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Aspects of CPT-even Lorentz-symmetry violating physics in a supersymmetric scenario

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Abstract Background fermion condensates in a landscape dominated by global supersymmetry are reassessed in connection with a scenario where Lorentz symmetry is violated in the bosonic sector (actually, the photon sector) by a CPT-even k_F term. An effective photonic action is discussed that originates from the supersymmetric background fermion condensates. Also, the photino mass emerges in terms of a particular condensate contrary to what happens in the case of k_{AF}-violation. Finally, the interparticle potential induced by the effective photonic action is investigated and a confining profile is identified.

1 Introduction

Models that realize the breaking of Lorentz symmetry have raised a great deal of interest after Kostelecký and Samuel had shown [1–7], in the context of bosonic strings, that condensation of tensor fields is dynamically possible, contrary to the physics of the standard model (SM), whose dynamics does not yield Lorentz-symmetry violation (LSV). However, models with LSV are to be considered as effective theories and the analysis of their phenomenological aspects at low energies may provide information and impose constraints on the more fundamental theory from which they stem.

A general framework for testing the low-energy manifestations of CPT-breaking and LSV is the so-called standard-model extension (SME). In this approach, the effective Lagrangian corresponds to the usual Lagrangian of the SM corrected by SM operators of any dimensionality contracted with suitable Lorentz-violating tensorial background coefficients. The effective Lagrangian is written in a Lorentz-invariant form so as to ensure what we refer to as observer's independence of the physics of the system under study. However, the physically relevant transformations are those that affect the dynamical variables (fields) that parametrize the system. These changes are named particle transformations, whereas the latter, the coordinate transformations (which include the background tensors) are called observer's transformations. We point out the work of Refs. [8,9] where these concepts are thoroughly analyzed.

Concerning the experimental searches for the CPT/LSV, the generality of the SME has provided the basis for many investigations. In the flat spacetime limit, phenomenological studies include electrons, photons, muons, mesons, baryons, neutrinos, and the Higgs [10] sector. Gravitational interaction has also been intensively investigated [10,11,16], and one can set current limits on the parameters associated to the breaking of relativistic covariance.

The violation of CPT invariance has also been extensively studied in the framework of a modified Dirac theory [17,18] and its non-relativistic regime, with the calculation and discussion of the spectrum of the non-relativistic hydrogen atom [19–23]. In the direction of fermionic models in the presence of LSV, there has been an effort to associate magnetic properties of spinless and/or neutral particles if a non-minimal coupling of the Lorentz-symmetry violating background to fermionic matter and gauge bosons is taken into account [24–35]. Still in the realm of atomic physics and optics, we should quote a set of works that set out to examine effects of LSV in electromagnetic cavities and optical systems [36–42], which have finally contributed to the setup of a new bound on the parameters associated to LSV.

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The breaking of Lorentz symmetry should be traced back to the dynamics of a more fundamental physics at energies much above our present accelerators' energies, for example, at very high energies in astrophysical and even cosmological phenomena. On the other hand, supersymmetry (SUSY) should be exact at these energy scales, or it may happen that it is broken at a scale very close to this primary physical environment. We claim that LSV breaking and SUSY breaking are not completely independent events in a high-energy regime. We then work with the hypothesis that LSV occurs in a world that is dominated by exact SUSY or still keeps track of a SUSY broken at a slightly higher scale. We highlight the works of Refs. [43–53], where we have listed papers that put SUSY in direct association with models of CPT-breaking and LSV. More recently, the relationship between SUSY breaking and LSV has been discussed in the works by Chkareuli [12] and in the article by Pospelov and Tamarit [13], where these authors consider the possibility that SUSY and Lorentz-symmetry breaking have a common origin if supersymmetric matter is coupled to Horava–Lifshitz gravity.

Our proposal here is to place LSV in a scenario where SUSY still holds as an exact symmetry. We shall then notice afterwards that the breaking of Lorentz symmetry naturally induces SUSY violation, as we shall show in detail throughout this paper. With the idea that SUSY is present from the very outset, we claim that the background vector (or tensor) that signals LSV must be a component of some particular SUSY multiplet. This is the key point of our proposal. In a previous paper [56], we have proposed a SUSY-dominated scenario to study LSV by considering the Carroll–Field–Jackiw model, and we have proposed that the background associated to LSV was sitting in a chiral scalar superfield. Our study has revealed that this situation is characterized by a set of fermion condensates that accompany the background vector of the Carroll–Field–Jackiw model. These fermionic pairs turn out to induce physical effects such as mass splitting for supersymmetric partners and a set of extended dispersion relations for the photon and photino sectors. In this direction, we would like to quote the interesting article by Tomboulis [54].

Motivated by the fact that SUSY reveals that LSV is realized with a bosonic background along with a whole set of fermions that condensate in the process, we pursue here another investigation to better understand the issue: we select the so-called k_F term, for which CPT is not broken, and we study the effect of the fermion condensates associated to this type of breaking on the physics of the photon and photino. In particular, we are very much concerned with the emergence of an effective photonic action that comes out as a by-product of LSV and the associated fermion condensates. Again, we are going to conclude that the LSV is accompanied by the emergence of a Goldstone fermion, which signals SUSY breaking, even though no F or D term is behind SUSY violation.

The effective photonic model we shall derive carries the fermionic condensates that are in this context messengers of LSV. The approach may be adopted of reassessing the discussion of the emergence of an interparticle potential with a confining piece along with a Yukawa profile whose parameters incorporate the contribution of the bosonic background and fermion condensates. This study reveals that LSV and the supersymmetric dynamics that induce the formation of pairs of fermions may be present in the electrostatic interacting energy of two particles with opposite charges. Our work is organized according to the following structure: Sect. 2 is simply the formulation of the component-field action for the supersymmetric version of the k_F term in the case that a single four-vector, ξ_μ, is the bosonic signal of LSV. We accommodate ξ_μ in a chiral scalar superfield and we identify the fermionic condensates that come out in the action with LSV.

Section 3 is devoted to a simplification of the LSV action by the elimination of the auxiliary field present in the gauge potential superfield. In Sect. 4, we actually start by deriving the physical effects we wish to discuss: photon–photino splitting, dispersion relations and the photon effective action inherited from LSV. Next, in Sect. 5, the effective photonic action is considered to discuss the electrostatic confining potential between two opposite charges. Finally, we present our concluding comments and future developments in Sect. 6. Two appendices follow: in Appendix A, we cast a primary component-field action in terms of Weyl spinors. In Appendix B, we present a term which is a key algebraic expression for the attainment of the field action that we shall be actually working with throughout our paper.

2 The k_F term, its reduction, and its supersymmetric extension

We start with the action for the CPT-even term for the abelian gauge sector of the standard model extension:

$$S_{\text{CPT-even}} = -\frac{1}{4}\int \mathrm{d}^4x\,(k_F)_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}. \tag{1}$$

The tensor k_F, from now on written as $K_{\mu\nu\alpha\beta}$, displays the properties

$$K_{\mu\nu\alpha\beta} = -K_{\nu\mu\alpha\beta} = -K_{\mu\nu\beta\alpha} = K_{\alpha\beta\mu\nu}, \tag{2}$$

it is double-traceless, and its fully anti-symmetric component is ruled out because it yields a total derivative. As is well known, it depends on 19 parameters.

If, moreover, we wish to suppress the components that yield birefringence, we end up with only nine independent components. We shall consider here a particular situation of the non-birefringent case, namely, the case in which we are left with only four coefficients that signal a violation of Lorentz symmetry; these are described by a four-vector (ξ_α). According to the ansatz discussed in [14,15], we may finally parametrize $K_{\mu\nu\alpha\beta}$ as follows:

$$K_{\mu\nu\alpha\beta} = \frac{1}{2}(\eta_{\mu\alpha}\tilde{\kappa}_{\nu\beta} - \eta_{\mu\beta}\tilde{\kappa}_{\nu\alpha} + \eta_{\nu\beta}\tilde{\kappa}_{\mu\alpha} - \eta_{\nu\alpha}\tilde{\kappa}_{\mu\beta}), \quad (3)$$

$$\tilde{\kappa}_{\alpha\beta} = \left(\xi_\alpha\xi_\beta - \eta_{\alpha\beta}\frac{(\xi_\rho\xi^\rho)}{4}\right), \quad (4)$$

and the essence of LSV is traced back to the constant background 4-vector ξ_μ, so that the k_F action becomes

$$S = \int \mathrm{d}^4x \frac{1}{4}\left(\frac{1}{2}\xi_\mu\xi_\nu F^\mu{}_\kappa F^{\kappa\nu} + \frac{1}{8}\xi_\rho\xi^\rho F_{\mu\nu}F{\mu\nu}\right). \quad (5)$$

In our proposal, this is a more reasonable situation. If we were to identify the whole tensor $K_{\mu\nu\alpha\beta}$ as a component of a given superfield, higher spins (actually, $s = \frac{3}{2}$) would be present in a global SUSY framework. Since we have ξ_μ as the signal of LSV, no risk of higher fermionic spins in the background is taken if the effects of the K-tensor are transferred to the ξ^μ-vector.

In this paper, we shall be working with supersymmetry formulated in superspace and in terms of superfields. To this aim, we refer the reader to notations and conventions adopted in Ref. [56].

In the work of Ref. [58], two ways have been suggested to implement a SUSY-extension for a 4-vector background: ξ_μ may appear as the gradient of a scalar (in this case, LSV is in a chiral superfield) or a complete vector (with transverse and longitudinal components); in the latter case, ξ_μ should be a vector component of what we call a vector superfield. To consider a simpler fermionic set of partners, we choose to place ξ^μ in the chiral superfield: in the first case the supersymmetry is implemented through a chiral multiplet and in the other by means of a vector multiplet. For simplicity, we work only in the chiral case. In this proposal, the extended action written in superfield formalism is

$$S^{(\mathrm{SUSY})}_{\text{CPT-even}} = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\left[(D^\alpha\Omega)W_\alpha(\bar{D}_{\dot{\alpha}}\bar{\Omega})\bar{W}^{\dot{\alpha}} + \text{h.c.}\right]$$
$$= S_{\text{ferm}} + S_{\text{boson}} + S_{\text{mixing}}, \quad (6)$$

where the supersymmetry covariant derivatives, the superspace action, and the superfields can be found in Ref. [56];

$$\begin{aligned} W_\alpha(x) &= \lambda_\alpha(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\lambda_\alpha(x) - \frac{1}{4}\bar{\theta}^2\theta^2\Box\lambda_\alpha(x) \\ &+ 2\theta_\alpha D(x) - i\theta^2(\bar{\theta}\sigma^\mu)_\alpha\partial_\mu D(x) \\ &+ (\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu}(x)x - \frac{1}{2}\theta^2(\sigma^{\mu\nu}\sigma^\rho)_\alpha\partial_\rho F_{\mu\nu}(x) \\ &- i(\sigma^\mu\partial_\mu\lambda[x])_\alpha\theta^2 \end{aligned} \quad (7)$$

is the well-known field-strength superfield (λ is the photino, $F_{\mu\nu}$ the usual gauge-field strength, and D the auxiliary field); the chiral background superfield, Ω, is θ-expanded as follows:

$$\begin{aligned} \Omega(x) &= S(x) + \sqrt{2}\theta\zeta(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu S(x) + \theta^2 G(x) \\ &+ \frac{i}{\sqrt{2}}\theta^2\bar{\theta}\bar{\sigma}^\mu\partial_\mu\zeta(x) - \frac{1}{4}\bar{\theta}^2\theta^2\Box S(x), \end{aligned} \quad (8)$$

where S and G are complex scalars and ζ is a Weyl component of a Majorana fermion. By projecting the action (6) into component fields, we readily get $\xi_\mu = \partial_\mu S$ and the S_{boson}, S_{ferm}, and S_{mixing} may be found, in terms of Weyl spinors in Appendix A. We prefer to quote below the component-field action directly in terms of Majorana spinors, for it is much simpler and one can control much more easily the various couplings present in the action.

At this point, we also make a special consideration about the background superfield Ω: taking S linear in x^μ ($S = \xi_\mu x^\mu$, ξ_μ constant), $\partial_\mu\zeta = 0$, and $G = 0$ is compatible with SUSY, in the sense that these properties are kept if global SUSY transformations are performed, and, moreover, we reproduce the k_F term as we wished from the very beginning. Now, we shall move on with two purposes:

(i) to rewrite the whole action in terms of 4-components Majorana spinors, $Z \equiv (\zeta\ \bar{\zeta})^t$ and $\Lambda \equiv (\lambda\ \bar{\lambda})^t$,
(ii) to Fierz rearrange the terms in S_{ferm} where the fermions ζ and λ are mixed.
This process selects for us three types of background fermion condensates (already written in terms of Majorana spinors):

$$\begin{aligned} \mathrm{X} &= \bar{Z}Z, \\ \tau &= \bar{Z}\gamma_5 Z, \\ C^\mu &= \bar{Z}\gamma^\mu\gamma_5 Z, \end{aligned} \quad (9)$$

for which

$$\mathrm{X}^2 = -\tau^2 = \frac{1}{4}C^\mu C_\mu,$$
$$\mathrm{X}\tau = 0 \quad \text{and} \quad \mathrm{X}C^\mu = \tau C^\mu = 0. \quad (10)$$

With all these considerations, the action (6) can be brought into a more readable form:

$$\begin{aligned} S_{\text{boson}} = \int \mathrm{d}^4x\Big[&D^2(32|G|^2 + 16\partial_\mu S\partial^\mu S^*) \\ &+ 8iDF^{\mu\nu}(\partial_\mu S\partial_\nu S^* - \partial_\mu S^*\partial_\nu S) \\ &- 8F^{\mu\kappa}F_\kappa{}^\nu(\partial_\mu S\partial_\nu S^* + \partial_\mu S^*\partial_\nu S) \\ &- 4F^{\mu\nu}F_{\mu\nu}\partial_\alpha S\partial^\alpha S^*\Big]; \end{aligned} \quad (11a)$$

$$S_{\text{ferm}} = \int \mathrm{d}x^4 (C^\mu \bar{\Lambda}\gamma^\nu\gamma_5\partial_\mu\partial_\nu\Lambda + yC_\mu\bar{\Lambda}\gamma^\mu\gamma_5\Box\Lambda),$$

$$\text{where y is a numerical factor } \left(y = \frac{4-\sqrt{2}}{16}\right); \tag{11b}$$

$$\begin{aligned} S_{\text{mixing}} = \int \mathrm{d}^4x \Big[&D\Big(10\sqrt{2}\mathrm{Re}(\partial_\mu S)(\bar{Z}\partial_\mu\Lambda) \\ &- 8\sqrt{2}i\mathrm{Re}(\partial_\mu S)(\bar{Z}\Sigma^{\mu\nu}\partial_\nu\Lambda) \\ &\times 8\sqrt{2}\mathrm{Im}(\partial_\mu S)(\bar{Z}\Sigma^{\mu\nu}\gamma_5\partial_\nu\Lambda) \\ &+ 10\sqrt{2}i\mathrm{Im}(\partial_\mu S)(\bar{Z}\gamma_5\partial^\mu\Lambda)\Big) \\ &- 3\sqrt{2}\mathrm{Im}(\partial_\nu S)[\partial_\mu F^{\mu\nu}]\bar{Z}\Lambda \\ &+ 3\sqrt{2}\mathrm{Re}(\partial_\nu S)[\partial_\mu F^{\mu\nu}]\bar{Z}\gamma_5\Lambda \\ &+ 4\sqrt{2}i\,\partial_{[\nu}F_{\mu]\alpha}\mathrm{Im}(\partial^\alpha S)\bar{Z}\Sigma^{\mu\nu}\Lambda \\ &+ 4\sqrt{2}i\,\partial_{[\nu}\tilde{F}_{\mu]\alpha}\mathrm{Re}(\partial^\alpha S)\bar{Z}\Sigma^{\mu\nu}\Lambda \\ &+ 4\sqrt{2}\partial_{[\nu}F_{\mu]\alpha}\mathrm{Im}(\partial^\alpha S)\bar{Z}\Sigma^{\mu\nu}\gamma_5\Lambda \\ &+ 4\sqrt{2}\partial_{[\nu}\tilde{F}_{\mu]\alpha}\mathrm{Re}(\partial^\alpha S)\bar{Z}\Sigma^{\mu\nu}\gamma_5\Lambda\Big], \end{aligned} \tag{11c}$$

where we adopt the conventions that the indices enclosed by square brackets stand for anti-symmetrization, $\tilde{F}_{\mu\nu}$ is the dual of $F_{\mu\nu}$, and $\Sigma_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]$. D appears as an auxiliary field and, in the next section, we are going to eliminate it upon using its corresponding equation of motion.

3 Eliminating the auxiliary field

The equations above are indeed more manageable to work with. In order to complete our model, we must add to Eqs. (11a)–(11c) the supersymmetric version of the Maxwell action. After this has been done, it is advisable to eliminate the auxiliary field, D, by means of the algebraic equation of motion. Notice that the total action can be written in terms of the auxiliary field in the form

$$\begin{aligned} S^{(\text{full})} &= S^{(\text{SUSY})}_{\text{Maxwell}} + S^{(\text{SUSY})}_{\text{CPT-even}} \\ &= S + \int \mathrm{d}^4x\,\beta D + \int \mathrm{d}^4x\,\alpha D^2, \end{aligned} \tag{12}$$

$$S^{(\text{full})} = S - \int \mathrm{d}x^4 \frac{\beta^2}{2(2+\alpha)}, \tag{13}$$

where α and β are expressed in terms of background and fields in the gauge sector as follows:

$$\begin{aligned} \alpha &= 16(\partial_\kappa S\partial^\kappa S^*), \\ \beta &= 10\sqrt{2}\mathrm{Re}(\partial_\mu S)(\bar{Z}\partial_\mu\Lambda) - 8\sqrt{2}i\mathrm{Re}(\partial_\mu S)(\bar{Z}\Sigma^{\mu\nu}\partial_\nu\Lambda) \\ &\quad + 8\sqrt{2}\mathrm{Im}(\partial_\mu S)(\bar{Z}\Sigma^{\mu\nu}\gamma_5\partial_\nu\Lambda) \\ &\quad + 10\sqrt{2}i\mathrm{Im}(\partial_\mu S)(\bar{Z}\gamma_5\partial^\mu\Lambda) + 16\,m_{\mu\nu}F^{\mu\nu}, \end{aligned} \tag{14}$$

where $m_{\mu\nu} = \mathrm{Re}(\partial_\mu S)\mathrm{Im}(\partial_\nu S) - \mathrm{Re}(\partial_\nu S)\mathrm{Im}(\partial_\mu S)$.

The calculation of β^2 involves again the use of Fierz identities and properties of bilinears formed by anticommuting Majorana spinors. The final result is somewhat cumbersome, so that, to keep the text in balance, we believe it is advisable to collect the result in an appendix. To this aim, we have included the Appendix B.

Then incorporating the β^2 term into the action, we have

$$\begin{aligned} S^{(\text{full})} = \int \mathrm{d}x^4 \Big[&-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}K_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} \\ &- \frac{64}{(1+8\partial_\rho S\partial^\rho S^*)}m_{\mu\nu}m_{\alpha\beta}F^{\mu\nu}F^{\alpha\beta} \\ &- \bar{\Lambda}\frac{\tilde{a}}{4(1+8\partial_\kappa S\partial^\kappa S^*)}\Lambda - \bar{\Lambda}\frac{\tilde{b}}{4(1+8\partial_\kappa S\partial^\kappa S^*)}\gamma_5\Lambda \\ &+ \bar{\Lambda}\Big[-(C\cdot\partial)\partial_\mu + y\Box C_\mu + C^\alpha d_{\alpha\mu}\Big] \\ &\times \gamma^\mu\gamma_5\Lambda + 2\bar{Z}N\Lambda\Big]. \end{aligned} \tag{15}$$

The $K_{\mu\nu\alpha\beta}$-tensor, appearing above, now in the supersymmetric background, is given in terms of the complex vectors ξ_μ according to the expression

$$K_{\mu\nu\alpha\beta} = -16(\eta_{\mu\alpha}\tilde{\kappa}_{\nu\beta} - \eta_{\mu\beta}\tilde{\kappa}_{\nu\alpha} + \eta_{\nu\beta}\tilde{\kappa}_{\mu\alpha} - \eta_{\nu\alpha}\tilde{\kappa}_{\mu\beta}), \tag{16}$$

with

$$\tilde{\kappa}_{\alpha\beta} = \frac{1}{2}(\xi_\alpha\xi^*_\beta + \xi^*_\alpha\xi_\beta) - \frac{\eta_{\alpha\beta}}{4}(\xi_\rho\xi^{\rho*}); \tag{17}$$

also we should point out that there is an extra contribution to the FF term given by the coefficients $m_{\alpha\beta}m_{\mu\nu}$. The latter contribution is intrinsic to supersymmetry. It is important here to remark that, even though ξ_μ is complex, as imposed by supersymmetry, the K-tensor is automatically real, as it should, to avoid dissipating solutions to the fields equations.

The coefficients $d_{\alpha\mu}$, $\tilde{a}$, and $\tilde{b}$ can all be found in Appendix B. The N-matrix above, which mixes the background fermion and the photino, is given by a lengthy expression that involves the photon field and its field strength, $F_{\mu\nu}$. This term mixes therefore the photon and the photino fields, and the explicit form of N follows:

$$N = I^{(1)} + iI^{(2)}\gamma_5 + iI_{\mu\nu}\Sigma^{\mu\nu}, \quad \text{where} \tag{18a}$$

$$\begin{aligned} I^{(1)} &= -\frac{3}{2}\sqrt{2}\mathrm{Im}(\partial_\mu S)\partial_\alpha F^{\alpha\mu} \\ &\quad + \frac{20\sqrt{2}}{(1+8\partial_\kappa S\partial^\kappa S^*)}m_{\alpha\beta}\mathrm{Re}(\partial_\rho S)\partial^\rho F^{\alpha\beta}, \end{aligned} \tag{18b}$$

$$\begin{aligned} I^{(2)} &= \frac{3}{2}\sqrt{2}\mathrm{Re}(\partial_\mu S)\partial_\alpha F^{\alpha\mu} \\ &\quad + \frac{20\sqrt{2}i}{(1+8\partial_\kappa S\partial^\kappa S^*)}m_{\alpha\beta}\mathrm{Im}(\partial_\rho S)\partial^\rho F^{\alpha\beta}, \end{aligned} \tag{18c}$$

$$I_{\mu\nu} = 2\sqrt{2}\Big[\mathrm{Im}(\partial^\alpha S)\partial_{[\nu} F_{\mu]\alpha} + \mathrm{Re}(\partial^\alpha S)\partial_{[\nu}\tilde{F}_{\mu]\alpha}\Big]$$
$$- \frac{16\sqrt{2}}{(1+8\partial_\kappa S\partial^\kappa S^*)} m_{\alpha\beta}\mathrm{Re}(\partial_\mu S)\partial_\nu F^{\alpha\beta}$$
$$\times \sqrt{2}\epsilon_{\alpha\beta\mu\nu}\Big[\mathrm{Re}(\partial_\rho S)\partial^\alpha \tilde{F}^{\beta\rho} - \mathrm{Im}(\partial_\rho S)\partial^\alpha \tilde{F}^{\beta\rho}\Big]$$
$$- \frac{8\sqrt{2}}{(1+8\partial_\kappa S\partial^\kappa S^*)}\epsilon_{\alpha\beta\mu\nu} m_{\kappa\lambda}\mathrm{Re}(\partial^\alpha S)\partial^\beta F^{\kappa\lambda}. \quad (18d)$$

Let us call the reader's attention to the fact that the $A^\mu - \Lambda$ mixed term appears in the form $\bar{Z}N\Lambda$; the N-matrix is written in terms of 1, γ_5, and $\Sigma_{\mu\nu}$, and the coefficients $I^{(1)}$, $I^{(2)}$ and $I_{\mu\nu}$ contain terms in the background field S (through $\partial_\mu S$) and $F_{\mu\nu}$. As a whole, the term $\bar{Z}N\Lambda$ is quadratic in the bosonic background and quadratic (but non-diagonal) in the degrees of freedom of the gauge sector (A^μ and Λ).

4 Dispersion relations and a purely photonic efective action

The N-matrix previously defined depends on the field strength, $F^{\alpha\beta}$, through terms of the form $\partial^\mu F^{\alpha\beta}$. For convenience, we factor out N according to the following splitting: $N = N'_\alpha A^\alpha$. N' is therefore a combination of differential operators acting on the gauge potential A_μ according to the expression for N. We do not present the explicit expression for N' because, besides being a lengthy combination of differential operators, it is not actually needed for the attainment of the dispersion relations we wish to write down for the photon and photino. This allows us to rewrite in a more compact form the quadratic action in the photon and photino fields. We unify the latter in a sort of doublet: $\Psi \equiv \begin{pmatrix}\Lambda \\ A_\nu\end{pmatrix}$, $\bar{\Psi} \equiv (\bar{\Lambda} \;\; A_\mu)$, so that the full action may be brought into the form

$$S^{(\mathrm{full})} = \frac{1}{2}\int \mathrm{d}x^4 \bar{\Psi}\mathcal{O}\Psi, \quad (19)$$

where the matrix operator $\mathcal{O}$ is given by

$$\mathcal{O} = \begin{pmatrix} M & N' \\ N' & Q\end{pmatrix}, \quad (20)$$

with the sub-matrices given by

$$M = -\frac{\tilde{a}}{4(1+8\partial_\kappa S\partial^\kappa S^*)}1_{4\times 4} - \frac{\tilde{b}}{4(1+8\partial_\kappa S\partial^\kappa S^*)}\gamma_5 + i\frac{\gamma^\mu\partial_\mu}{2} + \Big(-(C.\partial)\partial_\mu + y\Box C_\mu + C^\alpha d_{\alpha\mu}\Big)\gamma^\mu\gamma_5, \quad (21a)$$

$$Q_{\mu\nu} = -\frac{1}{2}\Box\theta_{\mu\nu} + (J_{\mu\alpha\beta\nu} - J_{\mu\alpha\nu\beta} + J_{\alpha\mu\nu\beta} - J_{\alpha\mu\beta\nu})\Box\omega^{\alpha\beta},$$

where

$$J_{\mu\alpha\beta\nu} = -\frac{1}{4}K_{\mu\alpha\beta\nu} - \frac{64}{(1+8\partial_\rho S\partial^\rho S^*)}m_{\mu\alpha}m_{\beta\nu}. \quad (21b)$$

In Eq. (21a) the quantities $\theta_{\mu\nu}$, $\omega_{\mu\nu}$, and $d_{\mu\nu}$ are defined in the Appendix B.

A conventional procedure would consist in explicitly calculating $\mathcal{O}^{-1}$ in order to get the $\bar{\Lambda}\Lambda$-, ΛA_μ-, and $A_\mu A_\nu$-propagators, whose pole structure corresponds to the dispersion relation. However, if we are simply interested in the dispersion relations for the photon and photino fields, we may concentrate only on the matrices M and Q, as was shown in more detail in the paper of Ref. [56]. Actually, the poles of the photon and photino propagators can be read off from $\det Q = 0$ and $\det N = 0$, respectively.

The photino propagator corresponds to the inverse matrix M^{-1}, whose pole structure is found in $\det M$:

$$M^{-1} = A + B\gamma_5 + v_\mu\gamma^\mu + \omega_\mu\gamma^\mu\gamma_5, \quad (22)$$

with the coefficients given by (in momentum space)

$$A = \frac{\tilde{a}p^2}{16(1+8\partial_\kappa S\partial^\kappa S^*)\Delta}, \quad (23)$$

$$B = -\frac{\tilde{b}p^2}{16(1+8\partial_\kappa S\partial^\kappa S^*)\Delta}, \quad (24)$$

$$v_\mu = \left[\frac{\tilde{a}^2 - \tilde{b}^2}{16(1+8\partial_\kappa S\partial^\kappa S^*)^2} - \frac{p^2}{4} - \tilde{w}^2\right]\frac{p_\mu}{2\Delta} + \frac{(\tilde{w}.p)\tilde{w}_\mu}{\Delta}, \quad (25)$$

$$\omega_\mu = (1-y)p^2(p.C)\frac{p_\mu}{2\Delta} + (C^\alpha p^\beta d_{\alpha\beta})\frac{p_\mu}{2\Delta} - \frac{p^2}{4\Delta}\Big[(p.C)p_\mu - yp^2 C_\mu + C^\alpha d_{\alpha\mu}\Big], \quad (26)$$

and

$$\Delta = \frac{p^4}{16} - (p.\tilde{w})^2 - \frac{p^2\tilde{a}^2}{32(1+8\partial_\kappa S\partial^\kappa S^*)^2} + \frac{p^2\tilde{b}^2}{32(1+8\partial_\kappa S\partial^\kappa S^*)^2} + \frac{p^2}{2}\tilde{w}^2, \quad (27)$$

where p^μ is the photino momentum and the coefficients $\tilde{a}$, $\tilde{b}$, and $\tilde{w}_\mu$ are explicitly stated in Eqs. (58)–(61) of Appendix B.

We can separate the denominator Δ in two parts: one part containing terms up to second order in powers of $\partial_\mu S$ and another piece that only contains higher powers in $\partial_\mu S$. This splitting is suitable if we recall that the LSV parameters are very tiny, so that we confine our considerations to terms which are second order in $\partial_\mu S$, and we collect higher terms in $\mathcal{O}(3)$:

$$\Delta = p^4\,\mathrm{X}^2\tilde{\Delta} = p^4\,\mathrm{X}^2\left(\frac{1}{16\,\mathrm{X}^2} + \left[C^{(1)}p^2 + C^{(2)}_{\mu\nu}p^\mu p^\nu\right] + \mathcal{O}(3)\right), \tag{28}$$

where

$$C^{(1)} = \left(y^2 - y - \frac{1}{2}\right) + \left[\frac{1}{(1+8\partial_\mu S\partial^\mu S^*)}\right](4y-2)(\eta_{\mu\nu}t^{\mu\nu}), \tag{29}$$

$$C^{(2)}_{\mu\nu} = \left[\frac{1}{2(1+8\partial_\mu S\partial^\mu S^*)}\right][42y-29]t_{\mu\nu}. \tag{30}$$

Since $K_{\mu\nu\alpha\beta}$ is a linear combination of bilinears in $\partial_\mu S$, terms of $\mathcal{O}(3)$ or higher in Eq. (15) are discarded. We also notice that the coefficient $C^{(2)}_{\mu\nu}$ is much smaller than $C^{(1)}$, since $|t_{\mu\nu}| << 1$, so, in this approximation, it is possible to remove the term that mixes the momenta and we find a very simple dispersion relation for the photino:

$$\Delta^{(\mathrm{approx})} = C^{(1)}\,\mathrm{X}^2 p^4(p^2 - m^2) = 0, \tag{31}$$

with

$$m^2_{\mathrm{photino}} = -\frac{1}{16\,\mathrm{X}^2 C^{(1)}}; \tag{32}$$

notice that $C^{(1)}$ is negative. Here, contrary to the Carrol–Field–Jackiw supersymmetrized model of Ref. [56], the photino mass carries an explicit dependence on the X-fermion condensate. This is a new feature of the k_F-model. We highlight here that even if the bosonic part of the background (the four-vector ξ_μ) is trivial, the photino mass does not vanish, because it is a natural consequence of the condensation of the fermionic sector of the background. This is a very salient aspect of the connection between supersymmetry and the violation of Lorentz invariance.

Following along analogous steps, we are able to find the dispersion relation for the photon:

$$p^0_\pm = (1 + \rho \pm \sigma)|\mathbf{p}|, \tag{33}$$

where $\rho = \frac{1}{2}\tilde{K}_\alpha{}^\alpha$ and $\sigma^2 = \frac{1}{2}(\tilde{K}_{\alpha\beta})^2 - \rho^2$, with $\tilde{K}^{\alpha\beta} = K^{\alpha\beta\mu\nu}\hat{p}_\mu\hat{p}_\nu$ and $\hat{p}^\mu = p^\mu/|\mathbf{p}|$ [57].

Finally, by eliminating the mixed $A_\mu\,\Lambda$ terms, we shall find an effective action for the purely photonic sector. In the action, the term that combines these fields is given by $2\bar{Z}N\Lambda$. We notice that this term can be removed by performing a convenient shift in the photino field. By redefining the fermion field according to $\Upsilon = \Lambda + M^{-1}\bar{N}Z$, we attain a new action that is totally diagonal in the fields Υ and A_μ. With the help of the properties of the fermionic condensates (7) and the gamma-matrix algebra, the redefinition of Λ suggested above yields an effective term for the photon sector which can be expressed as follows:

$$\begin{aligned} S^{(\mathrm{photon})}_{\mathrm{effective}} &= \int d^4x\,\bar{Z}(NM^{-1}\bar{N})Z \\ &= \int d^4x\Big[(I^{(1)}I^{(1)} - I^{(2)}I^{(2)} + \frac{1}{2}I_{\mu\nu}I^{\mu\nu}) \\ &\quad\times(A\,\mathrm{X} + B\tau) + i(2I^{(1)}I^{(2)} - \frac{1}{2}I_{\mu\nu}\tilde{I}^{\mu\nu}) \\ &\quad\times(A\tau + B\,\mathrm{X}) + (I^{(1)}I^{(1)} + I^{(2)}I^{(2)} \\ &\quad+\frac{1}{2}I_{\mu\nu}I^{\mu\nu})\omega_\rho C^\rho + 2I^{(1)}I^{\kappa\rho}\omega_\kappa C_\rho \\ &\quad-2I^{(2)}\tilde{I}^{\kappa\rho}\omega_\kappa C_\rho\Big], \end{aligned} \tag{34}$$

where the coefficients $I^{(1)}$, $I^{(2)}$, and $I_{\mu\nu}$ ($\tilde{I}^{\mu\nu}$ is the dual tensor of $I_{\mu\nu}$) only exhibit derivatives of the field strength.

Taking into account the previous discussion of the approximation we adopt to treat the LSV parameters, we can also ignore the terms of order $\mathcal{O}(3)$ in Eq. (16), so that the full effective Lagrangian density (in momentum space) for the photon is given by

$$\mathcal{L} = \mathcal{L}_{\mathrm{old}} + \mathcal{L}_{\mathrm{effective}}, \tag{35}$$

where

$$\mathcal{L}_{\mathrm{old}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - 16t_{\mu\nu}F^{\mu\kappa}F_\kappa{}^\nu - 4F_{\mu\nu}F^{\mu\nu}(t_{\alpha\beta}\eta^{\alpha\beta}) \tag{36}$$

and

$$\begin{aligned} \mathcal{L}_{\mathrm{effective}} &= \frac{1}{\tilde{\Delta}}\left(\frac{y}{4} - \frac{1}{8}\right)t_{\rho\lambda}\left[4p^2F_\mu{}^\rho F^{\mu\lambda} - \eta^{\rho\lambda}p^2F_{\mu\nu}F^{\mu\nu}\right] \\ &\quad+\frac{1}{\tilde{\Delta}}\left(\frac{5y}{8} + \frac{13}{16}\right)t_{\rho\lambda}\left[p_\mu p_\nu F^{\mu\rho}F^{\nu\lambda}\right]. \end{aligned} \tag{37}$$

A remarkable difference is to be highlighted between the effective photonic actions derived in the Carroll–Field–Jackiw and in the k_F-cases in a SUSY scenario: the supersymmetric version of the k_{AF} term induces a purely photonic action with CP-violating axionic terms; the k_F-case, on the other hand, does not induce CP-breaking terms in the effective photonic action. As we can check in Eq. (37), no CP-violating term of the form $F\tilde{F}$ shows up. It is also to be noticed that, in both the k_{AF}- and the k_F-cases, ∂F terms appear, so that the k_F- and the k_{AF}-models are equally sensitive in the high-frequency regime. The most remarkable difference is actually the absence of CP-violating terms in the photonic action of the k_F-case. Let us recall here that y has been given in Eq. (11b) and $t_{\mu\nu}$ is defined in Appendix B.

5 Interaction energy

We now examine the interaction energy from the viewpoint of the gauge-invariant but path-dependent variables formalism, along the lines of Refs. [55,56,59–63]. This can be done by computing the expectation value of the energy operator H in the physical state $|\Phi\rangle$ describing the sources, which we will denote by $\langle H\rangle_\Phi$. The starting point is the effective Lagrangian density:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}\left[1+16t^\alpha_\alpha-4\left(\frac{y}{4}-\frac{1}{8}\right)t^\alpha_\alpha\frac{\Delta}{\tilde{\Delta}}\right]F^{\mu\nu} - 16t_{\mu\nu}F^{\mu\lambda}F^{\nu}{}_{\lambda}-4\left(\frac{y}{4}-\frac{1}{8}\right)t_{\rho\lambda}F^{\mu\lambda}\frac{\Delta}{\tilde{\Delta}}F^\rho_\mu - \left(\frac{5y}{8}+\frac{13}{16}\right)t_{\rho\lambda}F^{\mu\lambda}\frac{\partial_\mu\partial_\nu}{\tilde{\Delta}}F^{\nu\rho}, \tag{38}$$

where $\Delta\equiv\partial_\mu\partial^\mu$. However, as mentioned before, this paper is aimed at studying the static potential of the above theory, and a consequence of this is that one may replace Δ by $-\nabla^2$ in Eq. (38). Furthermore, we recall that the only non-vanishing $t_{\mu\nu}$ terms are the diagonal ones, since, as already anticipated, $t_{\mu\nu}$ can be brought into a diagonal form. Without loss of generality, we may always choose $t_{00}\neq 0$.

Therefore, the effective Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}\gamma F_{\mu\nu}\frac{\left(\nabla^2-M^2\right)}{\left(\nabla^2-m^2\right)}F^{\mu\nu}+16t_{00}F_{i0}F^{i0} - \frac{A_4}{A_2}t_{00}F_{i0}\frac{\nabla^2}{\left(\nabla^2-m^2\right)}F^{i0} - \frac{A_5}{A_2}t_{00}F^{i0}\frac{\partial_i\partial_j}{\left(\nabla^2-m^2\right)}F^{j0}. \tag{39}$$

Here, $\gamma = \frac{A_3A_2-A_4t^\alpha_\alpha}{A_2}$, $M^2=\frac{A_3A_1}{A_3A_2-A_4t^\alpha_\alpha}$, $m^2=\frac{A_1}{A_2}$. Whereas that $A_1=\frac{1}{16\,\mathrm{X}^2}$, $A_2=-C^{(1)}$, $A_3=\left(1+16t^\alpha_\alpha\right)$, $A_4=4\left(\frac{y}{4}-\frac{1}{8}\right)$, and $A_5=\left(\frac{5y}{8}+\frac{13}{16}\right)$. Notice that these A_i are not to be confused with the photon field components.

To obtain the corresponding Hamiltonian we shall follow the Dirac method used in previous works [56,60–63]. The canonical momenta are found to be $\Pi^\mu=\frac{(\alpha\nabla^2-\beta)}{(\nabla^2-m^2)}F^{\mu 0}+\frac{2A_5}{A_2}t_{00}\frac{\partial^\mu\partial_j}{(\nabla^2-m^2)}F^{j0}$, where $\alpha=\gamma-32t_{00}+2\frac{A_4}{A_2}$ and $\beta=\gamma M^2-32t_{00}m^2$. Since Π^0 vanishes, we have the usual constraint equation, which according to Dirac's theory is written as a weak ($\approx$) equation: $\Pi^0\approx 0$. The remaining non-zero momenta must also be written as weak equations. In this case, $\Pi^i\approx\frac{(\alpha\nabla^2-\beta)}{(\nabla^2-m^2)}F^{i0}+\frac{2A_5}{A_2}t_{00}\frac{\partial^i\partial_j}{(\nabla^2-m^2)}F^{j0}$. Thus, the canonical Hamiltonian, which must be written as a weak equation, takes the form

$$H_C \approx \int d^3x\Bigg[-A_0\partial_i\Pi^i-\frac{1}{2}\Pi^i\frac{\left(\nabla^2-m^2\right)}{\left(\alpha\nabla^2-\beta\right)}\Pi_i + \frac{1}{4}F_{ij}\left(\frac{\nabla^2-M^2}{\nabla^2-m^2}\right)F^{ij} + \frac{1}{2}\partial^i\partial^k\Pi^k\frac{\left(\nabla^2-m^2\right)}{\left(\nabla^2+\Omega^2\right)^2\left(\alpha\nabla^2-\beta\right)}\partial_i\partial_j\Pi_j + \frac{A_5}{A_2}t_{00}\left(\frac{\left(\nabla^2-m^2\right)}{\left(\alpha\nabla^2-\beta\right)}\Pi_i+\frac{\left(\nabla^2-m^2\right)\partial_i\partial_k\Pi_k}{\left(\nabla^2+\Omega^2\right)\left(\alpha\nabla^2-\beta\right)}\right) \times\frac{\partial_i\partial_j}{\left(\nabla^2-m^2\right)}\left(\frac{\left(\nabla^2-m^2\right)}{\left(\alpha\nabla^2-\beta\right)}\Pi^j+\frac{\left(\nabla^2-m^2\right)\partial^j\partial^m\Pi^m}{\left(\nabla^2+\Omega^2\right)\left(\alpha\nabla^2-\beta\right)}\right)\Bigg], \tag{40}$$

where $\frac{1}{\Omega^2}=\frac{2A_5}{A_2}t_{00}\frac{1}{\alpha\nabla^2-\beta}$.

The primary constraint, $\Pi_0\approx 0$, must be satisfied for all times. By using the equation of motion of a dynamical variable, $\dot{\mathrm{Z}}\approx[\mathrm{Z},H_C]$, we obtain the usual secondary constraint, $\Gamma_1=\partial_i\Pi^i\approx 0$. The conservation of Γ_1 for all times does not give rise to any further constraints. Therefore, in this case there are two constraints, which are first class.

In accordance with the Dirac method we obtain the extended Hamiltonian that generates translations in time as an ordinary (or strong) equation by adding all the first-class constraints with arbitrary coefficients. Thus, we write $H=H_C+\int d^3x\left(c_0(x)\Pi_0(x)+c_1(x)\Gamma_1(x)\right)$, where $c_0(x)$ and $c_1(x)$ are arbitrary functions of space and time. It should be noted that when this new Hamiltonian is used, the equation of motion of a dynamical variable may be written as a strong equation, $\dot{\mathrm{Z}}=[\mathrm{Z},H]$. Since $\Pi^0\approx 0$ for all time and $\dot{A}_0(x)=[A_0(x),H]=c_0(x)$, which is completely arbitrary, we discard A^0 and Π^0. In fact, it the term containing A_0 is redundant, because it can be absorbed by redefining the function $c_1(x)$. In this case, the extended Hamiltonian takes the form

$$H = \int d^3x\Bigg[c(x)\left(\partial_i\Pi^i\right)-\frac{1}{2}\Pi^i\frac{\left(\nabla^2-m^2\right)}{\left(\alpha\nabla^2-\beta\right)}\Pi_i + \frac{1}{4}F_{ij}\left(\frac{\nabla^2-M^2}{\nabla^2-m^2}\right)F^{ij} + \frac{1}{2}\partial^i\partial^k\Pi^k\frac{\left(\nabla^2-m^2\right)}{\left(\nabla^2+\Omega^2\right)^2\left(\alpha\nabla^2-\beta\right)}\partial_i\partial_j\Pi_j + \frac{A_5}{A_2}t_{00}\left(\frac{\left(\nabla^2-m^2\right)}{\left(\alpha\nabla^2-\beta\right)}\Pi_i+\frac{\left(\nabla^2-m^2\right)\partial_i\partial_k\Pi_k}{\left(\nabla^2+\Omega^2\right)\left(\alpha\nabla^2-\beta\right)}\right) \times\frac{\partial_i\partial_j}{\left(\nabla^2-m^2\right)}\left(\frac{\left(\nabla^2-m^2\right)}{\left(\alpha\nabla^2-\beta\right)}\Pi^j+\frac{\left(\nabla^2-m^2\right)\partial^j\partial^m\Pi^m}{\left(\nabla^2+\Omega^2\right)\left(\alpha\nabla^2-\beta\right)}\right)\Bigg], \tag{41}$$

where $c(x)=c_1(x)-A_0(x)$.

Now we call attention to the fact that the presence of the arbitrary quantity $c(x)$ is undesirable, since we have no way of giving it a meaning in a quantum theory. To avoid this trouble, according to the standard procedure, we impose one gauge constraint such that the full set of constraints becomes second class. As was explained in [64], a particularly convenient condition is

$$\Gamma_2(x) \equiv \int\limits_{C_{\zeta x}} dz^{\nu} A_{\nu}(z) \equiv \int\limits_0^1 d\lambda x^i A_i(\lambda x) = 0, \tag{42}$$

where λ $(0 \leq \lambda \leq 1)$ is the parameter describing the space-like straight path $x^i = \zeta^i + \lambda(x-\zeta)^i$, and ζ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\zeta^i = 0$. In this case, the only non-vanishing equal-time Dirac bracket is

$$\left\{A_i(x), \Pi^j(y)\right\}^* = \delta_i^{\,j} \delta^{(3)}(x-y) - \partial_i^x \int\limits_0^1 d\lambda x^j \delta^{(3)}(\lambda x - y). \tag{43}$$

In passing we recall that the transition to a quantum theory is made by the replacement of the Dirac brackets by the operator commutation relations according to $\{A, B\}^* \to (-i/\hbar)[A, B]$.

We are now in a position to evaluate the interaction energy between point-like sources in the model under consideration. As already expressed, we will work out the expectation value of the energy operator H in the physical state $|\Phi\rangle$, where the physical states $|\Phi\rangle$ are gauge-invariant ones. To this end one recalls that the gauge-invariant state is given by [65]

$$|\Phi\rangle \equiv \left|\bar{\Psi}(\mathbf{y})\Psi(\mathbf{y}')\right\rangle = \bar{\psi}(\mathbf{y}) \exp\left(\frac{iq}{\hbar}\int_{\mathbf{y}'}^{\mathbf{y}} dz^i A_i(z)\right)\psi(\mathbf{y}')|0\rangle, \tag{44}$$

where $|0\rangle$ is the physical vacuum state and the line integral appearing in the above expression is along a space-like path starting at $\mathbf{y}'$ and ending at $\mathbf{y}$, on a fixed time slice. We see, therefore, that each of the states $(|\Phi\rangle)$ represents a fermion–antifermion pair surrounded by a cloud of gauge fields to maintain gauge invariance. Also it is important to point out that [64]

$$\{\Pi_k(\mathbf{x}), \Psi(\mathbf{y})\}^* = \frac{iq}{\hbar}\int_0^1 d\lambda y_k \delta^{(3)}(\mathbf{x} - \lambda\mathbf{y})\Psi(\mathbf{y}) \tag{45}$$

and

$$\left\{\Pi_k(\mathbf{x}), \bar{\Psi}(\mathbf{y})\right\}^* = -\frac{iq}{\hbar}\int_0^1 d\lambda y_k \delta^{(3)}(\mathbf{x} - \lambda\mathbf{y})\bar{\Psi}(\mathbf{y}). \tag{46}$$

With this at hand, we then consider the state $\Pi_i(\mathbf{x})|\Phi\rangle$, that is,

$$\Pi_i(\mathbf{x})|\Phi\rangle = \bar{\Psi}(\mathbf{y})\Psi(\mathbf{y}')\Pi_i(\mathbf{x})|0\rangle + \left(\left[\Pi_i(\mathbf{x}), \bar{\Psi}(\mathbf{y})\right]\Psi(\mathbf{y}') + \bar{\Psi}(\mathbf{y})\left[\Pi_i(\mathbf{x}), \Psi(\mathbf{y}')\right]\right)|0\rangle. \tag{47}$$

Now, by employing Eqs. (45) and (46), we can reduce Eq. (47) to

$$\Pi_i(\mathbf{x})\left|\overline{\Psi}(\mathbf{y})\Psi(\mathbf{y}')\right\rangle = \overline{\Psi}(\mathbf{y})\Psi(\mathbf{y}')\Pi_i(\mathbf{x})|0\rangle + q\int_{\mathbf{y}}^{\mathbf{y}'} dz_i \delta^{(3)}(\mathbf{z}-\mathbf{x})|\Phi\rangle. \tag{48}$$

Hence we see that our calculation is a semiclassical one.

We now proceed to determine the interaction energy. We further recall that the fermions are taken to be infinitely massive (static), which means that there is no magnetic field. In this case, the expectation value $\langle H\rangle_\Phi$ reads

$$\langle H\rangle_\Phi = \left\langle \Phi \middle| \int d^3x \left[-\frac{1}{2}\Pi^i \frac{(\nabla^2 - m^2)}{(\alpha\nabla^2 - \beta)}\Pi_i\right] \middle| \Phi \right\rangle. \tag{49}$$

Then using (48), we obtain explicitly

$$\langle H\rangle_\Phi = \langle H\rangle_0 + \langle H\rangle_\Phi^{(1)} + \langle H\rangle_\Phi^{(2)}, \tag{50}$$

where $\langle H\rangle_0 = \langle 0|H|0\rangle$, and the $\langle H\rangle_\Phi^{(1)}$ and $\langle H\rangle_\Phi^{(2)}$ terms are given by

$$\langle H\rangle_\Phi^{(1)} = -\frac{q^2}{2\alpha}\int d^3x \int_{\mathbf{y}}^{\mathbf{y}'} dz_i' \delta^{(3)}(\mathbf{x}-\mathbf{z}') \times \left(1 - \frac{\beta/\alpha}{\nabla^2}\right)_x^{-1} \int_{\mathbf{y}}^{\mathbf{y}'} dz^i \delta^{(3)}(\mathbf{x}-\mathbf{z}), \tag{51}$$

$$\langle H\rangle_\Phi^{(2)} = \frac{q^2 m^2}{2\alpha}\int d^3x \int_{\mathbf{y}}^{\mathbf{y}'} dz_i' \delta^{(3)}(\mathbf{x}-\mathbf{z}') \times \left(\frac{1}{\nabla^2 - \beta/\alpha}\right)_x \int_{\mathbf{y}}^{\mathbf{y}'} dz^i \delta^{(3)}(\mathbf{x}-\mathbf{z}). \tag{52}$$

The above integrals have been calculated in [66]; in view of this situation, we skip all the technical details and refer to [66]. We further note that the second and third term on the right-hand side of Eq. (50) are clearly dependent on the distance between the external static fields. Therefore the potential for two opposite charges located at $\mathbf{y}$ and $\mathbf{y}'$ is given by

$$V = -\frac{q^2}{4\pi\alpha}\frac{e^{-\sqrt{\beta/\alpha}L}}{L} + \frac{q^2 m^2}{8\pi\alpha}\ln\left(1 + \frac{\Lambda^2}{\beta/\alpha}\right)L, \tag{53}$$

where Λ is an ultraviolet cutoff, $|\mathbf{y}-\mathbf{y}'| \equiv L$, $\alpha = \gamma - 32t_{00} + 2\frac{A_4}{A_2}$, and $\beta = \gamma M^2 - 32t_{00}m^2$. We also point out that this cutoff arises when manipulating the ultraviolet divergent integral (51). At this stage of the calculations, we must decide on the choice of the cutoff, Λ. Following our chain of definitions for A_1, A_2, A_3, A_4, a, b, and γ, it is readily seen that the only pole that corresponds to a physical mass is exactly the photino mass, previously given in Eq. (32). This means that the interparticle potential above makes sense only for distances above the Compton wavelength of the photino, $\lambda_{\text{photino}} \equiv m_{\text{photino}}^{-1}$. We then are naturally led to make the identification $\Lambda = m_{\text{photino}}$. Therefore, our conclusion is that, whenever the particle–antiparticle pair is in static interaction at a regime of distances $r > \lambda_{\text{photino}}$, the form of V can be consistently taken as given in Eq. (53). With this identification, the potential of Eq. (53) takes the form

$$V = -\frac{q^2}{4\pi\alpha}\frac{e^{-\sqrt{\beta/\alpha}\,L}}{L} + \frac{q^2 m^2}{8\pi\alpha}\ln\left(1 + \frac{m_{\text{photino}}^2}{\beta/\alpha}\right)L. \tag{54}$$

It is worthwhile to note at this point the presence of a finite string tension (which is represented by the proportionality constant in the linear potential) in Eq. (54). Incidentally, it is of interest to notice that we can speak about a string tension because the usual qualitative picture of confinement, in terms of an electric flux linking quarks, emerges naturally in the gauge-invariant formalism used in this paper [56,60–63,66].

Also it is important to point out that our previous result (confining potential) may sound strange in an abelian gauge theory. It should, however, be recalled here that the existence of a phase structure for a continuum abelian $U(1)$ gauge theory has been obtained by including the effects due to the compactness of the $U(1)$ group, which dramatically changes the infrared properties of the model [67]. As is well known, this result has been ever since re-derived by many different techniques [68–70]; the main characteristic is the contribution of self-dual topological excitations of the theory. Nevertheless, the mechanism of confinement in this work is not condensation of topological excitations, but rather background fermions. In fact, this is what makes our work different from other studies of confinement in abelian gauge theories.

6 Concluding remarks

As mentioned in the Sect. 1 of the present contribution, there are in the literature a number of approaches to LSV that contemplate the introduction of SUSY in connection with the breaking of relativistic covariance in the sense of the so-called particle transformations.

The present work provides an investigation whose approach basically consists in assuming that LSV takes place in an environment dominated by SUSY, and we adopt the viewpoint that the bosonic background usually adopted to realize the breaking of Lorentz symmetry is part of a whole setup with fermionic SUSY partners. We then claim that LSV takes place through specific SUSY multiplets, so that the usual k_{AF} and k_F terms are accompanied by SUSY fermionic partners; in short, the background tensors that parametrize LSV are components of specific superfields.

In this paper, our main goal is to point out the salient aspects of the k_F-type LSV in association with $N = 1$ $D = 4$ SUSY, focusing specially on the background condensates that show up along with the $(k_F)_{\mu\nu\kappa\lambda}$ breaking term. The pattern of breaking is, in the present situation, much richer than the similar inspection carried out previously in Ref. [56].

Generally, what we have concluded with the studies reported in this work is that if we place the discussion of LSV in a SUSY framework, by assuming that the scale of breaking of Lorentz covariance is above the SUSY breaking scale, we actually find that, once Lorentz symmetry is violated, SUSY breaking also takes place and the photon and photino dispersion relations are split from one another with different profiles in terms of the SUSY background fermion condensates naturally induced in the process of LSV. While the photon dispersion relations that follow from LSV are not affected by the background fermions, the photino dispersion relations are strongly controlled by the whole set of fermionic condensates.

Particularly, the SUSY scenario for the k_F-LSV reveals the following.

(i) The photino mass now depends not only on the bosonic background (in this case, the scalar S) but also on the condensate $\mathrm{X} = \bar{Z}Z$:

$$m^2_{\text{photino}} = -\frac{1}{16X^2C^{(1)}}, \tag{55}$$

as given in Eq. (32). This means that the X-condensate (X has the canonical dimension of mass^{-1}) may be estimated if we take the photino mass in the TeV-scale. Recalling the experimental bounds on the components of k_F (and then on the components of the vector ξ^μ) [10], and the expression for $C^{(1)}$ in Eq. (29), it turns out that effectively only the condensate X fixes the photino mass; $C^{(1)}$ is actually of $\mathcal{O}(1)$. Thus, for a photino in the TeV-region, the condensate X is estimated to be $\mathcal{O}(\text{TeV}^{-1})$, corresponding to a length in the submillimetric scale. This result should be further explored, for it may point to an explicit SUSY breaking at an accelerator regime.

(ii) It is also remarkable that, like in the k_{AF}-case (Carroll–Field–Jackiw), the photon dispersion relation does not receive contributions from SUSY. This feature is common to both the k_{AF}- and the k_F-cases.

(iii) The effective photonic case associated to the supersymmetric k_F-model exhibits a quadratic dependence on the photon momentum, as occurs in the supersymmetric version of the Carroll–Field–Jackiw case; SUSY distinguishes these two models with LSV in that the purely photonic action in the k_{AF}-case exhibits CP-breaking terms, whereas the k_F-situation studied in this paper yields an effective photonic action which respects CP-symmetry.

(iv) The effects of the supersymmetric background fermion condensates are, moreover, felt through the photonic action. It is therefore not surprising that they become manifest in the interaction energy for the effective theory. In fact, we have obtained the effective theory for the condensed phase and computed the interaction energy between two static charges, in order to test the confinement versus screening properties of our effective model. Interestingly, we have explicitly shown that the static potential profile contains a Yukawa term and a linear term, leading to the confinement of static charges.

Finally, we would like to comment that we could also inspect this very same model (the k_F-model) by considering the ξ^μ-vector not given by the scalar supermultiplet as the 4-gradient of S. We could rather suppose that ξ^μ is placed in a (non-gauge) vector multiplet of $N = 1$ $D = 4$ SUSY, which would introduce a richer fermionic background. Moreover, ξ^μ would in this case become a complete vector, with a transverse part in addition to its gradient (longitudinal component). A wider class of condensates would emerge in such a situation and this might have an interesting consequence specially in the photon dispersion relations, always very sensitive to the particular choice of the multiplet that accommodates the background yielding LSV. We are already concentrating efforts in this direction and we shall be reporting our results in a forthcoming paper to better understand the influence of the particular supersymmetric structure on the physics of LSV.

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Appendix A

Below, we collect the three pieces of our component-field action corresponding to Eq. (6) in terms of (2-component) Weyl spinors:

$$
\begin{aligned}
S_{\text{boson}} = \int \mathrm{d}^4x \Big[& D^2(32|G|^2 + 16\partial_\mu S \partial_\mu S^*) \\
& + 8iDF^{\mu\nu}(\partial_\mu S \partial_\nu S^* - \partial_\mu S^* \partial_\nu S) \\
& - 8F^{\mu\kappa}F_\kappa{}^\nu(\partial_\mu S \partial_\nu S^* + \partial_\mu S^* \partial_\nu S) \\
& - 4F^{\mu\nu}F_{\mu\nu}\partial_\alpha S \partial_\alpha S^* \Big],
\end{aligned}
$$

$$
\begin{aligned}
S_{\text{ferm}} = \int \mathrm{d}^4x \Big[& \frac{1}{2}\partial_\lambda\zeta\sigma^\mu\partial_\mu\bar\zeta\lambda\sigma^\lambda\bar\lambda + \frac{1}{2}\partial_\lambda\zeta\sigma^\mu\bar\lambda\lambda\sigma^\lambda\partial_\mu\bar\zeta \\
& + 2\partial_\mu\zeta\partial^\mu\lambda\bar\zeta\bar\lambda \frac{1}{2}\partial_\lambda\zeta\sigma^\lambda\partial_\mu\bar\zeta\lambda\sigma^\mu\bar\lambda - 2\partial_\lambda\zeta\sigma^\lambda\bar\sigma^\mu\partial_\mu\lambda\bar\zeta\bar\lambda \\
& - \frac{1}{2}\lambda\sigma^\lambda\bar\sigma^\mu\partial_\lambda\zeta\bar\zeta\partial_\mu\bar\lambda - \frac{1}{2}\zeta\lambda\bar\zeta\Box\bar\lambda - \zeta\lambda\partial_\mu\bar\zeta\bar\sigma^\mu\sigma^\tau\partial_\tau\bar\lambda \\
& + \frac{1}{2}\zeta\lambda\partial_\mu\bar\lambda\bar\sigma^\nu\sigma^\mu\partial_\nu\bar\zeta + \frac{1}{2}\partial_\mu\zeta\sigma^\mu\bar\sigma^\nu\lambda\bar\zeta\partial_\nu\bar\lambda \\
& - \frac{1}{2\sqrt{2}}\zeta\Box\lambda\bar\zeta\bar\lambda - \frac{1}{2}\zeta\partial_\nu\lambda\partial_\mu\bar\lambda\bar\sigma^\mu\sigma^\nu\bar\zeta \\
& - \frac{1}{2}\zeta\partial_\nu\lambda\partial_\mu\bar\zeta\bar\sigma^\nu\sigma^\mu\bar\lambda + \zeta\partial_\mu\lambda\bar\zeta\partial^\mu\bar\lambda \\
& - 2\zeta\sigma^\mu\partial_\mu\bar\lambda\bar\zeta\bar\sigma^\nu\partial_\nu\lambda + \text{h.c.} \Big],
\end{aligned}
$$

$$
\begin{aligned}
S_{\text{mixing}} = \int \mathrm{d}^4x \Big[& -4iD^2\zeta\sigma^\mu\partial_\mu\bar\zeta - 2\sqrt{2}iDG^*\zeta\sigma^\mu\partial_\mu\bar\lambda \\
& + 2\sqrt{2}D\partial_\nu\lambda\sigma^\nu\bar\sigma^\mu\zeta\partial_\mu S^* + 2D\zeta\sigma^\nu\partial_\mu\bar\zeta F^\mu_\nu \\
& + iD\epsilon^{\tau\rho\mu\alpha}\zeta\sigma_\alpha\partial_\mu\bar\zeta F_{\tau\rho} + \sqrt{2}G^*\zeta\sigma^\mu\partial_\nu\bar\lambda F_\mu{}^\nu \\
& + \frac{i}{\sqrt{2}}G^*\epsilon^{\tau\rho\mu\alpha}\zeta\sigma_\alpha\partial_\mu\bar\lambda F_{\tau\rho} + \sqrt{2}i\zeta\sigma^\tau\bar\sigma^\nu\partial_\nu\lambda\partial_\mu S^* F^\mu_\tau \\
& - \frac{1}{\sqrt{2}}\epsilon^{\tau\rho\mu\alpha}\zeta\sigma_\alpha\bar\sigma^\nu\partial_\mu S^*\partial_\nu\lambda F_{\tau\rho} - 4\sqrt{2}iG^*D\zeta\sigma^\mu\partial_\mu\bar\lambda \\
& + 2\sqrt{2}D\zeta\partial_\mu\lambda\partial^\mu S^* - \frac{i}{\sqrt{2}}\epsilon^{\mu\nu\kappa\tau}\zeta\partial_\tau\lambda\partial_\mu S^* F_{\nu\kappa} \\
& + \frac{1}{2\sqrt{2}}\epsilon^{\mu\nu\kappa\tau}\zeta\partial_\tau\lambda\partial_\mu S^* F_{\nu\kappa} - 4iD^2\bar\zeta\bar\sigma^\mu\partial_\mu\zeta \\
& - 2D\bar\zeta\bar\sigma^\nu\partial_\mu\zeta F_\nu{}^\mu + iD\epsilon^{\nu\kappa\mu\alpha}\bar\zeta\bar\sigma_\alpha\partial_\mu\zeta F_{\nu\kappa} \\
& + 2\sqrt{2}iDG^*\partial_\mu\zeta\sigma^\mu\bar\lambda + 2D\partial_\mu\sigma^\tau\bar\zeta F_\tau{}^\mu \\
& + iD\epsilon^{\tau\rho\mu\alpha}\partial_\mu\zeta\sigma_\alpha\bar\zeta F_{\tau\rho} + 2\partial_\mu\zeta\sigma^\tau\bar\sigma^{\nu\kappa}\bar\zeta F_{\nu\kappa}F_\tau{}^\mu \\
& + i\epsilon^{\tau\rho\mu\alpha}\partial_\mu\zeta\sigma_\alpha\bar\sigma^{\nu\kappa}\bar\zeta F_{\tau\rho}F_{\tau\rho} + \sqrt{2}G^*\partial_\mu\zeta\sigma^\tau\bar\lambda F_\tau{}^\mu \\
& - 2iG\partial_\nu\lambda\sigma^\nu\bar\sigma^\mu\lambda\partial_\mu S^* + 4\sqrt{2}iGD\partial_\mu\lambda\sigma^\mu\bar\zeta \\
& - 2\sqrt{2}iGD\bar\zeta\bar\sigma^\mu\partial_\mu\lambda - \sqrt{2}G\bar\zeta\bar\sigma^\mu\partial_\tau\lambda F_\mu{}^\tau \\
& + \frac{i}{\sqrt{2}}G\epsilon^{\mu\nu\tau\alpha}\bar\zeta\bar\sigma_\alpha\partial_\tau\lambda F_{\mu\nu} - 2i|G|^2\bar\lambda\bar\sigma^\mu\partial_\mu\lambda \\
& + \frac{i}{\sqrt{2}}G^*\epsilon^{\tau\rho\mu\alpha}\partial_\mu\zeta\sigma_\alpha\bar\lambda F_{\tau\rho} - 2\sqrt{2}iGD\lambda\sigma^\mu\partial_\mu\bar\zeta \\
& + 2i|G|^2\lambda\sigma^\mu\partial_\mu\bar\lambda + 2\sqrt{2}D\partial_\mu S\bar\zeta\partial^\mu\bar\lambda \\
& + \sqrt{2}i\partial_\mu(\bar\lambda\bar\zeta)\partial_\lambda SF^{\lambda\mu} - \frac{1}{\sqrt{2}}\epsilon^{\mu\lambda\tau\rho}\partial_\mu(\bar\lambda\bar\zeta)\partial_\lambda SF_{\tau\rho} \\
& - 2\sqrt{2}D\bar\lambda\partial_\mu\bar\zeta\partial^\mu S + 2\sqrt{2}D\bar\zeta\bar\sigma^\mu\sigma^\nu\partial_\nu\bar\lambda\partial_\mu S \\
& - \sqrt{2}i\bar\zeta\bar\sigma^\nu\sigma^\mu\partial_\mu\bar\lambda F_{\nu\lambda}\partial^\lambda S - \frac{1}{\sqrt{2}}\epsilon^{\nu\kappa\lambda\alpha}\bar\zeta\bar\sigma_\alpha\sigma^\mu\partial_\mu\bar\lambda \\
& \times F_{\nu\kappa}\partial_\lambda S + 2G^*\bar\lambda\bar\sigma^\mu\sigma^\nu\partial_\nu\bar\lambda\partial_\mu S - 2\sqrt{2}\partial_\mu S\partial^\mu D\bar\zeta\bar\lambda \\
& + \sqrt{2}D\partial_\nu\zeta\sigma^\nu\bar\mu\lambda\partial_\mu S^* - \frac{i}{\sqrt{2}}\bar\sigma^\lambda\partial_\lambda\zeta\lambda\sigma^\nu\partial_\mu S^* F_\nu{}^\mu \\
& + \frac{1}{2\sqrt{2}}\epsilon^{\mu\nu\kappa\alpha}\lambda\sigma_\alpha\bar\sigma^\lambda\partial_\lambda\zeta\partial_\lambda\zeta\partial_\mu S^* F_{\nu\kappa} \\
& - \frac{1}{2}\partial_\lambda\zeta\sigma^\lambda\bar\lambda\lambda\sigma^\mu\partial_\mu\bar\zeta + \sqrt{2}D\lambda\sigma^\lambda\bar\sigma^\mu\partial_\lambda\zeta\partial_\mu S^* \\
& - \frac{i}{\sqrt{2}}\partial_\lambda\zeta\sigma^\nu\bar\lambda\lambda\partial_\mu S^* F_\nu{}^\mu \\
& - \frac{1}{2\sqrt{2}}\epsilon^{\mu\nu\kappa\alpha}\partial_\lambda\zeta\sigma_\alpha\bar\sigma^\lambda\lambda\partial_\mu S^* F_{\nu\kappa} + \text{h.c.} \Big]. \qquad (56)
\end{aligned}
$$

Appendix B

To render the text of Sect. 3 more fluent, we present, in this appendix, the full expression and details related to the β^2 term that yields our final expression for the action (6) after the D-auxiliary field is eliminated in favor of its equation of motion. We have

$$
\begin{aligned}
\beta^2 = & \bar\Lambda\left(\tilde a + \tilde b\gamma_5 + \tilde u^\rho\gamma_\rho\gamma_5\right)\Lambda \\
& + 16 m_{\alpha\beta}F^{\alpha\beta}\Big[10\sqrt{2}\mathrm{Re}(\partial_\mu S)(\bar Z\partial^\mu\Lambda) - 8\sqrt{2}i\mathrm{Re}(\partial_\mu S) \\
& \times (\bar Z\Sigma^{\mu\nu}\partial_\nu\Lambda) + 8\sqrt{2}\mathrm{Im}(\partial_\mu S)(\bar Z\Sigma^{\mu\nu}\gamma_5\partial_\nu\Lambda) \\
& + 10\sqrt{2}i\mathrm{Im}(\partial_\mu S)(\bar Z\gamma_5\partial^\mu\Lambda)\Big] + 256 m_{\mu\nu}m_{\alpha\beta}F^{\mu\nu}F^{\alpha\beta},
\end{aligned} \qquad (57)
$$

where the operators $\tilde a$, $\tilde b$, and $\tilde u^\rho$ are defined as

$$
\begin{aligned}
\tilde a = & 42\,\mathrm{X}s^{\alpha\beta}\Box\omega_{\alpha\beta} + 84i\tau\mathrm{Re}(\partial^\alpha S)\mathrm{Im}(\partial^\beta S)\Box\omega_{\alpha\beta} \\
& + 8\,\mathrm{X}s\Box + 16i\tau\mathrm{Re}(\partial_\rho S)\mathrm{Im}(\partial^\rho S)\Box, \qquad (58) \\
\tilde b = & 42\tau s^{\alpha\beta}\Box\omega_{\alpha\beta} + 84i\,\mathrm{X}\mathrm{Re}(\partial^\alpha S)\mathrm{Im}(\partial^\beta S)\Box\omega_{\alpha\beta} \\
& + 8\tau s\Box + 16i\,\mathrm{X}\mathrm{Re}(\partial_\rho S)\mathrm{Im}(\partial^\rho S)\Box, \qquad (59) \\
\tilde u^\rho = & -4(1 + 8\partial_\kappa S\partial^\kappa S^*)]C_\alpha \mathrm{d}^{\alpha\rho}; \qquad (60)
\end{aligned}
$$

also, it is useful to define the following operator:

$$
\tilde w^\rho = (p.C)p^\rho - y^2p^2C^\rho + C_\alpha\mathrm{d}^{\alpha\rho}, \qquad (61)
$$

where, in Eqs. (60) and (61),

$$\mathrm{d}^{\alpha\rho} = \frac{-1}{4(1+8\partial_\kappa S\partial^\kappa S^*)}\Big[-50\eta^{\rho\alpha}t^{\mu\nu}\Box\omega_{\mu\nu} + 40t^\alpha_\kappa(\eta^{\beta\alpha}\eta^{\kappa\rho}-\eta^{\kappa\alpha}\eta^{\beta\rho})\Box\omega_{\alpha\beta} + 40[\mathrm{Im}(\partial^\mu S)\mathrm{Re}(\partial^\nu S) - \mathrm{Re}(\partial^\mu S)\mathrm{Im}(\partial^\nu S)]\eta^{\kappa\alpha}\epsilon_{\kappa\nu}{}^{\beta\rho}\Box\omega_{\mu\beta} + 8\delta^\alpha_\kappa(r^{\kappa\rho\mu\nu}+u^{\kappa\mu\nu\rho})\Box\omega_{\mu\nu}\Big]. \quad (62)$$

To get the last line we have used

$$r^{\theta\alpha\beta\rho} = (\eta^{\theta\alpha}\epsilon^{\nu\mu\beta\rho}+\eta^{\rho\alpha}\epsilon^{\nu\mu\beta\theta} + \eta^{\rho\beta}\epsilon^{\nu\mu\alpha\theta}+\eta^{\theta\beta}\epsilon^{\nu\mu\alpha\rho})\mathrm{Re}(\partial_\nu S)\mathrm{Im}(\partial_\mu S), \quad (63)$$

$$u^{\theta\rho\alpha\beta} = 2t^{\theta\rho}\eta^{\alpha\beta}-2t^{\theta\alpha}\eta^{\beta\rho}-2t^{\beta\rho}\eta^{\theta\alpha} + t^{\alpha\beta}\eta^{\theta\rho}+t(2\eta^{\theta\alpha}\eta^{\beta\rho}-\eta^{\alpha\beta}\eta^{\theta\rho}), \quad (64)$$

$$s_{\alpha\beta} = \mathrm{Im}(\partial_\alpha S)\mathrm{Im}(\partial_\beta S)-\mathrm{Re}(\partial_\alpha S)\mathrm{Re}(\partial_\beta S), \quad (65)$$

$$t_{\alpha\beta} = \mathrm{Im}(\partial_\alpha S)\mathrm{Im}(\partial_\beta S)+\mathrm{Re}(\partial_\alpha S)\mathrm{Re}(\partial_\beta S), \quad (66)$$

$$t = \eta^{\alpha\beta}t_{\alpha\beta}, \quad (67)$$

$$s = \eta^{\alpha\beta}s_{\alpha\beta}, \quad (68)$$

$$\theta_{\alpha\beta} = \eta_{\alpha\beta}-\frac{\partial_\alpha\partial_\beta}{\Box}, \quad (69)$$

and

$$\omega_{\alpha\beta} = \frac{\partial_\alpha\partial_\beta}{\Box}. \quad (70)$$

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