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DSC and universal bit-level combining for HARQ systems

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Abstract

This paper proposes a Dempster -Shafer theory based combining scheme for single-input single-output (SISO) systems with hybrid automatic retransmission request (HARQ), referred to as DSC, in which two methods for soft information calculations are developed for equiprobable (EP) and non-equiprobable (NEP) sources, respectively. One is based on the distance from the received signal to the decision candidate set consisting of adjacent constellation points when the source bits are equiprobable, and the corresponding DSC is regarded as DSC-D. The other is based on the posterior probability of the transmitted signals when the priori probability for the NEP source bits is available, and the corresponding DSC is regarded as DSC-APP. For the diverse EP and NEP source cases, both DSCD and DSC-APP are superior to maximal ratio combining, the so-called optimal combining scheme for SISO systems. Moreover, the robustness of the proposed DSC is illustrated by the simulations performed in Rayleigh channel and AWGN channel, respectively. The results show that the proposed DSC is insensitive to and especially applicable to the fading channels. In addition, a DS detection-aided bit-level DS combining scheme is proposed for multiple-input multiple-output–HARQ systems. The bit-level DS combining is deduced to be a universal scheme, and the traditional log-likelihood-ratio combining is a special case when the likelihood probability is used as bit-level soft information.

Keywords: Basic probability assignment (BPA), Bit-level combining, Dempster -Shafer (D -S) evidence theory, Hybrid automatic retransmission request (HARQ), Multiple-input multiple-output (MIMO), Maximum-ratio combining (MRC)

I Introduction

A concern in packet data communication systems is how to control the transmission errors caused by the channel noise and interferences so that packets can be transmitted reliably. Automatic retransmission request (ARQ), as a fundamental approach, is intended to ensure an extremely low packet error rate. The efficiency of the system can be improved if the ARQ is combined with a forward-error-correcting (FEC) code, referred to as HARQ, which includes Chase combining [1] and incremental redundancy (IR) [2]. There are many HARQ strategies: including separating the HARQ process into HARQ sub-processes that operate over an isolated pairing of a transmitter and receiver antenna [3]; the constellation rearrangement technique [4] and the bit rearrangement scheme [5] that can provide a kind of diversity for performance improvement. In [4,5], authors developed effective HARQ strategies at the transmitter in order to improve the system reliability. Contrariwise, both [6,7] discussed combining algorithms at the receiver.

Three linear combining schemes [8], selection combining (SC), equal-gain combining (EGC), and maximal ratio combining (MRC), entail various trade-offs between performance and complexity, and comparatively MRC is deemed to be superior to the others by outputting the maximum signal-to-noise (SNR) ratio in SISO systems. Jang et al. [6] proposed an optimal combining scheme for MIMO systems with HARQ, which can be used in both symbol-level and bit-level. However, the complexity imposed in [6] increases exponentially with the number of both bits per symbol and transmit antennas [7] proposed an improved LLR combining scheme, invoking a new LLR calculation method. The traditional combining schemes are developed on the basis of Bayesian theory. This paper concentrates on the DST-based combining scheme [9,10]. DST has attracted

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much attention in many fields owing to its counteracting uncertainty merit, such as artificial intelligence research [11,12], data fusion [13,14], and has been shown to achieve a satisfactory performance. Xia et al. [9] first proposed a DS detection-aided bit-level DS combining scheme for MIMO -HARQ systems. Afterward, a symbol-level DS combining is proposed in [15]. Xia and Lv [10] aims to analyze the merits of the combining based on DST, in which DSC is justified to outperform the Bayesian theory based MRC for SISO systems.

The distance-based method for soft information calculations in the DSC scheme (termed as DSC-D) is proposed for equiprobable source bits [10], which are usually used in realistic applications. In the traditional MRC scheme, decisions-making is based on the maximal-likelihood (ML) rule (termed as MRC-ML) or the maximum-a-posterior-probability (MAP) rule (termed as MRC-MAP), both of which are equivalent for the equiprobable source [10] presents that the proposed DSC-D outperforms the traditional MRC-ML as well as MRC-MAP, and the performance gap becomes bigger with increasing SNR. This paper focuses on the performance of the proposed DSC scheme when the source bits are non-equiprobable for research integrity. For non-equiprobable source, this paper presents the system performance comparison between the DSC-D and MRC-ML as well as MRC-MAP, and shows that the DSC-D is inferior to the MRC-MAP because the distance-based soft information calculations do nothing with the priori probability of the transmitted signal, by which a new method for soft information calculations on the basis of the posterior probability of the transmitted signal (termed as DSC-APP) is inspired. DSC-APP is demonstrated to be superior to the other combining counterparts performed in Rayleigh fading channel, and both DSC-D and DSC-APP are insensitive to the channel state. However, the performance of MRC in AWGN channel is much degraded if it is employed in Rayleigh channel. Such conclusions validate the robustness of the proposed DSC. In addition, inspired by the DS detection-aided DS combining in [9], a universal bit-level combining scheme is proposed. It is deduced that the traditional LLR combining is a special case of the proposed universal bit-level DS combining scheme if the likelihood probability is used as the bit-level soft information.

The rest of this paper is organized as follows: The HARQ system model is introduced in Section II, followed by the proposed DSC combining scheme in Section III. The universal bit-level DS combining is proposed in Section IV. Simulations and comparisons are provided in Section V. At the end of this paper, conclusions are given in Section VI.

Notation: Transpose and Hermitian transpose of a vector or matrix are denoted by $(\cdot)^T$ and $(\cdot)^H$, respectively. Additionally, vectors and matrices are denoted by the bold lowercase and uppercase letters, respectively.

II System model

Figure 1 depicts the system model of interest, a packet-oriented ARQ system. This paper focuses on the BER performance after packets combined by diverse combining schemes, and the functional FEC code is therefore omitted from the system model for simplicity. As illustrated in Figure 1, original information bits are encoded by CRC, then modulated into transmissive signals suitable for the noisy and/or fading channel. If the receiver decodes the packet correctly, the recovered bits are output and an acknowledgment (ACK) signal is fed back to the transmitter. Otherwise, a negative acknowledgement (NACK) signal is fed back and the receiver simultaneously requests retransmission of the same packet.

In the tth (re)transmission with $t = 1, 2, \ldots, \bar{T}$, for MIMO systems with N_t transmit antennas and Nr receive antennas, the receiver obtains

$$\mathbf{y}^{(t)} = \mathbf{H}^{(t)}\mathbf{x}^{(t)} + \mathbf{n}^{(t)}, \quad t = 1, 2, \ldots, \bar{T}, \tag{1}$$

where $\mathbf{H}^{(t)} \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix in the tth (re)transmission with the entry $h_{ij}^{(t)}$ denoting the channel gain between the jth transmit antenna and the ith receive antenna. For additive white Gaussian noise (AWGN) channel, $h_{ij}^{(t)} = 1$, $i = 1,\ldots, N_r$; $j = 1,\ldots, N_t$; $t = 1, \ldots, \bar{T}$, but for Rayleigh fading channel, each entry of $\mathbf{H}^{(t)}$ is modeled as an independent complex Gaussian random variable with zero mean and unit variance. $\mathbf{x}^{(t)}$ denotes an N_t length modulated transmit symbols vector in the tth (re)transmission, whose elements are taken from the complex constellation set $U = \{s_1, s_2,\ldots, s_M\}$ with cardinality M. The component of U is obtained by invoking the mapping function $s_\alpha = \text{map}\,(s_{\alpha 1}s_{\alpha 2} \ldots s_{\alpha c})$ (e.g. Gray mapping), where $s_{\alpha k}(k = 1, 2,\ldots, c;\ c = \log_2 M$) represents the binary information bit. $\Pr(s_{\alpha k} = 0)$ and $\Pr(s_{\alpha k} = 1)$ denote the priori probability for the binary information bit 0 and 1, respectively, satisfying $\Pr(s_{\alpha k} = 0) + \Pr(s_{\alpha k} = 1) = 1$, with $\Pr(s_{\alpha k} = 0) = \Pr(s_{\alpha k} = 1) = 0.5$ indicating the equiprobable source bits. $\mathbf{n}^{(t)} \in \mathbb{C}^{N_r \times 1}$ is an independent and identical distributed (i.i.d.) Gaussian stationary noise vector with zero mean and variance matrix $\sigma^2\mathbf{I}$, where $\mathbf{I}$ is a $(N_r \times N_r)$-dimensional identity matrix. For SISO systems with $N_t = N_r = 1$, (1) is simplified as

$$y^{(t)} = h^{(t)}x^{(t)} + n^{(t)}, \quad t = 1, 2, \ldots, \bar{T}, \tag{2}$$

where $y^{(t)}$, $x^{(t)}$, $n^{(t)}$, and $h^{(t)}$ can be regarded as element of received signals vector $\mathbf{y}^{(t)}$, transmitted signals vector

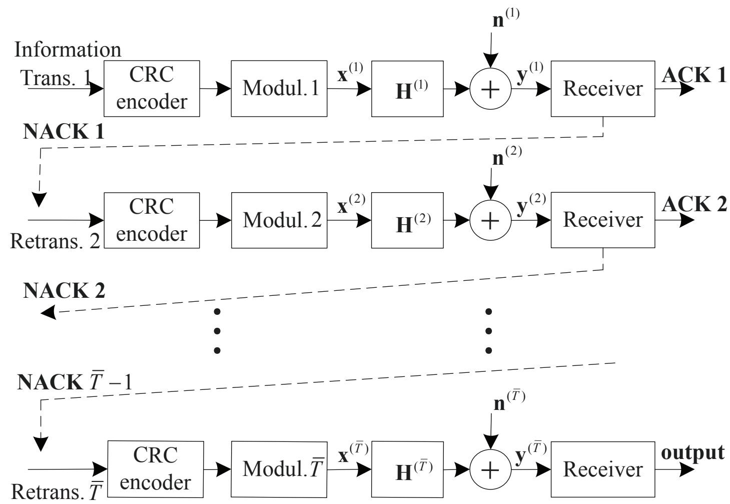


Figure 1 ARQ system block diagram with maximum $\bar{T}$ transmissions of the same data packet

$\mathbf{x}^{(t)}$, noise vector $\mathbf{n}^{(t)}$ and entry of channel matrix $\mathbf{H}^{(t)}$ in MIMO systems, respectively.

Although the information packets are identical in bit-level during all the transmissions, symbols $\mathbf{x}^{(t)}$ transmitted in a specific (re)transmission may be different, because different modulation schemes may be employed in each transmission, as presented in Figure 1.

III DST-based combining scheme and MRC

The traditional combining schemes, MRC as well as SC and EG, are based on Bayesian theory. DST as a generalization of the Bayesian theory has unique merits in uncertainty processing, based on which a novel combining scheme called DSC is proposed in [10].

A DSC

DSC refers to the modulation constellation set U as the frame of discernment with mutually exclusive and exhaustive hypotheses. Focal element set (FES) S_m is a subset $S_m \subset U$, in which the number of elements is denoted by m, e.g. $S_1 = \{s_\alpha\}$ or $S_2 = \{s_\alpha, s_\beta\}$ or $S_3 = \{s_\alpha, s_\beta, s_\gamma\}$..., where $\alpha \neq \beta \neq \gamma$ and $\alpha, \beta, \gamma = 1, 2, ..., M$. Set S_m reflects the uncertainty of decision judgements. For example, $S_2 = \{s_\alpha, s_\beta\}$ contains more uncertainty than $S_1 = \{s_\alpha\}$, which implies that the transmitted symbol may be s_α or s_β, but there is no convincing evidence for deciding which one must be the transmitted symbol. In wireless communication systems, the transmitted signals suffer from multipath fading channels and interferences, and the received signals thus contain much uncertainty. Therefore, it is reasonable to use FES S_m to characterize the uncertain decisions. In the proposed DSC scheme [10], the uncertain decision propositions S_m consist of the adjacent constellation points, since it is usually difficult to ensure which one is the transmitted symbol between the adjacent constellation points.

Basic probability assignment (BPA) denoted by Mas (S_m) characterizes the confidence reposed in the transmitted signal being contained in set S^m. Two methods for BPA calculations are proposed for equiprobable and non-equiprobable sources, respectively. One is based on the distance from the received signal to the decision candidate set, i.e. the nearer-distance-more-confidence rule, and the other is based on the posterior probability of the transmitted signals, both of which are introduced in detail as follows:

(1) *distance-based BPA calculations*: The nearer-distance-more-confidence principle is used for BPA calculations in [10], which is based on the distance between the received signal and the decision candidate set consisting of adjacent constellation points with the assumption that the source bits are equiprobable. The corresponding $\mathrm{Mas_D}(S_m\,|y^{(t)})$ function is expressed as

$$\mathrm{Mas_D}\left(S_m\left|y^{(t)}\right.\right)=\frac{R_{\mathrm{D}}^{(t)}-\left|y^{(t)}-h^{(t)}\cdot\frac{\sum_{s_\alpha\in S_m}s_\alpha}{m}\right|^2}{\left(\sum_{m=1}^{P}N(S_m)-1\right)R_{\mathrm{D}}^{(t)}},\quad m=1,2,\ldots,P;\ \ t=1,2,\ldots,\bar{T},\tag{3}$$

where $N(S_m)$ denotes the total number of the set S_m containing m adjacent constellation points, P is a key issue concerned with the trade-off between performance and complexity, and

$$R_{\mathrm{D}}^{(t)}=\sum_{m=1}^{P}\sum_{S_m}\left|y^{(t)}-h^{(t)}\cdot\frac{\sum_{s_\alpha\in S_m}s_\alpha}{m}\right|^2$$

is a normalization coefficient, satisfying

$$\sum_{m=1}^{P}\sum_{S_m}\mathrm{Mas_D}\left(S_m\left|y^{(t)}\right.\right)=1,\quad m=1,2,\ldots,M,\ \ 1\leq P\leq M.\tag{4}$$

From (3) it is obvious that the nearer it is from the received signal $y^{(t)}$ to the decision candidate set S_m, the more confidence (larger MasD Sm $\_y(t)$) is placed in the set.

(2) *a posterior probability-based BPA calculations*: When the source bits are non-equiprobable (NEP), of which the priori probability is available to the receiver, ML become suboptimal and MAP is the optimal method. In view of this, BPA calculations can thus be performed based on the posterior probability of the transmitted signals as

$$\text{Mas}_{\text{APP}}\left(S_m \left| y^{(t)}\right.\right) = \frac{\left(\prod_{s_\alpha \in S_m} \Pr(s_\alpha)\right)^{-N(S_m)} f\left(y^{(t)} \mid S_m\right)}{R_{\text{APP}}^{(t)}}, \quad (5)$$

where $f\, y^{(t)}|S_m$ is likelihood function,

$$f\left(y^{(t)} \middle| S_m\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\left|y^{(t)} - h^{(t)} \cdot \frac{\sum_{s_\alpha \in S_m} s_\alpha}{m}\right|^2}{2\sigma^2}\right) \quad (6)$$

with σ^2 denoting the AWGN noise power. The normalization coefficient $R_{\text{APP}}^{(t)}$ is expressed as

$$R_{\text{APP}}^{(t)} = \sum_{m=1}^{P} \sum_{S_m} \left(\prod_{s_\alpha \in S_m} \Pr(s_\alpha)\right)^{-N(S_m)} f\left(y^{(t)} \middle| S_m\right),$$

whereby the summation of 5 is unity as like 4.

If not specially pointed, the Mas (·) function has two expressions MasD (·) and MasAPP (·) as the above mentioned, both of which denote the soft information BPA but obtained by diverse calculation methods. For simplicity, only Mas (·) is used in the following context.

In addition, DST contains two new measure of “belief” or “credibility” that are foreign to Bayesian theory. These are the notions of support and plausibility [16], respectively. The support for the transmitted signal being in the set S_m is defined as the total BPA of all subsets implying the S_m set. Thus,

$$\text{Spt}\left(S_m \left| y^{(t)}\right.\right) = \sum_{S_{m'} \subseteq S_m} \text{Mas}\left(S_{m'} \left| y^{(t)}\right.\right). \quad (7)$$

The support is a kind of loose lower limit to the uncertainty. On the other hand, a loose upper limit to the uncertainty is the plausibility. This is defined, for the S_m set, as the total BPA of all subsets that do not contradict the S_m set. In other words,

$$\text{Pls}\left(S_m \left| y^{(t)}\right.\right) = \sum_{S_{m'} \cap S_m \neq \phi} \text{Mas}\left(S_{m'} \left| y^{(t)}\right.\right). \quad (8)$$

As a result, it can be inferred that the belief of the transmitted signal contained in set S_m lies in the interval [Spt S_m $|y(t)$, Pls S_m $|y(t)]$, which represents the uncertain propositions. The smaller the interval is, the clearer the evidence is to support the corresponding propositions. The more detailed explanations about the support and the plausibility functions refer to Shafer’s original work on DST in [17].

As the approach above mentioned, the similar belief interval as [Spt $S_m|y^{(t)}$), Pls$(S_m|(y^{(t)})]$ can be achieved for each $y^{(t)}, t = 1, 2, \ldots, \bar{T}$. The interval is gradually reduced along with making more use of the received signals as follows

$$\begin{aligned} \text{Spt}(S_m) &= \sup_{1 \le t \le \bar{T}} \left\{\text{Spt}\left(S_m \left| y^{(t)}\right.\right)\right\}, \\ \text{Pls}(S_m) &= \inf_{1 \le t \le \bar{T}} \left\{\text{Pls}\left(S_m \left| y^{(t)}\right.\right)\right\}, \\ & m = 1, 2, \ldots, P. \end{aligned} \quad (9)$$

At this time, Spt(S_m) and Pls(S_m) are two measures of the aggregate belief in the transmitted signal being contained in set S_m, which are achieved after combining multiple information sources by (9). These two measures of the aggregate belief need to be further merged before decision-making, since it is beneficial to make more reliable decisions by taking full advantage of them. The proposed DSC merges Spt(S_m) and Pls(S_m) in terms of the Dempster’s rule [18], which is a generalization of Bayes’ rule and is justified under many situations. The aggregation can be expressed as

$$\text{Spt\_Pls}(S_m) = \frac{\text{Spt}(S_m)\text{Pls}(S_m)}{1 - \text{Spt}(S_m)(1 - \text{Pls}(S_m))}, \quad m = 1, 2, \ldots, P, \quad (10)$$

where Spt_Pls(S_m) is regarded as the reliable belief in the transmitted signal that is included in set S_m and is applied to assist in making decisions. However, S_m is still a set containing m adjacent constellation points with m = 1, 2,..., P. The ultimate goal of the proposed scheme is to correctly judge which point of the constellation is the transmitted signal, thus the decision statistics are defined as

$$\text{De}(s_\alpha) = \sum_{s_\alpha \in S_m} \frac{\text{Spt\_Pls}(S_m)}{m}, \alpha = 1, 2, \ldots, M, \quad (11)$$

where the summation is carried out among all the sets (S_m) that contains the constellation s_α. Finally, the resulting decision is written as $\hat{s} = \arg\max_{s_\alpha \in U} \text{De}(s_\alpha)$.

B MRC

MRC receiver is deemed as the optimal since it results in a maximum likelihood receiver [8] when the source bits are equiprobable. If the same signal is transmitted $\bar{T}$ times, the corresponding channel fading coefficients, received signals and noise variables are concatenated as

$\tilde{\mathbf{n}} = [n^{(1)}\ n^{(2)} \cdots n^{(\bar{T})}]^T$, $\tilde{\mathbf{y}} = [y^{(1)}\ y^{(2)} \cdots y^{(\bar{T})}]^T$, $\tilde{\mathbf{n}} = [n^{(1)}\ n^{(2)} \cdots n^{(\bar{T})}]^T$, respectively. $\bar{T}$ transmissions for the signal x can thus be written in matrix expression as $\tilde{\mathbf{y}} = x\tilde{\mathbf{H}} + \tilde{\mathbf{n}}$, to which model the MRC scheme is applied, and the resulting decision statistics can be expressed as

$$x = \frac{\tilde{\mathbf{H}}^H \tilde{\mathbf{y}}}{||\tilde{\mathbf{H}}||^2} = x + \frac{1}{||\tilde{\mathbf{H}}||^2}\tilde{\mathbf{H}}^H \tilde{\mathbf{n}}, \tag{12}$$

where $\hat{x}$ is a Gaussian random variable with x mean and σ^2 variance.

If the source bits are equiprobable, the ML rule is equivalent to the minimum distance rule. The decision result of MRC is accordingly written as

$$\hat{s} = \arg\max_{s_\alpha \in U} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\hat{x} - s_\alpha)^2}{2\sigma^2}\right) = \arg\min_{s_\alpha \in U} (\hat{x} - s_\alpha)^2. \tag{13}$$

Otherwise, if the source bits are non-equiprobable and the priori probability of the source signals $\Pr(s_\alpha)$, $\alpha = 1, 2,..., M$, is available to the receiver, the decision result according to the maximum posterior probability rule can thus be achieved as

$$\hat{s} = \arg\max_{s_\alpha \in U} \frac{\Pr(s_\alpha)}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\hat{x} - s_\alpha)^2}{2\sigma^2}\right). \tag{14}$$

IV A universal bit-level combining scheme

The authors previously proposed a novel DS detection-aided bit-level DS combining scheme, called a soft-decision-soft-combining algorithm, for MIMO - HARQ systems in [9]. The scheme includes two important stages: DS detection and DS combining. In the DS detection stage, the symbol-level BPAs are assigned by the probability-density-function (PDF), i.e. the likelihood function (6), which is equivalent to the above-mentioned nearer-distance-more-confidence rule for BPA calculations. Soft information sources (symbol-level BPAs) from all receive antennas are aggregated by the Dempster's combination rule, and the uncertainty is counteracted during the combination process. In the DS combining stage, the bit-level BPAs are calculated according to the reliable aggregations that are induced from the DS detection stage. After receiving all (re)transmissions of the same packet, bit-level BPAs are combined, during which procedure the uncertainty is further counteracted. Such a soft-decision-soft-combining scheme improves system performance by characterizing and counteracting uncertainty, and the performance simulations demonstrate that the proposed DS detection-aided DS combining outperforms its conventional minimum-mean-square-error (MMSE) detection-aided LLR combining counterpart.

The ultimate aggregate symbol-level BPAs outputted by the DS detection algorithm are transformed to the plausibility values Pls (s_1), Pls (s_2),..., Pls (s_M), where s_α, $\alpha = 1, 2,..., M$, is a single constellation point, and the detailed algorithm flow refers to [9]. In the DS combining procedure, different from what happens in the DS detection, the frame of discernment $\Theta = \{1, 0\}$ is supposed, so there are only two choices for FES S, which is a set containing only one element, i.e. $S = \{1\}$ or $S = \{0\}$. The bit-level BPA $\text{Mas}_{\alpha k}^{(t)}(S)$ $(k = 1, 2, \ldots, c)$, satisfying $\text{Mas}_{\alpha k}^{(t)}(S) > 0$, represents the soft information for each bit of the transmited signal x_j, $j = 1, 2,..., N_t$ that is calculated by virtue of the plausibility values Pls (s_1), Pls (s_2),..., Pls (s_M) with most credibility obtained from the DS detection stage. Specifically, the bit-level BPA $\text{Mas}_{jk}^{(t)}(S)$ for the kth bit of xj is given by

$$\text{Mas}_{jk}^{(t)}(S) = \begin{cases} R_{jk}^{(t)} \cdot \sum\limits_{\substack{\forall s_\alpha \in U, \\ s_{\alpha k} = 1}} \text{Pls}_j^{(t)}(s_\alpha), & S = \{1\}, \\ R_{jk}^{(t)} \cdot \sum\limits_{\substack{\forall s_\alpha \in U, \\ s_{\alpha k} = 0}} \text{Pls}_j^{(t)}(s_\alpha), & S = \{0\}, \end{cases} \tag{15}$$

where the normalization coefficient $R_{jk}^{(t)}$ is represented as

$$R_{jk}^{(t)} = \frac{1}{\sum_{\substack{\forall s_\alpha \in U, \\ s_{\alpha k}=1}} \text{Pls}_j^{(t)}(s_\alpha) + \sum_{\substack{\forall s_\alpha \in U, \\ s_{\alpha k}=0}} \text{Pls}_j^{(t)}(s_\alpha)}. \tag{16}$$

The receiver obtains the maximum $\bar{T}$ transmissions of the same information packet and combines all the soft information sources of the packet in bit-level. As for the kth bit of the transmitted signal x_j, the aggregation can be expressed as

$$\text{Mas}_{jk}^{f}(S) = \text{Mas}_{jk}^{(1)}(S) \oplus \text{Mas}_{jk}^{(2)}(S) \oplus \cdots \oplus \text{Mas}_{jk}^{(\bar{T})}(S), \quad S = \{1\}, \{0\}. \tag{17}$$

The combining notion $\oplus$ here refers to the Dempster's combination operator, which is defined as the orthogonal sum (commutative and associative) as follows

$$m = m_1 \oplus m_2 \oplus \cdots \oplus m_m,$$

where the aggregation m is expressed as

$$m(A) = \left(\sum_{\substack{A_1, \cdots, A_m \in \Theta \\ A_1 \cap \cdots \cap A_m \neq \phi}} m_1(A_1) \cdots m_m(A_m) \right)^{-1} \cdot \sum_{\substack{A_1, \cdots, A_m \in \Theta \\ A_1 \cap \cdots \cap A_m = A}} m_1(A_1) \cdots m_m(A_m).$$

As the channel circumstance is stochastic in each (re) transmission, the obtained soft information sources $\text{Mas}_{jk}^{(t)}(S)$ $(t = 1, 2, \ldots, \bar{T})$ for each bit in all of $\bar{T}$ transmissions are independent. Equation (17) can make the most of such independent soft information and counteract the uncertainty contained in each information source, so that the aggregation $\mathrm{Mas}_{jk}^{f}(S)$ is more credible. After the DS combining (17), reliability of the soft information $\mathrm{Mas}_{jk}^{f}(S)$ for the kth bit of x_j is improved, then $\mathrm{Mas}_{jk}^{f}(S)$ is utilized to make decisions, and the decision output is written as

$$\hat{x}_{jk} = \begin{cases} 1, & \mathrm{Mas}_{jk}^{f}(S = \{1\}) \geq \mathrm{Mas}_{jk}^{f}(S = \{0\}), \\ 0, & \mathrm{Mas}_{jk}^{f}(S = \{1\}) < \mathrm{Mas}_{jk}^{f}(S = \{0\}). \end{cases} \tag{18}$$

Although it is first proposed as a DS detection-aided bit-level DS combining, the proposed DS combining is a universal scheme, in which the soft information sources $\mathrm{Mas}_{jk}^{(t)}(S)\ (t = 1, 2, \ldots, \bar{T})$ are achieved by (15), imposing the plausibility values from the DS detection. However, DS detection is not obligatory, and any other MIMO detection scheme is applicable. If only symbol-level BPA could be achieved according to the decision statistics of detection scheme, the DS combining algorithm needs only the symbol-level BPA for bit-level BPA calculations. Actually, the soft information $\mathrm{Mas}_{jk}^{(t)}(S)$ for each bit can be calculated by other ways. For example, in the LLR combining scheme [6], log-likelihood-probability-ratio for each bit is used. If the likelihood probability is invoked as a form of bit-level BPA, the conventional LLR combining is demonstrated to be equivalent to the proposed DS combining scheme with details as below.

Without loss of generality, this demonstration focuses on the kth bit of the transmitted signal x_j with k = 1, 2,..., c and j = 1, 2,..., N_t. The LLR of the kth bit of the transmit signal x_j in the tth (re)transmission is $\mathrm{LLR}^{(t)} = \ln \frac{p^{(t)}(x_{jk}=1)}{p^{(t)}(x_{jk}=0)}$ After receiving all of $\bar{T}$ transmissions for the same information packet, the receiver computes the final LLR^{f} for each bit by

$$\begin{aligned} \mathrm{LLR}^{f} &= \sum_{t=1}^{\bar{T}} \mathrm{LLR}^{(t)} \\ &= \sum_{t=1}^{\bar{T}} \ln \frac{p^{(t)}(x_{jk}=1)}{p^{(t)}(x_{jk}=0)} \\ &= \ln \prod_{t=1}^{\bar{T}} \frac{p^{(t)}(x_{jk}=1)}{p^{(t)}(x_{jk}=0)}. \end{aligned} \tag{19}$$

In the DS combining scheme, if choose $\mathrm{Mas}_{jk}^{(t)}(S = \{1\}) = p^{(t)}(x_{jk} = 1)$ and $\mathrm{Mas}_{jk}^{(t)}(S = \{0\}) = p^{(t)}(x_{jk} = 0)$ as a special case, the receiver combines all of $\bar{T}$ soft information sources and achieves the final aggregation as

$$\begin{aligned} &\mathrm{Mas}_{jk}^{f}(S = \{1\}) \\ &= \mathrm{Mas}_{jk}^{(t)}(S = \{1\}) \oplus \mathrm{Mas}_{jk}^{(2)}(S = \{1\}) \oplus \cdots \oplus \mathrm{Mas}_{jk}^{(\bar{T})}(S = \{1\}) \\ &= \frac{\prod_{t=1}^{\bar{T}} \mathrm{Mas}_{jk}^{(t)}(S=\{1\})}{\prod_{t=1}^{\bar{T}} \mathrm{Mas}_{jk}^{(t)}(S=\{1\}) + \prod_{t=1}^{\bar{T}} \mathrm{Mas}_{jk}^{(t)}(S=\{0\})}, \end{aligned} \tag{20}$$

and

$$\begin{aligned} &\mathrm{Mas}_{jk}^{f}(S = \{0\}) \\ &= \mathrm{Mas}_{jk}^{(1)}(S = \{0\}) \oplus \mathrm{Mas}_{jk}^{(2)}(S = \{0\}) \oplus \cdots \oplus \mathrm{Mas}_{jk}^{(\bar{T})}(S = \{0\}) \\ &= \frac{\prod_{t=1}^{\bar{T}} \mathrm{Mas}_{jk}^{(t)}(S=\{0\})}{\prod_{t=1}^{\bar{T}} \mathrm{Mas}_{jk}^{(t)}(S=\{1\}) + \prod_{t=1}^{\bar{T}} \mathrm{Mas}_{jk}^{(t)}(S=\{0\})}. \end{aligned} \tag{21}$$

In the decision-making stage, in terms of (19), the decision output in LLR combining scheme can be expressed as

$$\hat{x}_{jk} = \begin{cases} 1, & \mathrm{LLR}^{f} \geq 0, \\ 0, & \mathrm{LLR}^{f} < 0. \end{cases} \tag{22}$$

In the DS combining scheme, according to (20) and (21), the decision output can be expressed as (18). Comparing (18) with (22), it is concluded that the LLR combining is equivalent to the DS combining when the likelihood probability is chosen as a form of bit-level BPA. In other words, the LLR combining is a special case of the proposed bit-level DS combining scheme.

V Simulation and comparison

The system performance comparison between the proposed DS detection-aided DS combining and the conventional MMSE detection-aided LLR combining in MIMO -HARQ systems presented in [9] validates the proposed scheme. In addition, for SISO systems, system performance improvement of the proposed DSC over its MRC counterpart is demonstrated in [10] when the source bits are equiprobable, and the simulation result is shown in Figure 2. As MAP algorithm is equivalent to ML when source bits are equiprobable, thus MRC-MAP is equivalent to MRC-ML, which can be easily seen in Figure 2. The proposed DSC-D and DSC-APP outperform both MRC-ML and MRC-MAP, while gap between DSC-D and DSC-APP is very small. In the following context, we mainly focus on the situation when the source bits are non-equiprobable.

Three modulation schemes, BPSK, QPSK, and 8PSK are employed for the numerical results of both DSC and MRC when the source bits are non-equiprobable, respectively. The BER refers to the total BER, that is, the rejected bits are considered in BER calculation. Firstly, simulations are implemented in quasi-static at Rayleigh fading channels for an SISO system with the maximum retransmissions times $\bar{T} = 2$ for simplicity, and perfect channel estimation is assumed. This paper focuses on the performance of combining schemes at

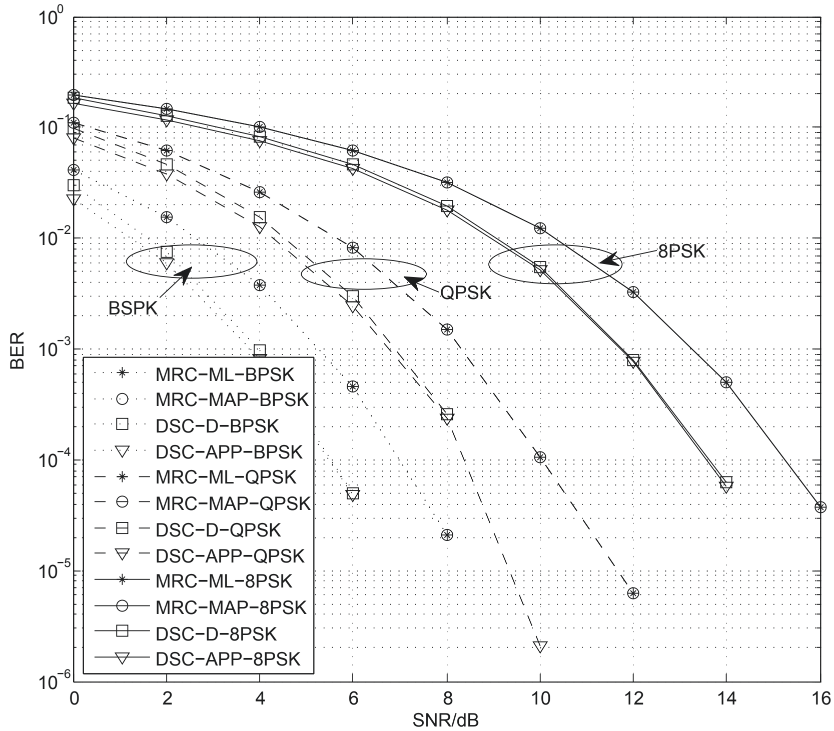


Figure 2 Performance comparison between MRC-ML, MRC-MAP, and the proposed DSC-D, DSC-APP in Rayleigh channel, when the source bits are equiprobable in diverse BPSK, QPSK, and 8PSK modulation schemes.

the receiver, so adaptive coded modulation as a common technique at the transmitter is not invoked in all simulation cases.

For the constellation set $U = \{s_1, s_2,..., s_M\}$, the element is achieved by the mapping function $s_\alpha = \text{map}\,(s_{\alpha 1} s_{\alpha 2} \ldots s_{\alpha c})$. When the source bits are non-equiprobable, let p and $1 - p$ denote the priori probability for bit 1 and 0, respectively, i.e. $\Pr(s_{\alpha k} = 1) = p$, $\Pr(s_{\alpha k} = 0) = 1 - p$. The corresponding priori probability for the modulated symbols for BPSK, QPSK, and 8PSK can be written as

$$\begin{aligned}
\text{BPSK}: &\Pr(s_1 = \text{map}(1)) = p, && \Pr(s_2 = \text{map}(0)) = 1 - p;\\
\text{QPSK}: &\Pr(s_1 = \text{map}(11)) = p^2, && \Pr(s_2 = \text{map}(10)) = p(1-p),\\
&\Pr(s_3 = \text{map}(01)) = p(1-p), && \Pr(s_4 = \text{map}(00)) = (1-p)^2;\\
\text{8PSK}: &\Pr(s_1 = \text{map}(111)) = p^3, && \Pr(s_2 = \text{map}(110)) = p^2(1-p),\\
&\Pr(s_3 = \text{map}(101)) = p^2(1-p), && \Pr(s_4 = \text{map}(100)) = p(1-p)^2,\\
&\Pr(s_5 = \text{map}(011)) = p^2(1-p), && \Pr(s_6 = \text{map}(010)) = p(1-p)^2,\\
&\Pr(s_7 = \text{map}(001)) = p(1-p)^2, && \Pr(s_8 = \text{map}(000)) = (1-p)^3;
\end{aligned}$$

A Performance comparison between DSC and MRC in Rayleigh channel, when the source bits are NEP and the priori knowledge is unavailable

When the source bits are non-equiprobable with $p = 0.1$, i.e. $\Pr(s_{\alpha k} = 1) = 0.1$, $\Pr(s_\alpha k = 0) = 0.9$, and such priori probability is not available to the receiver, the proposed DSC scheme calculates the BPAs on base of the distance from the received signal $y^{(t)}$ to the decision candidate set S_m, i.e. the nearer-distance-more-confidence rule (3). The system performance after combining two transmissions of the same packet by means of the proposed DSC scheme is shown in Figure 3, and the corresponding performance of MRC by the ML (13) as well as the MAP rule (14) is also provided in Figure 3 for the sake of comparison. From this figure, it is obvious that the proposed DSC (marked by DSC-D, denoting the distance-based BPA calculations for DSC) outperforms the ML-based MRC (marked by MRC-ML), both of which do not know the priori probability for the non-equiprobable source, and the gap for performance gains appears at low SNR region and becomes large as SNR increases. In addition, the corresponding performance of the MAP-based MRC (marked by MRC-MAP) plotted in Figure 3 is provided for reference. It is found that the significant performance gains of MRC-MAP over MRC-ML appear in low SNR region and the gap becomes small as SNR increases. As a result, the performance lines of DSC-D and MRC-MAP cross, but both outperform the MRC-ML scheme.

B Performance comparison between DSC-D and DSC-APP in Rayleigh channel, when the source bits are NEP and the priori knowledge is available

Following the last subsection, if the priori probability of the non-equiprobable source bits is available to the receiver, the posterior probability of the transmitted signal can be used for BPA calculations in the proposed

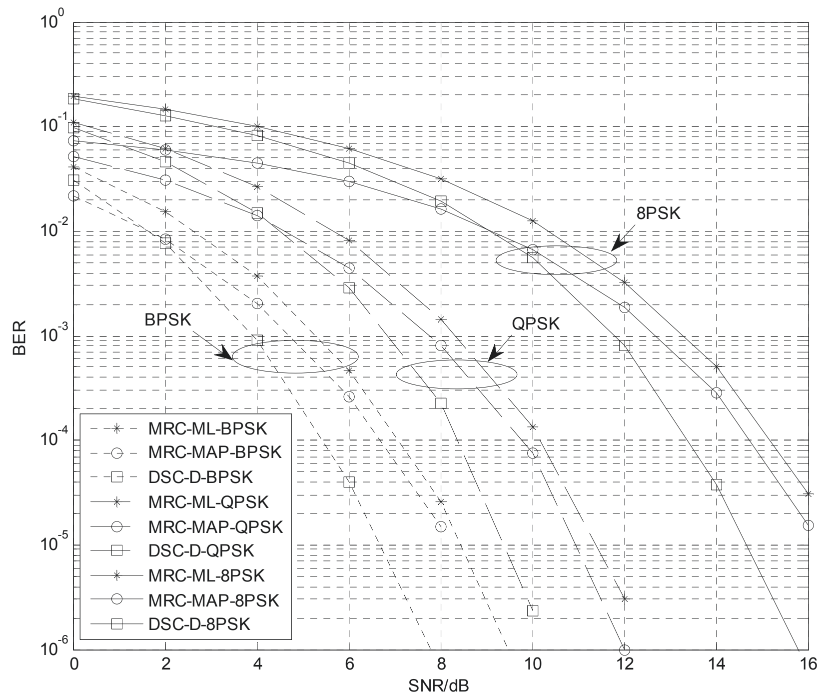


Figure 3 Performance comparison between MRC-ML, MRC-MAP, and the proposed DSC-D in Rayleigh channel, when the source bits are non-equiprobable with priori probability Pr ($s_{\alpha k}$ = 1) = 0.1 in diverse BPSK, QPSK, and 8PSK modulation schemes.

DSC scheme as (5), so as to improve the system performance compared to the distance-based method for BPA calculations (DSC-D). Assuming the priori probability is Pr ($s_{\alpha k}$ = 1) = 0.1, Pr ($s_{\alpha k}$ = 0) = 0.9, the DSC scheme makes use of such priori probability for BPA calculations, and the resulting system performance (marked by DSC-APP) in diverse BPSK, QPSK, and 8PSK modulation schemes is shown in Figure 4a, where the performance of DSC-APP in 8PSK is re-portrayed in Figure 4b for legible observation.

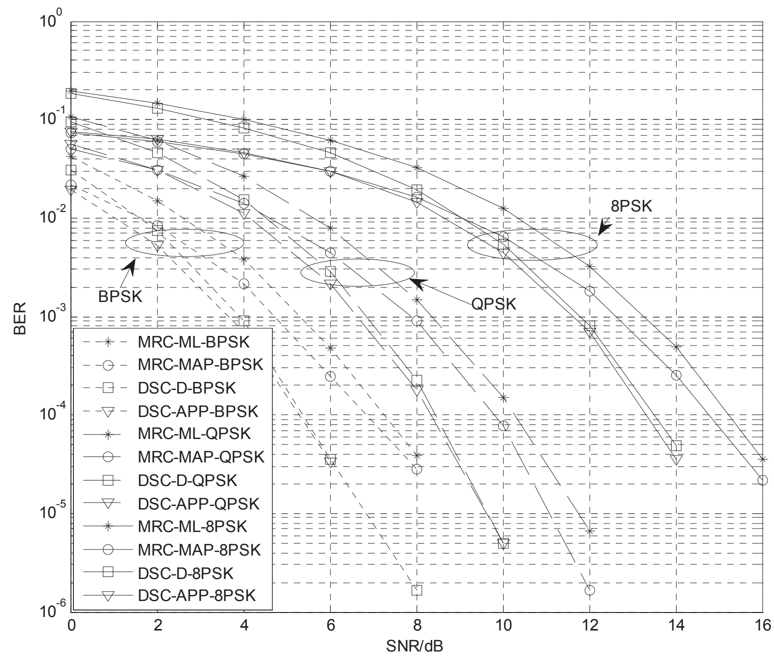
From Figure 4, it is concluded that DSC-APP outperforms DSC-D by making use of the priori probability of the non-equiprobable source, especially in low SNR region. Moreover, the performance of the proposed DSC-APP is almost equivalent to that of MRC-MAP in low SNR region, both of which employed the priori knowledge of the source, whereas when SNR increases, the superior performance of DSC-APP is becoming remarkable and the performance gap of DSC-APP over MRC-MAP is gradually enlarged. However, if the receiver cannot obtain the priori knowledge, the proposed DSC-APP degrades to be DSC-D that has been demonstrated to be superior to MRC-ML in previous subsection. In conclusion, whether the source bits are equiprobable or non-equiprobable, the proposed DSC outperforms its MRC counterpart.

C Performance comparison with Turbo codes, when the source bits are NEP and the priori knowledge is available

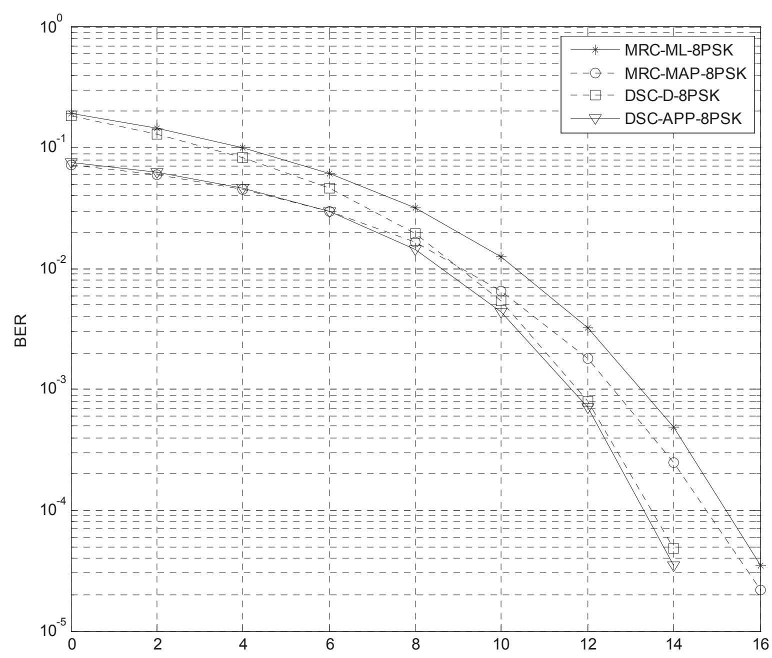
Since FEC coding schemes are incorporated in mainstream research about bit-level combining of HARQ retransmission mechanisms, we give the simulations with Turbo codes here. The frame size is 1024, code rate is $\frac{1}{2}$, maximum iteration number is 10, and MAP algorithm is adopted when decoding. The simulation result is given in Figure 5, as can be seen, the system performance is greatly improved when turbo code is applied, and the relationship between the proposed algorithms remains the same.

D System throughput comparison when the source bits are NEP and the priori knowledge is available

Since throughput is the main term in ARQ systems, we give the throughput comparison in Figure 5 besides the comparison of BER. The system throughput has the units bit/s/Hz and represents the amout of information correctly received at the receiver per channel use. The result is simulated in Rayleigh channel, when the source bits are non-equiprobable with priori probability Pr ($s_{\alpha k}$ = 1) = 0.1 in 8PSK modulation scheme, and we only give the result in 8PSK for the relationship among these algorithms are more significant in 8PSK modulation. From Figure 6, it can be easily seen that DSC-D outperforms



(a) Performance of DSC-APP in diverse BPSK, QPSK, and 8PSK modulation schemes.



(b) Performance of DSC-APP in 8PSK modulation scheme.

Figure 4 Performance comparison between DSC-D and DSC-APP in Rayleigh channel. **a** Performance of DSC-APP in diverse BPSK, QPSK, and 8PSK modulation schemes. **b** Performance of DSC-APP in 8PSK modulation scheme.

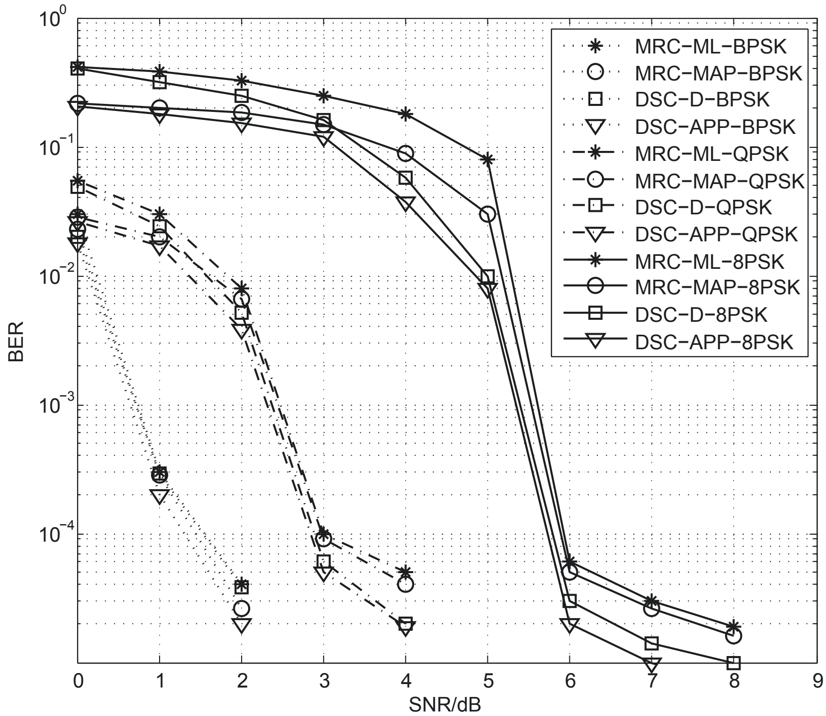


Figure 5 Performance comparison with Turbo codes, when the source bits are non-equiprobable with priori probability Pr ($s_{\alpha k}$ = 1) = 0.1.

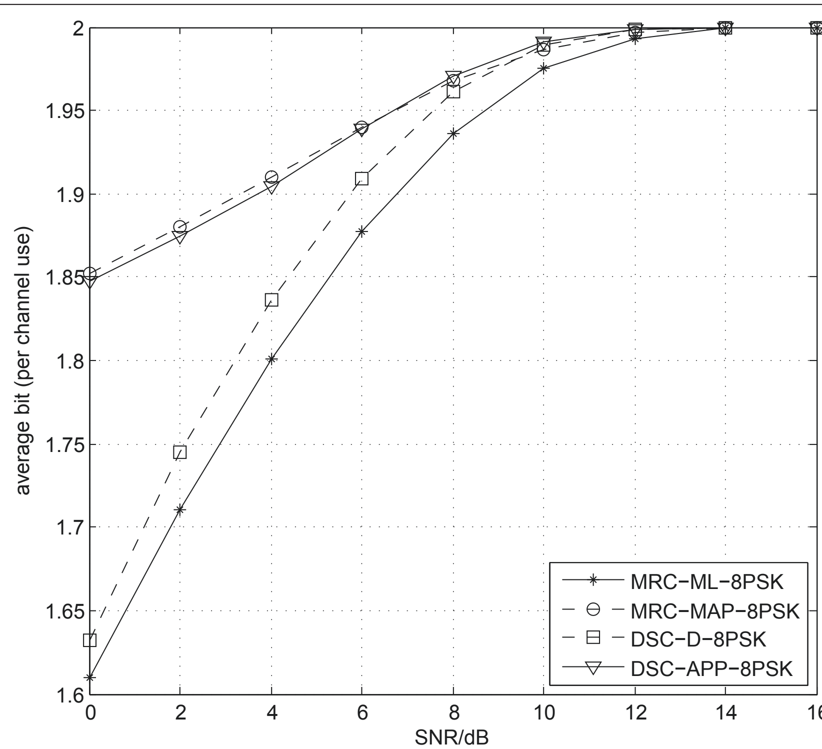


Figure 6 Throughput comparison in Rayleigh channel, when the source bits are non-equiprobable with priori probability Pr ($s_{\alpha k}$ = 1) = 0.1 in 8PSK modulation schemes.

MRC-ML, while DSC-APP slightly outperforms MRC-MAP. One can see that these relationships are largely related to that of BER, which is verified by comparing Figures 4 and 6.

E Performance comparison between DSC and MRC in AWGN channel

This simulations will demonstrate the robustness of the proposed DSC by comparing the performance of system in Rayleigh channel and in AWGN channel. Figure 7 shows the system performance of DSC-D and DSC-APP in QPSK and 8PSK modulation schemes, the corresponding performance lines of the MRC-ML and MRC-MAP in Rayleigh channel and in AWGN channel are provided for the sake of comparison.

The similar conclusion can be achieved from Figure 7 that the proposed DSC-APP outperforms DSC-D in low SNR region, but both of them as well as MRC-ML converges as SNR increases in AWGN channel. In addition, the system performance of MRC-MAP in AWGN channel is superior to all others in both QPSK and 8PSK modulation schemes. It is also observed that the performances of both MRC-ML and MRC-MAP are much degraded in Rayleigh channel compared to those in AWGN channel, which implies that the MRC scheme is sensitive to the channel state. It is obvious that the system performances of DSC-D and DSC-APP in Rayleigh channel degrade little compared to those in AWGN channel, which demonstrates that the proposed DSC scheme is robust to the channel state and is especially applicable to fading channels like the Rayleigh fading channel.

It is necessary to point out that the main term in an ARQ system is throughput, and the relationship between the throughput of the system with different algorithms is mostly related to that between the BER performance.

F Complexity analysis

Finally, to see how much additional computing effort is made by the proposed combining schemes to gain the BER improvement, we discuss the performance gain vs. computing complexity in this subsection. The ML and MAP combining schemes are shown in (13) and (14), respectively, and the complexity of them can be summarized as $O\ (M + t^2)$ due to the complexity of ML detector $O\left(M^{N_t} N_r N_t^2\right)$[19] and the complexity of (12). M is the modulation constellation set cardinality, t is the number of retransmission, N_t and N_r is simplified as 1 according to our simulations. As for the proposed DST-based schemes, the complexity is mainly decided by the complexity of BPAs calculations and combinations. In (3), $N\,(S_2) = \frac{1}{2} M\,(M-1)$ and $N\ (S_1)\ =\ M$, the complexity of (3) is thus approximately $O\ (M^2 t)$, and the complexity of (5) is almost the same with (3) except the computation

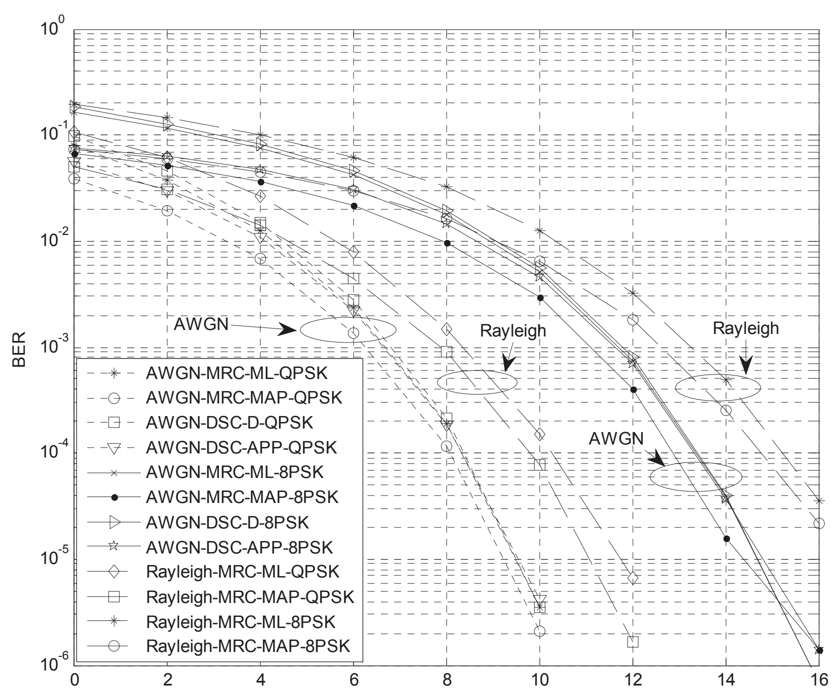


Figure 7 Performance comparison of DSC between in Rayleigh fading channel and in AWGN channel.

of exponential function. The complexity of (7), (8), and (9) is approximately $O\left(\frac{1}{2}M(M+1)t\right)$ and for (10) and (11) the complexity is approximately $O\left(\frac{1}{2}M(M+1)\right)$ and $O(3M)$ respectively. In summary, the total complexity of the proposed two combining schemes is both approximately $\frac{1}{2}O\left((3M+1)Mt+M^2+7M\right)$.

From the above analyses, although the total complexity of the proposed combining schemes is larger than ML and MAP, the total complexity is polynomial or non-exponential with M and t. And with this penalty of complexity, the performance of the proposed schemes is improved at a considerable extent, which can be seen in the above subsections.

VI Conclusions

The DS detection and DS combining schemes based on the DST are proposed, which are demonstrated to outperform the traditional ones. This paper proposes two methods for BPAs calculations of the DSC: distance-based DSC-D, and a posterior probability-based DSC-APP. The simulations are performed in BPSK, QPSK, and 8PSK modulation schemes to illustrate the impact of equiprobable or non-equiprobable source bits on the performance of diverse

HARQ systems based on the MRC and the proposed DSC schemes. The results justify the validity of the proposed DSC. Moreover, the comparison between the performance in Rayleigh channel and that in AWGN channel demonstrates the robustness of the proposed DSC that is insensitive to and especially applicable to the fading channels. In addition, this paper follows the research of the DS detection-aided DS combining scheme proposed previously and deduced that the bit-level DS combining is a universal scheme. If only the likelihood probability is used as the bit-level soft information, the LLR scheme is deduced to be a special case of the proposed bit-level DS combining. Whereas, the DSC-APP only exists in the case that the priori probability of the non-equiprobable source is available to the BPA calculator. But if the priori probability is only available to the combiner, but not available to the BPA calculator, the combining rule under the instruction of the priori probability is being studied and will be presented in another paper.

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Competing interests
The authors declare that they have no competing interests.

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