2 are the rate of inter-specific competition, respectively. The well-known model (1.1) and a lot of its generalized forms have been investigated widely (see [3–27] and the references cited therein).

In 1996, Chattopadhyay [5] introduced the effect of toxic substances into the competition model,

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\bigl[r_1 - a_1x_1(t) - b_1x_2(t) - c_1x_1(t)x_2(t)\bigr], \\ \dot{x}_2(t) &= x_2(t)\bigl[r_2 - a_2x_1(t) - b_2x_2(t) - c_2x_1(t)x_2(t)\bigr], \end{aligned} \tag{1.2}$$

where $c_1x_1^2(t)x_2(t)$ and $c_2x_1(t)x_2^2(t)$ describe the effect of toxic. Tineo [28] and He [6] studied the above autonomous or non-autonomous model and established some good results.

Furthermore, according to the experiments results, Ayala *et al.* [7] established the following competition model:

$$\dot{x}_i(t) = r_i x_i(t)\left[1 - \left(\frac{x_i(t)}{K_i}\right)^{\theta_i} - \sum_{j=1, j\neq i}^{n} a_{ij}\frac{x_j(t)}{K_j}\right], \quad i = 1, 2, \ldots, n, \tag{1.3}$$

where $x_i(t)$ are the population density of competing species X_i at time t, r_i represent the intrinsic exponential growth rate of competing species X_i, K_i denote the environment carrying capacity of competing species X_i in the absence of competition, θ_i provide a nonlinear measure of intra-specific interference, and a_{ij} $(i \neq j)$ measure the strength of inter-specific competition. For more excellent work on the system (1.3), see [8–14].

In some real life situations, one wishes to change the position of the existing periodic solution (or almost periodic solution) but to keep its stability. So, it is important to control the ecological balance of the system. One of the approaches for the realization of it is to introduce some feedback control variables so as to get a population stability at another periodic solution (or another almost periodic solution). For example, the implementation of the feedback control mechanism can be introduced by some biological control scheme or by the harvesting procedure. Recently, the feedback control method of the ecological system has been widely applied to control the ecological balance in theory and in practice; see [3, 4, 12, 13, 20, 21, 23, 29]. In [11], Chen proposed a periodic n-species Lotka-Volterra competition system with infinite delays and feedback controls,

$$\begin{aligned} \dot{x}_i(t) = x_i(t)\biggl[& r_i(t) - a_{ii}(t)x_i(t) - \sum_{j=1}^{n} a_{ij}(t)\int_0^{\infty} K_{ij}(\theta)x_j(t-\theta)\,d\theta \\ & - b_i(t)\int_0^{\infty} H_i(\theta)u_i(t-\theta)\,d\theta\biggr], \\ \dot{u}_i(t) = {} & -c_i(t)u_i(t) + d_i(t)\int_0^{\infty} R_i(\theta)x_i(t-\theta)\,d\theta, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{1.4}$$

here $u_i(t)$ denote the control variables. They obtained sufficient conditions for the global asymptotic stability of the system (1.4).

As we know, impulsive differential equations are more appropriate for characterizing ecological evolutionary process (for example, seasonal births of some wild animals). Many excellent results can be found in [14, 26, 30–33] and the references therein. In [14], Wang *et al.* studied the following generalized n-species Gilpin-Ayala impulsive competition system:

$$\begin{aligned} \dot{x}_i(t) = x_i(t)\biggl[& r_i(t) - \sum_{j=1}^{N} a_{ij}(t)x_j^{\alpha_{ij}}(t) - \sum_{j=1}^{N} b_{ij}(t)x_j^{\alpha_{ij}}\bigl(t - \tau_{ij}(t)\bigr) \\ & - \sum_{j=1}^{N} c_{ij}(t)x_i^{\alpha_{ii}}(t)x_j^{\alpha_{ij}}(t)\biggr], \quad t \neq t_k, \\ \Delta x_i(t_k) = {} & x_i\bigl(t_k^+\bigr) - x_i\bigl(t_k^-\bigr) = p_k^i x_i\bigl(t_k^-\bigr), \quad i = 1, 2, \ldots, N, k \in N. \end{aligned} \tag{1.5}$$

In the real world, time delay is common, because the process of a reproduction of the species is not instantaneous or the entire history of the species affects the present birth rate. So, time delay is introduced into the population models, which is a more realistic method to understand the population dynamics. For the effect of these kinds of delays on the asymptotic behavior of populations, we can refer to [10, 15–19, 23–27, 34–41].

Motivated by the above excellent work, in this paper, we investigated the following impulsive nonlinear competition model with infinitely distributed delays and feedback controls:

$$\begin{cases} \dot{x}_i(t) = x_i(t)[r_i(t) - \sum_{j=1}^n a_{ij}(t)x_j^{\alpha_{ij}}(t) - \sum_{j=1}^n b_{ij}(t)x_j^{\beta_{ij}}(t-\tau_{ij}(t)) \\ \qquad - \sum_{j=1}^n \int_{-\infty}^t c_{ij}(t,\theta)x_j^{\gamma_{ij}}(\theta)\,d\theta - \sum_{j=1}^n d_{ij}(t)x_i^{\alpha_{ii}}(t)x_j^{\alpha_{ij}}(t) \\ \qquad - \int_{-\infty}^t f_i(t,\theta)u_i(\theta)\,d\theta], \quad t \neq t_k, \\ \dot{u}_i(t) = -\alpha_i(t)u_i(t) + \int_{-\infty}^t g_i(t,\theta)x_i^{\alpha_{ii}}(\theta)\,d\theta, \quad t \geq 0, \\ \Delta x_i = x_i(t_k^+) - x_i(t_k^-) = p_{ik}x_i(t_k^-), \quad i = 1,2,\dots,n, k \in N, \end{cases} \tag{1.6}$$

where $x_i(t)$ are the density of the competing species X_i, $u_i(t)$ denote the control variables. The terms $b_{ij}(t)x_j^{\beta_{ij}}(t-\tau_{ij}(t))$ and $\int_{-\infty}^t c_{ij}(t,\theta)x_j^{\gamma_{ij}}(\theta)\,d\theta$ describe the negative feedback crowding and the effect of all the past life history of the species on its present birth rate, respectively. $p_{ik}x_i(t_k)$ represent the population $x_i(t)$ at t_k annual birth pulse. $x_i(t_k^+)$ and $x_i(t_k^-)$ are the right and the left limit of x_i at t_k, respectively. The model (1.6) incorporates many important competition models which have been extensively studied in the literature [12, 16, 20–27].

In this paper, for the system (1.6) we always assume that:

(H$_1$) $r_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $d_{ij}(t)$, $\alpha_i(t)$ are all nonnegative and continuous ω-periodic functions for all $t \in R^+$; α_{ij}, β_{ij}, γ_{ij} are all positive constants;

(H$_2$) $c_{ij}(t+\omega, s+\omega) = c_{ij}(t,s), f_i(t+\omega, s+\omega) = f_i(t,s), g_i(t+\omega, s+\omega) = g_i(t,s)$, $\int_{-\infty}^t c_{ij}(t,s)\,ds$, $\int_{-\infty}^t f_i(t,s)\,ds$, $\int_{-\infty}^t g_i(t,s)\,ds$ are continuous with respect to t; $c_{ij}(t+s,t), f_i(t+s,t), g_i(t+s,t)$ are integrable with respect to s on $[0,+\infty)$; and $\int_0^{+\infty}\int_{-s}^0 c_{ij}(u+s,u)\,du\,ds < +\infty$, $\int_0^{+\infty}\int_{-s}^0 f_i(u+s,u)\,du\,ds < +\infty$, $\int_0^{+\infty}\int_{-s}^0 g_i(u+s,u)\,du\,ds < +\infty$;

(H$_3$) $\tau_{ij}(t)$ is continuously differentiable for $t \geq 0$ such that $\tau_{ij}(t+\omega) = \tau_{ij}(t) \geq 0$, and $1 - \dot{\tau}_{ij}(t) > 0$ on $0 \leq t < +\infty$;

(H$_4$) t_k satisfies $t_k < t_{k+1}$ and $\lim_{k\to\infty} t_k = \infty$. $p_{ik} > -1$, and there exists a positive integer q such that $t_{k+q} = t_k + \omega$, $p_{i(k+q)} = p_{ik} \geq 0$.

Without loss of generality, we always assume that $t_k \neq 0$ and $[0,\omega] \cap t_k = \{t_l, t_2, \dots, t_m\}$, then $q = m$.

For the sake of convenience, we shall use some notations:

$$f^L = \min_{t\in[0,\omega]} f(t), \qquad f^M = \max_{t\in[0,\omega]} f(t), \qquad \bar{f} = \frac{1}{\omega}\int_0^\omega f(t)\,dt,$$

where $f(t)$ is a continuous ω-periodic function, and

$$\bar{c}_{ij} = \frac{1}{\omega}\int_0^\omega \int_{-\infty}^t c_{ij}(t,s)\,ds\,dt,$$

$$\bar{\Delta}_i = \frac{1}{\omega}\sum_{k=1}^m \ln(1+p_{ik}),$$

$$c_{ij}^M = \max_{t\in[0,\omega]} \int_0^{+\infty} c_{ij}(t+s,t)\,ds,$$

$$g_i^M = \max_{t\in[0,\omega]} \int_0^{+\infty} g_i(t+s,t)\,ds,$$

$$M_i = \left(\frac{\bar{r}_i + \bar{\Delta}_i}{\bar{a}_{ii} + \bar{b}_{ii} + \bar{c}_{ii}}\right)^{1/\alpha_{ii}} \exp\bigl\{[\bar{R}_i + \bar{r}_i + \bar{\Delta}_i]\omega + |\bar{\Delta}_i|\omega\bigr\},$$

$$A_i = \left(\frac{\bar{r}_i + \bar{\Delta}_i}{\bar{a}_{ii}}\right)^{1/\alpha_{ii}} \exp\bigl\{[\bar{R}_i + \bar{r}_i + \bar{\Delta}_i]\omega + |\bar{\Delta}_i|\omega\bigr\},$$

$$B_i = \left(\frac{\bar{r}_i + \bar{\Delta}_i}{\bar{b}_{ii}}\right)^{1/\beta_{ii}} \exp\bigl\{[\bar{R}_i + \bar{r}_i + \bar{\Delta}_i]\omega + |\bar{\Delta}_i|\omega\bigr\},$$

$$C_i = \left(\frac{\bar{r}_i + \bar{\Delta}_i}{\bar{c}_{ii}}\right)^{1/\gamma_{ii}} \exp\bigl\{[\bar{R}_i + \bar{r}_i + \bar{\Delta}_i]\omega + |\bar{\Delta}_i|\omega\bigr\},$$

$(\cdot)_{n\times m}$ is an $n \times m$ matrix,

$\sigma_{ij}^{-1}(t)$ is the inverse function of $t - \tau_{ij}(t)$.

The system (1.6) describes the multi species population dynamics. The existence and global asymptotic stability of positive periodic solutions of the ecological system are basic and important questions in the theory of mathematical ecology. Therefore, the main purpose of this paper is to obtain a set of sufficient conditions which guarantee the existence and globally asymptotic stability of a unique positive periodic solution of the system (1.6). To do this, the approach in this paper is based on coincidence degree theory and constructing a proper Lyapunov functional. Our results generalize and supplement those given by Chen [11], Yang and Xu [16], Xu *et al.* [18, 25], Gopalsamy [19], Weng [21], Fan *et al.* [22, 23], Zhao [24], Stamova [26], Li *et al.* [27].

The paper is organized as follows: In Section 2, with the help of Gaines and Mawhin's continuation theorem, some sufficient conditions are established, which guarantee the existence of positive periodic solutions of the system (1.6). In Section 3, by constructing a proper Lyapunov functional, some sufficient conditions are derived for the existence of a unique globally stable periodic solution of the system (1.6). In Section 4, some examples are given to show the feasibility and the effectiveness of the obtained results.

2 Existence of positive periodic solutions

With respect to some basic concepts of coincidence degree theory, one can refer to Gaines and Mawhin [42], and so, here we shall not restate these concepts, only we give some lemmas Gaines and Mawhin [42], which would be necessary for this section.

Lemma 2.1 ([42]) *Set L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$. Suppose*:

(i) *for each* $\lambda \in (0,1)$, $x \in \partial\Omega \cap \operatorname{Dom} L$, $Lx \neq \lambda N(x,\lambda)$;

(ii) $QN(x) \neq 0$ *for each* $x \in \partial\Omega \cap \operatorname{Ker} L$;

(iii) $\deg\{JQN(x), \Omega \cap \operatorname{Ker} L, 0\} \neq 0$.

Then the equation $Lx = Nx$ has at least one solution in $\operatorname{Dom} L \cap \bar{\Omega}$.

Theorem 2.1 *In addition to* (H_1)-(H_4), *assume further that*:

(H_5) *the system of algebraic equations*

$$\begin{cases} \bar{r}_i + \bar{\triangle}_i - \sum_{j=1}^{n}(\bar{a}_{ij}u_j^{\alpha_{ij}} + \bar{b}_{ij}u_j^{\beta_{ij}} + \bar{c}_{ij}u_j^{\gamma_{ij}} + \bar{d}_{ij}u_i^{\alpha_{ii}}u_j^{\alpha_{ij}}) - \bar{f}_i v_j = 0, \\ \bar{\alpha}_i v_i - \bar{g}_i u_i^{\alpha_{ii}} = 0, \end{cases}$$

has finite solutions $u^* = (u_1^*, \ldots, u_n^*, v_1^*, \ldots, v_n^*)^T \in R_+^{2n}$ *with* $u_i^* > 0$, $v_i^* > 0$ *and* $\sum_{u^*} \operatorname{sgn} J_g(u^*) \neq 0$;

(H_6) $\alpha_i^L > 0$, $\alpha_{ii} = \beta_{ii} = \gamma_{ii}$;

(H_7) $\bar{r}_i + \bar{\triangle}_i > \sum_{j=1,j\neq i}^{n}(\bar{a}_{ij}M_j^{\alpha_{ij}} + \bar{b}_{ij}M_j^{\beta_{ij}} + \bar{c}_{ij}M_j^{\gamma_{ij}}) + \sum_{j=1}^{n}\bar{d}_{ij}M_i^{\alpha_{ii}}M_j^{\alpha_{ij}}$.

Then the system (1.6) *has at least one positive* ω*-periodic solution, say* $x^* = (x_1^*, \ldots, x_n^*, u_1^*, \ldots, u_n^*)^T$, *and there exist positive constants* χ_i, μ_i *such that* $\chi_i \leq x_i^*(t)$, $u_i^*(t) \leq \mu_i$, $i = 1, 2, \ldots, n$.

Proof Let

$$x_i(t) = \exp\{y_i(t)\}, \quad i = 1, 2, \ldots, n.$$

On substituting the above equality into (1.6), we have

$$\begin{cases} \dot{y}_i(t) = r_i(t) - \sum_{j=1}^{n} a_{ij}(t)\exp\{\alpha_{ij}y_j(t)\} - \sum_{j=1}^{n} b_{ij}(t)\exp\{\beta_{ij}y_j(t-\tau_{ij}(t))\} \\ \qquad - \sum_{j=1}^{n}\int_{-\infty}^{t} c_{ij}(t,\theta)\exp\{\gamma_{ij}y_j(\theta)\}\,d\theta - \sum_{j=1}^{n} d_{ij}(t)\exp\{\alpha_{ii}y_i(t) + \alpha_{ij}y_j(t)\} \\ \qquad - \int_{-\infty}^{t} f_i(t,\theta)u_i(\theta)\,d\theta, \quad t \neq t_k, \\ \dot{u}_i(t) = -\alpha_i(t)u_i(t) + \int_{-\infty}^{t} g_i(t,\theta)\exp\{\alpha_{ii}y_i(\theta)\}\,d\theta, \quad t \geq 0, \\ \Delta y_i = y_i(t_k^+) - y_i(t_k^-) = \ln(1+p_{ik}), \quad i = 1, 2, \ldots, n, k \in N. \end{cases} \tag{2.1}$$

Set

$$y(t) = \big(y_1(t), y_2(t), \ldots, y_n(t)\big)^T, \qquad u(t) = \big(u_1(t), u_2(t), \ldots, u_n(t)\big)^T,$$

$$X = \big\{U(t) = \big(y(t)^T, u(t)^T\big)^T \in PC\big[R, R^{2n}\big] | U(t+\omega) = U(t)\big\},$$

$$\|U\|_X = \sup_{t\in[0,\omega]}\big\{\big\|y(t)\big\|\big\} + \sup_{t\in[0,\omega]}\big\{\big\|u(t)\big\|\big\},$$

here $\|\cdot\|$ is any norm in R^n, and

$$Z = X \times R^{2nq}, \qquad \|z\|_Z = \|U\|_X + \|v\|, \qquad z = (U, v) \in Z,$$

where $\|\cdot\|$ is any given norm of R^{2nq}, $U \in X$, $v \in R^{2nq}$. Then X and Z are both Banach spaces. Define

$$\operatorname{Dom} L = \big\{U(t) = \big(x(t)^T, u(t)^T\big)^T \in X \cap PC^1\big[R, R^{2n}\big]\big\},$$

$$L : \operatorname{Dom} L \to Z, \qquad U \to \big(\dot{U}, \Delta U(t_1), \ldots, \Delta U(t_q)\big),$$

$$N : X \to Z, \qquad NU = \big(\Phi(t), C_1, \ldots, C_q\big),$$

where

$$\Delta U(t_k) = \begin{pmatrix} x(t_k^+) - x(t_k) \\ 0 \end{pmatrix},$$

$$\Phi(t) = \big(\phi_1(t), \ldots, \phi_n(t), \varphi_1(t), \ldots, \varphi_n(t)\big)^T,$$
$$C_i = \big(\ln(1 + p_{1i}), \ldots, \ln(1 + p_{ni}), 0, \ldots, 0\big)^T \in R^{2n}, \quad i = 1, 2, \ldots, q,$$
$$\begin{aligned}\phi_i(t) = {} & r_i(t) - \sum_{j=1}^{n} a_{ij}(t)\exp\big\{\alpha_{ij}y_j(t)\big\} - \sum_{j=1}^{n} b_{ij}(t)\exp\big\{\beta_{ij}y_j\big(t - \tau_{ij}(t)\big)\big\} \\ & - \sum_{j=1}^{n}\int_{-\infty}^{t} c_{ij}(t,\theta)\exp\big\{\gamma_{ij}y_j(\theta)\big\}\,d\theta - \sum_{j=1}^{n} d_{ij}(t)\exp\big\{\alpha_{ii}y_i(t) + \alpha_{ij}y_j(t)\big\} \\ & - \int_{-\infty}^{t} f_i(t,\theta)u_i(\theta)\,d\theta,\end{aligned}$$
$$\varphi_i(t) = -\alpha_i(t)u_i(t) + \int_{-\infty}^{t} g_i(t,\theta)\exp\big\{\alpha_{ii}y_i(\theta)\big\}\,d\theta,$$

then

$$\operatorname{Ker} L = \big\{U \mid U \in X, U = h, h \in R^{2n}\big\},$$
$$\operatorname{Im} L = \left\{z \,\middle|\, z = (f, C_1, \ldots, C_q) \in Z : \int_0^{\omega} f(s)\,ds + \sum_{k=1}^{q} C_k = 0\right\},$$

and $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L$. Since $\operatorname{Im} L$ is closed in Z, L is a Fredholm mapping of index zero. Define

$$Px = \frac{1}{\omega}\int_0^{\omega} U(t)\,dt, \quad U \in X,$$
$$Qz = Q(f, C_1, C_2, \ldots, C_q) = \left(\frac{1}{\omega}\left[\int_0^{\omega} f(s)\,ds + \sum_{k=1}^{q} C_k\right], 0, 0, \ldots, 0\right).$$

It is easy to show that P, Q are continuous projectors such that $\operatorname{Im} P = \operatorname{Ker} L$, $\operatorname{Ker} Q = \operatorname{Im} L = \operatorname{Im}(I - Q)$. If $z = (f, C_1, C_2, \ldots, C_q) \in \operatorname{Im} L$, then there exists $U(t) \in X$ satisfying

$$\dot{U}(t) = f(t), \quad t \neq t_k, k \in N,$$
$$x\big(t_k^+\big) - x\big(t_k^-\big) = C_k.$$

Namely,

$$U(t) = \int_0^{t} f(s)\,ds + \sum_{t>t_k} C_k + U(0).$$

Since $U(t) \in \operatorname{Ker} P$, we have $\int_0^{\omega} U(s)\,ds = 0$. By the above equation, we have

$$\int_0^{\omega}\int_0^{t} f(s)\,ds\,dt + \int_0^{\omega}\sum_{t>t_k} C_k\,dt + \omega U(0) = 0,$$

so

$$U(t) = \int_0^{t} f(s)\,ds + \sum_{t>t_k} C_k - \frac{1}{\omega}\int_0^{\omega}\int_0^{t} f(s)\,ds\,dt - \frac{1}{\omega}\sum_{k=1}^{q}(\omega - t_k)C_k.$$

It follows that the generalized inverse (to L) $K_P : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ is given by

$$K_P z = \int_0^t f(s)\,ds + \sum_{t>t_k} C_k - \frac{1}{\omega}\int_0^\omega \int_0^t f(s)\,ds\,dt - \frac{1}{\omega}\sum_{k=1}^{q}(\omega - t_k)C_k.$$

Obviously, QN and $K_P(I-Q)N$ are continuous. It follows from the Ascoli-Arzela theorem that $\overline{K_P(I-Q)N(\bar{\Omega})}$ is compact for any open bounded $\Omega \subset X$, thus, N is L-compact on $\bar{\Omega}$. Now consider the operator equation $LU = \lambda NU$, $\lambda \in (0,1)$, that is,

$$\begin{cases} \dot{y}_i(t) = \lambda \phi_i(t), & t \neq t_k, \\ \dot{u}_i(t) = \lambda \varphi_i(t), & t \geq 0, \\ \Delta y_i = y_i(t_k^+) - y_i(t_k^-) = \lambda \ln(1 + p_{ik}), & i = 1,2,\ldots,n, k \in N. \end{cases} \tag{2.2}$$

Integrating on both sides of (2.2) over the interval $[0,\omega]$, we obtain

$$\begin{aligned} &\int_0^\omega \Biggl[\sum_{j=1}^n a_{ij}(t)\exp\{\alpha_{ij}y_j(t)\} + \sum_{j=1}^n b_{ij}(t)\exp\{\beta_{ij}y_j(t-\tau_{ij}(t))\} \\ &\quad + \sum_{j=1}^n \int_{-\infty}^t c_{ij}(t,\theta)\exp\{\gamma_{ij}y_j(\theta)\}\,d\theta + \sum_{j=1}^n d_{ij}(t)\exp\{\alpha_{ii}y_i(t) + \alpha_{ij}y_j(t)\} \\ &\quad + \int_{-\infty}^t f_i(t,\theta)u_i(\theta)\,d\theta\Biggr]\,dt = \omega[\bar{r}_i + \bar{\Delta}_i], \end{aligned} \tag{2.3}$$

$$\int_0^\omega \alpha_i(t)u_i(t)\,dt = \int_0^\omega \int_{-\infty}^t g_i(t,\theta)\exp\{\alpha_{ii}y_i(\theta)\}\,d\theta\,dt. \tag{2.4}$$

Since $U(t) \in X$, there exist $\xi_i, \eta_i, \bar{\xi}_i, \bar{\eta}_i \in [0,\omega]$, $i = 1,2,\ldots,n$ such that

$$\begin{aligned} y_i(\xi_i) &= \min_{t\in[0,\omega]}\{y_i(t)\}, & y_i(\eta_i) &= \max_{t\in[0,\omega]}\{y_i(t)\}, \\ u_i(\bar{\xi}_i) &= \min_{t\in[0,\omega]}\{u_i(t)\}, & u_i(\bar{\eta}_i) &= \max_{t\in[0,\omega]}\{u_i(t)\}. \end{aligned} \tag{2.5}$$

It follows from (2.3) that

$$\begin{aligned} &\int_0^\omega \Biggl[a_{ii}(t)\exp\{\alpha_{ii}y_i(t)\} + b_{ii}(t)\exp\{\beta_{ii}y_i(t-\tau_{ii}(t))\} \\ &\quad + \int_{-\infty}^t c_{ii}(t,\theta)\exp\{\gamma_{ii}y_i(\theta)\}\,d\theta\Biggr]\,dt \leq \omega[\bar{r}_i + \bar{\Delta}_i], \end{aligned}$$

which implies that

$$y_i(\xi_i) \leq \frac{1}{\alpha_{ii}}\ln\left\{\frac{\bar{r}_i + \bar{\Delta}_i}{\bar{a}_{ii} + \bar{b}_{ii} + \bar{c}_{ii}}\right\}. \tag{2.6}$$

By (2.2) and (2.3), we have

$$\int_0^\omega \bigl|\dot{y}_i(t)\bigr|\,dt \leq [\bar{R}_i + \bar{r}_i + \bar{\Delta}_i]\omega. \tag{2.7}$$

So, according to (2.6) and (2.7), we obtain

$$\begin{aligned} y_i(t) &\le y_i(\xi_i) + \int_0^{\omega} \left|\dot{y}_i(t)\right| dt + |\bar{\triangle}_i|\omega \\ &\le \frac{1}{\alpha_{ii}} \ln\left\{ \frac{\bar{r}_i + \bar{\triangle}_i}{\bar{a}_{ii} + \bar{b}_{ii} + \bar{c}_{ii}} \right\} + [\bar{R}_i + \bar{r}_i + \bar{\triangle}_i]\omega + |\bar{\triangle}_i|\omega := \ln M_i. \end{aligned} \tag{2.8}$$

From (2.4), it follows that

$$\begin{aligned} \int_0^{\omega} u_i(t)\,dt &\le \frac{1}{\alpha_i^L} \exp\left\{\alpha_{ii} y_i(\eta_i)\right\} \int_0^{\omega} \int_{-\infty}^{t} g_i(t,\theta)\,d\theta\,dt \\ &\le \frac{1}{\alpha_i^L} \exp\left\{\alpha_{ii} y_i(\eta_i)\right\} \bar{g}_i \omega, \end{aligned} \tag{2.9}$$

$$\int_0^{\omega} \alpha_i(t) u_i(t)\,dt \le M_i^{\alpha_{ii}} \int_0^{\omega} \int_{-\infty}^{t} g_i(t,\theta)\,d\theta\,dt \le M_i^{\alpha_{ii}} \bar{g}_i \omega, \tag{2.10}$$

that is,

$$u_i(\bar{\xi}_i) \le M_i^{\alpha_{ii}} \frac{\bar{g}_i}{\alpha_i^L}, \qquad \int_0^{\omega} \left|\dot{u}_i(t)\right| dt \le 2M_i^{\alpha_{ii}} \bar{g}_i \omega, \tag{2.11}$$

thus

$$u_i(t) \le u_i(\bar{\xi}_i) + \int_0^{\omega} \left|\dot{u}_i(t)\right| dt \le M_i^{\alpha_{ii}} \left[\frac{\bar{g}_i}{\alpha_i^L} + 2\bar{g}_i \omega \right] := L_i. \tag{2.12}$$

On the other hand, from (2.3), (2.6), and (2.9), we have

$$\begin{aligned} \omega[\bar{r}_i + \bar{\triangle}_i] \le{}& \left[\bar{a}_{ii} + \bar{b}_{ii} + \bar{c}_{ii} + \frac{f_i^M}{\alpha_i^L} \bar{g}_i \right] \omega \exp\left\{\alpha_{ii} y_i(\eta_i)\right\} + \sum_{j=1, j\neq i}^{n} \left(\bar{a}_{ij} \exp\left\{\alpha_{ij} y_j(\eta_j)\right\} \right. \\ & \left. + \bar{b}_{ij} \exp\left\{\beta_{ij} y_j(\eta_j)\right\} + \bar{c}_{ij} \exp\left\{\gamma_{ij} y_j(\eta_j)\right\} \right) \omega \\ & + \sum_{j=1}^{n} \bar{d}_{ij} \exp\left\{\alpha_{ii} y_i(\eta_i) + \alpha_{ij} y_j(\eta_j)\right\} \omega, \end{aligned}$$

then

$$\begin{aligned} & \left[\bar{a}_{ii} + \bar{b}_{ii} + \bar{c}_{ii} + \frac{f_i^M}{\alpha_i^L} \bar{g}_i \right] \exp\left\{\alpha_{ii} y_i(\eta_i)\right\} \\ & \quad \ge [\bar{r}_i + \bar{\triangle}_i] - \sum_{j=1, j\neq i}^{n} \left(\bar{a}_{ij} M_j^{\alpha_{ij}} + \bar{b}_{ij} M_j^{\beta_{ij}} + \bar{c}_{ij} M_j^{\gamma_{ij}} \right) \\ & \qquad - \sum_{j=1}^{n} \bar{d}_{ij} M_i^{\alpha_{ii}} M_j^{\alpha_{ij}} := P_i, \end{aligned}$$

that is,

$$y_i(\eta_i) \ge \frac{1}{\alpha_{ii}} \ln \frac{P_i}{\bar{a}_{ii} + \bar{b}_{ii} + \bar{c}_{ii} + \frac{f_i^M}{\alpha_i^L} \bar{g}_i}. \tag{2.13}$$

This together with (2.7), leads to

$$\begin{aligned} y_i(t) &\geq y_i(\eta_i) - \int_0^\omega \left|\dot{y}_i(t)\right| dt - |\bar{\triangle}_i|\omega \\ &\geq \frac{1}{\alpha_{ii}} \ln \frac{P_i}{\bar{a}_{ii} + \bar{b}_{ii} + \bar{c}_{ii} + \frac{f_i^M}{\alpha_i^L}\bar{g}_i} - [\bar{R}_i + \bar{r}_i + \bar{\triangle}_i]\omega - |\bar{\triangle}_i|\omega := \ln m_i. \end{aligned} \tag{2.14}$$

It follows from (2.4) that

$$u_i(\bar{\eta}_i)\bar{\alpha}_i\omega \geq \int_0^\omega \alpha_i(t)u_i(t)\,dt \geq m_i^{\alpha_{ii}}\bar{g}_i\omega,$$

then

$$u_i(\bar{\eta}_i) \geq \frac{\bar{g}_i}{\bar{\alpha}_i} m_i^{\alpha_{ii}}, \tag{2.15}$$

which, together with (2.11), leads to

$$u_i(t) \geq u_i(\bar{\eta}_i) - \int_0^\omega \left|\dot{u}_i(t)\right| dt \leq m_i^{\alpha_{ii}} \frac{\bar{g}_i}{\bar{\alpha}_i} - 2M_i^{\alpha_{ii}}\bar{g}_i\omega := l_i. \tag{2.16}$$

From (2.8), (2.12), (2.14), and (2.16), it follows that

$$\ln m_i \leq y_i(t) \leq \ln M_i, \qquad l_i \leq u_i(t) \leq L_i,$$

clearly, m_i, M_i, l_i, L_i are independent of λ. We take $D = \{U \in X | \|U\| < H\}$, $H = \max_{1\leq i\leq n}\{|\ln m_i| + |\ln M_i| + |l_i| + |L_i|\} + H_0$, H_0 is taken sufficiently large.

Now we check the conditions of Lemma 2.1. From (2.8), (2.12), (2.14), and (2.16), it is easily derive that, for each $\lambda \in (0,1)$, $U \in \partial D \cap \operatorname{Dom} L$, $LU \neq \lambda NU$. This satisfies condition (i) of Lemma 2.1.

Next let us consider the algebraic equations

$$\begin{cases} \bar{r}_i + \bar{\triangle}_i - \sum_{j=1}^n (\bar{a}_{ij}u_j^{\alpha_{ij}} + \bar{b}_{ij}u_j^{\beta_{ij}} + \bar{c}_{ij}u_j^{\gamma_{ij}} + \bar{d}_{ij}u_i^{\alpha_{ii}}u_j^{\alpha_{ij}}) - \mu\bar{f}_i v_j = 0, \\ \bar{\alpha}_i v_i - \bar{g}_i u_i^{\alpha_{ii}} = 0, \end{cases} \tag{2.17}$$

for $U \in R^{2n}$, $\mu \in [0,1]$. Similar to the argument of (2.8), (2.12), (2.14), and (2.16), we can derive

$$m_i \leq u_i(t) \leq M_i, \qquad l_i \leq v_i(t) \leq L_i. \tag{2.18}$$

When $U \in \partial D \cap \operatorname{Ker} L$, U is a constant vector in R^{2n} with $\|U\| = H$. Then

$$QNU = \left(\begin{pmatrix} \begin{pmatrix} \bar{r}_i + \bar{\triangle}_i - \sum_{j=1}^n (\bar{a}_{ij}u_j^{\alpha_{ij}} + \bar{b}_{ij}u_j^{\beta_{ij}} + \bar{c}_{ij}u_j^{\gamma_{ij}} \\ + \bar{d}_{ij}u_i^{\alpha_{ii}}u_j^{\alpha_{ij}}) - \bar{f}_i v_j \end{pmatrix}_{n\times 1} \\ (\bar{\alpha}_i v_i - \bar{g}_i u_i^{\alpha_{ii}})_{n\times 1} \end{pmatrix}, 0, \ldots, 0 \right) \neq 0,$$

it follows from (2.18) that $U \in \partial D \cap \operatorname{Ker} L$, $QNU \neq 0$. This proves that condition (ii) of Lemma 2.1 is satisfied. Define

$$H(\mu, U) = \mu QNU + (1 - \mu)G(U), \quad \mu \in [0,1],$$

$$G(U) = \left(\begin{array}{c} \left(\begin{array}{c} \bar{r}_i + \bar{\triangle}_i - \sum_{j=1}^{n} (\bar{a}_{ij} u_j^{\alpha_{ij}} + \bar{b}_{ij} u_j^{\beta_{ij}} + \bar{c}_{ij} u_j^{\gamma_{ij}} \\ + \bar{d}_{ij} u_i^{\alpha_{ii}} u_j^{\alpha_{ij}}) - \bar{f}_i v_j \end{array} \right)_{n\times 1} \\ (\bar{\alpha}_i v_i - \bar{g}_i u_i^{\alpha_{ii}})_{n\times 1} \end{array} \right),$$

from $U \in \partial D \cap \operatorname{Ker} L$ and $\mu \in [0,1]$, it follows that $H(\mu, U) \neq 0$. Moreover, we take $J = I$. According to Theorem 2.1, one has

$$\deg\bigl(JQN(U), D \cap \operatorname{Ker} L, 0\bigr) = \deg\bigl(H(U), D \cap \operatorname{Ker} L, 0\bigr) \neq 0.$$

Now we have to prove that D satisfy all the conditions of Lemma 2.1. Therefore, we know that the system (2.1) has at least one ω-periodic solution $(y_1^*(t), \dots, y_n^*(t), u_1^*(t), \dots, u_n^*(t))^T \in D$. By $x_i^*(t) = e^{y_i^*(t)}$, then we know that $(x_1^*(t), x_2^*(t), \dots, x_n^*(t), u_1^*(t), u_2^*(t), \dots, u_n^*(t))^T$ is a positive ω-periodic solution of the system (1.6). The proof of Theorem 2.1 is completed. □

Theorem 2.2 *In addition to* (H_1)-(H_4), *assume further that*:

(H_5) *the system of algebraic equations*

$$\begin{cases} \bar{r}_i + \bar{\triangle}_i - \sum_{j=1}^{n} (\bar{a}_{ij} u_j^{\alpha_{ij}} + \bar{b}_{ij} u_j^{\beta_{ij}} + \bar{c}_{ij} u_j^{\gamma_{ij}} + \bar{d}_{ij} u_i^{\alpha_{ii}} u_j^{\alpha_{ij}}) - \bar{f}_i v_j = 0, \\ \bar{\alpha}_i v_i - \bar{g}_i u_i^{\alpha_{ii}} = 0, \end{cases}$$

has finite solutions $u^* = (u_1^*, \dots, u_n^*, v_1^*, \dots, v_n^*)^T \in R_+^{2n}$ *with* $u_i^* > 0$, $v_i^* > 0$ *and* $\sum_{u^*} \operatorname{sgn} J_g(u^*) \neq 0$;

(H_6) $\alpha_i^L > 0$;

(H_7) *if one of the following conditions is satisfied*:

$$\bar{r}_i + \bar{\triangle}_i > \sum_{j=1, j\neq i}^{n} \bigl(\bar{a}_{ij} A_j^{\alpha_{ij}} + \bar{b}_{ij} A_j^{\beta_{ij}} + \bar{c}_{ij} A_j^{\gamma_{ij}}\bigr) + \sum_{j=1}^{n} \bar{d}_{ij} A_i^{\alpha_{ii}} A_j^{\alpha_{ij}},$$

$$\bar{r}_i + \bar{\triangle}_i > \sum_{j=1, j\neq i}^{n} \bigl(\bar{a}_{ij} B_j^{\alpha_{ij}} + \bar{b}_{ij} B_j^{\beta_{ij}} + \bar{c}_{ij} B_j^{\gamma_{ij}}\bigr) + \sum_{j=1}^{n} \bar{d}_{ij} B_i^{\alpha_{ii}} B_j^{\alpha_{ij}},$$

$$\bar{r}_i + \bar{\triangle}_i > \sum_{j=1, j\neq i}^{n} \bigl(\bar{a}_{ij} C_j^{\alpha_{ij}} + \bar{b}_{ij} C_j^{\beta_{ij}} + \bar{c}_{ij} C_j^{\gamma_{ij}}\bigr) + \sum_{j=1}^{n} \bar{d}_{ij} C_i^{\alpha_{ii}} C_j^{\alpha_{ij}}.$$

Then the system (1.6) *has at least one positive ω-periodic solution* $(x_1^*, \dots, x_n^*, u_1^*, \dots, u_n^*)^T$.

Proof The proof is the same as that of Theorem 2.1 with only slight changes, that is, (2.6) in the proof of Theorem 2.1 can be replaced by one of the following inequalities, respectively:

$$y_i(\xi_i) \le \frac{1}{\alpha_{ii}} \ln\left\{ \frac{\bar{r}_i + \bar{\triangle}_i}{\bar{a}_{ii}} \right\}, \qquad y_i(\xi_i) \le \frac{1}{\beta_{ii}} \ln\left\{ \frac{\bar{r}_i + \bar{\triangle}_i}{\bar{b}_{ii}} \right\}, \qquad y_i(\xi_i) \le \frac{1}{\gamma_{ii}} \ln\left\{ \frac{\bar{r}_i + \bar{\triangle}_i}{\bar{c}_{ii}} \right\},$$

so, the details of the following proof are omitted here. □

Remark 2.1 After the above proof of Theorem 2.1 and Theorem 2.2, we note that the criteria for the existence of positive periodic solutions of the system (1.6) are independent of the delays. Furthermore, it is not necessary for $\tau_{ij}(t)$ to remain nonnegative. Namely, the results of Theorem 2.1 and Theorem 2.2 are still valid for both advanced type systems and mixed type systems.

3 Global asymptotic stability

The aim of this section is to establish a set of sufficient conditions on the global asymptotic stability of a unique positive periodic solution of the system (1.6). We say a positive periodic solution $(x_1^*(t), x_2^*(t), \ldots, x_n^*(t), u_1^*(t), u_2^*(t), \ldots, u_n^*(t))^T$ of the system (1.6) is globally asymptotically stable if it attracts any other positive solution of the system (1.6). In addition, if the positive periodic solution $(x_1^*(t), x_2^*(t), \ldots, x_n^*(t), u_1^*(t), u_2^*(t), \ldots, u_n^*(t))^T$ is globally asymptotically stable, then it is unique.

The following lemma, borrowed from [43], would be basic to establish the main results.

Lemma 3.1 *Let h be a real number and f be a nonnegative function defined on $[h; +\infty)$ such that f is integrable on $[h; +\infty)$ and is uniformly continuous on $[h; +\infty)$, then* $\lim_{t\to+\infty} f(t) = 0$.

From Theorem 2.1 (or Theorem 2.2) we know that the system (1.6) has at least one positive periodic solution $(x_1^*(t), x_2^*(t), \ldots, x_n^*(t), u_1^*(t), u_2^*(t), \ldots, u_n^*(t))^T$ and there exist positive constants χ_i, μ_i such that $\chi_i \le x_i^*(t)$, $u_i^*(t) \le \mu_i$, $i = 1, 2, \ldots, n$. So, we take a positive constant λ which satisfies $0 < \lambda \le \min\{\chi_i\}$. Let

$$z_i(t) = x_i(t)/\lambda, \quad i = 1, 2, \ldots, n, \tag{3.1}$$

then the system (1.6) can be written as

$$\begin{cases} \dot{z}_i(t) = z_i(t)[r_i(t) - \sum_{j=1}^n \lambda^{\alpha_{ij}} a_{ij}(t) z_j^{\alpha_{ij}}(t) - \sum_{j=1}^n \lambda^{\beta_{ij}} b_{ij}(t) z_j^{\beta_{ij}}(t - \tau_{ij}(t)) \\ \qquad - \sum_{j=1}^n \int_{-\infty}^t \lambda^{\gamma_{ij}} c_{ij}(t,\theta) z_j^{\gamma_{ij}}(\theta)\, d\theta - \sum_{j=1}^n \lambda^{\alpha_{ii}+\alpha_{ij}} d_{ij}(t) z_i^{\alpha_{ii}}(t) z_j^{\alpha_{ij}}(t) \\ \qquad - \int_{-\infty}^t f_i(t,\theta) u_i(\theta)\, d\theta], \quad t \ne t_k, \\ \dot{u}_i(t) = -\alpha_i(t) u_i(t) + \lambda^{\alpha_{ii}} \int_{-\infty}^t g_i(t,\theta) z_i^{\alpha_{ii}}(\theta)\, d\theta, \quad t \ge 0, \\ \Delta z_i = z_i(t_k^+) - z_i(t_k^-) = p_{ik} z_i(t_k^-), \quad i = 1, 2, \ldots, n, k \in N. \end{cases} \tag{3.2}$$

It is clear that $z^*(t) = (z_1^*(t), z_2^*(t), \ldots, z_n^*(t), u_1^*(t), u_2^*(t), \ldots, u_n^*(t))^T$ is the periodic solution of the system (3.2). And if the periodic solution of the system (3.2) is globally asymptotically stable, then the periodic solution of the system (1.6) is also globally asymptotically stable.

Theorem 3.1 *In addition to the conditions in Theorem* 2.1 (*or in Theorem* 2.2), *assume further that*:

(H_8) $\alpha_{ii} \ge \{\alpha_{ji}, \beta_{ji}, \gamma_{ji}\}$, $1 \le j, i \le n$;

(H_9) *if there exist constants $\rho_i > 0$, $\delta_i > 0$ such that*

$$\inf_{t\in[0,+\infty)} \{\Phi_i(t), \Psi_i(t)\} > 0, \quad i = 1, 2, \ldots, n,$$

where

$$\Phi_i(t) = \rho_i\lambda^{\alpha_{ii}}a_{ii}(t) + \sum_{j=1}^{n}\rho_i\lambda^{\alpha_{ii}}M_j^{\alpha_{ij}}d_{ij}(t) - \delta_i\lambda^{\alpha_{ii}}\int_0^{+\infty}g_i(t+\theta,t)\,d\theta - \sum_{j=1,j\neq i}^{n}\rho_j\lambda^{\alpha_{ji}}a_{ji}(t) - \sum_{j=1}^{n}\rho_j\lambda^{\alpha_{ji}}M_j^{\alpha_{jj}}d_{ji}(t) - \sum_{j=1}^{n}\rho_j\lambda^{\beta_{ji}}\frac{b_{ji}(\sigma_{ji}^{-1}(t))}{1-\dot{\tau}_{ji}(\sigma_{ji}^{-1}(t))} - \sum_{j=1}^{n}\rho_j\lambda^{\lambda_{ji}}\int_0^{+\infty}c_{ji}(t+\theta,t)\,d\theta;$$

$$\Psi_i(t) = \delta_i\alpha_i(t) - \rho_i\int_0^{+\infty}f_i(t+\theta,t)\,d\theta.$$

Then the system (1.6) *has a unique globally asymptotically stable positive periodic solution* $(x_1^*(t), x_2^*(t), \ldots, x_n^*(t), u_1^*(t), u_2^*(t), \ldots, u_n^*(t))^T$.

Proof For any positive solution $(z_1(t), z_2(t), \ldots, z_n(t), u_1(t), u_2(t), \ldots, u_n(t))^T$ and positive periodic solution $(z_1^*(t), z_2^*(t), \ldots, z_n^*(t), u_1^*(t), u_2^*(t), \ldots, u_n^*(t))^T$ of the system (3.2). Now we construct a Lyapunov functional,

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t), \tag{3.3}$$

where

$$V_1(t) = \sum_{i=1}^{n}\rho_i\left|\ln z_i(t) - \ln z_i^*(t)\right|,$$

$$V_2(t) = \sum_{i=1}^{n}\delta_i\left|u_i(t) - u_i^*(t)\right|,$$

$$V_3(t) = \sum_{i=1}^{n}\rho_i\sum_{j=1}^{n}\lambda^{\beta_{ij}}\int_{t-\tau_{ij}(t)}^{t}\frac{b_{ij}(\sigma_{ij}^{-1}(s))}{1-\dot{\tau}_{ij}(\sigma_{ij}^{-1}(s))}\left|z_j^{\beta_{ij}}(s) - z_j^{*\beta_{ij}}(s)\right|ds,$$

$$V_4(t) = \sum_{i=1}^{n}\rho_i\sum_{j=1}^{n}\lambda^{\lambda_{ij}}\int_0^{+\infty}\int_{t-\theta}^{t}c_{ij}(s+\theta,s)\left|z_j^{\lambda_{ij}}(s) - z_j^{*\lambda_{ij}}(s)\right|ds\,d\theta,$$

$$V_5(t) = \sum_{i=1}^{n}\rho_i\int_0^{+\infty}\int_{t-\theta}^{t}f_i(s+\theta,s)\left|u_i(s) - u_i^*(s)\right|ds\,d\theta,$$

$$V_6(t) = \sum_{i=1}^{n}\delta_i\lambda^{\alpha_{ii}}\int_0^{+\infty}\int_{t-\theta}^{t}g_i(s+\theta,s)\left|z_i^{\alpha_{ii}}(s) - z_i^{*\alpha_{ii}}(s)\right|ds\,d\theta.$$

Calculating the upper right derivative of $V(t)$ along the solution of (3.2), it follows that, for $t \neq t_k$,

$$D^+V_1(t) = \sum_{i=1}^{n}\rho_i\left\{\operatorname{sgn}\bigl(z_i(t) - z_i^*(t)\bigr)\left(\frac{\dot{z}_i(t)}{z_i(t)} - \frac{\dot{z}_i^*(t)}{z_i^*(t)}\right)\right\}$$

$$= \sum_{i=1}^{n}\rho_i\left\{\operatorname{sgn}\bigl(z_i(t) - z_i^*(t)\bigr)\left[-\sum_{j=1}^{n}\lambda^{\alpha_{ij}}a_{ij}(t)\bigl(z_j^{\alpha_{ij}}(t) - z_j^{*\alpha_{ij}}(t)\bigr)\right.\right.$$

$$
\begin{aligned}
&- \sum_{j=1}^{n} \lambda^{\beta_{ij}} b_{ij}(t)\bigl(z_j^{\beta_{ij}}\bigl(t-\tau_{ij}(t)\bigr) - z_j^{*\beta_{ij}}\bigl(t-\tau_{ij}(t)\bigr)\bigr) \\
&- \sum_{j=1}^{n} \int_{-\infty}^{t} \lambda^{\gamma_{ij}} c_{ij}(t,\theta)\bigl(z_j^{\gamma_{ij}}(\theta) - z_j^{*\gamma_{ij}}(\theta)\bigr)\,d\theta \\
&- \sum_{j=1}^{n} \lambda^{\alpha_{ii}+\alpha_{ij}} d_{ij}(t)\bigl(z_i^{\alpha_{ii}}(t) z_j^{\alpha_{ij}}(t) - z_i^{*\alpha_{ii}}(t) z_j^{*\alpha_{ij}}(t)\bigr) \\
&- \int_{-\infty}^{t} f_i(t,\theta)\bigl(u_i(\theta) - u_i^*(\theta)\bigr)\,d\theta \Biggr]\Biggr\} \\
\le{}& - \sum_{i=1}^{n} \rho_i \lambda^{\alpha_{ii}} a_{ii}(t)\bigl|z_i^{\alpha_{ii}}(t) - z_i^{*\alpha_{ii}}(t)\bigr| \\
&+ \sum_{i=1}^{n} \rho_i \Biggl\{ \sum_{j=1, j\neq i}^{n} \lambda^{\alpha_{ij}} a_{ij}(t)\bigl|z_j^{\alpha_{ij}}(t) - z_j^{*\alpha_{ij}}(t)\bigr| \\
&+ \sum_{j=1}^{n} \lambda^{\beta_{ij}} b_{ij}(t)\bigl|z_j^{\beta_{ij}}\bigl(t-\tau_{ij}(t)\bigr) - z_j^{*\beta_{ij}}\bigl(t-\tau_{ij}(t)\bigr)\bigr| \\
&+ \sum_{j=1}^{n} \int_{-\infty}^{t} \lambda^{\gamma_{ij}} c_{ij}(t,\theta)\bigl|z_j^{\gamma_{ij}}(\theta) - z_j^{*\gamma_{ij}}(\theta)\bigr|\,d\theta \\
&+ \sum_{j=1}^{n} \lambda^{\alpha_{ii}+\alpha_{ij}} d_{ij}(t) z_i^{\alpha_{ii}}(t)\bigl|z_j^{\alpha_{ij}}(t) - z_j^{*\alpha_{ij}}(t)\bigr| \\
&- \sum_{j=1}^{n} \lambda^{\alpha_{ii}+\alpha_{ij}} d_{ij}(t) z_j^{*\alpha_{ij}}(t)\bigl|z_i^{\alpha_{ii}}(t) - z_i^{*\alpha_{ii}}(t)\bigr| \\
&+ \int_{-\infty}^{t} f_i(t,\theta)\bigl|u_i(\theta) - u_i^*(\theta)\bigr|\,d\theta \Biggr\},
\end{aligned}
$$

$$
D^+V_2(t) = \sum_{i=1}^{n} \delta_i \Biggl\{ -\alpha_i(t)\bigl|u_i(t) - u_i^*(t)\bigr| + \lambda^{\alpha_{ii}} \int_{-\infty}^{t} g_i(t,\theta)\bigl|z_i^{\alpha_{ii}}(\theta) - z_i^{*\alpha_{ii}}(\theta)\bigr|\,d\theta \Biggr\},
$$

$$
\begin{aligned}
D^+V_3(t) ={}& \sum_{i=1}^{n} \rho_i \sum_{j=1}^{n} \lambda^{\beta_{ij}} \frac{b_{ij}(\sigma_{ij}^{-1}(t))}{1 - \dot{\tau}_{ij}(\sigma_{ij}^{-1}(ts))}\bigl|z_j^{\beta_{ij}}(t) - z_j^{*\beta_{ij}}(t)\bigr| \\
&- \sum_{i=1}^{n} \rho_i \sum_{j=1}^{n} \lambda^{\beta_{ij}} b_{ij}(t)\bigl|z_j^{\beta_{ij}}\bigl(t-\tau_{ij}(t)\bigr) - z_j^{*\beta_{ij}}\bigl(t-\tau_{ij}(t)\bigr)\bigr|,
\end{aligned}
$$

$$
\begin{aligned}
D^+V_4(t) ={}& \sum_{i=1}^{n} \rho_i \sum_{j=1}^{n} \lambda^{\lambda_{ij}} \int_{0}^{+\infty} c_{ij}(t+\theta,t)\bigl|z_j^{\lambda_{ij}}(t) - z_j^{*\lambda_{ij}}(t)\bigr|\,d\theta \\
&- \sum_{i=1}^{n} \rho_i \sum_{j=1}^{n} \lambda^{\lambda_{ij}} \int_{0}^{+\infty} c_{ij}(t,t-\theta)\bigl|z_j^{\lambda_{ij}}(t-\theta) - z_j^{*\lambda_{ij}}(t-\theta)\bigr|\,d\theta,
\end{aligned}
$$

$$
\begin{aligned}
D^+V_5(t) ={}& \sum_{i=1}^{n} \rho_i \int_{0}^{+\infty} f_i(t+\theta,t)\bigl|u_i(t) - u_i^*(t)\bigr|\,d\theta \\
&- \sum_{i=1}^{n} \rho_i \int_{0}^{+\infty} f_i(t,t-\theta)\bigl|u_i(t-\theta) - u_i^*(t-\theta)\bigr|\,d\theta,
\end{aligned}
$$

$$
\begin{aligned}
D^{+}V_6(t) = {} & \sum_{i=1}^{n} \delta_i \lambda^{\alpha_{ii}} \int_0^{+\infty} g_i(t+\theta,t)\left|z_i^{\alpha_{ii}}(t) - z_i^{*\alpha_{ii}}(t)\right| d\theta \\
& - \sum_{i=1}^{n} \delta_i \lambda^{\alpha_{ii}} \int_0^{+\infty} g_i(t,t-\theta)\left|z_i^{\alpha_{ii}}(t-\theta) - z_i^{*\alpha_{ii}}(t-\theta)\right| d\theta.
\end{aligned}
$$

Substituting the above results into (3.3), and by easily computing, for $t \neq t_k$, we have

$$
\begin{aligned}
D^{+}V(t) \leq {} & \sum_{i=1}^{n} \rho_i \Bigg\{ -\lambda^{\alpha_{ii}} a_{ii}(t)\left|z_i^{\alpha_{ii}}(t) - z_i^{*\alpha_{ii}}(t)\right| \\
& + \sum_{j=1,j\neq i}^{n} \lambda^{\alpha_{ij}} a_{ij}(t)\left|z_j^{\alpha_{ij}}(t) - z_j^{*\alpha_{ij}}(t)\right| \\
& + \sum_{j=1}^{n} \lambda^{\beta_{ij}} \frac{b_{ij}(\sigma_{ij}^{-1}(t))}{1-\dot{\tau}_{ij}(\sigma_{ij}^{-1}(t))}\left|z_j^{\beta_{ij}}(t) - z_j^{*\beta_{ij}}(t)\right| \\
& + \sum_{j=1}^{n} \lambda^{\lambda_{ij}} \int_0^{+\infty} c_{ij}(t+\theta,t)\left|z_j^{\lambda_{ij}}(t) - z_j^{*\lambda_{ij}}(t)\right| d\theta \\
& + \sum_{j=1}^{n} \lambda^{\alpha_{ii}+\alpha_{ij}} d_{ij}(t) z_i^{\alpha_{ii}}(t)\left|z_j^{\alpha_{ij}}(t) - z_j^{*\alpha_{ij}}(t)\right| \\
& - \sum_{j=1}^{n} \lambda^{\alpha_{ii}+\alpha_{ij}} d_{ij}(t) z_j^{*\alpha_{ij}}(t)\left|z_i^{\alpha_{ii}}(t) - z_i^{*\alpha_{ii}}(t)\right| \\
& + \int_0^{+\infty} f_i(t+\theta,t)\left|u_i(t) - u_i^{*}(t)\right| d\theta \Bigg\} \\
& + \sum_{i=1}^{n} \delta_i \Bigg\{ -\alpha_i(t)\left|u_i(t) - u_i^{*}(t)\right| \\
& + \lambda^{\alpha_{ii}} \int_0^{+\infty} g_i(t+\theta,t)\left|z_i^{\alpha_{ii}}(t) - z_i^{*\alpha_{ii}}(t)\right| d\theta \Bigg\} \\
= {} & \sum_{i=1}^{n} \Bigg\{ -\rho_i \lambda^{\alpha_{ii}} a_{ii}(t)\left|z_i^{\alpha_{ii}}(t) - z_i^{*\alpha_{ii}}(t)\right| \\
& + \sum_{j=1,j\neq i}^{n} \rho_j \lambda^{\alpha_{ji}} a_{ji}(t)\left|z_i^{\alpha_{ji}}(t) - z_i^{*\alpha_{ji}}(t)\right| \\
& + \sum_{j=1}^{n} \rho_j \lambda^{\beta_{ji}} \frac{b_{ji}(\sigma_{ji}^{-1}(t))}{1-\dot{\tau}_{ji}(\sigma_{ji}^{-1}(t))}\left|z_i^{\beta_{ji}}(t) - z_i^{*\beta_{ji}}(t)\right| \\
& + \sum_{j=1}^{n} \rho_j \lambda^{\lambda_{ji}} \int_0^{+\infty} c_{ji}(t+\theta,t)\left|z_i^{\lambda_{ji}}(t) - z_i^{*\lambda_{ji}}(t)\right| d\theta \\
& + \sum_{j=1}^{n} \rho_j \lambda^{\alpha_{jj}+\alpha_{ji}} d_{ji}(t) z_j^{\alpha_{jj}}(t)\left|z_i^{\alpha_{ji}}(t) - z_i^{*\alpha_{ji}}(t)\right| \\
& - \rho_i \sum_{j=1}^{n} \lambda^{\alpha_{ii}+\alpha_{ij}} d_{ij}(t) z_j^{*\alpha_{ij}}(t)\left|z_i^{\alpha_{ii}}(t) - z_i^{*\alpha_{ii}}(t)\right|
\end{aligned}
$$

$$
\begin{aligned}
&+ \rho_i \int_0^{+\infty} f_i(t+\theta,t)\big|u_i(t)-u_i^*(t)\big|\,d\theta\Bigg\} \\
&+ \delta_i\Bigg\{-\alpha_i(t)\big|u_i(t)-u_i^*(t)\big| \\
&+ \lambda^{\alpha_{ii}} \int_0^{+\infty} g_i(t+\theta,t)\big|z_i^{\alpha_{ii}}(t)-z_i^{*\alpha_{ii}}(t)\big|\,d\theta\Bigg\} \\
\le{}& \sum_{i=1}^n\Bigg\{-\Bigg[\rho_i\lambda^{\alpha_{ii}}a_{ii}(t)+\sum_{j=1}^n\rho_i\lambda^{\alpha_{ii}}M_j^{\alpha_{ij}}d_{ij}(t) \\
&- \delta_i\lambda^{\alpha_{ii}}\int_0^{+\infty} g_i(t+\theta,t)\,d\theta\Bigg]\big|z_i^{\alpha_{ii}}(t)-z_i^{*\alpha_{ii}}(t)\big| \\
&+ \Bigg[\sum_{j=1,j\neq i}^n\rho_j\lambda^{\alpha_{ji}}a_{ji}(t)+\sum_{j=1}^n\rho_j\lambda^{\alpha_{ji}}M_j^{\alpha_{jj}}d_{ji}(t)\Bigg]\big|z_i^{\alpha_{ji}}(t)-z_i^{*\alpha_{ji}}(t)\big| \\
&+ \sum_{j=1}^n\rho_j\lambda^{\beta_{ji}}\frac{b_{ji}(\sigma_{ji}^{-1}(t))}{1-\dot{\tau}_{ji}(\sigma_{ji}^{-1}(t))}\big|z_i^{\beta_{ji}}(t)-z_i^{*\beta_{ji}}(t)\big| \\
&+ \sum_{j=1}^n\rho_j\lambda^{\lambda_{ji}}\int_0^{+\infty} c_{ji}(t+\theta,t)\,d\theta\big|z_i^{\lambda_{ji}}(t)-z_i^{*\lambda_{ji}}(t)\big|\,d\theta \\
&- \Bigg[\delta_i\alpha_i(t)-\rho_i\int_0^{+\infty} f_i(t+\theta,t)\,d\theta\Bigg]\big|u_i(t)-u_i^*(t)\big|\Bigg\}.
\end{aligned}
$$

From (3.1) we know $z_i^*(t)\ge 1$. Since $y=|a^x-b^x|$ is an increasing function when $a\ge 1$ and $x>0$. By $\alpha_{ii}\ge\{\alpha_{ji},\beta_{ji},\gamma_{ji}\}$, $1\le j,i\le n$, we have

$$
\begin{aligned}
&\big|z_i^{\alpha_{ji}}(t)-z_i^{*\alpha_{ji}}(t)\big|\le\big|z_i^{\alpha_{ii}}(t)-z_i^{*\alpha_{ii}}(t)\big|, \\
&\big|z_i^{\beta_{ji}}(t)-z_i^{*\beta_{ji}}(t)\big|\le\big|z_i^{\alpha_{ii}}(t)-z_i^{*\alpha_{ii}}(t)\big|, \\
&\big|z_i^{\gamma_{ji}}(t)-z_i^{*\gamma_{ji}}(t)\big|\le\big|z_i^{\alpha_{ii}}(t)-z_i^{*\alpha_{ii}}(t)\big|,
\end{aligned}
$$

so, for $t\neq t_k$, we get

$$
\begin{aligned}
D^+V(t)\le{}& -\sum_{i=1}^n\Bigg\{\Bigg[\rho_i\lambda^{\alpha_{ii}}a_{ii}(t)+\sum_{j=1}^n\rho_i\lambda^{\alpha_{ii}}M_j^{\alpha_{ij}}d_{ij}(t)-\delta_i\lambda^{\alpha_{ii}}\int_0^{+\infty} g_i(t+\theta,t)\,d\theta \\
&- \sum_{j=1,j\neq i}^n\rho_j\lambda^{\alpha_{ji}}a_{ji}(t)-\sum_{j=1}^n\rho_j\lambda^{\alpha_{ji}}M_j^{\alpha_{jj}}d_{ji}(t)-\sum_{j=1}^n\rho_j\lambda^{\beta_{ji}}\frac{b_{ji}(\sigma_{ji}^{-1}(t))}{1-\dot{\tau}_{ji}(\sigma_{ji}^{-1}(t))} \\
&- \sum_{j=1}^n\rho_j\lambda^{\lambda_{ji}}\int_0^{+\infty} c_{ji}(t+\theta,t)\,d\theta\Bigg]\big|z_i^{\alpha_{ii}}(t)-z_i^{*\alpha_{ii}}(t)\big| \\
&+ \Bigg[\delta_i\alpha_i(t)-\rho_i\int_0^{+\infty} f_i(t+\theta,t)\,d\theta\Bigg]\big|u_i(t)-u_i^*(t)\big|\Bigg\} \\
\le{}& -\sum_{i=1}^n\big\{\Phi_i(t)\big|z_i^{\alpha_{ii}}(t)-z_i^{*\alpha_{ii}}(t)\big|+\Psi_i(t)\big|u_i(t)-u_i^*(t)\big|\big\}.
\end{aligned}
$$

By the assumption (H_8), there exist enough small positive constants κ such that

$$\varphi_i(t) \geq \kappa, \qquad \phi_i(t) \geq \kappa.$$

Therefore,

$$D^+V(t) \leq -\kappa \sum_{i=1}^{n} \bigl(\bigl| z_i^{\alpha_{ii}}(t) - z_i^{*\alpha_{ii}}(t) \bigr| + \bigl| u_i(t) - u_i^*(t) \bigr| \bigr). \tag{3.4}$$

On the other hand, for $t = t_k$, $k \in N$,

$$\begin{aligned} V\bigl(t_k^+\bigr) - V\bigl(t_k^-\bigr) = {} & \sum_{i=1}^{n} \bigl[\bigl| \ln(1 + p_{ik})\bigl(z_i\bigl(t_k^-\bigr)\bigr) - \ln(1 + p_{ik})\bigl(z_i * \bigl(t_k^-\bigr)\bigr) \bigr| \\ & - \bigl| \ln\bigl(z_i\bigl(t_k^-\bigr)\bigr) - \ln\bigl(z_i^*\bigl(t_k^-\bigr)\bigr) \bigr| \bigr] = 0. \end{aligned}$$

Integrating both sides of (3.4) on interval $[0, t]$,

$$V(t) + \kappa \int_0^t \sum_{i=1}^{n} \bigl(\bigl| z_i^{\alpha_{ii}}(s) - z_i^{*\alpha_{ii}}(s) \bigr| + \bigl| u_i(s) - u_i^*(s) \bigr| \bigr) \, ds \leq V(0). \tag{3.5}$$

It follows from (3.5) that

$$\begin{aligned} & \int_0^t \sum_{i=1}^{n} \bigl(\bigl| z_i^{\alpha_{ii}}(s) - z_i^{*\alpha_{ii}}(s) \bigr| + \bigl| u_i(s) - u_i^*(s) \bigr| \bigr) \, ds \leq \frac{V(0)}{\kappa} < +\infty, \quad \text{for } t \geq 0, \\ & 0 \leq V(t) \leq V(0), \end{aligned} \tag{3.6}$$

which implies that

$$\sum_{i=1}^{n} \bigl(\bigl| z_i^{\alpha_{ii}}(s) - z_i^{*\alpha_{ii}}(s) \bigr| + \bigl| u_i(s) - u_i^*(s) \bigr| \bigr) \in L^1[0, +\infty).$$

By Theorem 2.1, (3.1), and (3.6), it is easy to derive that $z_i(t)$, $u_i(t)$, $i = 1, 2, \ldots, n$ are uniformly bounded on $[0, +\infty)$. This together with (3.2) leads to $\dot{z}_i(t)$, $\dot{z}_i^*(t)$, $\dot{u}_i(t)$, $\dot{u}_i^*(t)$, $i = 1, 2, \ldots, n$, being also uniformly bounded on $[0, +\infty)$. Thus, we know that $\sum_{i=1}^{n}(|z_i^{\alpha_{ii}}(t) - z_i^{*\alpha_{ii}}(t)| + |u_i(t) - u_i^*(t)|)$ are uniformly continuous on $[0, +\infty)$. According to Lemma 3.1, one has

$$\lim_{t \to +\infty} \sum_{i=1}^{n} \bigl(\bigl| z_i^{\alpha_{ii}}(t) - z_i^{*\alpha_{ii}}(t) \bigr| + \bigl| u_i(t) - u_i^*(t) \bigr| \bigr) = 0, \quad i = 1, 2, \ldots, n.$$

Therefore

$$\lim_{t \to +\infty} \bigl| z_i(t) - z_i(t) \bigr| = 0, \qquad \lim_{t \to +\infty} \bigl| u_i(t) - u_i^*(t) \bigr| = 0, \quad i = 1, 2, \ldots, n.$$

This completes the proof of Theorem 3.1. □

Corollary 3.1 *In addition to the conditions in Theorem* 2.1 (*or in Theorem* 2.2), *assume further that*:

$(\mathrm{H}_7)'$ $\alpha_{ii} \geq \{\alpha_{ji}, \beta_{ji}, \gamma_{ji}\}, 1 \leq j, i \leq n$;
$(\mathrm{H}_8)'$ *if there exist constants* $\rho_i > 0$, $\delta_i > 0$ *such that*

$$\begin{aligned} &\rho_i \lambda^{\alpha_{ii}} a_{ii}^L + \sum_{j=1}^{n} \rho_i \lambda^{\alpha_{ii}} M_j^{\alpha_{ij}} d_{ij}^L \\ &\quad > \delta_i \lambda^{\alpha_{ii}} g_i^M + \sum_{j=1, j\neq i}^{n} \rho_j \lambda^{\alpha_{ji}} a_{ji}^M + \sum_{j=1}^{n} \rho_j \lambda^{\alpha_{ji}} M_j^{\alpha_{ij}} d_{ji}^M \\ &\qquad + \sum_{j=1}^{n} \rho_j \lambda^{\beta_{ji}} \frac{b_{ji}^M}{1 - \dot{\tau}_{ji}^M} + \sum_{j=1}^{n} \rho_j \lambda^{\lambda_{ji}} c_{ji}^M, \\ &\delta_i \alpha_i^L > \rho_i f_i^M. \end{aligned}$$

Then the system (1.6) *has a unique globally asymptotically stable positive periodic solution* $(x_1^*(t), x_2^*(t), \ldots, x_n^*(t), u_1^*(t), u_2^*(t), \ldots, u_n^*(t))^T$.

4 Applications

In order to show the feasibility and the effectiveness of the results obtained, we will give some important competition models which have been well studied in the literature, and apply our main results to those examples, and we establish some new criteria to supplement and generalize some well-known results.

Example 4.1 Consider the following impulsive competition system with delays and feedback controls:

$$\begin{cases} \dot{x}_i(t) = x_i(t)[r_i(t) - \sum_{j=1}^{n} a_{ij}(t) x_j(t) - \sum_{j=1}^{n} b_{ij}(t) x_j(t - \tau_{ij}(t)) \\ \qquad - \sum_{j=1}^{n} c_{ij}(t) \int_{-\infty}^{t} K_{ij}(t-\theta) x_j(\theta)\, d\theta \\ \qquad - \eta_i(t) \int_{-\infty}^{t} H_i(t-\theta) u_i(\theta)\, d\theta], \quad t \neq t_k, \\ \dot{u}_i(t) = -\alpha_i(t) u_i(t) + \beta_i(t) \int_{-\infty}^{t} L_i(t-\theta) x_i(\theta)\, d\theta, \quad t \geq 0, \\ \Delta x_i = x_i(t_k^+) - x_i(t_k^-) = p_{ik} x_i(t_k^-), \quad i = 1, 2, \ldots, n, k \in N, \end{cases} \tag{4.1}$$

where $r_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $c_{ij}(t)$, $\alpha_i(t)$, $\beta_i(t)$, $\eta_i(t)$ are all nonnegative and continuous ω-periodic functions; $\tau_{ij}(t)$ is continuously differentiable such that $\tau_{ij}(t+\omega) = \tau_{ij}(t) \geq 0$, and $1 - \dot{\tau}_{ij}(t) > 0$. There exists a positive integer q such that $t_{k+q} = t_k + \omega$, $p_{i(k+q)} = p_{ik} \geq 0$; K_{ij}, H_i, L_i are integrable, ω-periodic and are normalized such that $\int_0^{+\infty} K_{ij}(\theta)\, d\theta = \int_0^{+\infty} L_i(\theta)\, d\theta = \int_0^{+\infty} H_i(\theta)\, d\theta = 1$ and $\int_0^{+\infty} \theta K_{ij}(\theta)\, d\theta < \infty$, $\int_0^{+\infty} \theta L_i(\theta)\, d\theta < \infty$, $\int_0^{+\infty} \theta H_i(\theta)\, d\theta < \infty$.

It is clear that the system (4.1) is a special case of the system (1.6), and by Theorem 2.1 and Theorem 3.1, we have the following results.

Theorem 4.1 *Assume that* $\alpha_i^L > 0$ *and*

(a) $\bar{r}_i + \bar{\Delta}_i > \sum_{j=1, j\neq i}^{n} \frac{\bar{a}_{ij} + \bar{b}_{ij} + \bar{c}_{ij}}{\bar{a}_{jj} + \bar{b}_{jj} + \bar{c}_{jj}} (\bar{r}_j + \bar{\Delta}_j) \exp\{[\bar{R}_j + \bar{r}_j + \bar{\Delta}_j]\omega + |\bar{\Delta}_j|\omega\}$.

Then the system (4.1) *has at least one positive* ω*-periodic solutions*. *Moreover*, *if there exist constants* $\rho_i > 0$, $\delta_i > 0$ *such that*

(b) $\inf_{t\in[0,+\infty)}\{\Phi_i(t),\Psi_i(t)\} > 0$, $i = 1,2,\dots,n$, *where*

$$\begin{aligned}\Phi_i(t) &= \rho_i a_{ii}(t) - \delta_i \int_0^{+\infty} \beta_i(t+\theta)L_i(\theta)\,d\theta \\ &\quad - \sum_{j=1,j\neq i}^{n} \rho_j a_{ji}(t) - \sum_{j=1}^{n} \rho_j \frac{b_{ji}(\sigma_{ji}^{-1}(t))}{1-\dot{\tau}_{ji}(\sigma_{ji}^{-1}(t))} \\ &\quad - \sum_{j=1}^{n} \rho_j \int_0^{+\infty} c_{ji}(t+\theta)K_{ji}(\theta)\,d\theta, \\ \Psi_i(t) &= \delta_i \alpha_i(t) - \rho_i \int_0^{+\infty} \eta_i(t+\theta)H_i(\theta)\,d\theta;\end{aligned}$$

or

(b)′ *if there exist constants* $\rho_i > 0$, $\delta_i > 0$, $i = 1,2,\dots,n$ *such that*

$$\begin{aligned}&\rho_i a_{ii}^L > \delta_i \beta_i^M + \sum_{j=1,j\neq i}^{n} \rho_j a_{ji}^M + \sum_{j=1}^{n} \rho_j \frac{b_{ji}^M}{1-\dot{\tau}_{ji}^M} + \sum_{j=1}^{n} \rho_j c_{ji}^M, \\ &\delta_i \alpha_i^L > \rho_i \eta_i^M.\end{aligned}$$

Then the system (4.1) *has a unique globally asymptotically stable positive periodic solution* $(x_1^*(t), x_2^*(t),\dots,x_n^*(t),u_1^*(t),u_2^*(t),\dots,u_n^*(t))^T$.

Remark 4.1 If $\tau_{ij}(t) = \tau_{ij}$, the system (4.1) is the system (1) in [16], our criteria on the existence of positive periodic solution are different from those in [16], which generalize one of the main results in [16].

Example 4.2 Consider the following n-species Lotka-Volterra competition system of integro differential equations:

$$\dot{x}_i(t) = x_i(t)\left[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t) - \sum_{j=1}^{n} b_{ij}(t) \int_{-\infty}^{t} K_{ij}(t-\theta)x_j(\theta)\,d\theta\right], \tag{4.2}$$

where $r_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $i = 1,2,\dots,n$, are all nonnegative and continuous ω-periodic functions; K_{ij} are integrable, ω-periodic and normalized functions such that $\int_0^{+\infty} K_{ij}(\theta)\,d\theta = 1$ and $\int_0^{+\infty} \theta K_{ij}(\theta)\,d\theta < \infty$.

According to Theorem 2.1 and Theorem 3.1, we have the following results.

Theorem 4.2 *Assume that*:

(a) $\bar{r}_i > \sum_{j=1,j\neq i}^{n} \frac{\bar{a}_{ij}+\bar{b}_{ij}}{\bar{a}_{jj}+\bar{b}_{jj}} \bar{r}_j \exp\{(\bar{R}_j + \bar{r}_j)\omega\}$;

(b) *there exist constants* $\rho_i > 0$, $i = 1,2,\dots,n$ *such that*

$$\inf_{t\in[0,+\infty)}\left\{\rho_i a_{ii}(t) - \sum_{j=1,j\neq i}^{n} \rho_j a_{ji}(t) - \sum_{j=1}^{n} \rho_j \int_0^{+\infty} b_{ji}(t+\theta)K_{ji}(\theta)\,d\theta\right\} > 0;$$

or

(b)′ *if there exist constants* $\rho_i > 0$, $i = 1, 2, \ldots, n$ *such that*

$$\rho_i a_{ii}^L > \sum_{j=1, j\neq i}^{n} \rho_j a_{ji}^M + \sum_{j=1}^{n} \rho_j b_{ji}^M.$$

Then the system (4.2) *has a unique globally asymptotically stable positive periodic solution* $(x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$.

Remark 4.2 If $a_{ij}(t) = 0$ $(i \neq j)$, then the system (4.2) is the system (1.1) considered by Xu *et al.* [18]. Xu *et al.* [18] studied the global asymptotic stability of the positive solution of the system. Obviously, our criteria on global asymptotic stability of the system are weaker than those in [18], which improve the main results in [18].

In particular, when $b_{ii}(t) = 0$, the system (4.2) reduced to the following system:

$$\dot{x}_i(t) = x_i(t)\left[r_i(t) - a_{ii}(t)x_i(t) - \sum_{j=1, j\neq i}^{n} b_{ij}(t) \int_{-\infty}^{t} K_{ij}(t-\theta)x_j(\theta)\,d\theta\right]. \tag{4.3}$$

Gopalsamy [19] also studied the existence of globally stable periodic solution of the above model and proved the following results.

(a) The delay kernels K_{ij} $(it = j)$ are piecewise (locally) continuous such that the series $\sum_{r=0}^{\infty} K_{ij}(u + r\omega)$ converges uniformly with respect to u on $[0, \omega]$.

(b) $r_i^L > 0$ and $b_{ij}^L > 0$.

(c) $r_i^L > \sum_{j=1, j\neq i}^{n} \frac{b_{ij}^M}{a_{jj}^L} r_j^M$.

(d) There exists a positive constant $m > 0$ such that $a_{ii}^L > \sum_{j=1, j\neq i}^{n} b_{ji}^M + m$.

Then the system (4.3) has a unique globally asymptotically stable positive ω-periodic solution.

It is clear that our conditions on the global asymptotic stability of the system (4.3) are different and are weaker than those in [19], as criterion (d) implies with $\rho_i = 1$. So Theorem 4.2 supplements and generalizes Theorem 2.1 and Theorem 3.1 obtained by [19].

Example 4.3 Consider the n-species non-autonomous Lotka-Volterra competition system with infinite delays and feedback controls

$$\begin{cases} \dot{x}_i(t) = x_i(t)[r_i(t) - a_i(t)x_i(t) - \sum_{j=1}^{n} a_{ij}(t) \int_{-\infty}^{t} K_{ij}(t-\theta)x_j(\theta)\,d\theta \\ \qquad - b_i(t) \int_{-\infty}^{t} H_i(t-\theta)u_i(\theta)\,d\theta], \\ \dot{u}_i(t) = -c_i(t)u_i(t) + d_i(t) \int_{-\infty}^{t} R_i(t-\theta)x_i(\theta)\,d\theta, \end{cases} \tag{4.4}$$

where $r_i(t)$, $a_i(t)$, $b_i(t)$, $a_{ij}(t)$, $c_i(t)$, $d_i(t)$, $i = 1, 2, \ldots, n$, are all nonnegative and continuous ω-periodic functions; K_{ij}, H_i, R_i are integrable, ω-periodic, and normalized such that $\int_0^{+\infty} K_{ij}(\theta)\,d\theta = \int_0^{+\infty} R_i(\theta)\,d\theta = \int_0^{+\infty} H_i(\theta)\,d\theta = 1$ and $\int_0^{+\infty} \theta K_{ij}(\theta)\,d\theta < \infty$, $\int_0^{+\infty} \theta R_i(\theta)\,d\theta < \infty$, $\int_0^{+\infty} \theta H_i(\theta)\,d\theta < \infty$.

Theorem 4.3 *Assume that* $c_i^L > 0$ *and*

(a) $\bar{r}_i > \sum_{j=1, j\neq i}^{n} \frac{\bar{a}_{ij}}{\bar{a}_j + \bar{a}_{jj}} \bar{r}_j \exp\{(\bar{R}_j + \bar{r}_j)\omega\}$.

Then the system (4.4) *has at least one positive* ω*-periodic solutions. Moreover, if there exist constants* $\rho_i > 0$, $\delta_i > 0$ *such that*

(b) $\inf_{t\in[0,+\infty)}\{\Phi_i(t),\Psi_i(t)\}>0$, $i=1,2,\ldots,n$, *where*

$$\begin{aligned}\Phi_i(t)&=\rho_i a_i(t)-\delta_i\int_0^{+\infty}d_i(t+\theta)R_i(\theta)\,d\theta\\&\quad-\sum_{j=1}^n\rho_j\int_0^{+\infty}a_{ji}(t+\theta)K_{ji}(\theta)\,d\theta,\\\Psi_i(t)&=\delta_i c_i(t)-\rho_i\int_0^{+\infty}b_i(t+\theta)H_i(\theta)\,d\theta;\end{aligned}$$

or

(b)′ *if there exist constants* $\rho_i>0$, $\delta_i>0$, $i=1,2,\ldots,n$ *such that*

$$\rho_i a_i^L>\delta_i d_i^M+\sum_{j=1}^n\rho_j a_{ji}^M,\qquad \delta_i c_i^L>\rho_i b_i^M.$$

Then the system (4.4) *has a unique globally asymptotically stable positive periodic solution* $(x_1^*(t),x_2^*(t),\ldots,x_n^*(t),u_1^*(t),u_2^*(t),\ldots,u_n^*(t))^T$.

Remark 4.3 Chen [20] investigated the global asymptotic stability of the model (4.4). It is easy to see our results supplement those in [20].

Furthermore, when $a_{ii}(t)=0$, Weng [21] also considered the existence and global stability of a positive periodic solution of a special model. If $\rho_i=1$, $\delta_i=1$, then the conditions (b) are equivalent to conditions (3.2) of [21]. Hence, Theorem 4.3 is more up to date, it generalizes the main results in [21].

Example 4.4 Consider the following Lotka-Volterra competition system with several deviating arguments:

$$\dot{x}_i(t)=x_i(t)\left[r_i(t)-\sum_{j=1}^n a_{ij}(t)x_j(t)-\sum_{j=1}^n b_{ij}(t)x_j\bigl(t-\tau_{ij}(t)\bigr)\right],\tag{4.5}$$

where $r_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $i=1,2,\ldots,n$, are all nonnegative and continuous ω-periodic functions; $\tau_{ij}(t)$ are continuously differentiable such that $\tau_{ij}(t+\omega)=\tau_{ij}(t)\geq 0$, and $1-\dot{\tau}_{ij}(t)>0$.

By Theorem 2.1 and Theorem 3.1, we have the following results.

Theorem 4.4 *Assume that*:

(a) $\bar{r}_i>\sum_{j=1,j\neq i}^n\frac{\bar{a}_{ij}+\bar{b}_{ij}}{\bar{a}_{jj}+\bar{b}_{jj}}\bar{r}_j\exp\{(\bar{R}_j+\bar{r}_j)\omega\}$;

(b) *there exist constants* $\rho_i>0$ *such that*

$$\inf_{t\in[0,+\infty)}\left\{\rho_i a_{ii}(t)-\sum_{j=1,j\neq i}^n\rho_j a_{ji}(t)-\sum_{j=1}^n\rho_j\frac{b_{ji}(\sigma_{ji}^{-1}(t))}{1-\dot{\tau}_{ji}(\sigma_{ji}^{-1}(t))}\right\}>0,\quad i=1,2,\ldots,n;$$

or

(b)′ *if there exist constants* $\rho_i>0$, $i=1,2,\ldots,n$ *such that*

$$\rho_i a_{ii}^L>\sum_{j=1,j\neq i}^n\rho_j a_{ji}^M+\sum_{j=1}^n\rho_j\frac{b_{ji}^M}{1-\dot{\tau}_{ji}^M}.$$

Then the system (4.5) *has an ω-periodic solution, which is globally asymptotically stable.*

Remark 4.4 When $\tau_{ij}(t) = \tau_{ij}$, the system (4.5) was investigated by Fan *et al.* [22, 23]. The conditions on global asymptotic stability in [22, 23] should be set with $\rho_i = 1$.

Remark 4.5 When $b_{ij}(t) = 0$, Zhao [24] studied the existence and global attractivity of a positive periodic solution of the model. Our results are more easily verified and more general than those in [24]. In particular, when $n = 1$, the special model reduced to the classical logistic equation. Our results generalize some well-known results.

Example 4.5 Consider the following Lotka-Volterra competition system with infinite delays:

$$\dot{x}_i(t) = x_i(t)\Biggl[r_i(t) - a_i(t)x_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j\bigl(t-\tau_{ij}(t)\bigr) - \sum_{j=1}^{n} b_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-\theta)x_j(\theta)\,d\theta\Biggr], \tag{4.6}$$

where $r_i(t)$, $a_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $i = 1, 2, \ldots, n$, are all nonnegative and continuous ω-periodic functions; $\tau_{ij}(t)$ is continuously differentiable such that $\tau_{ij}(t+\omega) = \tau_{ij}(t) \geq 0$, and $1 - \dot{\tau}_{ij}(t) > 0$; K_{ij} are integrable, ω-periodic, and normalized such that $\int_0^{+\infty} K_{ij}(\theta)\,d\theta = 1$ and $\int_0^{+\infty} \theta K_{ij}(\theta)\,d\theta < \infty$.

Theorem 4.5 *Assume that*:

(a) $\bar{r}_i > \sum_{j=1,j\neq i}^{n} \frac{\bar{a}_{ij}+\bar{b}_{ij}}{\bar{a}_j+\bar{a}_{jj}+\bar{b}_{jj}} \bar{r}_j \exp\{(\bar{R}_j + \bar{r}_j)\omega\}$;

(b) *there exist constants $\rho_i > 0$, $i = 1, 2, \ldots, n$, such that*

$$\inf_{t\in[0,+\infty)} \Biggl\{\rho_i a_i(t) - \sum_{j=1}^{n} \rho_j \frac{a_{ji}(\sigma_{ji}^{-1}(t))}{1-\dot{\tau}_{ji}(\sigma_{ji}^{-1}(t))} - \sum_{j=1}^{n} \rho_j \int_0^{+\infty} b_{ji}(t+\theta)K_{ji}(\theta)\,d\theta\Biggr\} > 0;$$

or

(b)′ *if there exist constants $\rho_i > 0$, $i = 1, 2, \ldots, n$ such that*

$$\rho_i a_i^L > \sum_{j=1,j\neq i}^{n} \rho_j \frac{a_{ji}^M}{1-\dot{\tau}_{ji}^M} + \sum_{j=1}^{n} \rho_j b_{ji}^M.$$

Then the system (4.6) *has an ω-periodic solution, which is globally asymptotically stable.*

Remark 4.6 Xu *et al.* [25] studied the global asymptotic stability of the system (4.6). Obviously, our criteria are more easily verifiable than those in [25].

Example 4.6 Consider the following n-species delay impulsive Lotka-Volterra competition system:

$$\begin{cases} \dot{x}_i(t) = x_i(t)[r_i(t) - a_{ii}(t)x_i(t) - \sum_{j=1,j\neq i}^{n} a_{ij}(t)x_j(t-\tau_{ij}(t))], & t \neq t_k, \\ \Delta x_i = x_i(t_k^+) - x_i(t_k^-) = b_{ik}x_i(t_k^-), & i = 1, 2, \ldots, n, k \in N, \end{cases} \tag{4.7}$$

where $r_i(t)$, $a_{ii}(t)$, $a_{ij}(t)$ are all nonnegative and continuous ω-periodic functions; $\tau_{ij}(t)$ is continuously differentiable such that $\tau_{ij}(t+\omega) = \tau_{ij}(t) \geq 0$, and $1 - \dot{\tau}_{ij}(t) > 0$. There exists a positive integer q such that $t_{k+q} = t_k + \omega$, $b_{i(k+q)} = b_{ik} \geq 0$.

Theorem 4.6 *Assume that*:

(a) $\bar{r}_i + \bar{\triangle}_i > \sum_{j=1,j\neq i}^{n} \frac{\bar{a}_{ij}}{\bar{a}_{jj}} (\bar{r}_j + \bar{\triangle}_j) \exp\{[\bar{R}_j + \bar{r}_j + \bar{\triangle}_j]\omega + |\bar{\triangle}_j|\omega\}$;
(b) *there exist constants* $\rho_i > 0$, $i = 1, 2, \ldots, n$ *such that*

$$\inf_{t\in[0,+\infty)} \left\{ \rho_i a_{ii}(t) - \sum_{j=1,j\neq i}^{n} \rho_j \frac{a_{ji}(\sigma_{ji}^{-1}(t))}{1 - \dot{\tau}_{ji}(\sigma_{ji}^{-1}(t))} \right\} > 0, \quad i = 1, 2, \ldots, n;$$

or

(b)′ *if there exist constants* $\rho_i > 0$, $i = 1, 2, \ldots, n$ *such that*

$$\rho_i a_{ii}^L > \sum_{j=1,j\neq i}^{n} \rho_j \frac{a_{ji}^M}{1 - \dot{\tau}_{ji}^M}.$$

Then the system (4.7) *has a unique globally asymptotically stable positive periodic solution*.

Remark 4.7 Stamova [26] explored the existence and global asymptotic stability of positive periodic solutions of the model (4.7). Our results are different from those in [26]. The case $r_i(t) < 0$ is considered by Li *et al.* [27]. Therefore, our results supplement some well-known results in [27].

5 Concluding remarks

In this paper, we study an impulsive nonlinear periodic competition model with delays and feedback controls. In mathematical ecology, the system (1.6) describes a system of the dynamics of an n-species model in which each individual competes with all the others of the model for a common resource, and the intra-species competition involves deviating arguments $\tau_{ij}(t)$ such that $0 \leq \tau_{ij}(t) \leq \tau$ where τ is a constant and time delays extend over the entire past as denoted by K_{ij}, H_i, R_i in (1.6). By means of coincidence degree theory, a set of sufficient conditions for the global existence of positive periodic solution of the system (1.6) are established, and constructing the suitable Lyapunov functional, some easily verifiable weaker sufficient conditions for the global asymptotic stability of positive periodic solution of the system (1.6) are obtained. Our results supplement and generalize some well-known results which have been well studied in the literature.

Competing interests
The author declares that they have no competing interests.

Acknowledgements
The author is grateful to the Editor and referees for a number of helpful suggestions that have greatly improved our original submission. This research is supported by the National Natural Science Foundation of China (71471030), MOE (Ministry of Education in China) Youth Foundation Project of Humanities and Social Sciences (13YJC790185), General Project for Scientific Research of Liaoning Educational Committee (L2014458) and General Project of Dongbei University of Finance and Economics (DUFE2015Y33).

Received: 3 June 2016 Accepted: 20 September 2016 Published online: 02 November 2016

References

1. Lotka, A: Elements of Physical Biology. Williams & Wilkins, Baltimore (1924)
2. Volterra, V: Variazioni e fluttuazioni del numero d'individui in specie d'animali conviventi. Mem. R. Accad. Naz. Lincei **2**, 31-113 (1926)
3. Cui, J, Chen, L: Asymptotic behavior of the solution for a class of time-dependent competitive system with feedback controls. Ann. Differ. Equ. **9**(1), 11-17 (1993)
4. Xia, YH, Cao, J, Zhang, H, Chen, FD: Almost periodic solutions of *n*-species competitive system with feedback controls. J. Math. Anal. Appl. **294**(2), 503-522 (2004)
5. Chattopadhyay, J: Effect of toxic substance on a two-species competitive system. Ecol. Model. **84**(1-3), 287-289 (1996)
6. He, C: On almost periodic solutions of Lotka-Volterra almost periodic competition systems. Ann. Differ. Equ. **9**(1), 26-36 (1993)
7. Ayala, FJ, Gilpin, ME, Eherenfeld, JG: Competition between species: theoretical models and experimental tests. Theor. Popul. Biol. **4**, 331-356 (1973)
8. Gilpin, ME, Ayala, FJ: Global models of growth and competition. Proc. Natl. Acad. Sci. USA **70**, 3590-3593 (1973)
9. Goh, BS, Agnew, TT: Stability in Gilpin and Ayala's model of competition. J. Math. Biol. **4**, 275-279 (1977)
10. Liao, XX, Li, J: Stability in Gilpin-Ayala competition models with diffusion. Nonlinear Anal. **28**, 1751-1758 (1997)
11. Chen, FD: Average conditions for permanence and extinction in nonautonomous Gilpin-Ayala competition model. Nonlinear Anal., Real World Appl. **4**, 895-915 (2006)
12. Ahmad, S: On the nonautonomous Lotka-Volterra competition equations. Proc. Am. Math. Soc. **117**, 199-204 (1993)
13. Weng, PX: Existence and global stability of positive periodic solution of periodic integrodifferential systems with feedback controls. Comput. Math. Appl. **40**(6-7), 747-759 (2000)
14. Wang, Q, Ding, MM, Wang, ZJ, Zhang, HY: Existence and attractivity of a periodic solution for an *N*-species Gilpin-Ayala impulsive competition system. Appl. Math. Comput. **170**, 1452-1468 (2005)
15. Ding, XH, Lu, C, Liu, MZ: Periodic solutions for a semi-ratio-dependent predator-prey system with nonmonotonic functional response and time delay. Nonlinear Anal., Real World Appl. **9**, 762-775 (2008)
16. Yang, ZC, Xu, DY: Periodic solutions and stability of impulsive competitive systems with infinitely distributed delays and feedback controls. Acta Math. Appl. Sin. **32**(1), 132-142 (2009)
17. Wang, WD, Chen, LS, Lu, ZY: Global stability of a competition model with periodic coefficients and time delay. Can. Appl. Math. Q. **3**, 365-378 (1995)
18. Xu, R, Chaplain, MAJ, Chen, LS: Global asymptotic stability in *n*-species nonautonomous Lotka-Volterra competitive systems with infinite delays. Appl. Math. Comput. **130**, 295-309 (2002)
19. Gopalsamy, K: Global asymptotic stability in a periodic integro differential system. Tohoku Math. J. **37**, 323-332 (1985)
20. Chen, FD: Global asymptotic stability in *n*-species non-autonomous Lotka-Volterra competitive systems with infinite delays and feedback controls. Appl. Math. Comput. **111**, 2675-2685 (2010)
21. Weng, PX: Global attractivity in a periodic competition system with feedback controls. Acta Math. Appl. Sin. **12**(1), 11-21 (1996)
22. Fan, M, Wang, K, Jiang, DQ: Existence and global attractivity of positive periodic solutions of periodic *n*-species Lotka-Volterra competitive systems with several deviating arguments. Math. Biosci. **160**, 47-61 (1999)
23. Fan, M, Wang, K: Existence and global attractivity of positive periodic solution of multispecies ecological system. Acta Math. Sin. **43**(1), 77-82 (2000)
24. Zhao, XQ: The qualitative analysis of *n*-species Lotka-Volterra periodic competition systems. Math. Comput. Model. **15**, 3-8 (1991)
25. Xu, R, Chaplain, MAJ, Chen, LS: Global asymptotic stability in *n*-species nonautonomous Lotka-Volterra competitive systems with delays. Appl. Math. Comput. **23**(B), 208-218 (2003)
26. Stamova, IM: Existence and global asymptotic stability of positive periodic solutions of *n*-species delay impulsive Lotka-Volterra type systems. J. Biol. Dyn. **5**(6), 619-635 (2011)
27. Li, M, Duan, Y, Zhang, W, Wang, M: The existence of positive periodic solutions of a class of Lotka-Volterra type impulsive systems with infinitely distributed delay. Comput. Math. Appl. **49**, 1037-1044 (2005)
28. Tineo, A: On the asymptotic behavior of some population model. J. Math. Anal. Appl. **167**, 516-529 (1992)
29. Yan, JR, Liu, GR: Periodicity and stability for a Lotka-Volterra type competition system with feedback controls and deviating. Nonlinear Anal. **74**(9), 2916-2928 (2011)
30. Xie, D, Jiang, Y: Global exponential stability of periodic solution for delayed complex-valued neural networks with impulses. Neurocomputing **207**(26), 528-538 (2016)
31. Zuo, W, Jiang, D: Periodic solutions for a stochastic non-autonomous Holling-Tanner predator-prey system with impulses. Nonlinear Anal. Hybrid Syst. **22**, 191-201 (2016)
32. Du, Z, Xu, M: Positive periodic solutions of *n*-species neutral delayed Lotka-Volterra competition system with impulsive perturbations. Appl. Math. Comput. **243**(15), 379-391 (2014)
33. Zhen, J, Ma, ZE, Han, MA: The existence of periodic solutions of the *n*-species Lotka-Volterra competition systems with impulsive. Chaos Solitons Fractals **22**(1), 161-176 (2004)
34. Meng, XZ, Chen, LS: Periodic solution and almost periodic solution for a nonautonomous Lotka-Volterra dispersal system with infinite delay. J. Math. Anal. Appl. **339**, 125-145 (2008)
35. Muroya, Y: Persistence and global stability in Lotka-Volterra delay differential systems. Appl. Math. Lett. **17**, 795-800 (2004)
36. Xu, CJ, Wu, YS: Dynamics in a Lotka-Volterra predator-prey model with time-varying delays. Abstr. Appl. Anal. **2013**, Article ID 956703 (2013)
37. Lu, HY, Wang, WG: Dynamics of a nonautonomous Leslie-Gower type food chain model with delay. Discrete Dyn. Nat. Soc. **2011**, Article ID 308279 (2011)
38. Yu, G, Lu, HY: Permanence and almost periodic solutions of a discrete ratio-dependent Leslie system with time delays and feedback controls. Abstr. Appl. Anal. **2012**, Article ID 358594 (2012)
39. Yang, P, Xu, R: Global attractivity of the periodic Lotka-Volterra system. J. Math. Anal. Appl. **233**, 221-232 (1999)
40. Wang, L, Zhang, L, Ding, X: Global dissipativity of a class of BAM neural networks with both time-varying and continuously distributed delays. Neurocomputing **152**(25), 250-260 (2015)

41. Lu, HY, Yu, G: Permanence of a Gilpin-Ayala predator-prey system with time-dependent delay. Adv. Differ. Equ. **2015**, 109 (2015)
42. Gaines, R, Mawhin, J: Coincidence Degree and Nonlinear Differential Equations. Springer, Berlin (1997)
43. Barbălat, I: Systèmes d'équations différentielles d'oscillations non linéaires. Rev. Roum. Math. Pures Appl. **4**(2), 267-270 (1959)