

# Effects of the dark energy and flat rotation curve on the gravitational time delay of particle with non-zero mass

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**Abstract** The effects of several dark energy models on gravitational time delay of particles with non-zero mass are investigated and analytical expressions for the same are obtained at the first order accuracy. Also the expression for gravitational time delay under the influence of conformal gravity potential that well describes the flat rotation curve of spiral galaxies is derived. The findings suggest that (i) the conformal gravity description of dark matter reduces the net time delay in contrast to the effect of normal dark matter, and therefore in principle the models can be discriminated using gravitational time delay observations, and (ii) the effect of dark energy/flat rotation curve may be revealed from high-precision measurements of gravitational time delay of particles involving the megaparsec and beyond distance scale.

## 1 Introduction

The explanation of the observed accelerated expansion of the Universe requires that the bulk of the energy density in the Universe is repulsive, which is termed ‘dark energy’ (DE). On the other hand, galactic rotation curves and few other observations require the existence of a non-luminous matter component, dubbed dark matter (DM). Several independent analyses of astrophysical and cosmological data now firmly suggest that DE, DM, and luminous matter constitute about 68, 27, and 5 % of the total energy budget of the Universe [1].

Out of the several wishful candidates for DE, the simplest candidate is the cosmological constant ( $\Lambda$ ). The model involving the cosmological constant, the so-called  $\Lambda$ CDM model with a value of  $\Lambda$  nearly  $10^{-56} \text{ cm}^{-2}$  and CDM, referring to cold DM, provides an excellent fit to the wealth of high-precision observational data, on the basis of a remark-

ably small number of cosmological parameters [2–4]. But the physical origin of cosmological constant remains a major problem. Besides its non-evolving nature, the  $\Lambda$ CDM model suffers from the so-called coincidence problem [5]. Alternative candidates of DE include scalar-field models like quintessence [4,6], k-essence [7,8] and phantom field [9]. There are also proposals for modifications of general relativity to account for the accelerated expansion without the need for DE which include scalar-tensor theories (or equivalently f(R) theories) [10–12,43], conformal gravity [13], massive gravity theories [14] including Dvali–Gabadadze–Porrati (DGP) braneworld gravity [15], etc. The nature of DM is also unknown at present but DM is a testable proposition in direct-detection experiments unlike DE. The gravitational effect of DM within a galaxy depends on the radial density profile of DM. While the flat rotation curve feature implies a logarithmic gravitational potential, the rotation curve data points for a large sample of spiral galaxies were also found to be described well by a gravitational potential linear in  $r$  [16].

Both DE and DM are expected to influence gravitational phenomena at all distance scales including those in the solar system. In solar system the influence of DE has been studied mainly through cosmological constant and is found to be maximum in the case of perihelion shift of mercury orbit where the  $\Lambda$  contribution is about  $10^{-15}$  of the total shift [17] and measurements of advances in the perihelia of Mercury impose an upper limit  $\Lambda < 10^{-42} \text{ m}^{-2}$  [18]. On the other hand, analysis of the perihelion precession of Mercury, Earth, and Mars give the upper bound on the density of DM  $\rho_{\text{dm}} < 3 \times 10^{-19} \text{ g/cm}^3$  [19]. Note that the rotation curve data suggests that the density of DM in the Milky Way at the location of solar system is  $\rho_{\text{dm}} = 0.5 \times 10^{-24} \text{ g/cm}^3$  [20]. DE is mainly effective at cosmological (megaparsec) scales and as a result the contribution of DE could be significant (larger than the second order term) even in a local gravitational phenomenon when kiloparsec (Kpc) to megaparsec (Mpc) scale distances are involved, such as the gravitational

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bending of light by cluster of galaxies [21] or the relativistic accretion phenomena around massive BHs (see [22] and references therein; [23]), whereas the effect of DM is significant at the outer part of galaxies. Consequently, for large distance scales, astrophysical and cosmological phenomena are likely to be dictated by DM/DE, and hence to probe DE/DM from local phenomena, one has to explore the local gravitational phenomena involving the Kpc to Mpc distance scale.

The phenomenon of gravitational time delay of an electrically neutral (henceforth just termed ‘neutral’ throughout the manuscript) particle with non-zero mass such as neutrino/neutron from an extra-galactic source may offer a possibility of studying the influence of DE/DM as it involves an Mpc distance scale. Here it is worthwhile to mention that measurements of gravitational time delay, for example, an extra time delay that light suffers while propagating in gravitational field over the time required for light transmission between two points in Minkowski space-time, through Doppler tracking of the Cassini spacecraft on its way to the Saturn, currently imposes the most stringent constraint on the first parameterized post-Newtonian parameter  $\gamma$  with  $\gamma - 1 < (2.1 \pm 2.3) \times 10^{-5}$  [24]. Note that  $\gamma$  is zero in the Newtonian theory, unity in general relativity, and  $\gamma - 1$  is considered as a measure of a deviation from general relativity. However, the effect of cosmological constant on gravitational time delay of photon is comparatively less and solar system measurements give only the restriction  $\Lambda \leq 10^{-24} \text{ m}^{-2}$  [17]. The effect of DE on gravitational time delay of photons has already been investigated by Asada [25]. Recently, the effect of DE/DM on gravitational time advancement (negative effective time delay) has been investigated by [26]. Considering a neutral particle for time delay measurement (as well as other similar effects) is advantageous over a photon due to the fact that the time delay for a particle depends also on the mass and energy of the particle, thereby offering additional control on the measurement [27] and we shall argue later in the discussion section that this additional control should be useful to study experimentally gravitational time delay involving the Kpc–Mpc distance scale.

In the present work we derive the analytical expression of gravitational time delay for particles having non-zero rest mass considering the presence of DE and DM, and we discuss the experimental feasibility to test DE/DM effects on the time delay experimentally in the future. The letter is organized as follows: in the next section we would formulate the problem mathematically for gravitational time delay corresponding to a neutral particle with non-zero rest mass for general spherically symmetric static space-time. In Sect. 3 we derive the analytical expression for time delay in the presence of DE/DM considering the case up to first order in  $M/r$  and  $\beta$ , where  $M$  is the mass of the gravitating object and  $\beta$  is the parameter describing the strength of DE/DM. In Sect. 4 we discuss our results stressing the possibilities of experimental

detection of such effects in a future experiment. Finally, we conclude in Sect. 5.

## 2 Gravitational time delay of a neutral particle with mass

For a general static spherically symmetric metric of the form

$$ds^2 = c^2 \mathcal{B}(r) dt^2 - \mathcal{A}(r) dr^2 - r^2 d\Omega^2 \tag{1}$$

where  $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$  and  $c$  the usual speed of light, the geodesic equations for a test particle motion in the equatorial plane around a spherical matter distribution having mass  $M$  leads to the following relation for the choice (of the affine parameter)  $dp = \mathcal{B}(r) dt$ :

$$\frac{\mathcal{A}(r)}{\mathcal{B}(r)^2} \left( \frac{dr}{dt} \right)^2 + \frac{\alpha_1}{r^2} - \frac{c^2}{\mathcal{B}(r)} = -\alpha_2 c^2, \tag{2}$$

where  $\alpha_1$  and  $\alpha_2$  are associated with the constants of motion.  $\alpha_1$  is related to the specific angular momentum of the particle [ $\alpha_1 \equiv r^4 (\frac{d\phi}{dp})^2$ ],  $\phi$  is the azimuthal angle, and  $\alpha_2$  is related to the specific energy of the particle. For a particle with non-zero rest mass  $m$ ,  $\alpha_2 > 0$ ; whereas for a particle with zero rest mass,  $\alpha_2 = 0$ . At the distance of closest approach  $r_p$ ,  $\frac{dr}{dt}$  must vanish, i.e.,  $\frac{dr}{dt}|_{r=r_p} = 0$ . This gives

$$\alpha_1 = c^2 \left[ -\alpha_2 + \frac{1}{\mathcal{B}(r_p)} \right] r_p^2. \tag{3}$$

From Eqs. (2) and (3), we obtain the time required by a particle to traverse a distance from  $r_p$  to  $r$ , which is given by

$$t(r, r_p) = \frac{1}{c} \int_{r_p}^r \sqrt{\mathcal{P}(r, \alpha_2)} dr, \tag{4}$$

where

$$\mathcal{P}(r, \alpha_2) \approx \frac{\mathcal{A}(r)/\mathcal{B}(r)}{\left[ 1 - \alpha_2 \mathcal{B}(r) + \frac{r_p^2}{r^2} \left( \alpha_2 \mathcal{B}(r) - \frac{\mathcal{B}(r)}{\mathcal{B}(r_p)} \right) \right]}. \tag{5}$$

Once the space-time geometry is given, the gravitational time delay can be computed from Eq. (4) through Eq. (5). Restricting our study up to first order in  $M$ , where  $M$  is mass of the gravitating object, the total travel time in the Schwarzschild geometry ( $\mathcal{B}(r) = \mathcal{A}(r)^{-1} = 1 - 2GM/c^2 r$ ) is given by [28]

$$t_{\text{Sch}}(r, r_p) \approx \frac{1}{c\sqrt{1-\alpha_2}} \times \left\{ \sqrt{r^2 - r_p^2} + \frac{GM(2-3\alpha_2)}{c^2(1-\alpha_2)} \ln \frac{r + \sqrt{r^2 - r_p^2}}{r_p} + \frac{GM}{c^2(1-\alpha_2)} \sqrt{\frac{r-r_p}{r+r_p}} \right\}, \tag{6}$$

where the subscript ‘Sch’ represents ‘Schwarzschild’. When  $\alpha_2 = 0$ , i.e. for photons, the above equation reduces to the well-known expression for gravitational time delay of photons.

### 3 Gravitational time delay of a neutral particle with mass in the presence of DE/DM

In the presence of DE/DM the exterior vacuum space-time will no longer be of the Schwarzschild geometry but of a modified one. Here we shall consider the following form of the metric tensor:

$$\mathcal{B}(r) = 1 - \frac{2GM}{c^2 r} - \beta_1 r^n \tag{7}$$

and

$$\mathcal{A}(r) = 1 + \frac{2GM}{c^2 r} + \beta_2 r^n, \tag{8}$$

where  $\beta_1$  and  $\beta_2$  are constants. Different choices of  $n$ ,  $\beta_1$ , and  $\beta_2$  lead to different models of DE/DM.

*Case 1* With  $n = 1/2$ ,  $\beta_1 = 2\beta_2 = \pm 2\sqrt{GM/r_c^2}$  the model represents the gravitational field of a spherically symmetric matter distribution on the background of an accelerating Universe in DGP braneworld gravity provided leading terms are only considered [29].  $r_c$  is the crossover scale beyond which gravity becomes five dimensional.

*Case 2* For the choice  $n = 1$ , with  $\beta_1 = \beta_2 = -\beta = -(5.42 \times 10^{-42} \frac{M}{M_\odot} + 3.06 \times 10^{-30}) \text{ cm}^{-1}$  the model well describes the flat rotation curves of spiral galaxies [13,16].

*Case 3* If  $n = 3/2$ , and  $\frac{2}{3}\beta_1 = -\beta_2 = m_g^2 \sqrt{\frac{2GM}{13c^2}}$ , the model corresponds to the non-perturbative solution of a massive gravity theory (an alternative description of accelerating expansion of the Universe) [30] where  $m_g$  is the mass of graviton.

*Case 4* When  $n = 2$ ,  $\beta_1 = \beta_2 = \Lambda/3$  and  $m = \mu$  the above metric describes the Schwarzschild–de Sitter (SDS) or Kotler space-time, which is the exterior space-time due to a static spherically symmetric mass distribution in the presence of the cosmological constant  $\Lambda$  with  $\Lambda \sim 10^{-56} \text{ cm}^{-2}$  [31].

Note that  $m$  and  $M$  have been defined in Sect. 2. Using DE/DM led by  $\mathcal{A}(r)$  and  $\mathcal{B}(r)$ , restricting our study up to first order correction due to  $\beta_i$  ( $i = 1, 2$ ), and neglecting the terms of the order  $M^2$  and beyond,  $\mathcal{P}(r, \alpha_2)$  given by Eq. (5) reduces to the form given by

$$\sqrt{\mathcal{P}(r, \alpha_2)} \approx \frac{1 + \frac{GM(2-3\alpha_2)}{c^2 r(1-\alpha_2)} + \frac{GM r_p}{c^2(1-\alpha_2)r(r+r_p)} + \left(\frac{\beta_1+\beta_2}{2} - \frac{\beta_1\alpha_2}{2(1-\alpha_2)}\right) r^n - \frac{\beta_1 r_p^2 (r^n - r_p^n)}{2(1-\alpha_2)(r^2 - r_p^2)}}{\sqrt{\left(1 - \frac{r_p^2}{r^2}\right)(1 - \alpha_2)}}. \tag{9}$$

Using Eqs. (4) and (9), we obtain the explicit expression to compute the time required by a particle to traverse a distance from  $r_p$  to  $r'$  (where  $r'$  is any arbitrary distance from the gravitational source) in the presence of space-time geometry defined by Eq. (1) with Eqs. (7) and (8), corresponding to different DE/DM models, given by

$$t_n(r', r_p) \approx t_{\text{Sch}}(r', r_p) + \frac{1}{2c\sqrt{1-\alpha_2}} \times \left\{ \left[ \beta_1 + \beta_2 - \frac{\beta_1\alpha_2}{(1-\alpha_2)} \right] \mathcal{I}_n^1 - \frac{\beta_1}{(1-\alpha_2)} \mathcal{I}_n^2 \right\}, \tag{10}$$

where  $\mathcal{I}_n^1$  and  $\mathcal{I}_n^2$  are integrals defined by  $\mathcal{I}_n^1 = \int_{r_p}^{r'}$   $\frac{r'^{n+1} dr}{\sqrt{(r'^2 - r_p^2)}}$  and  $\mathcal{I}_n^2 = r_p^2 \int_{r_p}^{r'} \frac{r'(r'^n - r_p^n) dr}{(r'^2 - r_p^2)\sqrt{(r'^2 - r_p^2)}}$ . Equation (10) is the general expression for the time required to traverse a distance from  $r_p$  to any arbitrary distance  $r'$  in the presence of a generic metric given by Eqs. (1), (7), and (8) corresponding to DE/DM models to first order corrections. For  $n = 1$  and  $n = 2$  corresponding to DM and DE, respectively, we have analytical solutions of  $\mathcal{I}_1^1, \mathcal{I}_1^2$  and  $\mathcal{I}_2^1, \mathcal{I}_2^2$ , which are given below

$$\begin{aligned} \mathcal{I}_1^1 &= \frac{r'}{2} \sqrt{r'^2 - r_p^2} + \frac{r_p^2}{2} \ln \frac{r' + \sqrt{r'^2 - r_p^2}}{r_p}, \\ \mathcal{I}_1^2 &= -r_p^2 \sqrt{\frac{r' - r_p}{r' + r_p}} + r_p^2 \ln \frac{r' + \sqrt{r'^2 - r_p^2}}{r_p}, \\ \mathcal{I}_2^1 &= \frac{1}{3} \sqrt{r'^2 - r_p^2} (r'^2 + 2r_p^2), \\ \mathcal{I}_2^2 &= r_p^2 \sqrt{r'^2 - r_p^2}. \end{aligned} \tag{11}$$

For general  $n$ , however,  $\mathcal{I}_n^1$  and  $\mathcal{I}_n^2$  can only be expressed through hypergeometric functions, which is not very useful. When  $r' \gg r_p$ , the integrals  $\mathcal{I}_n^1$  and  $\mathcal{I}_n^2$  for general  $n$  ( $n \neq 1$ ) reduce to

$$\begin{aligned} \mathcal{I}_n^1 &\approx \frac{r'^{n+1}}{n+1} + \frac{r_p^2 r'^{n-1}}{2(n-1)}, \\ \mathcal{I}_n^2 &\approx r_p^2 \frac{r'^{n-1}}{n-1}. \end{aligned} \tag{13}$$

For  $\alpha_2 = 0$ , Eq. (10) with Eq. (13) gives the gravitational time delay for a photon which agrees with the results obtained

by Asada [25] at the leading order in  $r'$ . In deriving the contribution of  $\beta_i$  on gravitational time delay we ignore the cross terms between  $M$  and  $\beta_i$  (and higher order terms in  $\beta_i$ ) since  $\beta_i$  is small. However, under some circumstances the cross terms may be relevant, which are given in the appendix.

#### 4 Discussions

Equation (10) through Eqs. (11)–(13) imply that DE enhances the time delay effect. Similar effects of DE were noted earlier for photons [17,25]. For gravitational time delay the influence of DE is somewhat counter intuitive; the repulsive nature of DE is expected to act differently from normal mass. Here it is worthwhile to mention that whether the gravitational bending angle of photon is reduced by the repulsive nature of DE or not is a controversial issue [22,32–34], mainly owing to the asymptotically non-flat geometry which requires one to consider a number of aspects including the position of the reference sources [35] as well as the finite-distance corrections [36]. The potential linear in the radial distance that describes the flat rotation curves of spiral galaxies well reduces the net time delay, which is opposite to the effect of normal dark matter on the gravitational time delay; hence the conformal gravity description of the galactic rotation curve can be discriminated from normal dark matter from a gravitational time delay measurement involving the Mpc distance scale, at least in principle.

Equation (10) with Eqs. (11) and (12) suggests that the DE and DM contribution to the time delay effect will be of the same order as the pure Schwarzschild contribution at a distance scale of roughly 30 and 300 Kpc, respectively, in our galaxy. Hence to detect the influences of DE/DM through gravitational time delay effects one needs to conduct the measurements involving the Kpc distance scale. Experimentally gravitational time delay is studied in the solar system by measuring the round-trip travel time of an electromagnetic signal emitted from the Earth past the Sun to a planet or satellite and returning to the Earth. Such a strategy is of course impractical for measuring gravitational time delay when the Kpc–Mpc scale distance is involved. Instead a feasible approach is to study the time difference of the arrival of neutral particles with the same mass at two (or more) different energies from a stellar collapse scenario such as extra-galactic gamma ray bursts (GRBs) or supernova explosions (SNe).

The theories of stellar collapse [37,38] require that neutrinos of different energies should be emitted in a short pulse of about 10 ms duration, which is also indicated experimentally [39,40]. The photons are expected to be emitted a few hours later than the neutrino emission [37,38]. Since gravitational time delay by the galaxy causes a time delay about 5 months [41], the difference in arrival times of neutrinos and photons from extra-galactic GRBs or SNe also may probe

the DE/DM influence on gravitational time delay. We, therefore, shall evaluate analytical expressions for the difference in arrival times between neutral particles having non-zero rest mass (such as neutrinos) of two different energies and that between particle and photon.

We consider the scenario that the particle/photon is emitted from the source  $S(R)$ , where ‘ $R$ ’ represents the radial coordinate of the source ‘ $S(R)$ ’, reaches the observer  $O(r_o)$  at  $r_o$  (here the Earth) with the distance of closest approach  $r_p$ . All the distances are measured taking the center of the spherical mass distribution with mass  $M$  as the center of the coordinate system. Hence the total transit time is  $T_n(r_o, R) = t_n(r_o, r_p) + t_n(r_p, R)$ . The particle/photon emitted from the source suffers a gravitational time delay due to the spherical mass distribution. For the test particle with mass  $m$  and energy  $\epsilon$  as received by the observer, the parameter  $\alpha_2$  (as described in Eq. (2)) and other subsequent equations in Sects. 2 and 3) is given by [28];  $\alpha_2 = \frac{m^2}{\mathcal{B}(r_o)\epsilon^2} = \frac{m^2}{\epsilon^2} \left(1 + \frac{2GM}{c^2 r_o} + \beta_1 r_o^n\right)$  (restricting our study up to a first order correction due to  $\beta_1$ ). As we wish to focus on highly relativistic particles, we consider  $\epsilon \gg m$ . Denoting by  $\Delta T_n|_{\epsilon_1}^{\epsilon_2}$  and  $\Delta T_n|_m^{m=0}$  the difference in arrival times between particles with the same mass  $m$  but two different energies  $\epsilon_1$  and  $\epsilon_2$  and the difference in arrival times between particle with mass  $m$  and energy  $\epsilon$  and photon, respectively, and restricting our study to the first order in the expansion of  $M$  and  $m^2/\epsilon^2$ , we get the most general expressions (ignoring the cross terms between  $M$  and  $\beta_i$ ); however, under some circumstances the cross terms may be relevant, which are given in the appendix.

For the DM model with  $n = 1$  and  $\beta_1 = \beta_2 = -\beta$ ,

$$\begin{aligned} \Delta T_1|_{\epsilon_1}^{\epsilon_2} \approx & \frac{D_S m^2}{2c} \left( \frac{1}{\epsilon_1^2} - \frac{1}{\epsilon_2^2} \right) \left\{ 1 + \frac{2GM}{c^2} \right. \\ & \times \left[ \frac{1}{r_o} + \frac{3}{2D_S} \left( \sqrt{\frac{R-r_p}{R+r_p}} + \sqrt{\frac{r_o-r_p}{r_o+r_p}} \right) \right] \\ & - \beta \left[ r_o + \frac{3}{2} \frac{r_p^2}{D_S} \left( \sqrt{\frac{R-r_p}{R+r_p}} + \sqrt{\frac{r_o-r_p}{r_o+r_p}} \right) \right. \\ & \left. \left. - \frac{3}{2} \frac{r_p^2}{D_S} \ln \frac{\left( R + \sqrt{R^2 - r_p^2} \right) \left( r_o + \sqrt{r_o^2 - r_p^2} \right)}{r_p^2} \right] \right\}, \end{aligned} \tag{14}$$

where  $D_S = \sqrt{R^2 - r_p^2} + \sqrt{r_o^2 - r_p^2}$ , the distance of the source  $S(R)$  from the observer  $O(r_o)$  on Earth. We have

$$\begin{aligned} \Delta T_1|_m^{m=0} \approx & \frac{D_S m^2}{2c\epsilon^2} \\ & \times \left\{ 1 + \frac{2GM}{c^2} \left[ \frac{1}{r_o} + \frac{3}{2D_S} \left( \sqrt{\frac{R-r_p}{R+r_p}} + \sqrt{\frac{r_o-r_p}{r_o+r_p}} \right) \right] \right\} \end{aligned}$$



$$\begin{aligned}
 & -\beta \left[ r_o + \frac{3 r_p^2}{2 D_S} \left( \sqrt{\frac{R-r_p}{R+r_p}} + \sqrt{\frac{r_o-r_p}{r_o+r_p}} \right) \right. \\
 & \left. - \frac{3 r_p^2}{2 D_S} \ln \frac{(R + \sqrt{R^2 - r_p^2})(r_o + \sqrt{r_o^2 - r_p^2})}{r_p^2} \right] \Bigg\}, \tag{15}
 \end{aligned}$$

whereas, corresponding to DE (described by the cosmological constant), with  $n = 2$  and  $\beta_1 = \beta_2 = \frac{\Lambda}{3}$  in SDS geometry,

$$\begin{aligned}
 \Delta T_2|_{\varepsilon_1}^{\varepsilon_2} & \approx \frac{D_S m^2}{2c} \left( \frac{1}{\varepsilon_1^2} - \frac{1}{\varepsilon_2^2} \right) \left\{ 1 + \frac{2GM}{c^2} \right. \\
 & \times \left[ \frac{1}{r_o} + \frac{3}{2D_S} \left( \sqrt{\frac{R-r_p}{R+r_p}} + \sqrt{\frac{r_o-r_p}{r_o+r_p}} \right) \right] \\
 & \left. + \Lambda \left( \frac{r_o^2}{3} - \frac{r_p^2}{2} \right) \right\}, \tag{16}
 \end{aligned}$$

$$\begin{aligned}
 \Delta T_2|_m^{m=0} & \approx \frac{D_S m^2}{2c \varepsilon^2} \left\{ 1 + \frac{2GM}{c^2} \right. \\
 & \times \left[ \frac{1}{r_o} + \frac{3}{2D_S} \left( \sqrt{\frac{R-r_p}{R+r_p}} + \sqrt{\frac{r_o-r_p}{r_o+r_p}} \right) \right] \\
 & \left. + \Lambda \left( \frac{r_o^2}{3} - \frac{r_p^2}{2} \right) \right\}, \tag{17}
 \end{aligned}$$

and for general  $n$ , with condition  $R \gg r_p$ ; equivalently,  $D_S \sim R, r_p \sim r_o$ , we have

$$\begin{aligned}
 \Delta T_n|_{\varepsilon_1}^{\varepsilon_2} & \approx \frac{D_S m^2}{2c} \left( \frac{1}{\varepsilon_1^2} - \frac{1}{\varepsilon_2^2} \right) \\
 & \times \left\{ 1 + \frac{2GM}{c^2} \frac{1}{r_o} + \frac{(\beta_2 - \beta_1) R^n}{2} \left[ \frac{1}{n+1} + \frac{r_p^2}{2(n-1)R^2} \right] \right. \\
 & \left. + \beta_1 \left[ r_o^n - \frac{3 r_o^2 R^{n-2}}{2(n-1)} \right] \right\}, \tag{18}
 \end{aligned}$$

$$\begin{aligned}
 \Delta T_n|_m^{m=0} & \approx \frac{D_S m^2}{2c \varepsilon^2} \\
 & \times \left\{ 1 + \frac{2GM}{c^2} \frac{1}{r_o} + \frac{(\beta_2 - \beta_1) R^n}{2} \left[ \frac{1}{n+1} + \frac{r_p^2}{2(n-1)R^2} \right] \right. \\
 & \left. + \beta_1 \left[ r_o^n - \frac{3 r_o^2 R^{n-2}}{2(n-1)} \right] \right\}, \tag{19}
 \end{aligned}$$

respectively.

Since our objective is to explore the effect of dark matter and dark energy on the gravitational time delay of particles with mass, we restrict our study to the gravitating systems involving the large (Kpc to Mpc) distance scale, such as our galaxy, local group or even local supercluster (see Tables 1 and 2 for different characteristic parameters of these systems). For such gravitating systems the magnitude of  $M/R, \beta R$  and  $\Lambda R^2$  are small, much less than 1. For instance, the

magnitude of  $M/R, \beta R$  and  $\Lambda R^2$  are of the order of  $10^{-6}, 10^{-8}$  and  $10^{-11}$ , respectively, for our galaxy whereas for local group they become  $2 \times 10^{-7}, 5 \times 10^{-6}$  and  $10^{-8}$ . For local supercluster the stated parameters take the values  $10^{-5}, 10^{-1}$  and  $10^{-5}$ , respectively. Hence to estimate the effect of dark matter/energy on the time delay, we can safely neglect the second order terms as well as the cross terms.

To find the DM/DE contribution to gravitational time delay that the signal (like the neutrino with mass ‘ $m$ ’) suffers while traveling from a distant source to the observer on Earth about gravitating mass distribution with mass  $M$ , it is necessary to estimate in actual seconds the quantities  $\Delta T_1|_m^{m=0}$  or  $\Delta T_1|_{\varepsilon_1}^{\varepsilon_2}, \Delta T_2|_m^{m=0}$  or  $\Delta T_2|_{\varepsilon_1}^{\varepsilon_2}$ , explicitly for terms corresponding to DM and DE, respectively. To compute these terms explicitly due to the contribution of DM/DE as well as to clearly reveal the effects of DM and DE we focus on two (mathematically) simple but practically feasible scenarios as described below.

In one scenario, the signal originates from a distant source, and the observer  $O(r_o)$  is located on the Earth at a distance  $r_o$  from the center of the spherical mass distribution. This can be represented by the conditions  $R \gg r_p$  and  $r_p \sim r_o$ ; which imply  $D_S \gg r_p, D_S \gg r_o$ . Such a scenario will arise, for example, if a signal originates from a distance local extra-galactic source due to supernova explosion like that in the case of SN 1987A and suffers gravitational time delay by our galaxy. Other examples of such a scenario are that the source is an extra-galactic one, situated far away from our local group or local supercluster and the signal from the source suffers gravitational time delay due to the contribution of the local group or local supercluster itself while reaching to the observer on the Earth.

Corresponding to this scenario, it can be seen from Eqs. (16) and (17) that the difference in arrival times between particles with the same mass but different energies  $\varepsilon_1$  and  $\varepsilon_2$  with  $\varepsilon_2 > \varepsilon_1$  or between particle with mass  $m$  and energy  $\varepsilon$  and photon is reduced due to the cosmological constant; for the flat rotation curve a similar aspect is also noticed from Eqs. (14) and (15). To ascertain the effective contribution to gravitational time delay due to DM/DE as compared to the pure Schwarzschild contribution for the cases described by Eqs. (14)–(17), it is necessary to compute the magnitude of the ratio ( $\chi$ ) between these two contributing terms given in Eqs. (14)–(17). For the scenario described here, for the DM model, this ratio  $\chi \approx \left| \frac{-\beta c^2 r_o^2}{2GM} \right|$ . Similarly, for the DE model (described by  $\Lambda$ ),  $\chi \approx \left| \frac{-\Lambda c^2 r_o^3}{12GM} \right|$ . The differences in times of arrival would then be as follows;

for the DM model:

$$\Delta T_1|_m^{m=0} \approx \left| \frac{-\beta r_o D_S m^2}{2c \varepsilon^2} \right| \tag{20}$$

**Table 1** Values of  $\chi$  for potential astrophysical events

Scenarios	G.M	$M/M_\odot$	$r_o$	$\chi$ (DM)	$\chi$ (DE)
$R \gg r_p$	Our galaxy	$\sim 10^{11}$	$\sim 10$ Kpc	$\sim 10^{-1}$	$\sim 10^{-6}$
	Local group	$\sim 10^{12}$	$\sim 0.52$ Mpc	$\sim 7$	$\sim 2 \times 10^{-2}$
	Local supercluster	$\sim 10^{15}$	$\sim 16$ Mpc	$\sim 7000$	$\sim 0.6$
$R \sim r_p$	Our galaxy	$\sim 10^{11}$	$\sim 10$ Kpc	$\sim 4 \times 10^{-2}$	$\sim 8 \times 10^{-7}$
	Local group	$\sim 10^{12}$	$\sim 0.52$ Mpc	$\sim 2.8$	$\sim 1.6 \times 10^{-2}$
	Local supercluster	$\sim 10^{15}$	$\sim 16$ Mpc	$\sim 2800$	$\sim 0.5$

$$\Delta T_1|_{\varepsilon_1}^{\varepsilon_2} \approx \left| \frac{-\beta r_o D_S m^2}{2c} \left( \frac{1}{\varepsilon_1^2} - \frac{1}{\varepsilon_2^2} \right) \right|, \tag{21}$$

and for the DE model:

$$\Delta T_2|_m^{m=0} \approx \left| \frac{A r_o^2 D_S m^2}{12 c \varepsilon^2} \right| \tag{22}$$

$$\Delta T_2|_{\varepsilon_1}^{\varepsilon_2} \approx \left| \frac{A r_o^2 D_S m^2}{12 c} \left( \frac{1}{\varepsilon_1^2} - \frac{1}{\varepsilon_2^2} \right) \right|. \tag{23}$$

In the other scenario, the source  $S(R)$  may be situated close to the center of the spherical mass distribution, however,  $S(R)$  is located far away from the observer on the Earth. This can be represented by the condition  $R \sim r_p$ ; which implies  $D_S \sim r_o, r_o \gg r_p$ . Such a scenario will arise when a core-collapse extra-galactic SNe/GRB occurs close to the center of our galaxy or local group or our local supercluster and the signal originating from SNe/GRB is detected by the observer on the Earth. For the scenario described here,  $\chi \approx \left| \frac{-\beta c^2 r_o^2}{5GM} \right|$  for the DM model, whereas for the DE model (described by  $\Lambda$ ),  $\chi \approx \left| \frac{\Lambda c^2 r_o^3}{15GM} \right|$ .

For this scenario, corresponding to the DM model describing the galactic rotation curves, the expressions of the difference in arrival times between particles with the same mass but different energies  $\varepsilon_1$  and  $\varepsilon_2$  with  $\varepsilon_2 > \varepsilon_1$  or between particle with mass  $m$  and energy  $\varepsilon$  and photon are identical to those in the earlier mentioned scenario as given by Eqs. (20) and (21), whereas for cosmological constant model describing DE, the differences in arrival times are just twice those in the previous scenario as given in Eqs. (22) and (23). Note that the galactic rotation curve feature reduces the net time delay, whereas for the cosmological constant, the reverse (enhance) aspect is noticed from Eqs. (16) and (17), which should be a distinguishing signature between DE and DM.

In Table 1, we display the numerical estimate of the quantity  $\chi$  for few potential astrophysical events (different signal sources and different gravitational mass distributions ( $G.M$ ) about which the signal suffers gravitational time delay), corresponding to the two scenarios described here, for both DM and DE (described by  $\Lambda$ ) models. It is to be noted that corresponding to the two scenarios described here, the ratio of the contributing terms to the gravitational time delay ( $\chi$ ) for both

DM and DE (described by  $\Lambda$ ) models is independent of the distance of source from the observer, as well as of the closest distance of approach, however, only depends on the distance of the observer on Earth from the gravitational mass distribution about which the signal suffers time delay. In Table 1 we choose appropriate gravitating systems ( $G.M$ ) and values of the masses of gravitating systems about which the signal (like neutrino signal) suffers gravitational time delay, and the corresponding values of  $r_o$ .

In Table 2, we estimate the values of the differences in arrival times for a similar choice of gravitating systems as in Table 1. For the case  $R \gg r_p$ , corresponding to our local group, we choose a distant core-collapse SNe emitting neutrinos, located at a typical distance of  $\sim 10$  Mpc from the observer on Earth, whereas for a local supercluster we choose the source at 50 Mpc away. Similarly for the case  $R \sim r_p$ , corresponding to our local group, we choose a core-collapse SNe emitting neutrinos from close to the center of the mass of the local group; for which  $D_S \sim r_o$ . Corresponding to our galaxy about which the signal suffers gravitational time delay, for the case  $R \gg r_p$ , we choose a SNe neutrino emitting source to be located at a typical distance of  $\sim 50$  Kpc from the observer on Earth; whereas, for the case  $R \sim r_p$ , we choose a SNe emitting source located close to the center of our galaxy. In Table 2, we display the estimated values of differences in arrival time corresponding to all three neutrino flavors assuming their masses to be equal to the experimentally obtained upper bound limits [42] and between the photon (i.e.,  $\Delta T_n|_m^{m=0}, \{n = 1, 2\}$ ) in seconds, for the explicit contribution of DM and DE. For  $\nu_e$  and  $\nu_\mu$ , we choose the typical value of the energy of the observed signal  $\varepsilon = 10$  MeV, however, for  $\nu_\tau$ , we choose the energy of the observed signal to be  $\varepsilon = 100$  MeV, owing to the large upper bound limit of its mass.

When we consider difference in arrival times of two neutrinos with different energies we should get the time delay of the same order as displayed in Table 2 for difference in arrival times of photon and neutrino unless two energies are very close. Measurement of the differences in arrival times of neutrinos with different energies should provide information as regards gravitational delay, as neutrinos are expected to be emitted within a second in explosions whereas the time

**Table 2** Estimated values of  $\Delta T_n|_{m=0}$ ,  $\{n = 1, 2\}$  for potential astrophysical events when  $\varepsilon = 100$  MeV for  $\nu_\tau$  and 10 MeV for other neutrino flavors and assuming  $m_{\nu_e} \sim 2$  eV,  $m_{\nu_\mu} \sim 0.19$  MeV,  $m_{\nu_\tau} \sim 18.2$  MeV

Scenarios	G.M	$M/M_\odot$	$r_o$	$D_S$	$\nu$	$\Delta T_1 _{m=0}$ (s)	$\Delta T_2 _{m=0}$ (s)
$R \gg r_p$	Our galaxy	$\sim 10^{11}$	$\sim 10$ Kpc	$\sim 50$ Kpc	$\nu_e$	$\sim 1.1 \times 10^{-8}$	$\sim 1.6 \times 10^{-13}$
					$\nu_\mu$	$\sim 103$	$\sim 1.4 \times 10^{-3}$
					$\nu_\tau$	$\sim 9.5 \times 10^3$	$\sim 0.13$
	Local group	$\sim 10^{12}$	$\sim 0.52$ Mpc	$\sim 10$ Mpc	$\nu_e$	$\sim 2.8 \times 10^{-4}$	$\sim 8.8 \times 10^{-8}$
					$\nu_\mu$	$\sim 2.5 \times 10^6$	$\sim 797$
					$\nu_\tau$	$\sim 2.3 \times 10^8$	$\sim 7.3 \times 10^4$
	Local supercluster	$\sim 10^{15}$	$\sim 16$ Mpc	$\sim 50$ Mpc	$\nu_e$	$\sim 45$	$\sim 4.5 \times 10^{-4}$
					$\nu_\mu$	$\sim 4 \times 10^{11}$	$\sim 4.1 \times 10^6$
					$\nu_\tau$	$\sim 3.7 \times 10^{13}$	$\sim 3.7 \times 10^8$
$R \sim r_p$	Our galaxy	$\sim 10^{11}$	$\sim 10$ Kpc	$\sim 10$ Kpc	$\nu_e$	$\sim 2.3 \times 10^{-9}$	$\sim 3.3 \times 10^{-14}$
					$\nu_\mu$	$\sim 20.6$	$\sim 2.9 \times 10^{-4}$
					$\nu_\tau$	$\sim 1.9 \times 10^3$	$\sim 0.03$
	Local group	$\sim 10^{12}$	$\sim 0.52$ Mpc	$\sim 0.52$ Mpc	$\nu_e$	$\sim 1.4 \times 10^{-5}$	$\sim 4.6 \times 10^{-9}$
					$\nu_\mu$	$\sim 1.3 \times 10^5$	$\sim 41$
					$\nu_\tau$	$\sim 1.2 \times 10^7$	$\sim 3.8 \times 10^3$
	Local supercluster	$\sim 10^{15}$	$\sim 16$ Mpc	$\sim 16$ Mpc	$\nu_e$	$\sim 14$	$\sim 1.5 \times 10^{-4}$
					$\nu_\mu$	$\sim 1.3 \times 10^{11}$	$\sim 1.4 \times 10^6$
					$\nu_\tau$	$\sim 1.2 \times 10^{13}$	$\sim 1.2 \times 10^8$

difference between the emission of the neutrinos and the optical brightening at the source is somewhat controversial. The results as displayed in Table 2 suggest that owing to the small upper bound mass of  $\nu_e$  measuring the gravitational time delay of electron neutrinos caused by DE/DM is only feasible when the gravitating system is a local supercluster or an even larger system. The distance scale involved is a few tens of Mpc, which should be detectable by the low energy extension of ICECUBE [43] and some other upcoming/proposed neutrino telescopes and thereby the results found here is also physically meaningful. For the mentioned scenarios, the ratios of gravitational time delays caused by DM and DE to the Minkowskian time delay are  $\beta r_o$  and  $\Lambda r_o^2/6$ , respectively, which numerically are  $\sim 10^{-7}$  and  $\sim 10^{-11}$  when the time delay is caused by our galaxy. If the time delay is caused by the local group, these ratios become  $\sim 5 \times 10^{-6}$  and  $\sim 5 \times 10^{-10}$ , respectively, and for a local supercluster they take the values  $\sim 0.16$  and  $\sim 5 \times 10^{-7}$ , respectively. So a good idea as regards the source distance is needed to discriminate the DE/DM contribution on the time delay from Minkowskian and pure gravitational time delay.

### 5 Conclusion

We obtain an analytical expression for gravitational time delay of particles with non-zero rest mass in the presence of the dark energy/matter. We found that a measurement

of the gravitational time delay involving the Mpc distance scale should detect the contribution of galactic rotation curve description under conformal gravity as well as dark energy in terms of cosmological constant. Hence if such a measurement can be realized in the future, it should cross check the validity of the potential linear in radial distance that describes the flat rotation curve of spiral galaxies consistently. The magnitude of the cosmological constant also may be verified from such measurements. An interesting observation is that for a cosmological constant description of the dark energy, the source distance does not appear in the difference of arrival times between particles with the same mass but different energies or between a particle with mass  $m$  and a photon, but for some other dark energy models such as the DGP braneworld gravity or massive gravity description the stated differences of arrival times do contain a source distance term and hence can be very large. This feature, therefore, might distinguish the alternative dark energy models from cosmological constant, at least in principle.

An important question is which particle should be used for a gravitational time delay measurement. Probing gravity through gravitational time delay effect of high energy neutrons originating outside of our solar system is not feasible owing to their short mean life ( $\sim 15$  min); even ultra high energy neutrons ( $> 10^{18}$  eV) with a Lorentz boosted lifetime would not travel a distance scale of not more than 10 Kpc. Neutrinos seem to be the only viable candidate for the stated purpose. Being a weakly interacting parti-

cle, the neutrino can provide deeper information as regards both the relic and the distant Universe. They are messengers of extreme conditions inside SN cores. Core-collapse SNe's, both galactic and extra-galactic, are predictably rich sources of neutrinos. Despite the fact that till date SN 1987A is the only detectable SN source of neutrinos, the present generation neutrino detectors e.g., Icecube neutrino telescope, under certain conditions, might be able to detect SNe beyond 10 Mpc, while furnishing between 10 and 41 regular core-collapse SN detections per decade. Besides, high energy cosmic rays (HECRs) which plausibly originate from active galactic nuclei (AGNs), relativistic extra-galactic jets or GRBs located mostly at cosmological distances are other profuse sources of very high energy neutrinos ( $> \text{TeV}$  range).

A major problem with the neutrino, however, is the uncertainty of its mass as well as its small magnitude. So far the exact mass of any neutrino flavor is not known; experiments provide only the upper bound of its mass. However, the electron neutrino mass is expected to be pinned down by the future neutrino experiments such as JUNO [44] and KATRIN [45]. Also the study of cosmology leads to some useful information on the mass scale of light neutrinos [1]. Moreover, the lower bound mass of electron neutrino (few meV) may put some constraint on the dark energy parameters from a high-precision gravitational time delay measurements. Conversely, the supernova neutrino observation may put some restriction on the mass of the neutrinos, particularly the muon and tau neutrinos. There are two (model independent) approaches of the measurement of the neutrino mass: time-of-flight measurements and precision investigations of weak decays. Owing to our imprecise knowledge about the pion mass, the investigation of weak decays puts much looser upper bounds on the masses of muon and tau neutrinos. The study of neutrinos from the supernova SN1987a in the Large Magellanic Cloud, employing the time of flight approach yields an upper limit of 5.7 eV (95 % CL) on electron neutrino mass [46]. Future detection of neutrinos from supernova explosions is expected to improve the stated limit as well as to impose a new limit on the masses of muon and tau neutrinos. Obviously the special relativistic term of Eqs. (14)–(17) will play a dominant role for imposing the mass constraint. The dark matter contribution, being the second largest contributor, also may be important, particularly when the gravitating system is a local supercluster or supercluster and thereby may set the precision limit of the mass determination.

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## Appendix A

Here we furnish the correction to the gravitational time delay due to the coupling terms to the first order corresponding to DM/DE models in Eqs. (10)–(17) (i.e., the cross term between  $M$  and  $\beta_i$ ). As the correction due to cross terms will not affect the total transit time or the gravitational time delay considerably, here we only show the expressions (due to the algebraic simplicity) considering the source ‘ $S(R)$ ’ to be situated far away from the spherical mass distribution about which the signal suffers gravitational time delay, i.e.,  $R \gg r_p$ , the scenario already described in Sect. 4 after Eq. (19).

Corresponding to  $n = 1, 2$  for DM/DE, the total transit time  $T_1(r_o, R)$  and  $T_2(r_o, R)$  and their corresponding gravitational time delay will be enhanced considering the coupling terms to the first order by the quantity  $\delta t_1(r_o, R)|_{\beta M}$  and  $\delta t_2(r_o, R)|_{\Lambda M}$ , respectively, given by

$$\begin{aligned} \delta t_1(r_o, R)|_{\beta M} &\approx \frac{D_S G M \beta r_o}{c^2} \\ &\times \left[ \frac{4}{r_o} \left( 1 + \frac{m^2}{2\varepsilon^2} \right) - \frac{\sqrt{D_S^2 + r_o^2} + 2r_o}{2 \left( \sqrt{D_S^2 + r_o^2} + r_o \right)^2} \right], \quad (\text{A.1}) \\ \delta t_2(r_o, R)|_{\Lambda M} &\approx \frac{D_S G M \Lambda}{6c^2} \left[ 4\sqrt{D_S^2 + r_o^2} - 2r_o \frac{m^2}{\varepsilon^2} \right]. \quad (\text{A.2}) \end{aligned}$$

Accordingly in Eqs. (14) and (15), corresponding to  $n = 1$ , for DM, the correction term due to coupling to the first order will reduce the corresponding gravitational time delay by quantities  $\sim \frac{2D_S G M \beta m^2}{c^2 \varepsilon^2}$  and  $\sim \frac{2D_S G M \beta m^2}{c^2} \left( \frac{1}{\varepsilon_1^2} - \frac{1}{\varepsilon_2^2} \right)$ , using the condition described above. Similarly, corresponding to  $n = 2$ , for DE, the correction term due to coupling to the first order will also reduce the corresponding gravitational time delay by quantities  $\sim \frac{D_S G M \Lambda r_o m^2}{3c^2 \varepsilon^2}$  and  $\sim \frac{D_S G M \Lambda r_o m^2}{3c^2} \left( \frac{1}{\varepsilon_1^2} - \frac{1}{\varepsilon_2^2} \right)$ , accordingly in Eqs. (16) and (17), respectively.

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