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Research Article

Reverse Link Outage Probabilities of Multicarrier CDMA Systems with Beamforming in the Presence of Carrier Frequency Offset

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The outage probability of reverse link multicarrier (MC) code-division multiple access (CDMA) systems with beamforming in the presence of carrier frequency offset (CFO) is studied. A conventional uniform linear array (ULA) beamformer is utilized. An independent Nakagami fading channel is assumed for each subcarrier of all users. The outage probability is first investigated under a scenario where perfect beamforming is assumed. A closed form expression of the outage probability is derived. The impact of different types of beamforming impairments on the outage probability is then evaluated, including direction-of-arrival (DOA) estimation errors, angle spreads, and mutual couplings. Numerical results show that the outage probability improves significantly as the number of antenna elements increases. The effect of CFO on the outage probability is reduced significantly when the beamforming technique is employed. Also, it is seen that small beamforming impairments (DOA estimation errors and angle spreads) only affect the outage probability very slightly, and the mutual coupling between adjacent antenna elements does not affect the outage probability noticeably.

1. INTRODUCTION

Future wireless communication systems demand high-data-rate multimedia transmissions in diverse mobile environments. The underlying wideband nature makes the overall system vulnerable to the hostile frequency-selective multipath fading. Code-division multiple access (CDMA) has received tremendous attentions because it offers various attractive features such as high spectrum efficiency, narrow-band interference rejection, and soft capacity [1, 2]. Recently, the multicarrier (MC) CDMA system, which is a combination of orthogonal frequency division multiplexing (OFDM) and CDMA, has gained significant interests as a powerful candidate for future wireless broadband communications [3]. Multicarrier CDMA inherits distinct advantages from both OFDM and CDMA. By dividing the full available bandwidth into a large number of small orthogonal narrow bands or subcarriers each having bandwidth much less than the channel coherent bandwidth, the transmission over each subcarrier will experience frequency nonselective fading. Also, it can be interpreted as CDMA with spreading taking place in the frequency domain rather than temporal domain, achieving enhanced frequency diversity. MC-CDMA is basically a multicarrier transmission scheme and its receiver is vulnerable to carrier frequency offset (CFO) which is due to the mismatch in frequencies between the local oscillators in the transmitter and the receiver.

Antenna array techniques are used to reduce interference to meet increased capacity requirements without sacrificing the frequency spectrum [4, 5], which can be realized through space diversity, beamforming, and spatial multiplexing [6]. In this paper, the use of conventional uniform linear array (ULA) beamformer [16] is to provide performance improvements in MC-CDMA systems, especially with the consideration of CFO.

The outage probability is an important performance measure in the design of wireless communication systems, which represents the probability of unsatisfactory reception

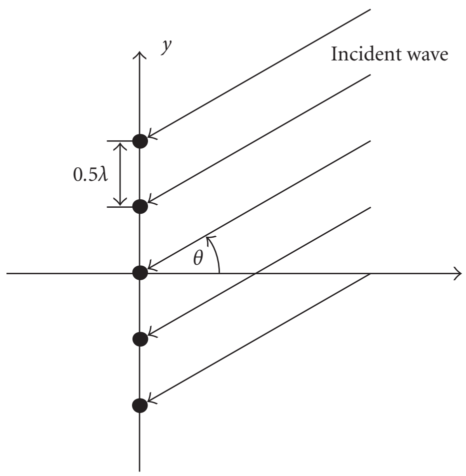


Figure 1: ULA antenna array.

over an intended coverage area. The performance in terms of the bit-error rate (BER) for MC-CDMA systems has been investigated in a number of literatures, either assuming perfect carrier frequency synchronization [8, 9] or with CFO [10–13]. There have been several papers studying the outage probability performance in various CDMA systems [14, 15, 18]. However, MC-CDMA systems have not been examined in such studies.

In this paper, the reverse link of an MC-CDMA system with the beamforming technique in the presence of CFO is considered, and we concentrate the analysis on the outage probability performance. A Nakagami fading channel is assumed throughout the paper. Based on a newly developed simplified beamforming model [18], a closed-form expression is derived for the outage probability when perfect beamforming is considered. The impact of CFO and beamforming is modeled in signal and interference expressions. Furthermore, the effect of various beamforming impairments is examined, including direction-of-arrival (DOA) estimation errors, angle spreads, and mutual couplings. To summarize, this paper differs from previous research mainly in two aspects: first, we develop signal and interference models to characterize the beamforming gain and CFO in MC-CDMA systems; second, outage probabilities are derived for MC-CDMA systems with either perfect or imperfect beamforming in the presence of CFO.

The remainder of the paper is organized as follows. The system model is described in Section 2. The outage probability for MC-CDMA with beamforming in the presence of CFO is presented in Section 3. The effect of impairments in beamforming is investigated in Section 4. Numerical results are presented and discussed in Section 5. Conclusions are given in Section 6.

2. SYSTEM MODEL

2.1. Beamforming

Due to the space limitation of mobile terminals, few antenna elements can be employed at the mobile station (MS). While at the base station (BS), a large number of antenna elements can be implemented in an array. Considering receive beamforming in reverse-link transmissions, signals from these antenna elements are combined to form a movable beam pattern that can be steered to a desired direction to track the MS as it moves [17, 18]. When beamforming is used at the MS, the transmit beam pattern can be adjusted to minimize interference to unintended receivers. At the BS, receive beamforming for each desired user could be implemented independently without affecting the performance of other links [17, 18]. A ULA beamformer is considered and shown in Figure 1, in which θ is an arrival angle. In this paper, a two-dimension (2D) single-cell environment is considered. The distance d between elements of the ULA array is assumed to be 0.5λ, where λ is the carrier wavelength. In the ULA array system, a combining network connects an array of low-gain antenna elements and could generate an antenna pattern [17, 19]:

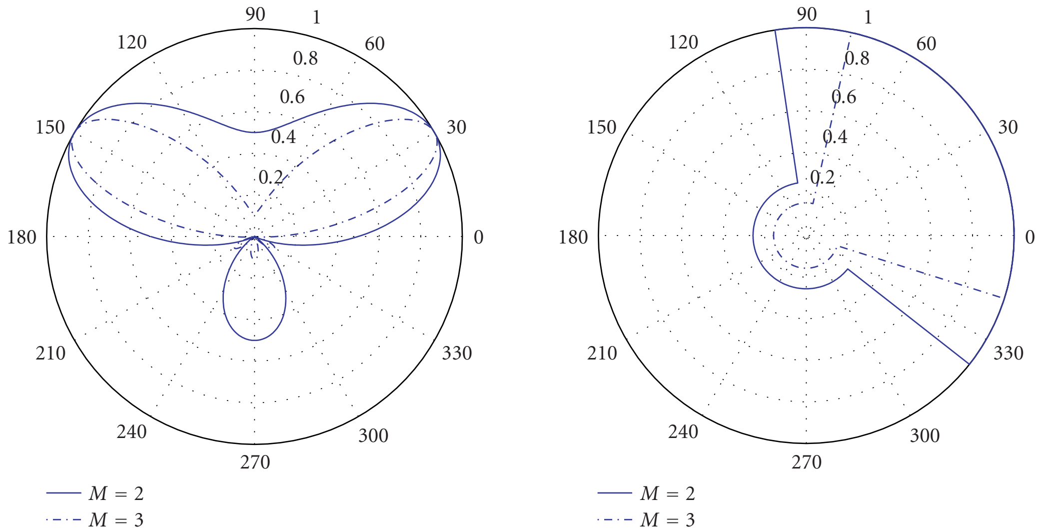
$$G(\theta,\psi) = \left| \frac{\sin\left(0.5M\pi(\sin\theta - \sin\psi)\right)}{M\sin\left(0.5\pi(\sin\theta - \sin\psi)\right)} \right|^2, \tag{1}$$

where M is the number of antenna elements and ψ is a scan angle. The beam could be steered to a desired direction by varying ψ, that is to say, setting ψ equal to the arrival angle θ of the desired signal. Hereafter, we will use the antenna pattern specified in (1) to evaluate the outage probability for MC-CDMA systems with beamforming in reverse link transmissions.

2.2. Simplified beamforming model

The analytical complexity in evaluating the exact beam pattern is very high when a large number of interfering users are present in the MC-CDMA system, especially for the investigation of effects of beamforming impairments such as DOA estimation errors, angle spreads, and mutual couplings. A simplified Bernoulli model is introduced in [20] where the signal is considered to be either within the mainlobe ($G = 1$) or out of the mainlobe ($G = 0$) and the half-power beamwidth is defined as the beamwidth. This model is easy to use but it neglects the impact of sidelobes and the effect of any specific beam patterns. Spagnolini provides a beamforming model in [21] with a triangular pattern to characterize the beam head. A beamforming model that takes into account the impact of sidelobes and the actual beam patterns is introduced in [18]. The beamwidth is assumed to be B which is normalized by 2π. The gain of the mainlobe is normalized to unity, while the gain in sidelobe is α. This implies that one interferer stays in the mainlobe with probability B. Considering an exact beam pattern and normalizing the pattern by the gain at the desired direction, these two parameters α and B are determined by

$$\alpha = \frac{E\{G^2(\theta,\psi)\} - E\{G(\theta,\psi)\}}{E\{G(\theta,\psi)\} - 1},$$
$$B = \frac{E\{G^2(\theta,\psi)\} - E^2\{G(\theta,\psi)\}}{E\{G^2(\theta,\psi)\} + 1 - 2E\{G(\theta,\psi)\}}, \tag{2}$$



(a) Signal model

(b) Interference mode

Figure 2: A simplified beamforming model with arrival angle $\theta = 30^{\circ}$.

where $E\{G(\theta, \psi)\}$ and $E\{G^2(\theta, \psi)\}$ are the first and second moments of the antenna gain, respectively, averaged with respect to uniformly distributed random variables (RVs) θ and ψ from 0 to 2π. We have to point out that throughout the paper the desired user still uses the exact beam pattern as illustrated in Figure 2(a), nevertheless, multiuser interferers will use the above simplified beam pattern with parameters α and B as shown in Figure 2(b) in performance evaluations.

2.3. MC-CDMA

A reverse link MC-CDMA system with beamforming in the presence of CFO is considered. The number of subcarriers is chosen so that the bit duration is assumed to be much longer than channel delay spread such that the signal in each subcarrier will undergo flat fading. Suppose that there are K asynchronous users, each employing L subcarriers and using binary phase-shift keying (BPSK) with the same power S and bit duration T_b. The signal is spread in the frequency domain with the spreading gain L which is also equal to the number of subcarriers. Δf_k is the CFO between oscillators of the kth user's transmitter and the receiver of the BS. The Nakagami-m fading channel is assumed over each subcarrier with its probability density function (PDF):

$$f_{\beta_{k,l}}(\beta_{k,l}) = \frac{2m^m \beta_{k,l}^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m\beta_{k,l}^2}{\Omega}\right) \quad l = 0, 1, \ldots, L-1, \tag{3}$$

where $\beta_{k,l}$ is the channel fading gain on the lth subcarrier of the kth user and is assumed to be independent for different l and k, m is the Nakagami-m fading parameter which ranges from 1/2 to ∞, $\Omega = E\{\beta_{k,l}^2\}$, and $\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt$ is a gamma function.

Assuming that the maximum ratio combining (MRC) technique is used, and following [10, 11, 18], the received signal can be expressed as

$$U = \sum_{l=0}^{L-1} \sqrt{\Xi}\, \beta_{0,l}^2 + I, \tag{4}$$

where $\Xi = 2[SG_t(\theta_0 - \pi, \theta_0 - \pi) G_r(\theta_0, \psi)] \cdot \mathrm{sinc}^2(\varepsilon)$, and I represents the interference and noise items. Hence, the received power from desired 0th user can be expressed as

$$E_b = 2[SG_t(\theta_0 - \pi, \theta_0 - \pi) G_r(\theta_0, \psi)] \cdot \mathrm{sinc}^2(\varepsilon) \left(\sum_{l=0}^{L-1} \beta_{0,l}^2\right)^2, \tag{5}$$

where $G_t(\theta_0 - \pi, \theta_0 - \pi)$ and $G_r(\theta_0, \psi)$ are the transmit and receive beamforming gain, respectively; $\theta_0 - \pi$ and θ_0 are the transmit angle and arrival angle from the 0th user to the BS, respectively; ψ is the estimated arrival angle that is used to steer the beam to the desired 0th user and is assumed to be equal to θ_0, that is, $\psi = \theta_0$; sinc $(x) = \sin(\pi x)/\pi x$ and $\varepsilon = \Delta f_0 T_b$ is the normalized CFO (NCFO) for the desired 0th user, and assume that $\varepsilon \in [0, 1]$; denote $\varepsilon_k = \Delta f_k T_b (k = 1, \ldots, K-1)$ as the NCFO for the kth interfering user which is uniformly distributed over $[0, \varepsilon]$. Figure 3 indicates angle notations in transmit beamforming at the MS and receive beamforming at the BS.

The interference power E_I can be divided into three parts [10], self-interference (SI) from other subcarriers E_{so}, multiuser interference (MUI) from the same subcarriers E_{ms},

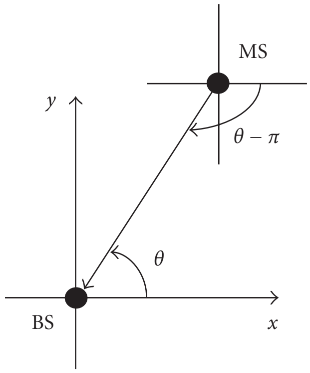


Figure 3: Angle notations for transmit beamforming and receive beamforming.

and MUI from other subcarriers E_{mo}. Hence, we have $E_I = E_{so} + E_{ms} + E_{mo}$. The SI power E_{so} can be written as

$$E_{so} = [SG_t(\theta_0 - \pi, \theta_0 - \pi) G_r(\theta_0, \psi)]\Omega \cdot \sum_{l=0}^{L-1} \sum_{h=0, h\neq l}^{L-1} \operatorname{sinc}^2(l - h - \varepsilon) \cdot \beta_{0,l}^2. \tag{6}$$

The interference power E_{ms} can be expressed as

$$E_{ms} = \sum_{k=1}^{K-1} [SG_t(\theta_k - \pi, \theta_k - \pi) G_r(\theta_k, \psi)] \frac{\Omega}{\pi^2 \varepsilon^2} \cdot \left[-1 + {}_pF_q \left(\left\{ -\frac{1}{2} \right\}; \left\{ \frac{1}{2}, \frac{2}{3} \right\}; -\pi^2 \varepsilon^2 \right) \right] \sum_{l=0}^{L-1} \beta_{0,l}^2, \tag{7}$$

where $G_t(\theta_k - \pi, \theta_k - \pi)$ and $G_r(\theta_k, \psi)$ are the transmit and receive beamforming gain, respectively; $\theta_k - \pi$ and θ_k are the transmit angle and arrival angle from the kth user to the BS, respectively; ${}_pF_q(\mathbf{a}; \mathbf{b}; z)$ is a generalized hypergeometric function [22], and the interference power E_{mo} is given by

$$E_{mo} = \sum_{k=1}^{K-1} [SG_t(\theta_k - \pi, \theta_k - \pi) G_r(\theta_k, \psi)] \frac{\Omega}{\pi^2 \varepsilon} \cdot \sum_{l=0}^{L-1} \sum_{h=0, h\neq 1}^{L-1} [g(l-h, \varepsilon) - g(l-h, 0)] \cdot \beta_{0,l}^2, \tag{8}$$

where

$$g(x, y) = \frac{1}{2(x-y)} \{2 - \cos[2\pi(x-y)] - \operatorname{sinc}[2 - (x-y)] - 2\pi(x-y)\operatorname{Si}[2\pi(x-y)]\}, \tag{9}$$

and $\operatorname{Si}[z] = \int_0^z (\sin(t)/t) dt$.

Due to the use of the MRC diversity combining technique, the received signal at each subcarrier is multiplied by the conjugate of channel fading coefficient. This also applied to the noise in each subcarrier. The noise power can thus be expressed as

$$\eta = \frac{N_0}{2T_b} \sum_{l=0}^{L-1} \beta_{0,l}^2, \tag{10}$$

where N_0 is the power spectral density (PSD) of the additive white Gaussian noise (AWGN).

In the remainder of this paper, only receive beamforming is considered. The antenna gain of transmit elements is set to 1, that is, $G_t(\theta_k - \pi, \theta_k - \pi) = 1$. Apply the lemma in [10, 11], the conditioned signal to interference and noise ratio (SINR) can be obtained by

$$\gamma = \frac{E_b}{E_I + \eta} \cong \frac{c}{a \sum_{k=1}^{K-1} G_r(\theta_k, \psi) + b} \sum_{l=0}^{L-1} \beta_{0,l}^2, \tag{11}$$

where

$$\begin{aligned} a &= \frac{\Omega}{\pi^2 \varepsilon^2} \left[-1 + {}_pF_q \left(\left\{ -\frac{1}{2} \right\}; \left\{ \frac{1}{2}, \frac{3}{2} \right\}; -\pi^2 \varepsilon^2 \right) \right] \\ &\quad + \frac{\Omega}{\pi^2 \varepsilon L} \left[\sum_{l=0}^{L-1} \sum_{h=0, h\neq l}^{L-1} (g(l-h, \varepsilon) - g(l-h, 0)) \right], \\ b &= \frac{\Omega}{L} \sum_{l=0}^{L-1} \sum_{h=0, h\neq l}^{L-1} \operatorname{sinc}^2(l-h-\varepsilon) + \frac{N_0}{2T_b L}, \\ c &= 2 \operatorname{sinc}^2(\varepsilon). \end{aligned} \tag{12}$$

3. OUTAGE PROBABILITY ANALYSIS

An important performance measure that characterizes the system quality is the outage probability, which is defined as the probability that the instantaneous error rate exceeds a specified value or, equivalently, that the instantaneous SINR γ falls below a certain specified threshold γ_0. Mathematically the outage probability P_{out} is expressed as

$$P_{\text{out}} = \int_0^{\gamma_0} f_\gamma(\gamma) d\gamma. \tag{13}$$

In this section, the outage probability of MC-CDMA systems in the presence of CFO with perfect beamforming is evaluated. To start the analysis of the outage probability, the SINR in (11) can be rewritten as

$$\gamma = \sum_{l=0}^{L-1} \gamma_l, \tag{14}$$

where

$$\gamma_l = \mu \beta_{0,l}^2, \quad \mu = \frac{c}{a \sum_{k=1}^{K-1} G_r(\theta_k, \psi) + b}. \tag{15}$$

Since $\beta_{0,l}$ is a Nakagami-m distributed RV defined in (3), then γ_l has a gamma distribution with its PDF given by

$$f_{\gamma_l}(\gamma_l) = \frac{1}{\Gamma(m)} \left(\frac{m}{\bar{\gamma}_c} \right)^m \gamma_l^{m-1} \exp\left(-\frac{m}{\bar{\gamma}_c} \gamma_l \right), \tag{16}$$

where

$$\overline{\gamma}_c = \mu\Omega. \quad (17)$$

Its characteristic function (CHF) can be obtained by

$$\Psi_{\gamma_l}(jw) = \left(1 - jw\frac{\overline{\gamma}_c}{m}\right)^{-m}. \quad (18)$$

Since $\gamma = \sum_{l=o}^{L-1} \gamma_l$ and γ_l is independent for different l, the CHF of γ can be expressed as

$$\Psi_{\gamma}(jw) = \left(1 - jw\frac{\overline{\gamma}_c}{m}\right)^{-mL}. \quad (19)$$

The PDF of SINR γ can be obtained through the inverse transformation of its CHF. Using [23], we have

$$\begin{aligned} f_\gamma(\ gamma) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi_\gamma(jw)\exp(-jw\gamma)\,dw \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(1 - jw\frac{\overline{\gamma}_c}{m}\right)^{-mL}\exp(-jw\gamma)\,dw \\ &= \frac{1}{\Gamma(mL)}\left(\frac{m}{\overline{\gamma}_c}\right)^{mL}\gamma^{mL-1}\exp\left(-\frac{m}{\overline{\gamma}_c}\gamma\right). \end{aligned} \quad (20)$$

The conditioned outage probability on the interfering user's angle of arrival $\theta_k (k = 1, 2, \ldots, K-1)$ and the scan angle ψ is obtained as [23]

$$\begin{aligned} &p_{\text{out}}(\theta_1, \theta_2, \ldots, \theta_{K-1}, \psi) \\ &\quad = \int_0^{\gamma_0} \frac{1}{\Gamma(mL)}\left(\frac{m}{\overline{\gamma}_c}\right)^{mL}\gamma^{mL-1}\exp\left(-\frac{m}{\overline{\gamma}_c}\gamma\right)d\gamma \\ &\quad = 1 - \frac{\Gamma(mL, m\gamma_0/\overline{\gamma}_c)}{\Gamma(mL)}, \end{aligned} \quad (21)$$

where $\Gamma(z, x) = \int_x^{\infty} e^{-t}\, t^{z-1} dt$ is an incomplete gamma function. Since RV θ_k and ψ are assumed to be uniformly distributed over $[0, 2\pi]$, the average outage probability is given by

$$\begin{aligned} P_{\text{out}} = &\iint_0^{2\pi} \cdots \iint_0^{2\pi} \frac{1}{(2\pi)^K} P_{\text{out}}(\theta_1, \theta_2, \ldots, \theta_{K-1}, \psi) \\ &\times d\theta_1 d\theta_2, \ldots, d\theta_{K-1} d\psi. \end{aligned} \quad (22)$$

Due to the complexity of the actual beamforming pattern, a closed-form expression to evaluate the average outage probability in (22) could not be derived. While, a numerical approach can be used to evaluate (22), the computation complexity of calculating above multi-dimensional integration is significant when the number of users presented in the system is large.

It is necessary to introduce a method to reduce the computation complexity of the average outage probability expression. Hereafter, we start the evaluation of the outage probability in (22) based on the simplified beamforming model described in Section 2.2. Assume that there are K_n interfering users within the mainlobe having a unit antenna gain with the probability B, and $K - K_n - 1$ interfering users within the sidelobe having the antenna gain α with the probability $1 - B$, respectively. With this model, $\overline{\gamma}_c$ in (17) can be simplified as

$$\overline{\gamma}_c(K_n) = \frac{c\Omega}{a(K_n + \alpha(K - K_n - 1)) + b}. \quad (23)$$

Assume that K_n is uniformly distributed over $[0, K-1]$ in all direction, the average outage probability can be easily obtained by

$$\begin{aligned} P_{\text{out}} = &\sum_{K_n=0}^{K-1} \binom{K-1}{K_n} B^{K_n}(1-B)^{K-K_n-1} \\ &\cdot \left(1 - \frac{\Gamma(mL, m\gamma_0/\overline{\gamma}_c(K_n))}{\Gamma(mL)}\right), \end{aligned} \quad (24)$$

where α and B are determined based on the actual beam pattern.

4. OUTAGE PROBABILITY WITH IMPERFECT BEAM FORMING

In practice, a variety of beamforming impairments, such as DOA estimation errors, spatial spreads, and mutual couplings, exist in the system. However, the outage probability analysis in previous section is just based on perfect beamforming. In this section, we will evaluate the outage probability by considering those beamforming impairments. All impairments will affect the shape of the beam pattern and antenna gain. We need to point out that in the simplified beamforming model, only parameters α and B need to be modified according to the change of the beam pattern due to impairments. The outage probability can still be obtained through (24) but with revised parameters α and B accordingly.

4.1. Effect of DOA estimation errors

For practical systems, DOA is usually estimated through certain algorithm. The estimated arrival angle $\hat{\psi}$ for the desired user can be characterized as an RV with a uniform distribution or normal distribution [16]. The PDF of $\hat{\psi}$ is expressed as

$$f_{\hat{\psi}}(\hat{\psi}) = \begin{cases} \dfrac{1}{2\sqrt{3}\Delta}, -\sqrt{3}\Delta \le (\hat{\psi} - \theta_0) \le \sqrt{3}\Delta & \text{uniform}, \\ \dfrac{1}{\sqrt{2\pi}\Delta}\exp\left(-\dfrac{(\overline{\psi} - \theta_0)}{2\Delta^2}\right)^2, & \text{norm}, \end{cases} \quad (25)$$

where θ_0 is the actual arrival angle, Δ^2 represents the variance of the estimation error for uniform or normal distribution.

Hence, parameters α and B which determine the simplified beam pattern in (2) are modified to

$$\alpha = \frac{E_{\theta,\hat{\psi}}\{G^2(\theta,\hat{\psi})\} - E_{\theta,\hat{\psi}}\{G(\theta,\hat{\psi})\}}{E_{\theta,\hat{\psi}}\{G(\theta,\hat{\psi})\} - 1},$$
$$B = \frac{E_{\theta,\hat{\psi}}\{G^2(\theta,\hat{\psi})\} - E^2_{\theta,\hat{\psi}}\{G(\theta,\hat{\psi})\}}{E_{\theta,\hat{\psi}}\{G^2(\theta,\hat{\psi})\} + 1 - 2E_{\theta,\hat{\psi}}\{G(\theta,\hat{\psi})\}}, \quad (26)$$

respectively, where $E_{\theta,\hat{\psi}}\{\cdot\}$ is the expectation with respect to RV θ and $\hat{\psi}$. The standard deviation Δ is normalized by $\Delta_{\max}$, where $\Delta_{\max}$ is the standard deviation of a DOA estimation error that is uniformly distributed from null to null when θ is equal to 0° (toward the broadside direction), and $\Delta_{\max}$ can be obtained by

$$\Delta_{\max} = \frac{\arcsin(2/M)}{\sqrt{3}}. \quad (27)$$

4.2. Effect of angle spreads

The angle spread refers to the spread of angles of arrival of multipaths at the antenna array, and the signal is spread in space. The angle spread has been measured and investigated in [24, 25]. For rural environments, angular spreads between $1-5°$ have been observed in [24]. For urban and hilly terrain environments, considerably larger angular spreads, as large as 20°, have been found in [25]. Angle spreads not only reduce the received signal power, but also cause DOA estimation uncertainty as the DOA estimation becomes random in the interval of arrival angles. Assume that the angle spread follows the same distribution as (25). The expected receive power should be averaged by considering both arrival angle estimations and angle spreads. Therefore, parameters α and B are changed to

$$\alpha = \frac{E_{\theta,\hat{\psi},\overline{\theta},\overline{\psi}}\{G^2(\theta,\hat{\psi})\} - E_{\theta,\hat{\psi},\overline{\theta},\overline{\psi}}\{G(\theta,\hat{\psi})\}}{E_{\theta,\hat{\psi},\overline{\theta},\overline{\psi}}\{G(\theta,\hat{\psi})\} - 1},$$
$$B = \frac{E_{\theta,\hat{\psi},\overline{\theta},\hat{\psi}}\{G^2(\theta,\hat{\psi})\} - E^2_{\theta,\hat{\psi},\overline{\theta},\overline{\psi}}\{G(\theta,\hat{\psi})\}}{E_{\theta,\hat{\psi},\overline{\theta},\hat{\psi}}\{G^2(\theta,\hat{\psi})\} + 1 - 2E_{\theta,\hat{\psi},\overline{\theta},\overline{\psi}}\{G(\theta,\hat{\psi})\}}, \quad (28)$$

respectively, where $E_{\theta,\hat{\psi},\overline{\theta},\overline{\psi}}\{\cdot\}$ is the expectation with respect to all the RVs $\theta, \hat{\psi}, \overline{\theta}$, and $\overline{\psi}$. $\overline{\theta}$ and $\overline{\psi}$ are the mean of RV θ and ψ, respectively.

4.3. Effect of mutual couplings

The mutual coupling between antenna elements also has impact on beam patterns. It affects the estimation of arrival angles, resulting in the disturbance of the weighting vector in beamforming. Assume thin half-wavelength dipoles, mutual coupling is characterized by an impedance matrix [18, 26, 27]:

$$\mathbf{C} = (Z_T + Z_A)(\mathbf{Z} + Z_T\mathbf{I})^{-1}, \quad (29)$$

where Z_A is the antenna impedance, Z_T is the terminating impedance, $\mathbf{I}$ is an identity matrix and $\mathbf{Z}$ is the mutual impedance matrix. Assume perfect arrival angles, the beam pattern is given by

$$A = \sum_{n=-N}^{N} \exp\{-jn\pi\sin\theta\} \sum_{m=-N}^{N} \mathbf{C}_{n,m} \exp\{jm\pi\sin\psi\}, \quad (30)$$

where $\mathbf{C}_{n,m}$ is the (n,m)th element of the matrix $\mathbf{C}$ given in (29), and the normalized beamforming gain can be obtained by

$$G(\theta,\psi) = \frac{|AA^*|}{M^2}. \quad (31)$$

Substitute (31) into (2) , the modified α and B can be obtained.

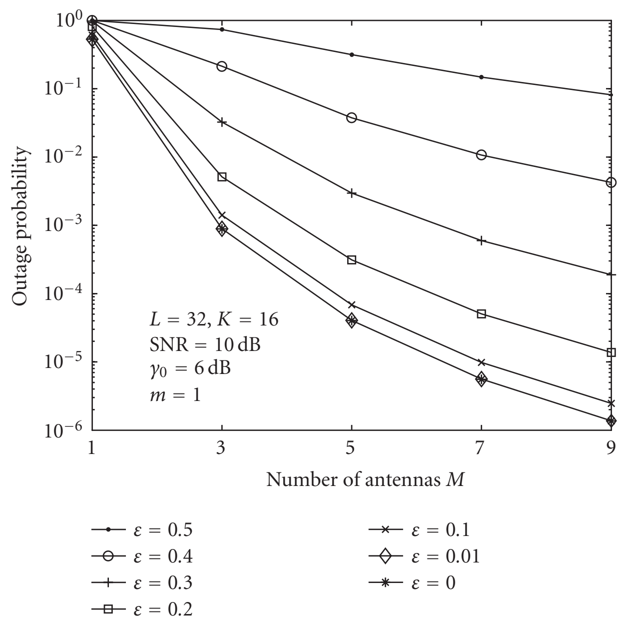


Figure 4: Outage probability versus number of antennas M and NFCO ε.

5. NUMERICAL RESULTS

The numerical investigation of the outage probability for a reverse link MC-CDMA wireless cellular system with either ideal beamforming or imperfect beamforming in the presence of CFO is given in this section. The spreading gain L (or total number of subcarriers) for each user is set to $L = 32$. There are total $K = 16$ active users in the system. The Nakagami-m channel fading is assumed over each subcarrier for all users. The required SINR threshold γ_0 is set to 6 dB. The signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = \frac{LST_b}{N_0}. \quad (32)$$

The actual beam pattern is used for the desired user, while for the interference users, the simplified beam pattern described in Section 2.2 is used.

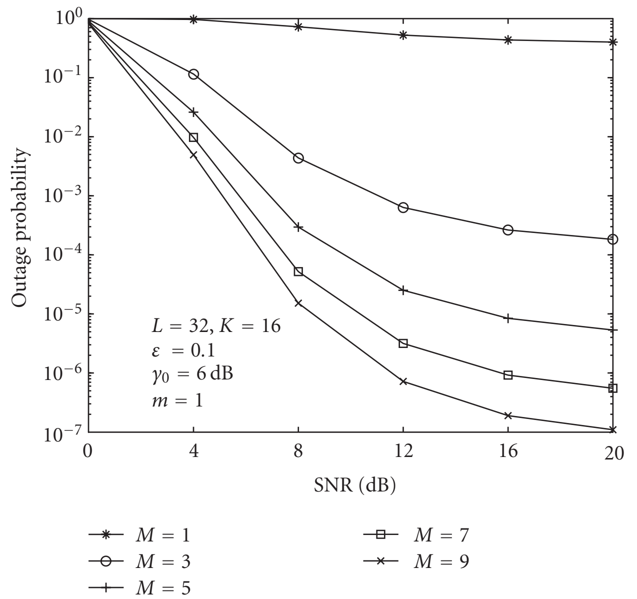


Figure 5: Outage probability versus SNR and number of antennas M.

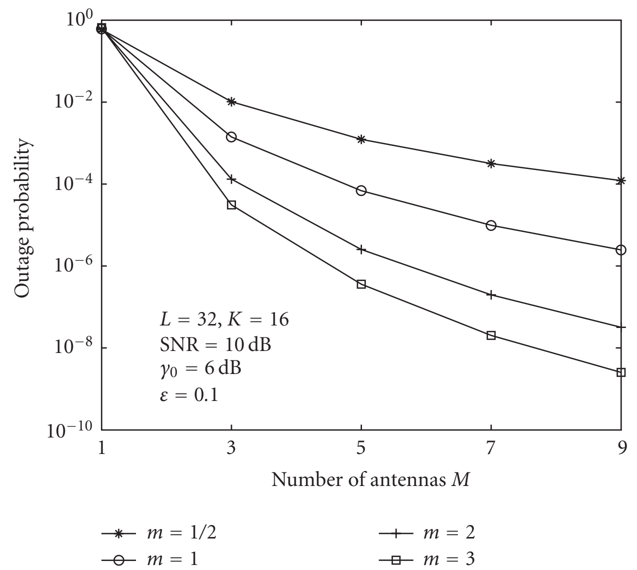


Figure 6: Outage probability versus number of antennas M and Nakagami m.

From Figure 4 to Figure 6, the outage probability is evaluated when perfect beamforming is assumed at the BS. Figure 4 shows the effect of receive beamforming on the outage probability for reverse link MC-CDMA systems when CFO is present. The Nakagami fading parameter m is set to 1; SNR is assumed to be 10 dB. It can be observed from Figure 4 that the outage probability improves significantly as the number of receive antenna elements increases. The beamforming technique has brought a noticeable benefit for the system performance. The larger the number of receive antenna elements, the lower the outage probability of the system. It is also seen from Figure 4 that the beamforming plays an important role in mitigating the impact of the CFO. The outage probability is approximately 0.1% when the NCFO $\varepsilon = 0$ and the number of antenna elements $M = 3$. When the CFO increases to 30%, the outage probability deteriorates to 4%, which could be improved to 0.1% through the use of a larger number of antenna elements $M = 7$. This illustrates the significant benefit of using the beamforming technique.

Figure 5 presents the outage probability versus SNR with different number of receive antenna elements. The NCFO ε and Nakagami fading parameter m are set to 0.1 and 1, respectively. We observe that as SNR increases, the outage probability decreases gradually. It can be seen from Figure 5 that the outage probability remains at a very high level no matter how much SNR increases when the system does not employ beamforming (the number of receive antennas $M = 1$). This is due to the fact that the MUI contributes most of the impairments to the system in this situation, and there is no beamforming technique to mitigate the MUI. Hence it is difficult to achieve the required SINR threshold γ_0. However, when beamforming is used ($M > 1$), it will combat the MUI efficiently; as a result, the outage probability decreases greatly.

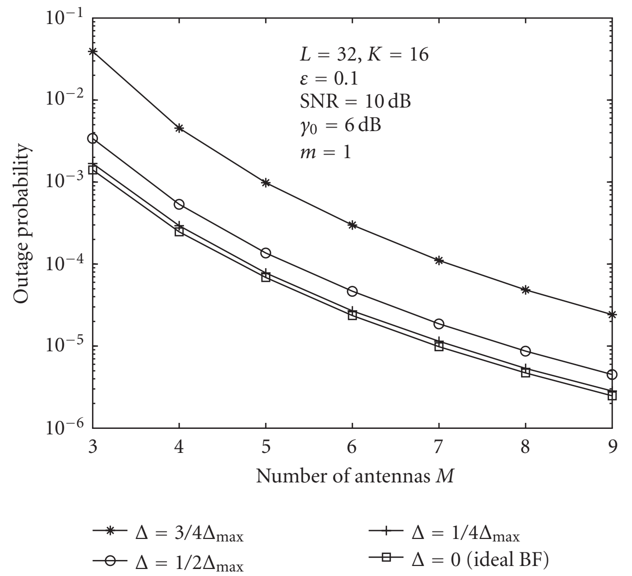


Figure 7: Outage probability with DOA estimation errors. Δ is the standard deviation of uniformly distributed DOA estimation errors. M is the number of antennas.

Figure 6 gives the outage probability under different Nakagami fading parameter m. Again, the SNR is set to 10 dB. The figure shows that the outage probability decreases as the parameter m increases. That is because that the better channel environment the system experiences, the larger the parameter m. Better channel conditions definitely improve the system performance.

From Figure 7 to Figure 9, we investigate the impact of beamforming impairments on the outage probability of the system.

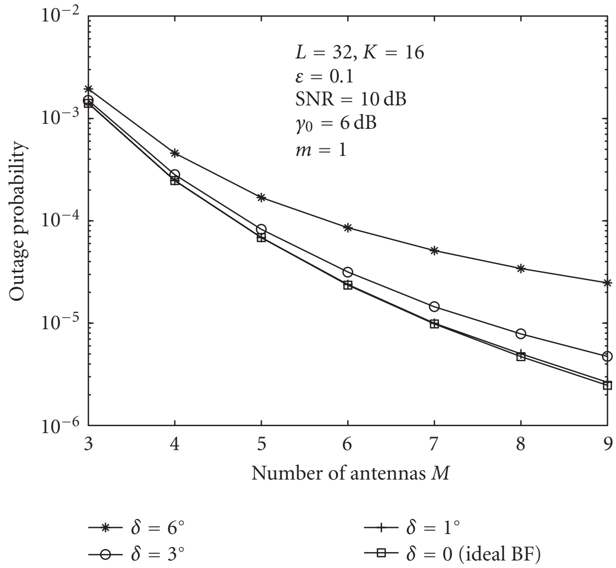


FIGURE 8: Outage probability with angle spreads. δ is the standard deviation of uniformaly distributed angle spreads. M is the number of antennas.

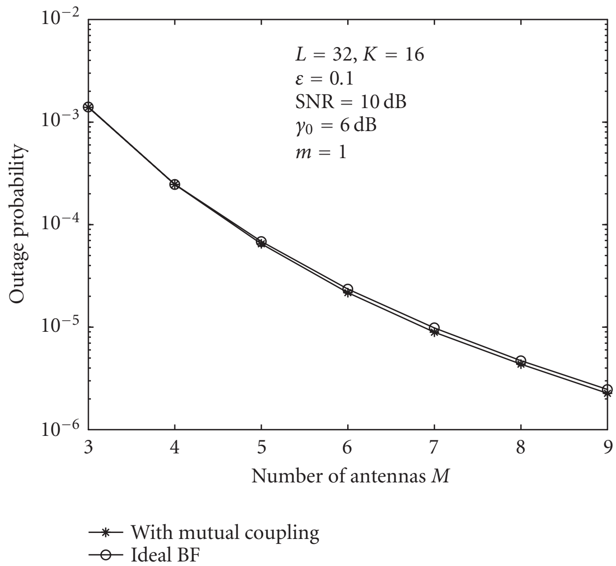


FIGURE 9: Outage probability with mutual coupling. M is the number of antennas.

In the following, a small CFO is assumed in the system, that is, $\varepsilon = 0.1$; the SNR is set to 10 dB and all users experience Nakagami fading ($m = 1$) over each subcarrier. Figure 7 shows the effect of DOA estimation errors. The DOA error is assumed to follow a uniform distribution with a standard deviation Δ, and $\Delta_{\max}$ is given in (27). It can be seen from Figure 7 that the DOA estimation error does not impact much on the outage probability when the error is within the half null-to-null beam width ($\Delta \le (1/2)\Delta_{\max}$). When a larger DOA estimation error is present, that is, the case of $\Delta \ge (3/4)\Delta_{\max}$ in Figure 7, it leads to a significant increase of the outage probability.

Figure 8 plots the outage probability when different angle spreads are present in the system. The angle spread is assumed to follow a uniform distribution with a standard deviation δ. We observe that the outage probability does not vary much when δ is small, that is, $\delta < 3^\circ$. However, a noticeable deterioration of the outage probability can be seen if the angle spread is large, that is the case of $\delta = 6^\circ$ in Figure 8.

Figure 9 illustrates the impact of the mutual coupling among antenna elements on the outage probability. From Figure 9, only a very small change of the outage probability is observed when the mutual coupling exists in the system. This is because the distance between adjacent antenna elements is $\lambda/2$ which is large enough to eliminate any noticeable coupling.

6. CONCLUSION

The outage probability of reverse link MC-CDMA systems with beamforming in the presence of CFO over Nakagami fading channels is evaluated in this paper. A simplified beamforming model is utilized to reduce the complexity of the analysis. A closed-form expression of the outage probability is obtained to examine the effect of CFO and beamforming. First, the outage probability is evaluated when perfect beamforming is assumed. It can be concluded that the outage probability improves significantly as the number of antenna elements increases; second, the outage probability is investigated when different types of beamforming impairments are present in the system. It is seen that small DOA estimation errors and angle spreads have only a slight impact on the outage probability of the system; however, as those impairments become large, the outage probability deteriorates significantly. Also it is observed that the outage probability changes very slightly when there is mutual coupling in the antenna array.

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