Chapter 3
The Wave Energy Resource

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3.1 Introduction to Ocean Waves

Nobody who has spent time looking at the waves from the beach and seen the damage that they can cause to coastal structures can doubt that they can contain large amounts of energy. However, to convert this wave energy resource into a useable form requires an understanding of its characteristics. Of course, the wave energy industry is not the first industry that has had the need to understand and characterise ocean waves; however, it is important to recognise that the required ocean wave characteristics for wave energy converters are somewhat different from the typical characteristics used for other industries.

3.1.1 Origin of Ocean Waves

A combination of a variety of different disturbing and restoring forces can create the waves on the ocean surface, as shown in Fig. 3.1. Thus, in general terms, tides could be considered as very long period waves and disturbances such as earthquakes that cause tsunamis could also legitimately be called waves. However, the waves that are exploited by wave energy converters are generally generated by wind blowing across the surface of the ocean.

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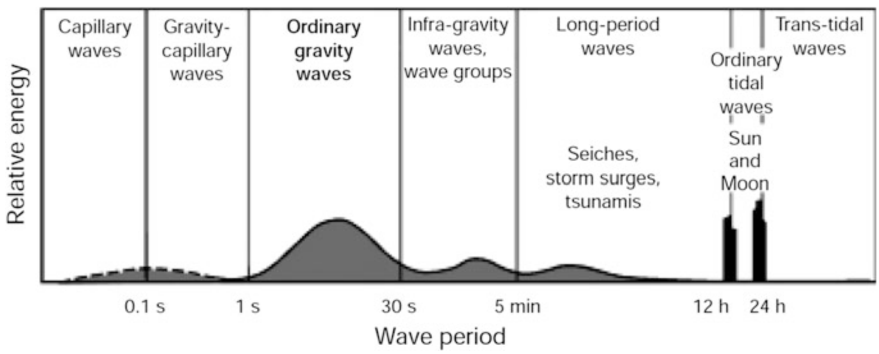


Fig. 3.1 The types of waves that may occur in the ocean [1]

If the process of waves generated by the wind is considered in greater detail, the waves always start as small ripples, but increase in size due to the sustained energy input from the wind. Provided that the wind continues to blow then the waves reach a limit, beyond which they do not grow due to energy losses such as white-capping balancing out the energy input from the wind. In this case the waves are considered to be fully developed. Whether or not a sea is fully developed will depend on both the wind speed and also the distance, or fetch, over which the wind has been blowing. However, when the wind stops blowing the waves will continue to exist and can travel for very large distances with virtually no loss of energy. In this state, they are typically called swell waves because the wind responsible for their

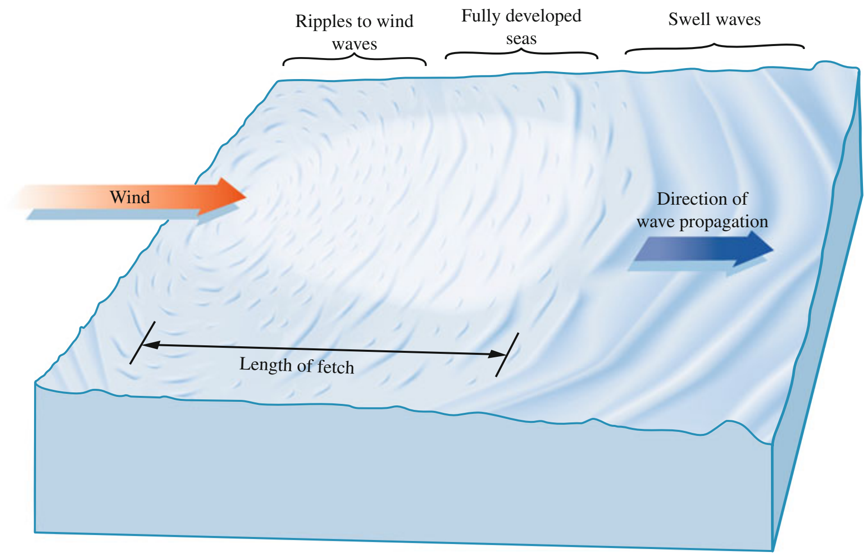


Fig. 3.2 The generation of ocean waves. Adapted from [2]

generation is no longer present. Figure 3.2 shows a simplified representation of these processes.

Using this representation of wave generation, it is common to separate waves into wind waves created by local winds and swell waves, created by to winds that are no longer blowing. Whilst the separation of waves into wind and swell waves may be useful for discussion, it should be recognised that they are essentially two extremes of a continuum of waves. In actuality, all waves are both created by the effect of previous wind and affected by the local wind. Thus, although separation of waves into wind and swell waves may be a useful tool for describing the conditions of a particular location in the ocean, this is simply an abstraction and there is no fundamental difference in the hydrodynamics of wind and swell waves.

3.1.2 Overview of the Global Wave Energy Resource

Figure 3.3 shows the global variation in the annual average omni-directional wave power density. This figure shows that the main areas of wave energy resource occur in bands across the Northern and Southern hemispheres, with less energetic regions close to the equator and poles. However, as discussed in more depth in Sect. 3.3, the annual average omni-directional wave power density does not include significant amounts of information that are vital to determining the utility of a particular wave energy resource for a particular wave energy converter. It is possible to produce a range of other figures that show how other important factors vary across the globe, and some of these factors have been reported by others [3, 4]; however, the specificity of individual wave energy converters means that it is not possible to be highly prescriptive regarding the appropriate important factors. Indeed, as discussed in Sect. 3.3, particular care must be taken when reducing the wave climate

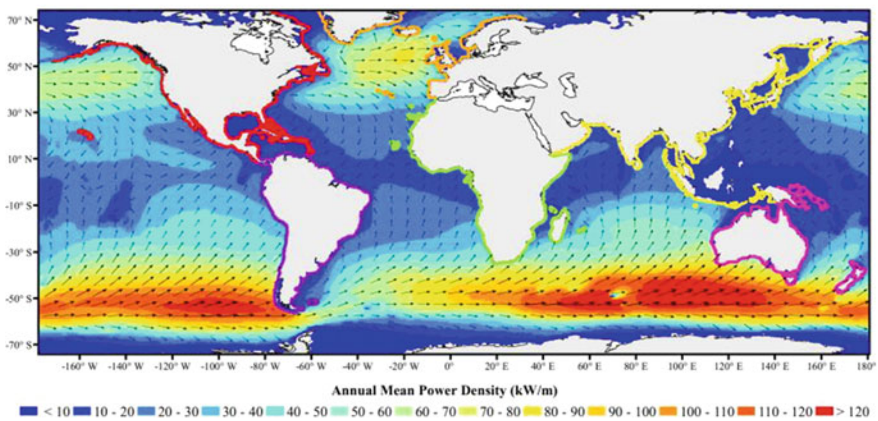


Fig. 3.3 The global wave energy resource [5]

to a finite set of factors since this data reduction always results in a loss of information, which could distort the representation of the resource and thus potentially the conclusions that may be drawn from it.

As an alternative to using factorisations that define the global wave resource, it is possible to use an understanding of the local meteorology and geography to estimate the expected characteristics of the wave resource. Although the results of such an analysis may be more qualitative than quantitative, it also provides a more direct understanding of the conditions and minimises the potential of obtaining a distorted view of the wave resource. Using the understanding of how waves develop, it is possible to state that the most energetic wave climates may be expected to occur where there is a large body of water, with weather systems that track in the same direction as the direction of wave propagation. In these cases, the winds associated with the weather system will cause the waves to continue growing across the whole body of water resulting in the largest waves. Thus, the waves that reach the western coast of Europe are typically larger than those that reach the eastern coast of the USA because not only do the winds normally blow west to east, but the typical track of the weather systems in the North Atlantic is west to east.

Knowledge of the typical wind directions and fetch lengths can also be used to provide an initial indication of the type of waves that may be expected at a particular location. For example, waves in the Mediterranean Sea are typically small because the fetch lengths are also relatively small. Conversely, waves in the South Pacific Ocean are typically large because of the large fetch lengths and relatively high winds in this region, especially in the higher latitudes. Finally, waves in the equatorial regions are typically relatively small because the wind speeds in this region are also typically small.

The seasonal weather variations can also be used to understand the consistency of the wave resource. A significant factor here is that the wind in the Southern hemisphere is significantly more consistent than the winds in the Northern hemisphere so that the wave resource is much less variable. Thus, it is possible to produce reasonable qualitative estimates of the wave resource with some knowledge of the local geography and meteorological conditions.

3.2 Water Wave Mechanics

3.2.1 Definition and Symbols

A basic wave is typically considered as a sinusoidal variation at the water surface elevation and can be defined as having a height, H, which is the vertical distance from the wave crest to the wave trough, a wavelength, λ, which is the distance between two similar points of the wave and the wave period, T, which is the time taken for the wave to repeat (Fig. 3.4).

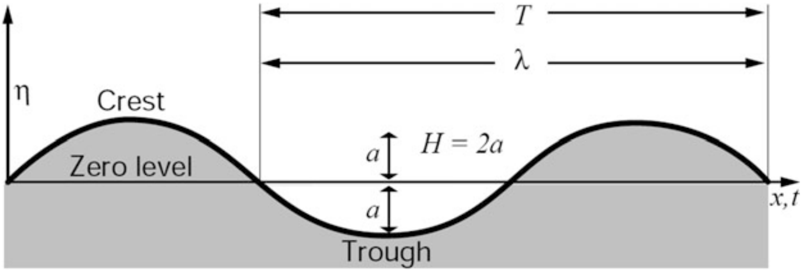


Fig. 3.4 Definition of wave parameters over a sinusoidal wave [1]

In addition it is useful to define a number of other wave parameters

$$\text{Wave steepness}, s = H/\lambda \tag{3.1}$$

$$\text{Wave number}, k = 2\pi/\lambda \tag{3.2}$$

$$\text{Wave frequency}, \omega = 2\pi/T \tag{3.3}$$

Of these additional parameters, the wave steepness is often used to distinguish between linear and non-linear waves. Typically, if the steepness is less than 0.01 then the linear wave relationships are valid, but as the steepness increases then linear theory becomes less accurate and higher-order wave models such as the 5th order Stokes waves are more appropriate [6]. However, in actuality it is very difficult to use the higher-order wave models for analysing anything other than regular waves and so linear wave theory is often used for waves much steeper than 0.01.

Figure 3.5 shows the suitability of the different wave theories based on the wave steepness $s = H/gT^2$ and the relative water depth h/gT^2. Although useful to note the applicability of different wave theories, it is worth noting that it is extremely complex to apply any theory, except linear theory, to irregular waves. Consequently, it is common to use linear wave theory beyond the bounds shown in Fig. 3.5, but recognising that it is not entirely correct.

3.2.2 Dispersion Relationship

An important characteristic of ocean waves is that they are generally dispersive, which means that the energy in the wave does not travel at the same velocity as the wave profile [6]. The effect of dispersion can be seen when a stone is dropped into water or the wave paddles in a wave tank stop generating waves. In this case waves appear to be left behind the main wave and are travelling at a slower velocity than the wave crests due to the wave energy. The velocity of a wave crest is typically

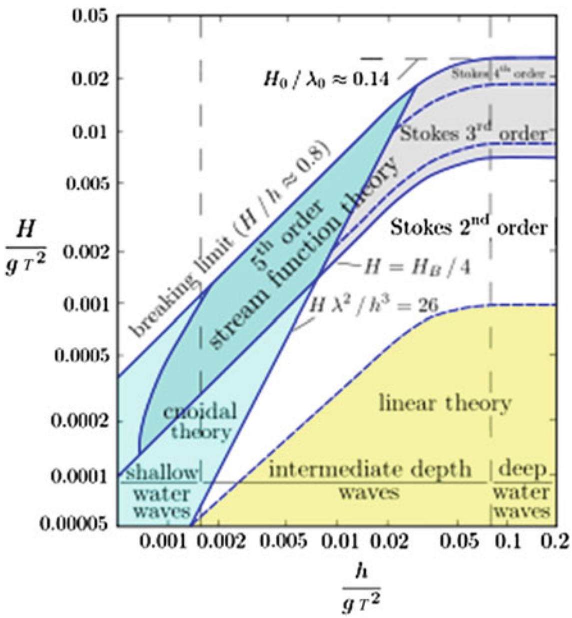


Fig. 3.5 Chart of wave model suitability [7]

called the wave celerity, c, and the velocity of the energy propagation is typically called the group velocity, C_g. In deep water the group velocity is equal to a half of the wave celerity, but in general the group velocity is given by

$$C_g = \frac{1}{2}\left[1 + \frac{4\pi d/\lambda}{\sinh(4\pi d/\lambda)}\right]c \tag{3.4}$$

Moreover, not only does the group velocity vary with water depth, but the wave celerity also varies with water depth and is given by

$$c = \frac{\lambda}{T} = \frac{gT}{2\pi}\tanh\left(\frac{2\pi d}{\lambda}\right) \tag{3.5}$$

This is called the dispersion equation and defines the wavelength based on the wave period and water depth.

3.2.3 Water Particle Path and Wave Motions

In this case the water surface elevation ζ is given by

$$\zeta = \frac{H}{2}\cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] \tag{3.6}$$

However, this variation in water surface elevation is actually the result of an elliptical motion of the water particles, which also extends far below the water surface, with the amplitude of motion decreasing exponentially with depth as shown in Fig. 3.6. Thus, the vertical displacement of the water particles $\zeta(z)$ is given by

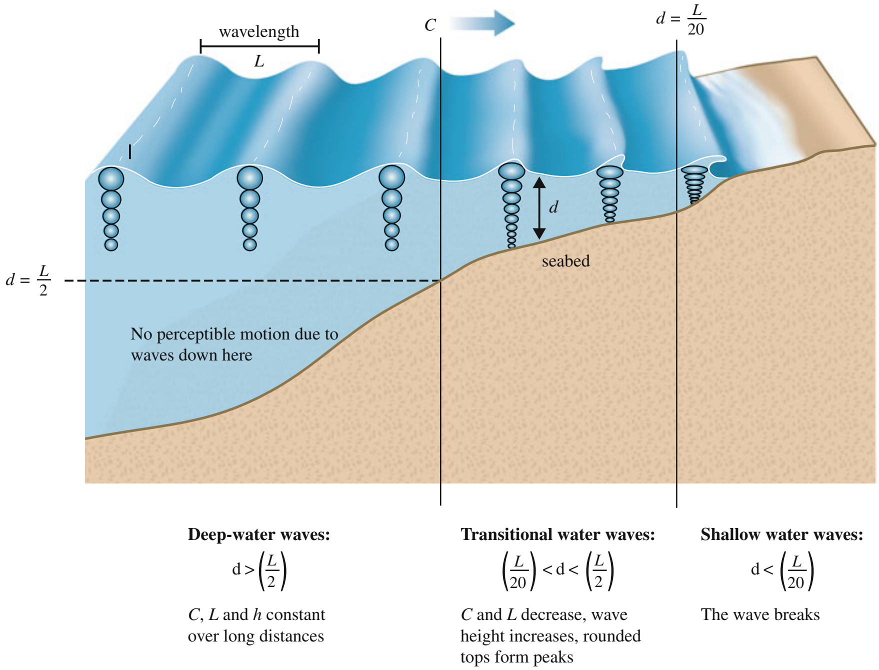


Fig. 3.6 Motion of water particles in deep and shallow water. Adapted from [2]

$$\zeta(z) = \frac{H}{2}\cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]\frac{\sinh[2\pi(z+d)/\lambda]}{\sinh[2\pi d/\lambda]} \tag{3.7}$$

and the horizontal displacement $\xi(z)$ is given by

$$\xi(z) = -\frac{H}{2}\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]\frac{\cosh[2\pi(z+d)/\lambda]}{\sinh[2\pi d/\lambda]} \tag{3.8}$$

Thus, in deep water the water particle motions are circular, but they become more elliptical as the water depth decreases as shown in the Fig. 3.6.

In particular, it can be seen that the variation in water particle motion is dependent on the water depth relative to the wavelength, and this is often used to define three regions of water depth: 1) deep water where the seabed does not affect the waves and typically requires the water depth to be greater than half the wavelength, 2) shallow water where there is no variation in horizontal water particle motion with water depth and typically requires the water depth to be less than 1/20th of a wavelength and 3) intermediate depth that exists between these two extremes [6].

At a depth of half a wavelength, the wave-induced motions are only approximately 4 % of those at the surface and thus could be considered insignificant. However, it should always be remembered that these limits are somewhat arbitrary, and since they depend on the wavelength this means that the definition of water depth is not fixed. That is, a site may be defined as being in deep water for a short wave, whilst the same site for a different wave may be in intermediate water. Thus, care should always be taken to determine which reference wavelength should be used to define the relative depth. For wave energy, it is particularly important to recognise this condition, because many wave energy converters defined as "deep-water" devices, such as Pelamis, are typically deployed in what many oceanographers would define as intermediate water depths.

3.3 Characterisation of Ocean Waves and the Wave Climate

3.3.1 Introduction

Traditionally, sea-states have been characterised using a representative wave height, which before any method for recording waves existed was based solely on observation. That is, the representative wave height was defined as the wave height as reported by an "experienced observer", whom we must assume had spent many years listening to the estimates of other experienced observers so that a relatively consistent estimate of the wave height could be made. This was called the Significant Wave Height, symbolized by H_s. However, it is clear that the accuracy of this method is highly dependent on the experience of the observer and as such subject to significant error. When it became possible to record the variation in the water surface elevation, an alternative method of defining wave height was developed. With a record of the variation in the water surface elevation, it is possible to measure the height of individual waves and thus produce a more reliable estimate of the wave height. In order to be consistent with historical reports, it was decided that the new records of surface elevation should be analysed so as to produce an estimate equivalent to the H_s. It was found that a good estimate of H_s was given using the average height of the third highest waves.

In modern times, the variation in wave surface elevation is typically recorded digitally, which provides the potential for significant analysis of the wave record. The most significant development in the representation of the sea is the definition of the sea using a spectrum. To understand the concept of the wave spectrum, it is first necessary to accept that the variation in water surface can be represented as the linear super-position of sinusoidal waves of different frequencies, amplitudes, directions and phases. Although this representation could be viewed as simply a change in the co-ordinate system (from the time-domain to the frequency-domain), it actually appears to be a reasonably good representation of the underlying physics. Indeed, the wave spectrum is now generally used to fully define any sea-state, with the assumption that there is a random phase between all of the individual wave components, which is a natural consequence of the assumption of linear super-position.

Although the linear super-position and random phase assumptions are commonly applied, it is important to recognise that they are not universally valid. The most obvious example of the breakdown of linear super-positioning is in sea-states with steep and breaking waves. The super-positioning of the waves means that the waves will become unstable and break at particular times and locations, resulting in a variation in the water surface elevation that is poorly represented by a random phase, linear super-position of the wave components. Another case where linear super-positioning does not provide an accurate representation of the water surface elevation is in shallow water, where a single frequency wave would be more accurately represented by a cnoidal or higher-order Stokes wave than by a sinusoidal wave. Notwithstanding these cases, it is still typical to use a spectrum and linear super-positioning to represent the sea-state because no reasonable alternative currently exists. However, it is important to recognise that, in general, this representation of the sea is, to some extent, simply an abstraction that has been found to be very useful in defining the sea.

3.3.2 Temporal, Directional and Spectral Characteristics of the Wave Climate

To understand how the ocean waves may influence the performance of a wave energy converter, it is useful to consider the temporal, directional and spectral characteristics of the ocean waves and how these may influence the relationship between the average omni-directional wave power and the average power generation.

Firstly, the **temporal characteristic of a wave climate** is how the sea-states that make up a wave climate vary in time as illustrated in Fig. 3.7 for the significant wave height. In general, the more consistent the wave climate is, the more attractive it becomes (for a particular average wave power) because the WEC and power generating plant can remain closest to its optimal operating conditions and thus maximise the system efficiency. However, the sea-states will vary due to changes in the metrological conditions that generate the winds and associated waves. Not surprisingly, the stability of metrological conditions varies across the world so that the wave climate is more consistent in some locations than others. This variability may be primarily associated with daily, seasonal and/or annual variations in the sea-states, each of which will have a slightly different impact on the power generation and its utility. Thus, it is clear that for all locations the temporal characteristics are an important element of the wave climate, which can result in different power generations for the same average incident wave power.

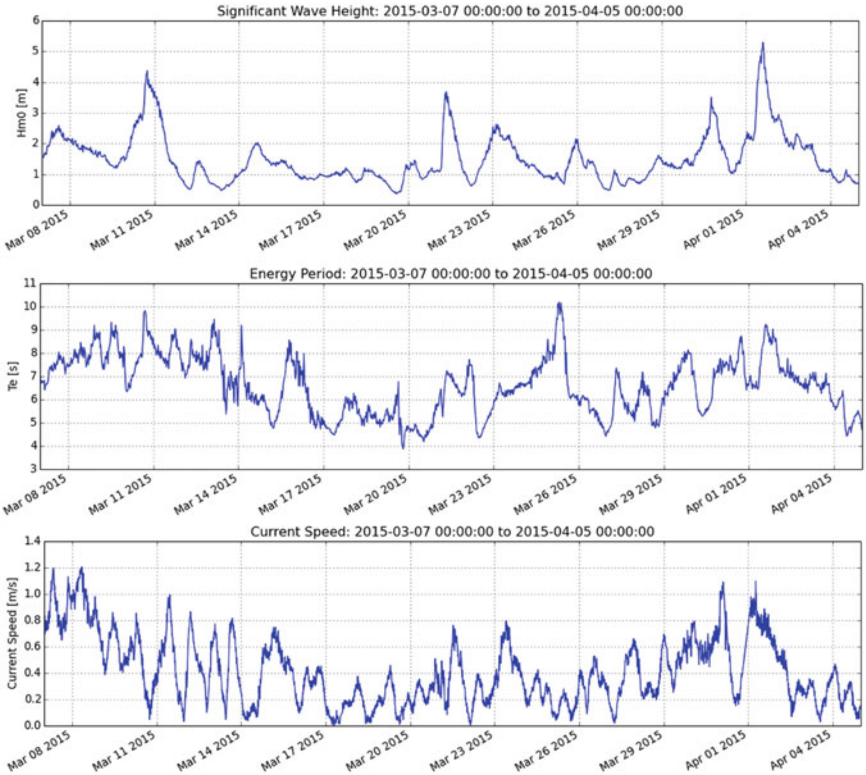


Fig. 3.7 Example of the variation of the significant wave height, energy wave period and tidal current over one month at DanWEC

Secondly, the **directional characteristics of a wave climate** are associated with not only the directional spreading of individual sea-states, but also the directional variation of all the sea-states. The only situation where this may not be critical is for an isolated omni-directional WEC such as a heaving buoy. However, as soon as this buoy is put into a wave farm, the directional characteristics of the sea-state become significant, with the significance increasing with the size of the wave farm. In general, an increase in the directional variation of the wave climate will lead to a reduction in the average power generation because the WECs and/or wave farm will typically be less optimally aligned. As would be expected, the directional characteristics of a wave climate is dependent on its location, which defines the range of weather systems that produce winds, and subsequently waves that contribute to the local wave climate. Thus, the directional characteristics are an important element of the wave climate that should be considered when assessing a potential site.

Finally, the spectral **characteristics of a wave climate** are associated with the wave spectrum of individual sea-states, together with the spectral variation for all the sea-states as illustrated in Fig. 3.8 for the average directional spectra at EMEC. The spectral characteristics of the wave climate can be particularly important because the "efficiency"

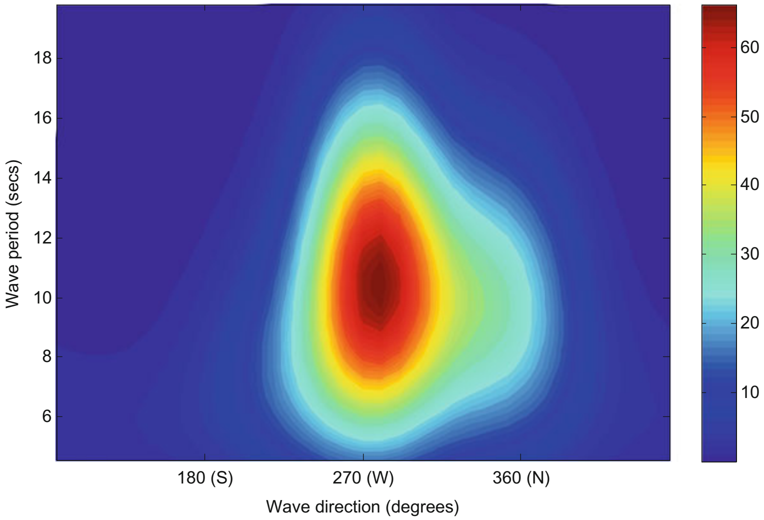


Fig. 3.8 Average directional spectral variance density (m^2/Hz/rad) at the European Marine Energy Centre, Orkney, Scotland

of many WECs is frequency-dependent. Thus, wave power associated with particular wave frequencies may be more significant than wave power at other frequencies. For example, the power capture per unit wave height of many surging/pitching devices tends to increase with wave frequency because the incident wave force/torque also tends to increase with wave frequency [8, 9], whilst the incident wave power decreases with wave frequency (see Eq. 3.18). Thus, in this case, incident wave power is clearly not a good proxy for power generation. Thus, in assessing a potential site, it is important consider the spectral characteristics of a potential site, especially in relation to the spectral response of the WECs being considered for deployment at the site.

When a particular site of interest has been identified, then a scatter diagram is often used to characterise it. Figure 3.9 shows an example of a scatter diagram which consists of a frequency of occurrence table indexed by a representative wave period, typically a peak period, zero-crossing period or energy period and a representative wave height, almost always the Significant Wave Height. The scatter diagram clearly provides significantly more information about the wave climate than the average omni-directional wave power; however, it is not without issues. Firstly, depending on the table resolution, the sea-states can vary significantly within any single cell in the scatter table, especially for the cells indexed by a small significant wave height. For example, there is a potential 4:1 variation in wave power between sea-states in a cell defined by Significant Wave Heights of between 0.5 and 1.0 m. Of course, any contribution from these sea-states to either the average incident wave power or power generation is likely to be small and so the impact on any estimate of WEC performance is also likely to be small; however, this may not always be the case and any potential distortion should be considered when using scatter diagrams. Secondly, typically there are no details on the temporal/ directional distribution or spectral shape of the sea-states that are all contained in a single cell, both of

Scatter diagram

Hs \ Tz	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5
0.25	0.0066	0.0056	0.0030	0.0023	0.0011	0.0007	0.0003	0.00005				
1	0.0453	0.1650	0.0906	0.0347	0.0131	0.0047	0.0019	0.00069	0.0001	0.00004	0.00007	0.00005
2	0.0018	0.0368	0.1604	0.0650	0.0229	0.0099	0.0032	0.00121	0.00009	0.00005	0.00005	
3		0.0003	0.0187	0.1084	0.0335	0.0071	0.0033	0.00171	0.0004	0.00007		0.00002
4			0	0.01021	0.05565	0.01163	0.00209	0.00052	0.00034	0.00021	0.00005	
5				0.00002	0.00729	0.02391	0.00301	0.00069	0.00031	0.00014	0.00005	0.00005
6					0.00012	0.00603	0.00691	0.00052	0.00007			
7				0.00002	0.00009	0.00026	0.00352	0.00152	0.00016	0.00005		
8							0.00062	0.00288	0.00017			
9								0.00086	0.00073	0.00002		
10								0.00002	0.00043	0.00016		
11									0.00011	0.00014		
12										0.00004		

Fig. 3.9 Example of a scatter diagram

which can have a significant impact on the power generation of a WEC as discussed above. This second issue is sometimes reduced by producing multiple scatter diagrams for a particular site, which are used to separate the wave climate by peak wave direction or season, but there is clearly a practical limit to the number and range of scatter diagrams that can be produced and used effectively.

Another representation of the wave climate that is often used is the **wave rose** as shown in Fig. 3.10. A wave rose is a graphical representation of the average wave power or Significant Wave Height from different directional sectors. Similar to a set of wave roses may be produced based on season in order to provide additional information that may be useful in understanding the wave climate, especially where different meteorological conditions are responsible for different wave conditions at different times of the year.

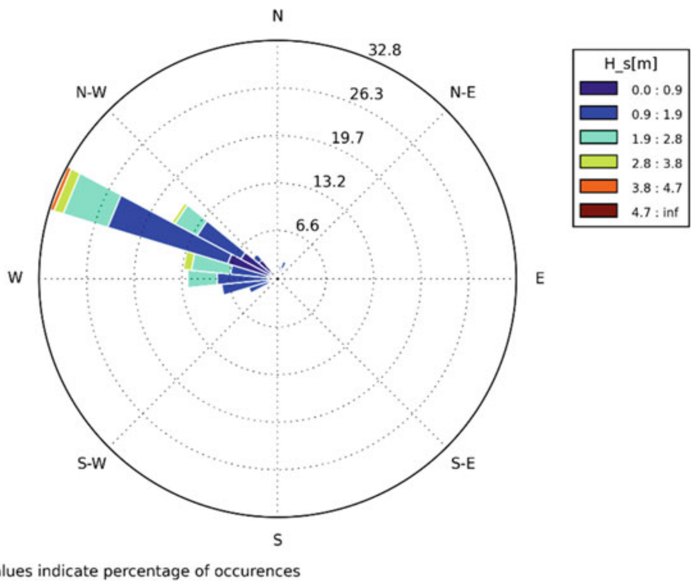


Fig. 3.10 Example wave rose of significant wave height

In summary, it can be seen that the characterisation of the wave climate using single parameters such as the average omni-directional wave power, scatter diagrams and wave roses present only a partial picture of the wave climate. Moreover, care must be taken in translating this partial picture into an estimate of the power generation of a WEC. Indeed, whenever possible it is recommended to use the full time-series of directional wave spectra to estimate the average power generation of a WEC [10], and if this is not possible, either because the full data set is not available or it would require too much effort, then it is important to recognise that not only is there an increased uncertainty in the estimate of power generation, but relative performance of the WEC at different locations may also not be a simple function of the average omni-directional wave power or other parameterised characteristic of the wave climate. However, although the use of wave climate characterisations for estimating power generation is not recommended, they do provide an overview that can be useful for understanding the performance of a WEC. Furthermore, as understanding of a WEC increases with identification of the most appropriate characterisations of the wave climate for the particular WEC, it is possible that the WEC's performance could be reasonably estimated from the wave climate characteristics. However, until that point is reached it remains prudent to recognise the limitations of any characterisation of the wave climate and the potential distortion in the estimate of a WEC's power that it may cause.

3.3.3 Spectral Representation of Ocean Waves

In the preceding sections, the wave spectra has been discussed without any clear definition of exactly what it represents. Representation of the ocean using a wave spectrum assumes that it is possible to represent the water surface as the sum of sinusoidal waves with a range of frequencies, amplitudes and directions. The variation of the wave energy with frequency (and direction) is called the wave spectrum. Figure 3.11 shows an illustration of this super-positioning of waves, together with an example of a typical wave spectrum.

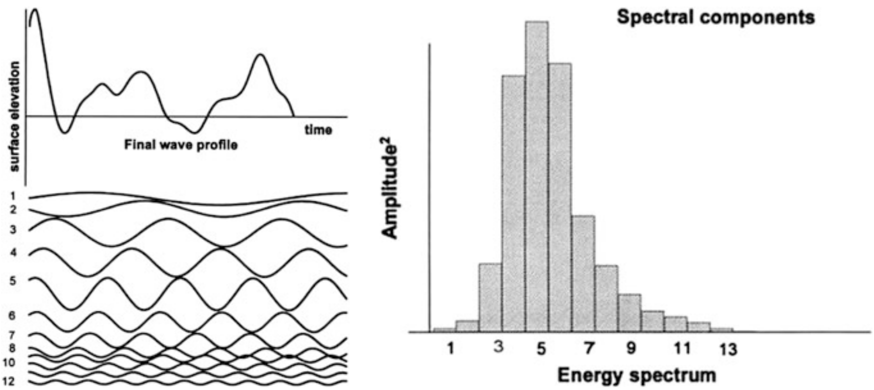


Fig. 3.11 Super-positioning of waves (corresponding to spectral components) to create water surface elevation (*left*) and the resulting spectrum (*right*) [11]

A variety of idealised spectra have been suggested to represent a fully developed sea-state. Perhaps the most commonly used spectrum was developed by Pierson and Moscowitz in 1964 and is called the Pierson-Moscowitz (PM) Spectrum [12]. This spectrum assumes that the wind has been blowing across a sufficiently large expanse of water for sufficiently long that the waves are in equilibrium with the wind, i.e. the sea-state is fully developed and so that the spectrum is dependent on only the wind speed.

Subsequent research completed by Hasselman et al. in the Joint North Sea Wave Observation Project (JONSWAP) identified a refinement to this wave spectrum for when the sea is not fully developed and is based on the wind speed and fetch length [13]. This spectrum is called the JONSWAP spectrum and is commonly used to represent the sea-state that is not fully-developed.

$$S(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left[-\beta\left(\frac{\omega_p}{\omega}\right)^4\right] \tag{3.9}$$

$$S(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right] \gamma^{\exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\right]} \tag{3.10}$$

PM spectrum (Eq. 3.9)	JONSWAP spectrum (Eq. 3.10)
$\alpha = 0.0081$	$\alpha = 0.076\left(\frac{U_{10}^2}{Fg}\right)^{0.22}$
$\beta = 0.74$	$\omega_p = 22\left(\frac{g^2}{U_{10}F}\right)^{1/3}$
$\omega_p = \frac{g}{U_{19.5}}$	$\gamma = 3.3$
	$\sigma = \begin{cases} 0.07 \omega \le \omega_p \\ 0.09 \omega > \omega_p \end{cases}$

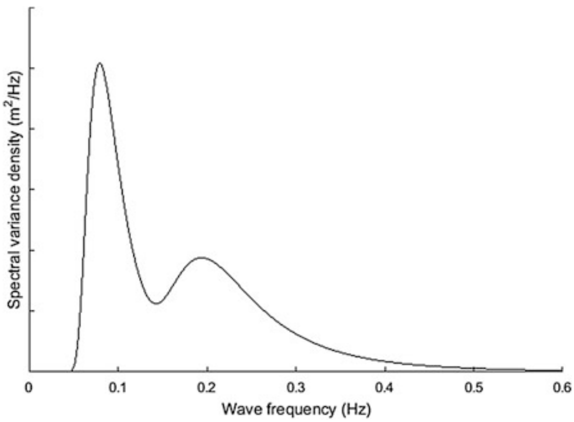
where $S(\omega)$ is the spectral variance density,ω_p is the peak frequency, g is the gravitational acceleration, U_{10} is the wind speed at a height of 10 m, F is the fetch length and ω is the wave component frequency.

In addition to the wind speed and fetch length, the JONSWAP spectrum is also defined by the peak enhancement factor γ. This parameter defines how the peakiness of the spectrum as seen in Fig. 3.11.

Comparison of Eqs. 3.9 and 3.10 reveals that the spectral shapes of the JONSWAP and Pierson-Moscowitz spectra are identical when the peak enhancement factor of the JONSWAP spectrum equals 1.0. Thus, it can be inferred that the bandwidth of the spectrum is dependent on its state of development with new and developing seas having a narrower bandwidth, so that the wave components are all at similar frequencies and fully-developed seas having a broader bandwidth, with the wave energy spread over a larger range of frequencies.

To facilitate understanding, the discussion above only considers sea-states that have been generated by a single source of wind. However, in reality the sea-state at a single location may have waves generated from a number of different sources of winds from different directions with different speeds and fetch lengths. Where there are two distinct sources of waves then the sea-state is called bi-modal and has two peaks with different peak directions and frequencies. Figure 3.12 shows an example of a bimodal sea-state. Cases where there are more than two sources of wind result are called multi-modal sea-states. Although there will be some interaction between the waves from the different sources, typically this interaction is small and the spectra can generally be linearly superimposed without too much loss of accuracy (at least when they are not close to breaking).

Fig. 3.12 Example spectrum of a bimodal sea-state



3.3.4 Characterization Parameters

Recalling the introduction to this section, it was noted that there are three different definitions of the Significant Wave Height: the first based on observation, the second based on analysis of a record of the surface elevation time-series and the third based on the wave spectrum. It is important to be aware of which one is being used. The first method is never used nowadays; however, wave data from both the other methods is still commonly used. Thus, it is good practice when referencing the Significant Wave Height to use a subscript to identify the method, with the subscript '1/3' used when the Significant Wave Height is based on the average height of the third highest waves and the subscript 'm0' when the Significant Wave Height is based on the wave spectrum. Unfortunately, in many cases the Significant Wave Height is identified by the subscript 's' and the method used to generate it is unknown. As noted above, the difference between the methods in deep water for a moderate sea-state is relatively small, typically about 1 %; however, the difference increases progressively as the waves steepen and/or water depth decreases.

In wave energy, the preferred representative wave height is the Significant Wave Height derived from the spectral moments of the wave spectrum, H_{m0}:

$$H_{m0} = 4\sqrt{\int_0^\infty S(\omega)d\omega} \tag{3.11}$$

This is because it is effectively based on the energy in the waves and as such is directly related to the average wave power density. To show this, it is necessary to recognise that with linear super-position the power in each wave can be considered independently and then summed together to give the total average wave power density. Thus, consider a single wave component then the wave power is given by

$$J(\omega) = \rho g S(\omega) \cdot C_g(\omega) \tag{3.12}$$

where the first half of the right-hand side of the equation is the energy in the wave and the second half of the right-hand side of the equation is the velocity at which the energy is propagating, the group velocity [6]. The speed that the wave energy propagates depends on the wave frequency ω and water depth h and is given by

$$C_g(\omega) = \frac{1}{2}\frac{\omega}{k(\omega)}\left(1 + \frac{2k(\omega)h}{\sinh 2k(\omega)h}\right) \tag{3.13}$$

where $k(\omega)$ is defined by the dispersion equation

$$\omega^2 = gk(\omega)\tanh k(\omega)h \tag{3.14}$$

Using the assumption of linear super-positioning, the average wave power density for the sea-state is given by the integral of the wave components

$$J = \int_0^\infty \rho g S(\omega) \cdot \frac{1}{2}\frac{\omega}{k(\omega)}\left(1 + \frac{2k(\omega)h}{\sinh 2k(\omega)h}\right)d\omega \tag{3.15}$$

To progress further, it is useful to define the moments of the spectrum m_n as

$$m_n = \int_0^\infty S(\omega)\omega^n d\omega \tag{3.16}$$

Then, the wave energy period T_e can be defined as the ratio of the first negative moment of the spectrum to the zeroth moment of the spectrum as given by Eq. 3.17

$$T_e = \frac{m_{-1}}{m_0} \tag{3.17}$$

and the Significant Wave Height ($H_{m0} = 4\sqrt{m_0}$, see Eq. 3.11) can be used directly in the calculation of the wave power density. Consequently, the omnidirectional wave power J can be defined in deep water as

$$J = \frac{\rho g^2}{64\pi} H_{m0}^2 T_e \tag{3.18}$$

In addition to defining the Significant Wave Height and Energy Period, the moments of the spectrum can also be usefully used to define other characteristics of a sea-state. For example, the relative spreading of the energy with wave frequency, often termed the spectral bandwidth ϵ_0, can be defined as the standard deviation of the period variance density, normalized by the energy period, which is

$$\epsilon_0 = \sqrt{\frac{m_0 m_{-2}}{m_{-1}^2} - 1} \tag{3.19}$$

In addition, it is possible to make a spectral estimate of the mean zero-crossing period of the waves T_z, which is given by

$$T_z \cong T_{02} = \sqrt{\frac{m_0}{m_2}} \tag{3.20}$$

This spectral estimate of the mean zero-crossing period of a sea-state is useful because it allows a spectrum to be scaled using assumptions regarding the spectral shape and the mean zero-crossing period, which is a common parameter used to define historical wave resource data. Similarly, the spectrum can be scaled using the peak period T_p, which has also been commonly used to define wave resource data.

Using these expressions, it is also possible to calculate the ratio between different measures of the wave period for particular spectral shapes. This can be especially useful when it is considered necessary to convert between representations of the wave period. For example, for a JONSWAP spectrum with a peak enhancement factor, $\gamma = 3.3$, the ratios of the wave periods are

$$1.12\,T_e = 1.29\,T_z = T_p \tag{3.21}$$

For many devices, and for all wave farms, the directional characteristics of the sea-state will also be important. The directionally resolved wave power density $J(\theta)$ is a key directional characteristic of the sea-state as it defines the wave power propagation in a particular direction. The directional wave spectrum can be used to calculate the variation in the directionally resolved wave power density $J(\theta)$ as given by

$$\begin{gathered} J(\theta) = \rho g \int_{-\pi}^{+\pi} \int_0^{\infty} S(\omega, \varphi) C_g(\omega) \cos(\theta - \varphi) \delta \cdot d\omega \cdot d\varphi \\ \begin{cases} \delta = 1, & \cos(\theta - \varphi) \geq 0 \\ \delta = 0, & \cos(\theta - \varphi) < 0 \end{cases} \end{gathered} \tag{3.22}$$

Other direction parameters that can be derived from this and provide additional characteristics of a sea-state include the direction of maximum directionally resolved wave power density and the directionality coefficient, which is the ratio of the maximum directionally resolved wave power density and the omni-directional wave power density as defined in Eq. 3.15.

A further characterisation of the waves, which may be particularly important when considering transient effects, is the tendency for larger waves to be grouped together; this characteristic is called wave groupiness. Non-linear processes can play an important part in the creation of wave groups, especially in shallow water; however, it is also dependent on the spectral bandwidth, with narrow-banded spectra generally having higher levels of wave groupiness than broad-banded spectra. A common measure of wave groupiness is the *average run length*, which is the average number of consecutive waves that exceed a specified threshold such as the significant or mean wave height.

The characterisations of a sea-state described above have generally proved useful for analysing the wave climate for wave energy converters; however, it must always be appreciated that all these characterisations reduce the amount of information available on the wave climate. When available, it is almost always better to use the directional wave spectrum of a sea-state in any analysis rather than a characterisation of the sea-state. Unfortunately, this is not always possible either because the directional wave spectrum is not available or because it would require excessive amounts of computational effort. However, whenever only the sea-state characteristics are presented, or used in an analysis, it is important to be cautious with any conclusions because of the potential distortions that can occur.

3.3.5 Challenges in Wave Climate Characterisation

A wave climate can be reasonably approximated as a long-term series of sea-states that are defined by the directional wave spectrum. Together with other metocean parameters such as water depth, marine current speed/direction and wind speed/direction, this can be used to estimate the power capture and design parameters for any wave energy converter deployed at the location. However, typically it is not possible to work with this amount of data (or the data is not available) and so a characterisation of the wave climate is used. The wave climate characterisation can essentially be one of two types: the characterisation of the **wave climate at a single point or the characterisation of the wave climate over an area**. However, it is important to recognise that in either case the characterisation results in a compression of the details on the wave climate and so does not contain all the information that may be relevant to the performance of a wave energy converter.

The **average omni-directional wave power** is probably the most common characterisation of the wave resource for the assessment of wave energy. This seems, and likely is, a reasonable characterisation since it is clear that to extract significant amounts of wave energy the incident wave power must also be significant; without waves there is no wave power. Figure 3.3 shows an example of this characterisation illustrating the variation in the average omni-directional wave power around the world. As would be expected, the areas with higher average omni-directional wave power, such as the north-west coast of Europe, are also the areas with the most interest in the deployment of wave energy converters. The implicit assumption is that a wave energy converter's power capture is proportional to the average omni-directional wave power thus a larger average omni-directional wave power equates to a larger power capture. However, whilst it may be reasonable to assume that a wave energy converter will produce more power at a site with an average omni-directional wave power of 40 kW/m compared to an alternative site with 2 kW/m, it is less clear that this will be true if the alternative site had an average omni-directional wave power of 30 kW/m.

The key factor to consider is that when comparing potential sites the use of the average omni-directional wave power obscures information regarding the temporal, directional and spectral characteristics of the wave climate (see Sect. 3.2) that may be important to the average power capture. Of course, how these characteristics may affect the average power generation will vary with the WEC and so it is difficult to be overly prescriptive regarding the extent of distortion that may be due to using average omni-directional wave power as a proxy for average power generation. One method to compensate for the potential distortion is to provide information on other aspects of the wave climate simultaneously with the average omni-directional wave power. Examples of this additional information could include the ratio of maximum wave power to average wave power, the average directionality coefficient, the average spectral width, and/or the average energy period. Unfortunately, whilst this additional information does provide more details of the characteristics of the wave resource that may suggest the relative strengths and weaknesses of particular sites, it still does not provide a clear indication of how a WEC's power generation may differ between locations.

Whilst it is frustrating that a single parameter, or even set of parameters, cannot be used to assess the suitability of a potential WEC deployment site, this is the state of the wave energy industry at the moment. The rich diversity of WEC concepts currently being developed means that there are a multiple of relationships between the wave resource and power generation. Moreover, it is possible that a particular WEC concept may be most suitable at one location, whilst another WEC concept is more suitable at another location. Thus, there may not be the complete convergence onto a single concept as in wind energy, with the three-bladed horizontal-axis turbine, due to the potentially greater diversity of wave resource characteristics compared to wind resource characteristics, which is generally successfully characterised simply by the average wind speed.

Although not associated with a particular WEC concept, a useful illustration of the dangers of using the average omni-directional wave power as a proxy for power generation is in assessing the effect of water depth on the incident wave power. Off of the west coast of Scotland, the average omni-directional wave power decreases as it approaches the shore and the water depth reduces, so that in 10–20 m of water it is only typically 50 % of its offshore value. To assess the extent that this reduction in average omni-directional wave power may translate to a reduction in potential power generation, it is necessary to consider how the change in average omni-directional wave power has occurred. Consideration of the wave propagation process indicates that there are **six main processes responsible for the change in average omni-directional wave power**, namely: shoaling, refraction, diffraction, depth-induced wave breaking, bottom friction and wind growth, which are each considered in detail below.

3.3.6 Coastal Processes

3.3.6.1 Shoaling

Shoaling can be understood by considering a wave propagating into shallower water. When a wave propagates into shallower water, the wave group velocity changes, but the change in group velocity is not accompanied by a change in energy flux. Thus, conservation of energy means that the wave height must get larger in order to keep the total energy flux constant. It can be visualised as a bunching up of the incident waves so that they increase in height as illustrated in Fig. 3.13.

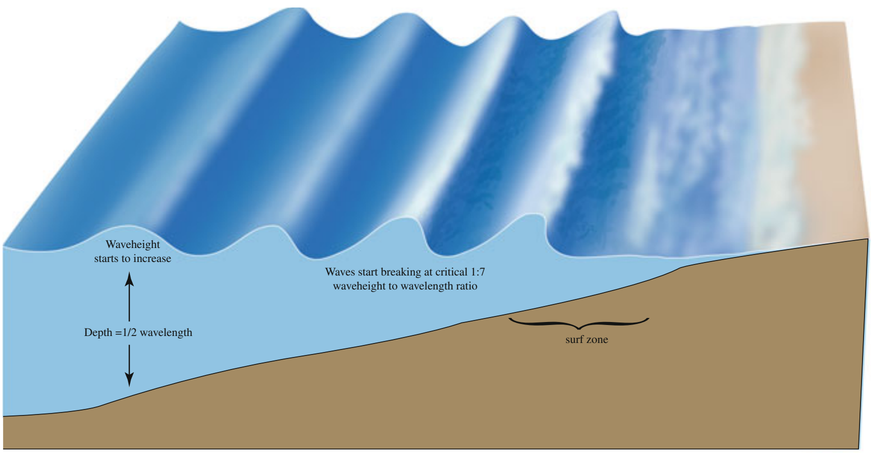


Fig. 3.13 Change in wave shape due to water depth. Adapted from [7]

3.3.6.2 Refraction

To understand refraction, consider a wave propagating at an angle to the depth contours. In this case, the dispersion equation tells us that the part of the wave crest in shallow water will travel slower resulting in a turning of the direction of wave propagation. This effect explains why on the beach all the waves appear to come from a direction approximately orthogonal to the coastline (Fig. 3.14).

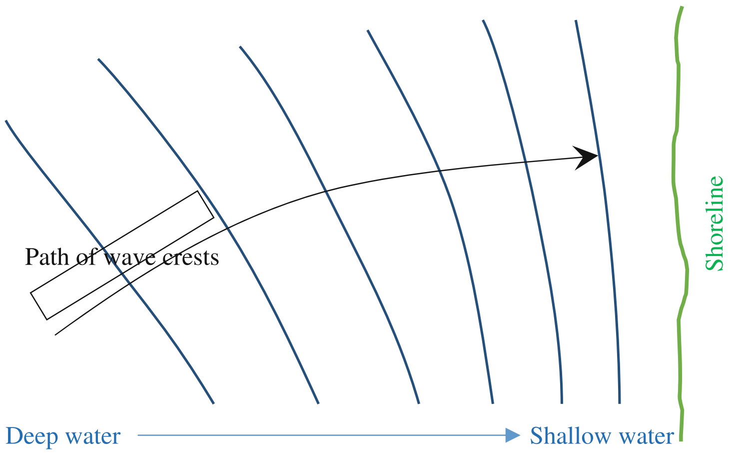


Fig. 3.14 Wave refraction near the shore due to a change in water depth

Refraction causes the waves to change direction so that their propagation direction is more orthogonal to the seabed depth contours. The net effect of this on all the waves is to reduce the directional spreading in the waves so that as the water depth reduces their approach from a more concentrated direction. In addition, any single wave component refraction causes a reduction in the wave height as it is spread out over a larger distance; however, it is important to remember that the refraction process is energy conserving and thus does not change the amount of energy travelling orthogonally to the depth contours. Using the west coast of Orkney again as an indicative site, it can be seen that refraction causes a significant change in the average omni-directional wave power between an offshore and nearshore site, with a reduction of 30–40 %. Thus, the key to assessing whether or not the change in average omni-directional wave power due to refraction will similarly affect the power generation depends on the directional sensitivity of the WEC or wave farm. An isolated heaving buoy may be insensitive to wave direction and so the reduction in average omni-directional wave power will result in a similar reduction in power generation; however, a wave farm will have a directional sensitivity so that the incident wave power is defined by the wave power incident on the wave farm. If this is the case and the wave farm is aligned with the depth contours, then refraction would have no effect on the relevant incident wave power as refraction is energy conserving. In reality, it is likely that the wave farm will not be aligned with the depth contours and so refraction will change the power incident on the wave farm, with the effect due to refraction increasing as the angle between the wave farm and depth contours increases. Thus, again it can be seen that the suitability of using average omni-directional wave power as a proxy for power generation depends on the WEC characteristics and deployment configuration.

3.3.6.3 Diffraction

Diffraction occurs when waves meet a surface-piercing obstacle such as an island, headland or breakwater. Without diffraction the waves would continue to travel in the same direction leaving a region of calm water in the lee of the obstacle. However, diffraction means that the waves will bend so that there are waves behind the obstacle. The amount of diffraction depends on the wavelength, with the longer waves diffracting to a greater extent than the shorter waves. If there is more than one source of diffraction, e.g. either side of an island, then a diffraction pattern may form where there are areas of increased and decreased wave height due to constructive and destructive interference. Although diffraction means that waves will occur on the leeward side of an obstacle, generally these waves will be smaller than the incident waves (except in the special case of constructive interference) so the wave resource behind an obstacle, is likely to be smaller than the seaward wave resource.

3.3.6.4 Depth-Induced Wave Breaking

Wave breaking occurs when the horizontal wave particle velocity becomes greater than the wave celerity. When this occurs the wave will spill energy in the form of breaking waves. Depth-induced wave breaking is related to the steepening of the waves in shallow water due to shoaling. When the wave height is greater than about 0.8 of the water depth (or about 0.14 of the wavelength), then the waves break. There are three different types of breaking waves: spilling, plunging and surging as shown in Fig. 3.15, depending on the wave and seabed steepnesses (or more specifically the Iribarren number [14]).

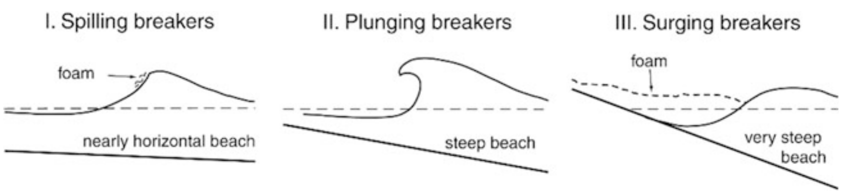


Fig. 3.15 Classification of breaking waves [15]

In water depths greater than about 10 m, the vast majority of waves will not break and so it is tempting to consider that this process is not significant in assessing the suitability of using average omni-directional wave power to compare offshore and nearshore sites. However, the average omni-directional wave power includes energy from all events irrespective of its exploitability. In particular, it includes the wave energy in storms, which at the offshore site, in deep water, may have 40–50 times the wave power of the average wave power. Thus, although storms may only occur infrequently, they may make a relatively large contribution to the average wave power and account for perhaps 15–20 % of the total wave energy. On the contrary, at the nearshore site the wave energy in a storm is a much smaller multiple of the average wave power because depth-induced wave breaking has limited the wave energy in a storm that reaches the nearshore, but not affected the wave power in the most commonly occurring seas. The proportion of the total wave energy contained in storms is important since it is largely un-exploitable, either because the WEC power generation is limited by the plant rating, or because it has to shut down in order to survive the storm. Thus, because the average omni-directional wave power does not distinguish whether or not the wave energy is exploitable, it distorts the relative potential power generation at the offshore and nearshore sites.

3.3.6.5 Bottom Friction

The reduction of average omni-directional wave power has often been primarily attributed to bottom friction. However, as illustrated above, a significant proportion of the reduction is caused by other factors and in particular refraction [16]. Indeed, for a typical seabed bottom friction only accounts for about 5 % of the reduction in average omni-directional wave power. The reduction in spectral wave energy due to bottom friction is complex and varies with depth so that the wave spectrum changes as a result of bottom friction, although the small amount of energy reduction means that the change in spectrum will also be small. However, as different WEC concepts have different spectral responses, it is possible that the change in spectral shape will be more significant for one WEC concept than another. Thus, it is possible that the change in average omni-directional wave power due to bottom friction has a different impact on average power generation for different devices because of their different response characteristics.

3.3.6.6 Wind Growth

As there is a larger fetch to the open ocean for the nearshore, it may be expected that wind growth will increase the wave power at this site. Unfortunately, in many cases the offshore waves are already in equilibrium with the wind because of the large fetch and so they cannot grow significantly between offshore and nearshore. However, when the wind blows from the land there will be minimal fetch for the nearshore site, but the fetch may be a significant for the offshore site. At a location 40 km off of the west coast of Orkney, Scotland, the waves travelling away from the shore typically account for about 15 % of the average omni-directional wave power, which is not an insignificant proportion of the total wave power. The key to assessing whether this 15 % of wave power should or should not be included depends on whether or not the WEC can capture energy travelling in the opposite direction to the majority of the waves. For example, a heaving buoy, such as the Wavebob, may be expected to exploit this wave power, but an overtopping device, such as the WaveDragon, is less likely to be able to exploit it so that the omni-directional wave power is less appropriate as a proxy for power generation in this case.

3.3.7 Case Study—Incident Wave Power

Figure 3.16 shows an example of the effect of considering these factors for the wave climate at the European Marine Energy Centre (EMEC), where gross power refers to the omni-directional wave power, net power includes the effect of wave refraction and exploitable power also includes the effect of wave breaking and bottom friction. It can be seen that the difference in exploitable resource from a 50 m "deep water" site to a 10 m "shallow water" site is around 20 %, significantly less than the 50 % reduction in resource that can arise from an inappropriate use of the omni-directional wave power. More details on this case study are available in [17].

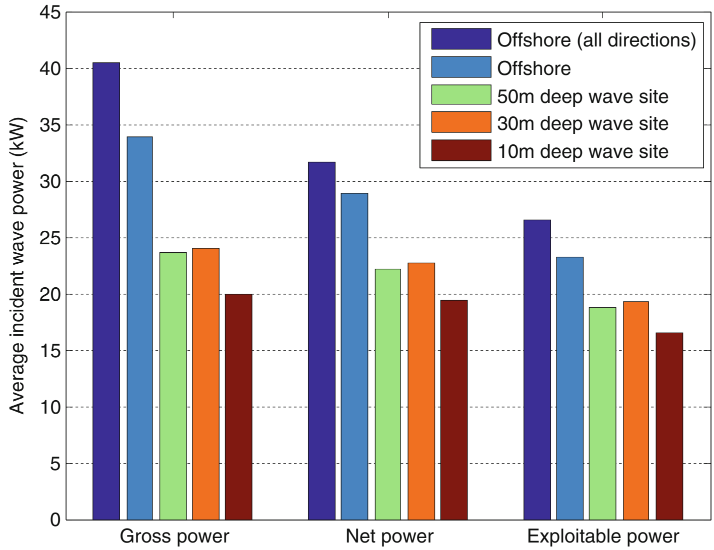


Fig. 3.16 Average incident wave power at the European Marine Energy Centre, Orkney, Scotland [17]

The analysis and discussion above illustrate that the change in average omni-directional wave power from offshore to nearshore means that it is unlikely to be a good representation of the change in average power generation. Essentially, the wave climate in the nearshore cannot be considered as simply a less energetic version of the offshore wave climate (if this were the case then there would be more justification for the use of the average omni-directional wave power as a proxy for average power generation). Alternative representations have been proposed to compensate for this potential distortion. Thus, there is the average directionally-resolved wave power, which is the wave power resolved to a particular direction and also the average exploitable wave power, which is where the directionally-resolved wave power is limited to a fixed multiple (typically four) of the average.

3.4 Measurement of Ocean Waves

3.4.1 Overview

Measurement of the ocean waves is clearly a very important part of understanding the wave energy resource. Although it is possible to estimate the expected waves at a particular location by analysing wave fields and modelling wave propagation, actual field measurements are required to both validate these estimates as well as provide information on the sea-state where an accurate model cannot currently be easily constructed. An example of the latter case would be measuring the impact of a wave energy converter on the down-wave conditions.

Table 3.1 contains a summary of the available systems and their characteristics.

Table 3.1 Wave measurement systems

System	Key characteristics
Surface-following buoy	Relatively expensive, accuracy well established, affected by currents, limited accuracy in steep waves, suitable for long-term deployment
Seabed pressure sensor	Relatively cheap, only suitable for shallow water, deployed in an array, can give directional characteristics, also measures variation in water depth
Acoustic current profiler	Relatively expensive, suitable for water depths up to 50 m, also measures marine currents, recovery required to extract data
Radar (land-based)	Deployed on land away for aggressive environment, typically requires calibration for each site, often limited to wave height measurement
Radar (satellite)	Large geographical coverage with low spatial and temporal resolution, typically limited to wave height measurement

The first three wave measuring instruments described above are deployed at the location of interest and use a time-series analysis to produce the wave spectrum. The duration used to define the wave spectrum depends on a number of competing factors. The first factor is that shorter sampling durations result in a larger uncertainty in the estimated wave spectrum and sea-state parameters. For example, the standard uncertainty in the significant wave height based on a 15-min sample in the North Atlantic is about 15 %, with the standard uncertainty reducing proportionally with the square root of the sample duration [18]. The second factor is that the sea-state is continually changing, with the waves growing or subsiding with the varying wind fields. Consequently, excessively long sample durations would potentially result in extreme sea-states being "smoothed out". The typical 15–30 min sample duration is generally considered to be a reasonable compromise between these two factors. The last of the factors is battery life and deployment time since powering the measuring instrument will limit the product of the sampling direction and number of sea-state records. For example, a wave-measuring buoy may be able to be used for a year deployment with 3 hourly records using 15 min sample durations, or a 2-month deployment reporting every hour using 30 min sample durations. The final choice of sample duration and frequency will depend on the purpose of the deployment, which should be carefully designed to ensure that the required information is obtained.

3.4.2 Surface-Following Buoy

Currently, the most common way of measuring ocean waves is with a **surface-following buoy** that is slackly moored to the seabed as shown in Fig. 3.17. In these buoys, the vertical motion is typically measured using an accelerometer (although GPS systems are now becoming more common), which can then be double integrated to provide a time-series of the water surface elevation [19]. The recorded surface elevation is then used to estimate the wave spectrum and from that the sea-state parameters. Wave measurement buoys may also contain instruments to measure the inclination of the buoy so that the direction of the waves can be inferred and used to produce a directional wave spectrum, with associated directional sea-state parameters. A major benefit of using wave measuring buoys is that they have been used for a long time and thus their accuracy is well established. Their limitations are also relatively well recognised in that strong currents and steep

Fig. 3.17 A surface-following wave measurement buoy

waves both reduce the accuracy of the measurements because the buoy does not exactly follow the water surface. In addition, they are relatively expensive and there is a relatively high risk of loss of the instrument.

3.4.3 Sea-Bed Pressure Sensor

A cheaper alternative to wave measuring buoys are **sea-bed pressure sensors** as shown in Fig. 3.18. These instruments measure the variation in pressure and from that infer the water surface elevation [20]. The attenuation of wave pressure with depth means that they are only suitable for relatively shallow water, with the high-frequency waves being attenuated the most and thus sea-bed pressure sensors are most suitable for measuring swell waves. The exact water depth limit for sea-bed pressure sensors depends on the signal-to-noise ratio of the instrument, but a depth limit of about 10–20 m is typical. In addition to the waves, a sea-bed pressure transducer can also be used to determine the tide level, although in this case care should be taken to ensure that changes to the atmospheric pressure are accounted for. Finally, sea-bed pressure sensors may be deployed in an array to provide information on the directional distribution of the waves.

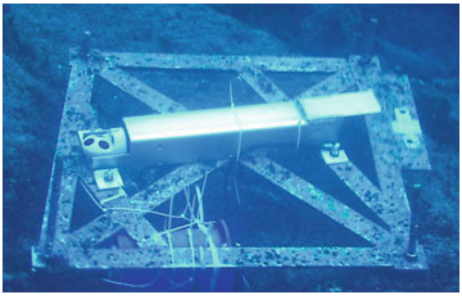


Fig. 3.18 A seabed pressure transducer for wave measurement in shallow water

3.4.4 Acoustic Current Profiler

A more recently developed method of measuring waves is the use of **acoustic current profilers**, and in particular multi-beam acoustic current profilers as shown in Fig. 3.19. Acoustic current profilers measure the water velocity using the red/blue shift in acoustic pulses for the instrument [21]. The water velocities as determined from each beam are combined and processed to produce a time-series of the 3D wave-induced water velocities and from that the directional wave spectrum. Although these instruments can be deployed in any orientation, it is normal for wave measurement to deploy them on the seabed where they are less susceptible to damage and bio-fouling. In this case the instrument must be deployed in less than about 50 m of water since they become less capable of detecting the wave-induced

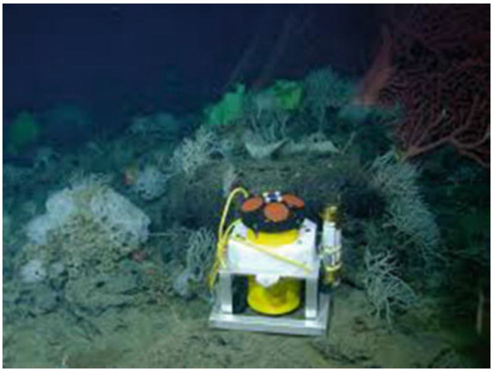


Fig. 3.19 An acoustic current profiler

water velocities as they are further from the water surface. However, acoustic current profilers are developing rapidly, and it is possible that future improvements mean that they will be suitable for deployment in deeper water. An additional advantage of acoustic current profilers is that they also provide information on the local marine currents; however, a disadvantage that they share with sea-bed pressure transducers is that they typically store the data on board. This means that the existence or quality of the wave resource data is not known until the instrument is recovered, which could be a problem if the data is critical for project development.

3.4.5 Land-Based and Satellite Radar

In addition to local wave measurement, it is also possible to use remote sensors, principally **radar**, to measure the waves, for example the 16-element HF radar array set up on the dike at Petten (NL) as shown in Fig. 3.20. The main advantage of using a remote sensor is that the instrument does not need to be deployed in an aggressive environment, that the data is readily available and that the costs are typically lower [22]. The radar provides information on the sea-state by analysing the backscatter from the waves over an area. Coupled analysis from multiple locations means that the directional wave spectrum can often be calculated, together with sea-state parameters. Typically the radar system is calibrated using data from a

Fig. 3.20 A land-based radar system for measurement of waves

local wave measuring instrument [23]. The use of radar for wave measurement is a rapidly developing area, with a range of different systems being developed and continually improved.

A radar deployment of particular note is **in satellites**. Radar altimeters are available in a number of satellites that are circling the earth and can provide estimates of the Significant Wave Height along the track of the satellite [24]. Although the temporal resolution of the wave resource data may be very low (a satellite may only pass over a point every 10–35 days) it does provide wave resource data over a large geographical area. Although current satellite-based radar altimeter systems typically only provide information on the Significant Wave Height, as with other radar systems, the potential for satellite measurement of waves is increasing rapidly. Thus, current developments exist to use satellite data to produce estimates of the wave period as well as potentially other sea-state parameters.

Interestingly, the satellite data can be used to compare the output from local wave measuring instruments and the consistency of these assessed. Whilst the Canadian and UK wave buoy networks share a common calibration factor, the US buoy network has a slightly different calibration factor. This implies that there is some difference in the calibration, deployment and/or operation of the wave buoys between the US and Canada/UK that results in a difference in the estimated sea-state. This illustrates the challenges in wave measurement and that there are sources of uncertainty and error in all measurement systems. Consequently, there is no definitive “gold standard” for wave measurement, which is important to bear in mind for the calibration of wave propagation models that are discussed in the next section.

3.5 Modelling of Ocean Waves

3.5.1 Introduction

There are two main reasons for modelling ocean waves. The first reason is that until it is known where a wave energy converter may be deployed, it is unlikely that data from a wave measuring instrument for the desired location will exist. In this case a model is required to propagate the waves from where the wave resource data is known to the points of interest. The second reason is that knowledge of the average wave climate requires many years of data and it is not typically possible to deploy a wave measuring instrument for the time required to produce the required information, so a wave model is used to generate this long-term data.

Although other wave propagation models exist, such as Boussinesq [25] and mild-slope models [26], it is generally accepted that third generation spectral wave models are most suitable for modelling the propagation of waves from long-term wave measurement points or validated deep-water models to the potential locations of wave energy converters [27]. Examples of third generation spectral wave models include SWAN [28], TOMAWAC [29] and Mike21 SW [30], of which the first two are open source. Third generation spectral wave models solve the action balance equation shown in Eq. 3.23

$$\frac{\partial N}{\partial t} + \nabla_x \cdot \left[C_g + U\right]N + \frac{\partial C_\sigma N}{\partial \sigma} + \frac{\partial C_\theta N}{\partial \theta} = \frac{S_{tot}}{\sigma} \tag{3.23}$$

The action balance equation is essentially a conservation equation that states that the total action is conserved (left-hand side of the equation) except when there is an input of wave action (right-hand side of the equation) from sources/sinks of wave action. The wave action, which is the wave energy divided by the intrinsic wave frequency, is used because it is conserved in the presence of background currents whilst the wave energy is not. However, in the absence of background currents, the wave action balance equation reduces to the wave energy balance equation. The wave action is allowed to propagate between four dimensions, geographic space, frequency space, directional space and time space (dependent on the physical processes involved), but the change in the sum of these must equal the source/sink of wave action, which may generate, dissipate and/or re-distribute the wave action.

3.5.2 General Spectral Wave Models

In the absence of sources/sinks of wave action, then Eq. 3.23 simply defines the wave kinematics and includes the processes of both shoaling and refraction.

- Depth-induced shoaling is the change in wave height with water depth and, in the absence of marine currents, it occurs due to the change in group velocity with water depth. When a wave enters shallow water, the group velocity changes, initially increasing slightly and then decreasing as the water depth decreases. The consequence of this is that the wave height initially decreases and then increases due to the conservation of energy as the energy gets stretched out and subsequently bunched up (note, it is commonly assumed that wave height only increases in shallow water since the initial decrease in wave height is only small and often not considered; however, it does occur). Depth-induced shoaling normally refers to the case when the wave is propagating orthogonally to the depth contours and so the wave does not change direction. Assuming a time-invariant condition, depth-induced shoaling is all included in the second term of the wave energy balance equation.
- When the wave is not travelling orthogonally to the depth contours, then depth-induced **refraction** occurs. Depth-induced refraction is also related to the change in wave group velocity with water depth. Indeed, it is possible to consider depth-induced shoaling as simply a special case of depth-induced refraction where the direction of wave propagation is parallel to the water depth gradient. If this is not the case, then the direction of wave propagation changes as the group velocity changes. Thus, when the wave group velocity starts to

reduce in shallow water, the wave tends to rotate so that the wave crests become more parallel to the depth contours. The rate of change on direction is defined by Snell's law, which relates the change in direction of propagation with the change in wave group velocity. The consequence is that when the waves reach the shore, they are nearly parallel to the shoreline (they do not get to be completely parallel to the shoreline because non-linear processes cause them to break before they get to that point). Assuming a time-invariant condition, this process is modelled with the second and fourth terms of the wave energy balance equation.

- Where there are marine currents, then **current-induced shoaling** occurs when there is a change in the background current. The kinematics are more complex in this case because the change in current also causes a change in the relative (or intrinsic) frequency, which is the frequency of the waves as observed from a reference frame travelling at the same velocity as the current. In addition, there is an exchange of energy between the current and the waves, which is the reason Eq. 3.23 is not defined in terms of wave energy, but in terms of wave action, which is conserved in the presence of changing currents. The exact current-induced shoaling can be determined by solving Eq. 3.23 which shows that a following current tends to reduce the wave height as the energy is spread out, whilst an opposing current causes the wave height to increase as the energy bunches. If the opposing current is sufficiently strong, this can stop the waves propagating, and the energy increases until the waves start to break and thus lose energy. This can be seen commonly at the mouth of rivers where there is area of breaking waves as the current stops the waves from travelling upstream. For a time-invariant system, this is modelled with the second and third terms of the wave energy balance equation.
- As would be expected, marine currents can also be responsible for **current-induced refraction**. The explanation for this is similar to that of the depth-induced refraction, except that the change in velocity of the wave crest is due to spatial variation of the marine currents, rather than the change in water depth. In this final case, a time-invariant system the second, third and fourth terms of the wave energy balance equation are required to solve the wave kinematics.
- A major limitation of spectral wave models is that they assume a random **phase between the wave components**, although this is also the assumption that allows them to be computationally efficient for modelling wave transformations over large distances. Consequently, phase-dependent processes such as harbour resonance and diffraction cannot be modelled explicitly. However, whilst no approximation for processes such as harbour resonance exist for spectral wave models, it has been found that diffraction can be modelled using a phase-decoupled refraction-diffraction approximation [31]. This approximation is based on the mild-slope equation where the turning rate of the wave due to diffraction is dependent on the slope of the wave amplitude, and has been found to be a reasonable approximation in the majority of circumstances except locally close to the body causing the diffraction field.

3.5.3 Third Generation Spectral Wave Models

In third generation spectral wave models (for example SWAN, Mike21SW, TOMAWAC), there are typically six types of **wave energy source/sink**. These are wind input, quadruplet wave-wave interactions, white-capping, bottom friction, triad wave-wave interactions and depth-induced wave breaking, cf. Fig. 3.21.

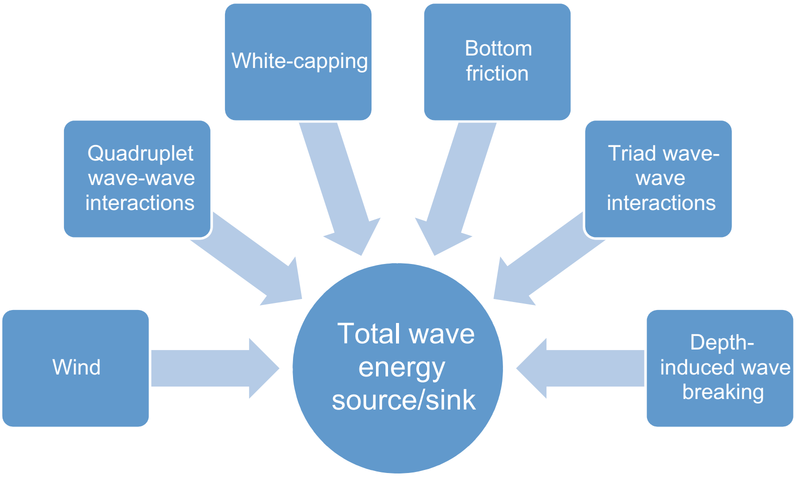


Fig. 3.21 Source terms used in a third-generation spectral wave model

These sources/sinks are, in general, represented by semi-empirical approximations. In many cases, a number of alternative representations can be used in the wave models; the choice being somewhat dependent on the preferences of the modeller. A brief description of each of these source/sinks of wave energy is provided; however, further details may be found in the literature [27].

- The wind input source term represents the energy that is transferred from the **wind into the waves**. Energy is primarily transferred through the propagation of pressure fluctuations that travel in the same direction as the waves. In the initial generation of waves, these pressure fluctuations cause small ripples on the surface of the water. Subsequently, the waves influence the air-flow so that there is a positive pressure on the windward side of the wave crest and a negative pressure on the leeward side. Thus, the net force on the wave and the wave velocity are in-phase. There is a transfer of energy from the wind to the wave and so the waves grow. Moreover, this net force increases with the size of the wave so that there is a positive feedback mechanism in the growth of the waves. However, whilst the general processes of wind to wave energy transfer may be understood, estimates of the wind input source term are based on a semi-empirical representation of the processes discussed above.

- The energy input from the wind to the waves occurs towards the high frequency side of the spectral peak, with the energy subsequently transferred to lower frequency waves (together with a small proportion of the energy being transferred to higher frequency waves) via **non-linear quadruplet wave-wave interactions**. The quadruplet wave-wave interactions are associated with multiple resonant couplings between sets of four wave components that cause energy transfer via non-linear interactions. Thus, quadruplet non-linear wave-wave interactions are responsible for the increase in the average wave period of the waves so that as the wind blows both the wave height and period increase. However, quadruplet wave-wave interactions also tend to stabilise the high frequency components of the spectrum so that in the absence of wave breaking, a fourth-order frequency tail (f^{-4}) is produced. Unfortunately, calculation of the quadruplet wave-wave interactions is computationally very expensive and so it is usual to use an approximation to calculate the strength of the quadruplet wave-wave interactions source term. Finally, it is worth noting that the explicit calculation of the quadruplet wave-wave interactions is the key distinguishing feature of third-generation spectral wave models compared to first and second order spectral wave models that either assume or parameterise the spectral shape.
- The final process that controls the spectral shape in deep water is **white-capping**. White-capping is wave breaking that occurs in deep water and may be expected to be associated with the steepness of the waves. That is, when the steepness of a wave becomes too large, the top of the wave becomes unstable and the wave breaks resulting in white-capping. However, as the wave steepens, the hydrodynamics become highly non-linear, and a complete theoretical understanding of white-capping has not yet been developed. Notwithstanding this lack of understanding, the strength of the white-capping source term is typically based on a model where the weight of the white-cap extracts energy from the waves in the opposite sense of the wind input source term, i.e. the net force on the wave due to the white-cap extracts energy from the waves. The effect of white-capping tends to be strongest on the high frequency wave components as these are typically steeper and result in a fifth-order frequency tail (f^{-5}) as seen in the JONSWAP and many other standard spectra.
- As the waves enter shallow water, they will begin to be affected by the seabed, with the most obvious (although not necessarily most significant) source term being **bottom friction**. The bottom friction source term represents the energy transfer from the waves to turbulence induced by shear stress from fluid flow over the bottom. A quadratic relationship between shear stress and fluid flow is typically assumed, as in Morison's Equation, so that the strength of the bottom friction source term is proportional to the square of the wave-induced velocity at the seabed. The coefficient of proportionality, or bottom-friction coefficient, is dependent on the characteristics of the seabed, with rocky, sandy and vegetated seabeds all having different bottom-friction coefficients.
- Another process that becomes more significant as the waves enter shallow water is the **triad wave-wave interaction**. In a similar way to quadruplet wave-wave interactions, triad wave-wave interactions are associated with multiple resonant

couplings between wave components, but in this case they are between sets of three wave components that cause energy transfer via non-linear interactions. When the waves are dispersive these interactions cannot be created, which is why they only occur in very shallow water. The effect of triad wave-wave interactions is to generate a second peak in the wave spectrum at twice the frequency of the original spectrum, which is bound to the main frequency peak in the sense that it travels with the same phase velocity. Unfortunately, currently third generation wave models are unable to correctly model these bound waves, and the triad wave-wave interaction source term is estimated based on the wave spectrum and water depth; however, these source terms have been found to be reasonable approximations in most cases.

- The final source term typically included in wave models is the **depth-induced wave breaking** (surf-breaking) source term. This source term typically assumes that a fixed proportion of the energy in any wave is lost when it breaks. Thus it is necessary to make an estimate on the proportion of waves that break at any particular water depth for any particular wave spectrum. This may be done by making some assumptions about the distribution of wave heights and the relative water depth in which these waves will break. Perhaps surprisingly, laboratory observations suggest that the spectral shape is not affected by wave breaking and so the energy removed due to wave breaking, and thus the strength of the depth-induced wave breaking source term, is typically assumed to be proportional to the wave energy spectrum, with the coefficient of proportionality dependent on the proportion of breaking waves.

3.5.4 Grid Definition

With the definition of the conservation equation for the action density, together with the source terms that add and remove energy, it is only necessary to propagate the action density from one point to another in order to define the wave resource. This may be done using a regular or irregular grid, using a number of different propagation schemes, each of which have their own particular set of advantages and disadvantages. The resolution of the grid will depend on the desired accuracy of the model, with higher resolution grids typically resulting in more accurate models, but at the expense of greater computational effort. However, the change in accuracy with grid resolution also depends on the relative magnitude of the components in the action density equation, so that where the action density changes little a low resolution grid may be used, but higher resolution grids are required where the changes in action density are greater. This naturally leads to the use of irregular (or nested regular) grids as shown in Fig. 3.22, which can have a higher resolution where required (although the areas requiring higher resolution grids and the required resolution may not be immediate obvious).

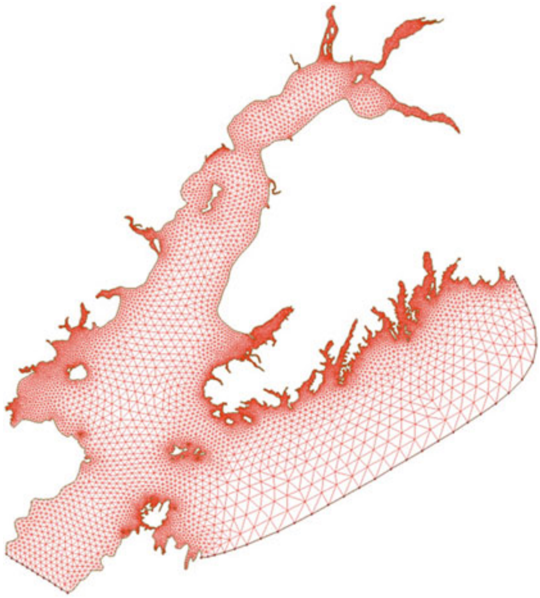


Fig. 3.22 An example grid for a spectral wave model

Combining the uncertainty in the grid design with the uncertainties in the model boundary data (e.g. the waves on the model boundary, the bathymetry, the variation in the seabed bottom friction coefficient, the choice of source term models), it can be seen why setting up a spectral wave model has been likened to a “black act”. Unfortunately, there is no simple procedure that can be followed to guarantee the accuracy of the numerical wave model. A good match between the model prediction and measured data may provide some confidence in the model. However, this is strictly limited to the particular sea-state and the model may be less accurate when applied to different wind and wave conditions. Thus, it is generally recommended that any numerical model should be validated using a wide range of different conditions that include what may be expected over the whole year. Unfortunately, this data is not always available, and the wave resource model may be validated against only a limited data-set or even no data-set at all. In general, and in these cases in particular, it is clearly necessary to be aware that the wave resource is not fully validated and the WEC performance based on this data should be treated as such.

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