Published for SISSA by Springer

Received: October 26, 2009
Accepted: December 2, 2009
Published: January 4, 2010

Fine tuning in general gauge mediation

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Abstract: We study the fine-tuning problem in the context of general gauge mediation. Numerical analyses toward for relaxing fine-tuning are presented. We analyse the problem in typical three cases of the messenger scale, that is, GUT (2×10^{16} GeV), intermediate (10^{10} GeV), and relatively low energy (10^{6} GeV) scales. In each messenger scale, the parameter space reducing the degree of tuning as around 10% is found. Certain ratios among gluino mass, wino mass and soft scalar masses are favorable. It is shown that the favorable region becomes narrow as the messenger scale becomes lower, and tachyonic initial conditions of stop masses at the messenger scale are favored to relax the fine-tuning problem for the relatively low energy messenger scale. Our spectra would also be important from the viewpoint of the $\mu - B$ problem.

Keywords: Supersymmetry Breaking, Beyond Standard Model, Supersymmetric Standard Model

ArXiv ePrint: 0910.3477

JHEP01(2010)003

Open Access

doi:10.1007/JHEP01(2010)003

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1 Introduction

Low-energy supersymmetric extension of the standard model is one of promising candidates for a new physics at a TeV scale. The supersymmetry (SUSY) can stabilize the huge hierarchy between the weak scale and the Planck scale. That is a motivation for the low-energy SUSY. In addition, the three gauge couplings are unified at the grand unified theory (GUT) scale, 2×10^{16} GeV, in the minimal supersymmetric standard model (MSSM). Also, supersymmetric standard models have candidates for the dark matter.

Although low-energy SUSY solves the (huge) hierarchy problem between the weak scale and Planck/GUT scale, a few percent of fine-tuning is required in the MSSM as follows. The lightest CP-even Higgs mass m_h is predicted as $m_h \lesssim M_Z$ at the tree level in the MSSM, but that is smaller than the experimental bound $m_h \gtrsim 114.4\,\text{GeV}$. However, the Higgs mass receives a large radiative correction depending on the averaged stop mass $m_{\tilde{t}}$ [1–5]. The experimental bound $m_h \gtrsim 114.4\,\text{GeV}$ requires $m_{\tilde{t}} \gtrsim 1\,\text{TeV}$ when $|A_t|/m_{\tilde{t}} \lesssim 1.0$, where A_t is the so-called A-term corresponding to the top Yukawa coupling. On the other hand, the stop mass also has a renormalization group (RG) effect on the soft scalar mass m_{H_u} of the up-sector Higgs field as [6–12]

$$\Delta m_{H_u}^2 \sim -\frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln \frac{\Lambda}{m_{\tilde{t}}}, \tag{1.1}$$

where y_t is the top Yukawa coupling and Λ denotes a cut-off scale of the MSSM such as the Planck scale or GUT scale. This RG effect $|\Delta m_{H_u}^2|$ would be comparable to the stop mass with a negative sign. Furthermore, the successful electroweak (EW) symmetry breaking requires

$$\frac{1}{2} M_Z^2 \sim -\mu^2 - m_{H_u}^2, \tag{1.2}$$

where μ denotes the supersymmetric mass of the up-sector Higgs field H_u and the down-sector Higgs field H_d. If $m^2_{H_u} \sim -m^2_{\tilde{t}}$ and $m_{\tilde{t}} = \mathcal{O}(1)\,\text{TeV}$, one needs a few percent of fine-tuning between μ^2 and $m^2_{H_u}$ in order to derive the correct value of M_Z. That is the so-called little hierarchy problem [13–18]. Several works have been done to address this issue [19]–[45]. Some of them include extensions of the MSSM.

In the bottom-up approach [46], it is found that non-universal gaugino masses with a certain ratio are favorable to improve fine-tuning in the MSSM when the messenger scale of SUSY breaking is the Planck/GUT scale. Such a favorable ratio of gaugino masses can be realized in the TeV scale mirage mediation [33, 34, 47, 49] and gravity mediation, e.g. moduli mediation [43, 51–53] and the SUSY breaking scenario, where F-components of gauge non-singlets are sizable [42, 54, 59].[1] On the other hand, the spectrum of the constrained MSSM with the universal gaugino mass would be unfavorable. It is also pointed out that a negative value of the stop mass squared at the Planck/GUT scale would also be favorable [39, 40].

Since the minimal gauge mediation [64] leads to the universal gaugino mass, that would be unfavorable from the viewpoint of fine-tuning [29, 63]. Recently, Meade, Seiberg and Shih have extended the gauge mediation to general gauge mediation (GGM) [65]. (See also [66]–[77].) That leads to non-universal gaugino and soft scalar masses. Thus, it is important to study fine-tuning in the GGM. That is our purpose.[2] The important difference of the gauge mediation (including GGM) from other mediation scenarios such as gravity mediation is that the messenger scale can vary from the GUT scale to a TeV scale and predicted A-terms are very small in most of models. These would also lead to an important difference in the fine-tuning behavior.

This paper is organized as follows. In section 2, we briefly review on the fine-tuning problem in the MSSM. Section 3 is also a brief review on the GGM. In section 4, we analyse numerically on fine-tuning in the GGM. In section 5, we give a comment on the $\mu - B$ problem. Section 6 is devoted to conclusion.

2 Fine tuning in the MSSM

Here, we briefly review the fine-tuning problem in the MSSM by showing explicitly equations. In our analysis, we neglect the Yukawa couplings except the top Yukawa coupling y_t. Then, the Higgs sector in the MSSM is described as the following superpotential,

$$W_{\text{Higgs}} = \mu H_u H_d + y_t Q_3 U_3 H_u, \tag{2.1}$$

where Q_3, and U_3 are the chiral superfields corresponding to the left- and right-handed top quarks, respectively. The Higgs fields and top-stop multiplets as well as the gaugino fields play an important role in the fine-tuning problem. Thus, we concentrate on these fields. Their soft SUSY breaking terms are given by

$$\begin{aligned} V_{\text{soft}} &= m^2_{H_u}|H_u|^2 + m^2_{H_d}|H_d|^2 + m^2_{Q_3}|Q_3|^2 + m^2_{U_3}|U_3|^2 \\ &\quad + (\mu B H_u H_d + y_t A_t Q_3 U_3 H_u + \text{h.c.}), \end{aligned} \tag{2.2}$$

[1] Those spectra with less fine-tuning also have interesting aspects on the dark matter physics [60–62].
[2] See also [63, 72].

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where m_X $(X = H_{u,d}, Q_3, U_3)$ are the soft scalar masses for X, respectively, μB is the SUSY breaking mass, i.e. the so-called B-term. Note that we utilize the same notation for denoting a chiral superfield and its lowest scalar component.

The soft SUSY breaking mass for the up-type Higgs m_{H_u} is subject to relatively large logarithmic radiative correction (1.1) from mainly stop loops. The radiative correction $\Delta m^2_{H_u}$ is comparable to the stop mass with the negative sign, i.e. $\Delta m^2_{H_u} \sim -m^2_{\tilde{t}}$. Such a large and negative correction leads to the EW symmetry breaking at the weak scale. Here, we define the averaged top squark mass $m_{\tilde{t}}$ as

$$m^2_{\tilde{t}} \equiv \sqrt{m^2_{Q_3}(M_Z) m^2_{U_3}(M_Z)}. \tag{2.3}$$

A stationary condition of the Higgs potential gives the relation among the Z boson mass M_Z, the μ parameter and soft scalar masses, $m^2_{H_u}$ and $m^2_{H_d}$, as

$$\frac{M_Z^2}{2} = -\mu^2(M_Z) - \frac{m^2_{H_u}(M_Z)\tan^2\beta - m^2_{H_d}(M_Z)}{\tan^2\beta - 1}, \tag{2.4}$$

where $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$. The lightest Higgs boson mass is constrained by

$$\begin{aligned} m_h^2 \leq\ & M_Z^2 \cos^2 2\beta \left(1 - \frac{3m_t^2}{8\pi^2 v^2} \ln \frac{m_{\tilde{t}}^2}{m_t^2}\right) \\ & + \frac{3m_t^4}{4\pi^2 v^2}\left[\ln \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{\tilde{A}_t^2}{m_{\tilde{t}}^2}\left(1 - \frac{\tilde{A}_t^2}{12 m_{\tilde{t}}^2}\right)\right. \\ & \left. + \frac{1}{16\pi^2}\left(\frac{3m_t^2}{2v^2} - 32\pi\alpha_3\right)\left\{\frac{2\tilde{A}_t^2}{m_{\tilde{t}}^2}\left(1 - \frac{\tilde{A}_t^2}{12 m_{\tilde{t}}^2}\right)\ln \frac{m_{\tilde{t}}^2}{m_t^2} + \left(\ln \frac{m_{\tilde{t}}^2}{m_t^2}\right)^2\right\}\right], \end{aligned} \tag{2.5}$$

within the 2-loop approximation [4, 5], where $v = 174\,\mathrm{GeV}$, $\tilde{A}_t \equiv A_t(M_Z) - \mu\cot\beta$ and m_t is the running top squark mass at M_Z.

The current experimental lower bound for the Higgs mass is given by the LEP experiment as $m_h \geq 114.4\,\mathrm{GeV}$. In order to realize $m_h \geq 114.4\,\mathrm{GeV}$, a large top squark mass is required as $m_{\tilde{t}} \gtrsim 1\,\mathrm{TeV}$ when $|A_t(M_Z)/m_{\tilde{t}}| \lesssim 1.0$. The soft scalar mass of the up-sector Higgs field, m_{H_u} suffers from a large radiative correction according to such a large top squark mass through (1.1). Therefore, a few percent of fine-tuning between $m^2_{H_u}$ and μ^2 is required in (2.4) in order to realize the EW symmetry breaking with the experimentally observed Z boson mass, $M_Z \simeq 91.2\,\mathrm{GeV}$. That is the so-called little hierarchy problem. We investigate this fine-tuning problem in the context of the GGM. Furthermore, when $|A_t(M_Z)/m_{\tilde{t}}| \lesssim 1.5$, the condition $m_h \geq 114.4\,\mathrm{GeV}$ requires $m_{\tilde{t}} \gtrsim 500\,\mathrm{GeV}$. Hence, the stop mixing $A_t/m_{\tilde{t}}$ is important [4, 5, 78].

3 General gauge mediation

Before considering the fine-tuning problem in the GGM, we also give a brief review on the GGM. Recently, Meade, Seiberg and Shih have presented the most general spectrum which can be obtained in gauge mediated SUSY breaking model [65]. A careful definition

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of gauge mediation mechanism has been given in the work, that is, in the limit that the MSSM gauge couplings $\alpha_i \to 0$, the theory decouples into the MSSM and a separate hidden sector which breaks SUSY. Following the convention, we label the gauge groups, SU(3), SU(2) and U(1) of the MSSM by $a = 3, 2, 1$, respectively. Within the framework of the GGM, the three gaugino masses M_a $(a = 1, 2, 3)$ of the MSSM are given at the messenger scale M as,

$$M_a = 2g_a^2 B_a. \tag{3.1}$$

In general, B_a $(a = 1, 2, 3)$ are three independent complex parameters. If CP phases of B_a are not aligned each other, that would lead to a serious CP problem. Thus, we use B_a as three real parameters. The soft scalar masses squared are also given in the GGM as

$$m_{\tilde{f}}^2 = g_1^2 Y_f \zeta + \sum_{a=1}^{3} g_a^4 c_2(f;a) A_a, \tag{3.2}$$

at M, where $c_2(f;a)$ is the quadratic Casimir of the representation of fermion f under the gauge group corresponding to the label a. Here, A_a $(a = 1, 2, 3)$ are three independent real parameters. Hereafter, we concentrate on the models with $\zeta = 0$.[3] In this case, there are the mass relations at M

$$m_{Q_f}^2 + m_{D_f}^2 + m_{E_f}^2 - m_{L_f}^2 - 2m_{U_f}^2 = 0, \qquad m_{H_u} = m_{H_d}, \tag{3.3}$$

where m_{Q_f}, m_{U_f}, m_{D_f}, m_{L_f}, and m_{E_f} denote soft scalar masses for the f-th generation of the left-handed squarks, up-sector right-handed squarks, down-sector right-handed squarks, left-handed sleptons and right-handed sleptons. Thus, the $\mathrm{U}(1)_Y$ D-term S, i.e.,

$$S = m_{H_u}^2 - m_{H_d}^2 + \sum_{f=1}^{3} (m_{Q_f}^2 + m_{D_f}^2 + m_{E_f}^2 - m_{L_f}^2 - 2m_{U_f}^2), \tag{3.4}$$

vanishes at the messenger scale M. Furthermore, its RG equation is given as

$$(4\pi)^2 \frac{dS}{dt} = -b_1 g_1^2(t) S(t), \tag{3.5}$$

where $t \equiv 2\log(M_Z/\bar{\mu})$, $\bar{\mu}$ is an arbitrary energy scale, and $b_1 = 33/5$ (and $b_2 = 1$, $b_3 = -3$ for references). Thus, when S is vanishing at M, it vanishes at any scale. For concreteness, we show explicitly the initial conditions of the soft scalar masses, m_{Q_3}, m_{U_3}, m_{H_u} and m_{H_d} as

$$\begin{aligned} m_{Q_3}^2(M) &= \frac{4}{3} g_3^4(M) A_3 + \frac{3}{4} g_2^4(M) A_2 + \frac{3}{5}\left(\frac{1}{6}\right)^2 g_1^4(M) A_1 \\ &= (4\pi)^4 B_3^2 \left[\frac{4}{3}\tilde{\alpha}_3^2(M) a_3 + \frac{3}{4}\tilde{\alpha}_2^2(M) a_2 + \frac{1}{60}\tilde{\alpha}_1^2(M) a_1\right], \\ m_{U_3}^2(M) &= \frac{4}{3} g_3^4(M) A_3 + \frac{3}{5}\left(-\frac{2}{3}\right)^2 g_1^4(M) A_1 \\ &= (4\pi)^4 B_3^2 \left[\frac{4}{3}\tilde{\alpha}_3^2(M) a_3 + \frac{4}{15}\tilde{\alpha}_1^2(M) a_1\right], \end{aligned} \tag{3.6}$$

[3] This situation, $\zeta = 0$, can be realized by invoking messenger parity.

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$$
\begin{aligned}
m_{H_u}^2(M) &= m_{H_d}^2(M) \\
&= \frac{3}{4} g_2^4(M) A_2 + \frac{3}{5}\left(\pm\frac{1}{2}\right)^2 g_1^4(M) A_1 \\
&= (4\pi)^4 B_3^2 \left[\frac{3}{4}\tilde{\alpha}_2^2(M) a_2 + \frac{3}{20}\tilde{\alpha}_1^2(M) a_1\right],
\end{aligned}
$$

where $\tilde{\alpha}_a \equiv \alpha_a/(4\pi) \equiv g_a^2/(4\pi)^2$. Here, we have defined the ratios

$$
a_a \equiv \frac{A_a}{B_3^2}, \tag{3.7}
$$

for convenience. Similarly, we define the ratios of gaugino masses to the gluino mass,

$$
b_a \equiv \frac{B_a}{B_3}. \tag{3.8}
$$

The initial condition of the A-term in the GGM is given as

$$
A_t = 0, \tag{3.9}
$$

at M. Thus, the A-term A_t at the weak scale is given only by the RG effect between the weak scale and the messenger scale M. This initial condition is important because the stop mixing $A_t/m_{\tilde{t}}$ at the weak scale has a significant effect on the Higgs mass (2.5).

By utilizing these gaugino and sfermion masses given in the GGM, we numerically analyze the fine-tuning problem in the next section.

4 Numerical analyses

We study the fine-tuning problem in the GGM and present numerical analyses. In gauge mediated SUSY breaking models, phenomenological consequences at the EW scale generally depend on the messenger scale M. We present our analyses for three typical messenger scales, that is (i) GUT scale $M = \Lambda_{\rm GUT} \equiv 2 \times 10^{16}\,{\rm GeV}$, (ii) intermediate scale $M = 10^{10}\,{\rm GeV}$, and (iii) relatively low energy scale $M = 10^6\,{\rm GeV}$.

Firstly, we give the soft parameters at the EW scale by integrating the 1-loop RG equations [6–10]. The gaugino mass at the EW scale are

$$
M_1(M_Z) \simeq 0.428 B_1, \tag{4.1}
$$

$$
M_2(M_Z) \simeq 0.859 B_2, \tag{4.2}
$$

$$
M_3(M_Z) \simeq 3.00 B_3. \tag{4.3}
$$

In this analysis, we use the values of gauge couplings at the EW scale as $\tilde{\alpha}_1(M_Z) \simeq 1.36 \times 10^{-3}$, $\tilde{\alpha}_2(M_Z) \simeq 2.72 \times 10^{-3}$, and $\tilde{\alpha}_3(M_Z) \simeq 9.50 \times 10^{-3}$. These couplings in the MSSM would be unified at the GUT scale within a good accuracy. In addition, we use the running top mass $m_t = 164.5\,{\rm GeV}$ at M_Z and $\tan\beta = 10$ for numerical analysis.

The scalar masses such as m_{Q_3}, m_{U_3}, $m_{H_{u,d}}$, and A_t, which are important to discuss the fine-tuning problem, are given for each typical messenger scale as

(i) $M = \Lambda_{\rm GUT}$,

$$\begin{aligned}
m^2_{Q_3}(M_Z) \simeq\ & 6.07B_3^2 - 0.0120B_1B_3 - 0.00754B_1^2 - 0.0834B_2B_3 \\
& -0.00245B_1B_2 + 0.437B_2^2 \\
& -0.116m^2_{H_u}(M) + 0.884m^2_{Q_3}(M) - 0.116m^2_{U_3}(M), \qquad (4.4)\\
m^2_{U_3}(M_Z) \simeq\ & 5.11B_3^2 - 0.0240B_1B_3 + 0.0495B_1^2 - 0.167B_2B_3 \\
& -0.00490B_1B_2 - 0.202B_2^2 \\
& -0.232m^2_{H_u}(M) - 0.232m^2_{Q_3}(M) + 0.768m^2_{U_3}(M), \qquad (4.5)\\
m^2_{H_u}(M_Z) \simeq\ & -2.90B_3^2 - 0.0361B_1B_3 + 0.00505B_1^2 - 0.250B_2B_3 \\
& -0.00735B_1B_2 + 0.235B_2^2 \\
& +0.652m^2_{H_u}(M) - 0.348m^2_{Q_3}(M) - 0.348m^2_{U_3}(M), \qquad (4.6)\\
m^2_{H_d}(M_Z) \simeq\ & 0.538B_2^2 + 0.0415B_1^2 + m^2_{H_d}(M), \qquad (4.7)\\
A_t(M_Z) \simeq\ & 2.20B_3 + 0.278B_2 + 0.0352B_1, \qquad (4.8)
\end{aligned}$$

(ii) $M = 10^{10}\,\text{GeV}$,

$$\begin{aligned}
m^2_{Q_3}(M_Z) \simeq\ & 5.43B_3^2 - 0.00327B_1B_3 - 0.000940B_1^2 - 0.0331B_2B_3 \\
& -0.000404B_1B_2 + 0.227B_2^2 \\
& -0.0958m^2_{H_u}(M) + 0.904m^2_{Q_3}(M) - 0.0958m^2_{U_3}(M), \qquad (4.9)\\
m^2_{U_3}(M_Z) \simeq\ & 4.76B_3^2 - 0.00654B_1B_3 + 0.0142B_1^2 - 0.0661B_2B_3 \\
& -0.000807B_1B_2 - 0.0701B_2^2 \\
& -0.192m^2_{H_u}(M) - 0.192m^2_{Q_3}(M) + 0.808m^2_{U_3}(M), \qquad (4.10)\\
m^2_{H_u}(M_Z) \simeq\ & -2.03B_3^2 - 0.00981B_1B_3 + 0.00405B_1^2 - 0.0992B_2B_3 \\
& -0.00121B_1B_2 + 0.157B_2^2 \\
& +0.712m^2_{H_u}(M) - 0.288m^2_{Q_3}(M) - 0.288m^2_{U_3}(M), \qquad (4.11)\\
m^2_{H_d}(M_Z) \simeq\ & 0.262B_2^2 + 0.0103B_1^2 + m^2_{H_d}(M), \qquad (4.12)\\
A_t(M_Z) \simeq\ & 1.93B_3 + 0.181B_2 + 0.0167B_1, \qquad (4.13)
\end{aligned}$$

(iii) $M = 10^{6}\,\text{GeV}$,

$$\begin{aligned}
m^2_{Q_3}(M_Z) \simeq\ & 4.24B_3^2 - 0.000733B_1B_3 - 0.0000353B_1^2 - 0.00879B_2B_3 \\
& -0.0000603B_1B_2 + 0.111B_2^2 \\
& -0.0669m^2_{H_u}(M) + 0.933m^2_{Q_3}(M) - 0.0669m^2_{U_3}(M), \qquad (4.14)\\
m^2_{U_3}(M_Z) \simeq\ & 3.90B_3^2 - 0.00147B_1B_3 + 0.00563B_1^2 - 0.0176B_2B_3 \\
& -0.000121B_1B_2 - 0.0198B_2^2 \\
& -0.134m^2_{H_u}(M) - 0.134m^2_{Q_3}(M) + 0.866m^2_{U_3}(M), \qquad (4.15)\\
m^2_{H_u}(M_Z) \simeq\ & -1.03B_3^2 - 0.00220B_1B_3 + 0.00234B_1^2 - 0.0264B_2B_3 \\
& -0.000181B_1B_2 + 0.0914B_2^2 \\
& +0.799m^2_{H_u}(M) - 0.201m^2_{Q_3}(M) - 0.201m^2_{U_3}(M), \qquad (4.16)
\end{aligned}$$

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$$m_{H_d}^2(M_Z) \simeq 0.121 B_2^2 + 0.00366 B_1^2 + m_{H_d}^2(M), \tag{4.17}$$

$$A_t(M_Z) \simeq 1.47 B_3 + 0.105 B_2 + 0.00850 B_1. \tag{4.18}$$

Here, we have used the initial conditions, $A_t(M) = S(M) = 0$. The change of RG effects between the cases (ii) and (iii) is rather drastic compared with one between (i) and (ii).

If all soft parameters are taken as the same order, $B_a \sim m_X(M)$, the averaged top squark mass is approximated for each messenger scale as

$$m_{\tilde{t}}^2 \sim \begin{cases} 6.0 B_3^2 & \text{in the case (i)} \\ 5.7 B_3^2 & \text{in the case (ii)} \\ 4.8 B_3^2 & \text{in the case (iii)} \end{cases} . \tag{4.19}$$

For a fixed value of $|A_t(M_Z)/m_{\tilde{t}}|$, a large value of $m_{\tilde{t}}^2$ would be favorable to realize the Higgs mass $m_h \geq 114.4\,\text{GeV}$. That implies that a higher messenger scale would be favorable for a fixed value of the gluino mass, i.e. B_3. In order to satisfy the experimental bound for the Higgs mass, the lower bound for B_3 is roughly estimated as

$$B_3 \gtrsim \begin{cases} 200\ (410)\ \text{GeV for } |A_t(M_Z)/m_{\tilde{t}}| \lesssim 1.5\ (1.0) \text{ in the case (i)} \\ 210\ (420)\ \text{GeV for } |A_t(M_Z)/m_{\tilde{t}}| \lesssim 1.5\ (1.0) \text{ in the case (ii)} \\ 230\ (460)\ \text{GeV for } |A_t(M_Z)/m_{\tilde{t}}| \lesssim 1.5\ (1.0) \text{ in the case (iii)} \end{cases} . \tag{4.20}$$

Furthermore, we can estimate the stop mixing $|A_t(M_Z)/m_{\tilde{t}}|$. For example, for $B_a \sim m_X(M)$ we estimate

$$|A_t(M_Z)/m_{\tilde{t}}| \sim \begin{cases} 1.0 & \text{in the case (i)} \\ 0.89 & \text{in the case (ii)} \\ 0.72 & \text{in the case (iii)} \end{cases} . \tag{4.21}$$

A large value of $|A_t(M_Z)/m_{\tilde{t}}|$ would be favorable to realize the Higgs mass $m_h \geq 114.4\,\text{GeV}$. That implies that a higher messenger scale would be favorable.

On the other hand, the dominant part of the RG effects in $m_{H_d}^2$ (4.6), (4.11) and (4.16) is due to the gluino mass, i.e. B_3^2. If $B_3 \sim 500\,\text{GeV}$, we need fine-tuning between $m_{H_u}^2$ and μ^2 to realize M_Z. The absolute value of coefficient of B_3^2 in $m_{H_d}^2(M_Z)$ decreases as the messenger scale M decreases. Thus, for a fixed value of B_3, the degree of fine-tuning is reduced as the messenger scale becomes lower.

Thus, the tension between the fine-tuning and the lower bound of the Higgs mass $m_h \geq 114.4\,\text{GeV}$ depends non-trivially on the messenger scale M. Also that would depend on ratios among gaugino masses and scalar masses, although we have used $B_a \sim m_X(M)$ in the above estimation.

Toward the numerical analyses of the fine-tuning problem, we introduce fine-tuning parameters [13–18],

$$\Delta_Y \equiv \frac{1}{2} \frac{Y}{M_Z^2} \frac{\partial M_Z^2}{\partial Y}, \tag{4.22}$$

which indicates that we need $100/\Delta_Y$ percent of fine-tuning for Y to derive M_Z. A larger value of Δ_Y means more severe fine-tuning to be required.

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If B_a and A_a are independent of each other, fine-tuning for B_3 would be most severe, because $m_{H_d}^2(M_Z)$ depends dominantly on B_3. For example, we can calculate

(i) $M = \Lambda_{\rm GUT}$

$$\Delta_{B_3} = 5.85\hat{M}_3^2 + (0.0364\hat{M}_1 + 0.253\hat{M}_2)\hat{M}_3, \tag{4.23}$$

(ii) $M = 10^{10}\,{\rm GeV}$

$$\Delta_{B_3} = 4.10\hat{M}_3^2 + (0.00990\hat{M}_1 + 0.100\hat{M}_2)\hat{M}_3, \tag{4.24}$$

(iii) $M = 10^{6}\,{\rm GeV}$

$$\Delta_{B_3} = 2.08\hat{M}_3^2 + (0.00222\hat{M}_1 + 0.0266\hat{M}_2)\hat{M}_3, \tag{4.25}$$

where $\hat{M}_a \equiv B_a/M_Z$. It is found that the coefficients of the terms become small as the messenger scale becomes lower. If $\Delta_{B_3} \leq 10$ is required under the condition $B_1 = B_2 = B_3$, the allowed value of B_3 are (i) $B_3 \leq 110\,{\rm GeV}$, (ii) $B_3 \leq 140\,{\rm GeV}$, and (iii) $B_3 \leq 190\,{\rm GeV}$. They could not satisfy the bounds on the Higgs mass (4.20). On the other hand, when we take $B_3 \simeq 500$ GeV, we find that severe fine-tunings such as (i) $\Delta_{B_3} \simeq 200$, (ii) $\Delta_{B_3} \simeq 140$, (iii) $\Delta_{B_3} \simeq 70$ are needed.

We have assumed that B_a and A_a are independent of each other. However, in a definite theory, they are not independent, but certain ratios are predicted in each theory. That is, in a definite theory there is one parameter, which determines the overall size of soft SUSY breaking terms. We choose B_3 as such a parameter and the ratios a_a and b_a are fixed in a theory. Then, we consider the fine-tuning only for B_3, i.e. Δ_{B_3} under fixed ratios of a_a and b_a. Varying a_a and b_a means that we compare different theories in the theory space of the GGM. Then, the fine-tuning parameter can be rewritten as

(i) $M = \Lambda_{\rm GUT}$

$$\begin{aligned}\Delta_{B_3} = \hat{M}_3^2(&5.85 + 0.506b_2 - 0.465b_2^2 + 0.508a_3 - 0.122a_2 + 0.0728b_1 + 0.0148b_1b_2 \\ &- 0.00936b_1^2 + 0.00132a_1),\end{aligned} \tag{4.26}$$

(ii) $M = 10^{10}\,{\rm GeV}$

$$\begin{aligned}\Delta_{B_3} = \hat{M}_3^2(&4.10 + 0.200b_2 - 0.311b_2^2 + 0.825a_3 - 0.143a_2 + 0.0198b_1 + 0.00245b_1b_2 \\ &- 0.00798b_1^2 - 0.00495a_1),\end{aligned} \tag{4.27}$$

(iii) $M = 10^{6}\,{\rm GeV}$

$$\begin{aligned}\Delta_{B_3} = \hat{M}_3^2(&2.08 + 0.0533b_2 - 0.182b_2^2 + 1.04a_3 - 0.183a_2 + 0.00444b_1 + 0.000365b_1b_2 \\ &- 0.00465b_1^2 - 0.00821a_1).\end{aligned} \tag{4.28}$$

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Coefficients of b_1 and a_1 in the above equations are very small. Thus, those terms would not be important unless $b_1 = \mathcal{O}(10)$ or $a_1 = \mathcal{O}(100)$. Therefore, we concentrate on others and throughout our numerical analyses we take $b_1 = a_1 = 1$ as a typical value. It is found that the coefficients of a_2 and b_2^2, which determines the wino mass, are negative. Hence, it would be favorable to cancel the dominant term by relatively large b_2 and/or a_2. That is, models satisfying

(i) $M = \Lambda_{\rm GUT}$

$$|5.85 + 0.506b_2 - 0.465b_2^2 + 0.508a_3 - 0.122a_2| \ll 1, \tag{4.29}$$

(ii) $M = 10^{10}\,\text{GeV}$

$$|4.10 + 0.200b_2 - 0.311b_2^2 + 0.825a_3 - 0.143a_2| \ll 1, \tag{4.30}$$

(iii) $M = 10^{6}\,\text{GeV}$

$$|2.08 + 0.0533b_2 - 0.182b_2^2 + 1.04a_3 - 0.183a_2| \ll 1, \tag{4.31}$$

would be interesting in the theory space. For fixed values of a_2 and a_3, a favorable value of b_2 is determined. That means a favorable ratio between the gluino and wino masses such as ref. [46]. For a fixed value of b_2, a linear correlation between a_2 and a_3 is required. On the other hand, for a fixed value of a_2 (a_3) a quadratic relation between b_2 and a_3 (a_2) is required.

The results of numerical analyses are shown in figures 1–4. Figure 1 (a) and (b) show three curves corresponding to $\Delta_{B_3} = 5, 10, 15$ for $B_3 = 500$ and $300\,\text{GeV}$ in the case (i), respectively. The darkest (darker) solid lines correspond $\Delta_{B_3} = 5$ (10). The dotted (red) curve is $m_h = 114.4.\,\text{GeV}$ and the shaded (yellow) region corresponds to the region with $m_h \geq 114.4\,\text{GeV}$. In these figures, we fix $b_1 = a_1 = 1$ and $b_2 \simeq 4.19$ for (a) and $b_2 \simeq 4.01$ for (b). These values of b_2 lead to $\Delta_{B_3} = 10$ when $b_1 = a_1 = a_2 = a_3 = 1$.[4] These figures mean how much stable the region with $\Delta_{B_3} = 10$ is in the (a_2, a_3) plane, when b_2 is fixed such that $\Delta_{B_3} = 10$ is realized for $b_1 = a_1 = a_2 = a_3 = 1$. We find from figure 1 (a) that $a_2 \lesssim 5$ and $a_3 \lesssim 2$ are required to realize $\Delta_{B_3} \sim 10$. Figure 1 (b) shows that these upper bounds of both a_2 and a_3 are raised for $B_3 = 300\,\text{GeV}$. It is seen from (4.26) that the widths among three lines become wider as the B_3 becomes lower. The lower bound for a_2 and a_3 are evaluated as $(a_2, a_3) \gtrsim (-45, -10)$ for $B_3 = 500\,\text{GeV}$ and $(-40, -9)$ for $300\,\text{GeV}$. These results are insensitive to a value of a_1, even if a_1 is larger such as $a_1 \sim \mathcal{O}(10)$. Figure 1 (c) and (d) correspond to the case of $(b_2, B_3) = (1, 500\ \text{GeV})$ and $(b_2, B_3) = (1, 300\ \text{GeV})$, respectively. The lower bounds for a_2 are raised to $a_2 \gtrsim -10$. It can be also found that the favorable region is $a_3 \gtrsim -15$.

Figures 2 and 3 show the results of the same analyses as case (i) for the cases (ii) and (iii), respectively, but (a) and (c) for $B_3 = 1\,\text{TeV}$ and (b) and (d) for $B_3 = 500\,\text{GeV}$

[4] There is another value of b_2, which is negative and its absolute value is similar for positive one, to lead to $\Delta_{B_3} = 10$ when $b_1 = a_1 = a_2 = a_3$=1. In this work, we focus on only a positive value of solution but our results are not modified for a negative one.

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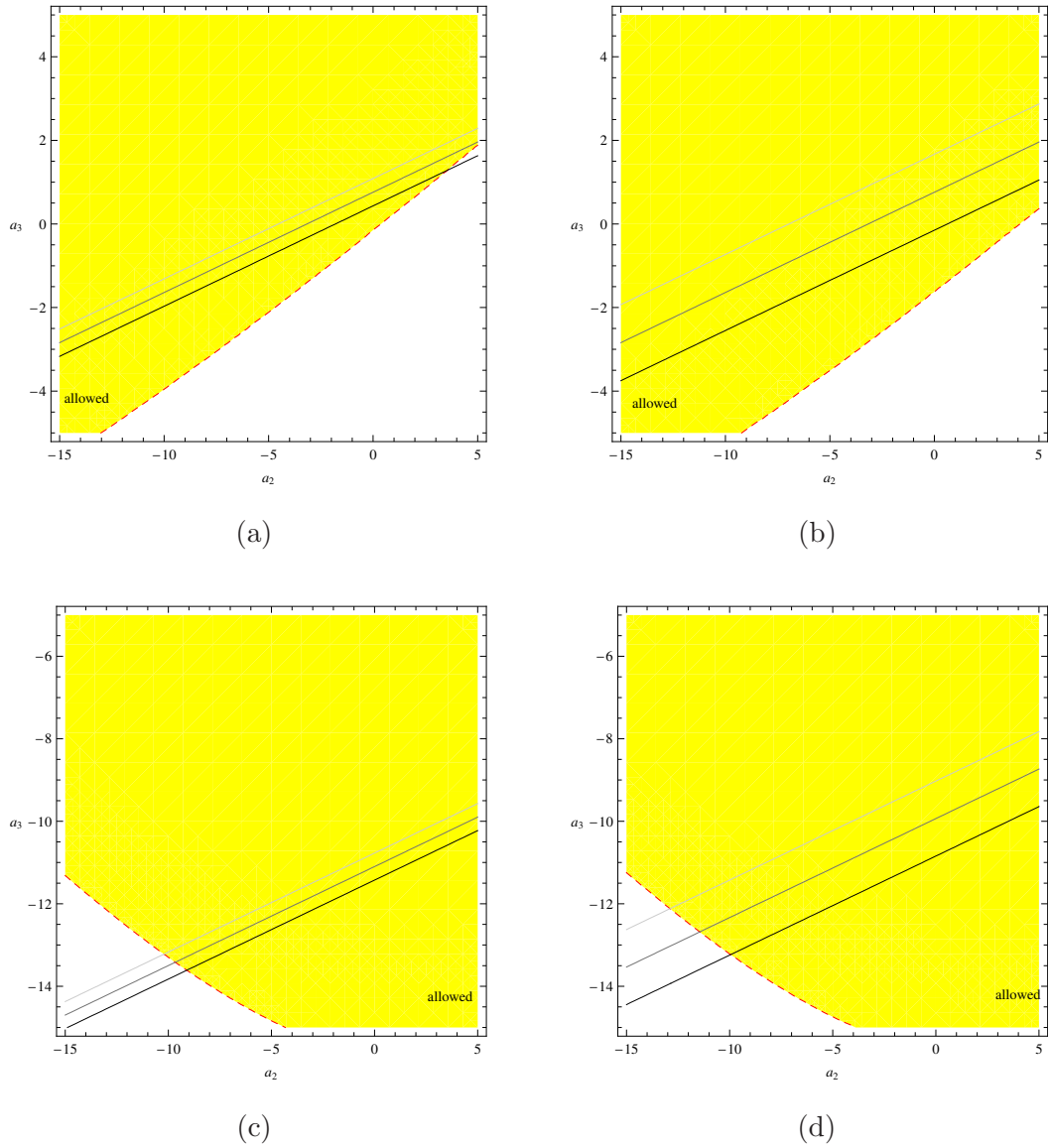

Figure 1. Lines and curve for the case (i) determined by constraints from $\Delta_{B_3} = 5, 10, 15$ (solid lines), $m_h = 114.4\,\mathrm{GeV}$ (dashed (red) curve), and experimentally allowed region $m_h \geq 114.4\,\mathrm{GeV}$ (shaded (yellow) region). The darker and darkest solid lines correspond $\Delta_{B_3} = 10$ and 5, respectively. We take as $b_1 = a_1 = 1$ in all figures. (a) for $B_3 = 500\,\mathrm{GeV}$ and $b_2 \simeq 4.19$. (b) for $B_3 = 300\,\mathrm{GeV}$ and $b_2 \simeq 4.01$. These values of b_2 lead to $\Delta_{B_3} = 10$ when $b_1 = a_1 = a_2 = a_3 = 1$ in each value of B_3. (c) for $B_3 = 500\,\mathrm{GeV}$ and $b_2 = 1$. (b) for $B_3 = 300\,\mathrm{GeV}$ and $b_2 = 1$.

in figure 3. For the messenger scale of $M = 10^6\,\mathrm{GeV}$, there is no region corresponding to $\Delta_{B_3} \sim 10$ and $m_h \geq 114.4\,\mathrm{GeV}$ when the gluino mass is relatively light such as $B_3 \sim 300\,\mathrm{GeV}$. Figure 4 corresponds to the enlargement of figure 3. All favorable regions shown in figures also satisfy the experimental bound of the top squark mass, $m_{\tilde{t}_1} \geq 95.7\,\mathrm{GeV}$. The allowed regions become generally narrow as the messenger scale becomes lower. Especially, the values of a_2 and a_3 are constrained to only negative values

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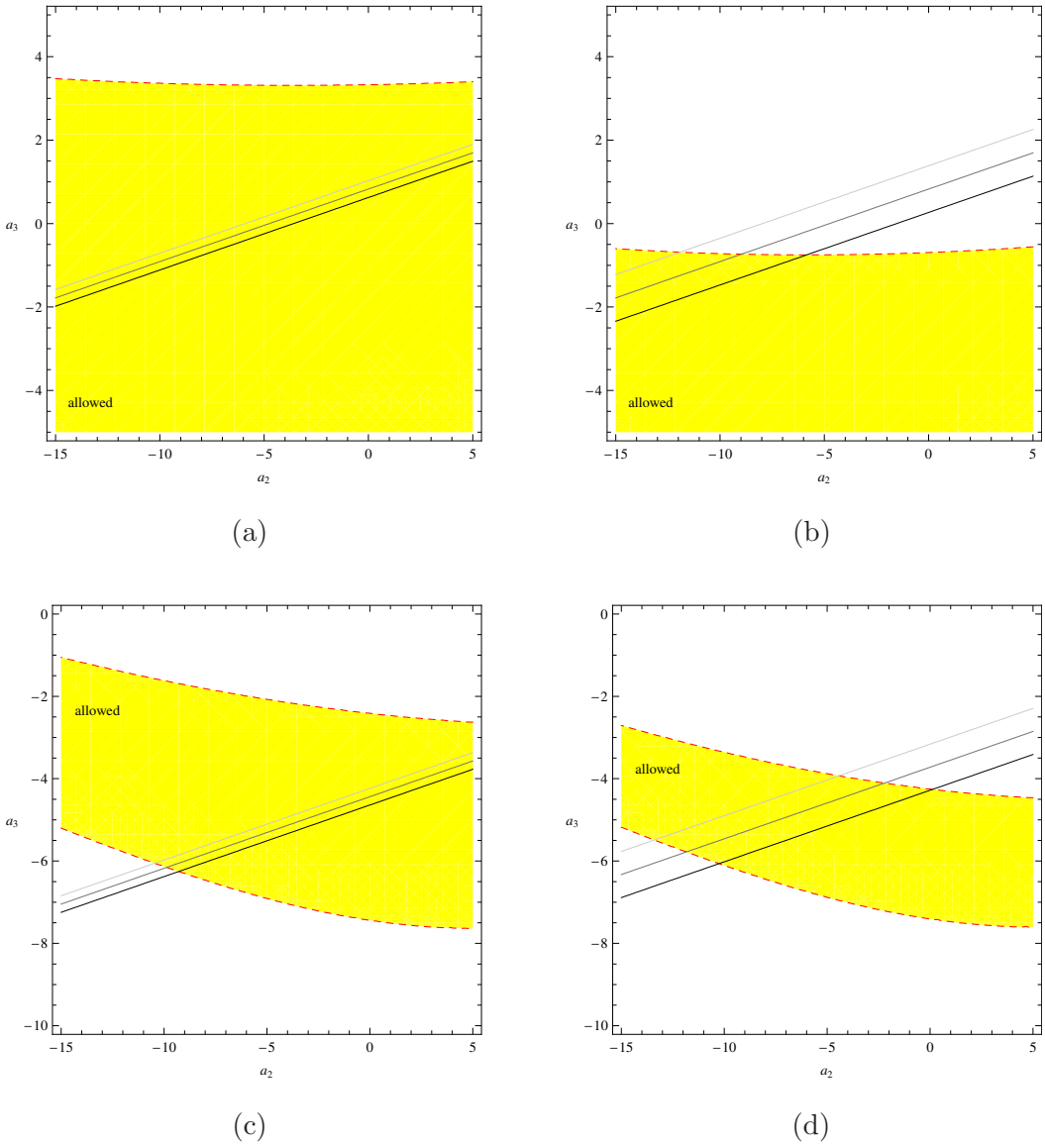

Figure 2. The same lines and curve as figure 1 but in the case (ii)

for $(M, B_3, b_2) = (10^6 \text{ GeV}, 500 \text{ GeV}, 1)$ shown in figure 3 (d). This means that tachyonic scalar masses are required at the messenger scale to reduce the fine-tuning in the context of the GGM.

Toward for future model building of the GGM to relax the fine-tuning problem, we present a summary of a typical parameter space in tables 1, 2, 3. When we fix as $a_3 = 1$, which is always allowed in all cases of the messenger scale, the favorable regions are obtained as

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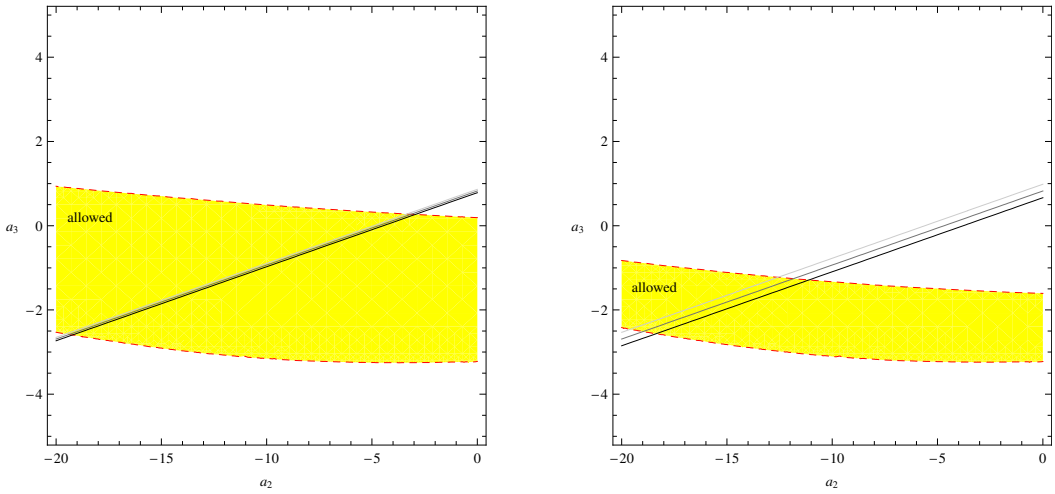
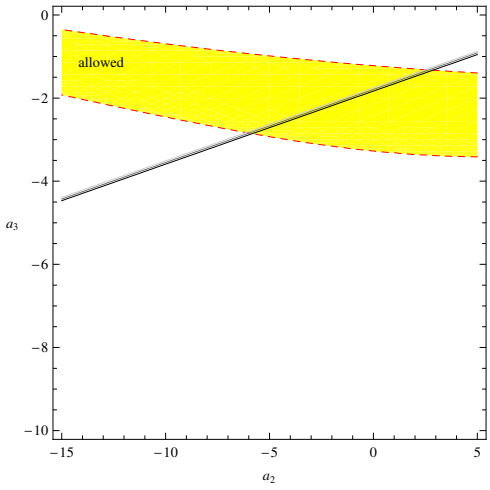
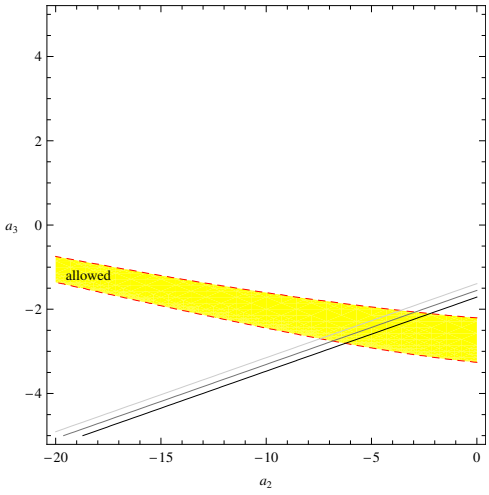


(a)

(b)



(c)



(d)

Figure 3. The same lines and curve as figure 1 but (a) and (c) for $B_3 = 1\,\text{TeV}$ and (b) and (d) for $B_3 = 500\,\text{GeV}$ in the case (iii).

(i) $M = \Lambda_{\text{GUT}}$

$$0 \lesssim b_2 \lesssim 7 \text{ for } -100 \lesssim a_2 \lesssim 40 \text{ and } B_3 = 500 \text{ GeV}, \tag{4.32}$$

$$0 \lesssim b_2 \lesssim 6.5 \text{ for } -100 \lesssim a_2 \lesssim 40 \text{ and } B_3 = 300 \text{ GeV}, \tag{4.33}$$

(ii) $M = 10^{10}\,\text{GeV}$

$$2 \lesssim b_2 \lesssim 8 \text{ for } a_3 = 1, \ -100 \lesssim a_2 \lesssim 20, \text{ and } B_3 = 500 \text{ GeV}, \tag{4.34}$$

$$0 \lesssim b_2 \lesssim 6.5 \text{ for } a_3 = 1, \ -100 \lesssim a_2 \lesssim -10, \text{ and } B_3 = 300 \text{ GeV}, \tag{4.35}$$

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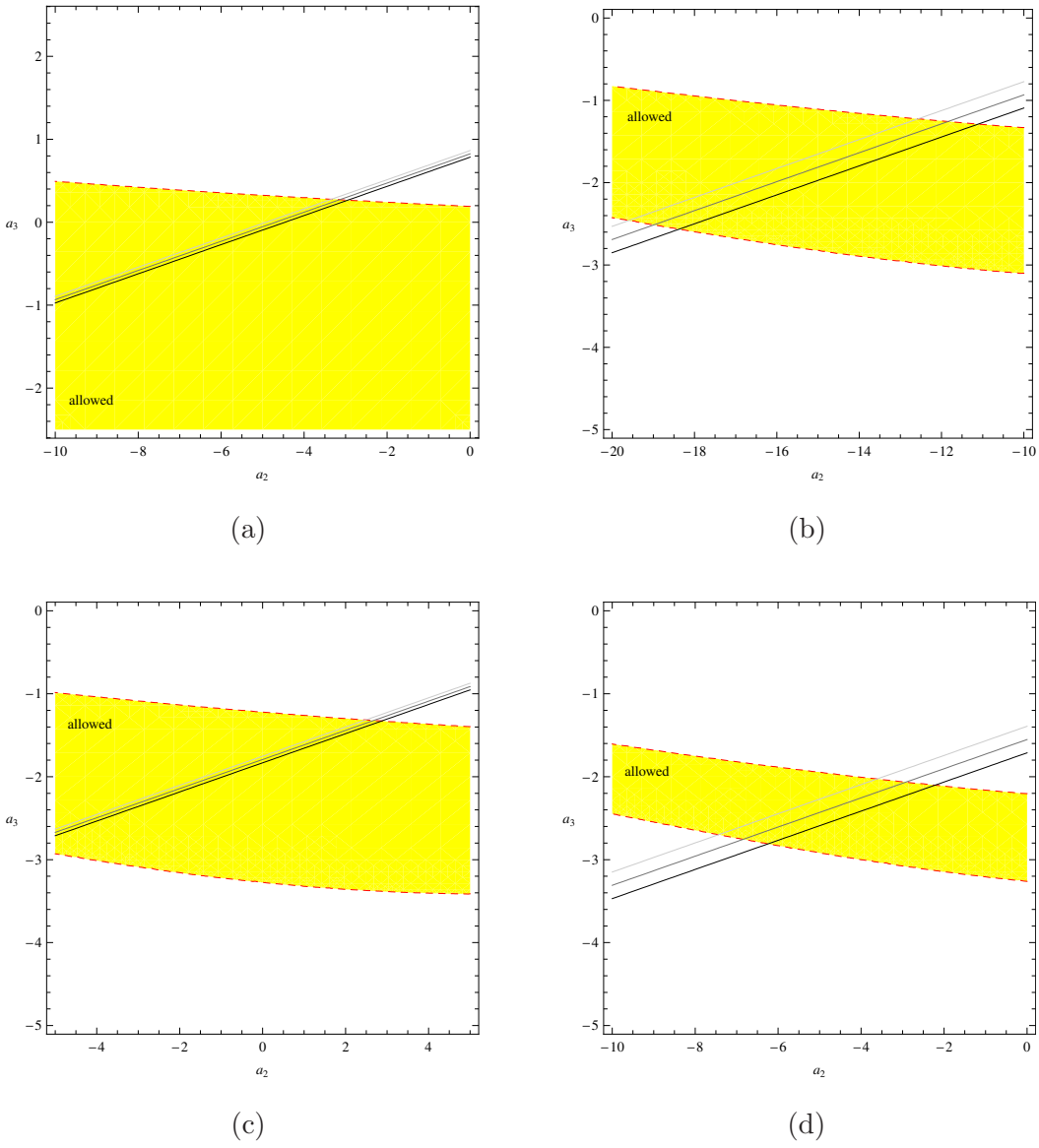

Figure 4. The enlargements of figure 3.

(iii) $M = 10^6\,\mathrm{GeV}$

$$5 \lesssim b_2 \lesssim 11 \text{ for } a_3 = 1,\ -100 \lesssim a_2 \lesssim 5, \text{ and } B_3 = 1000\ \mathrm{GeV}, \tag{4.36}$$

$$5 \lesssim b_2 \lesssim 10 \text{ for } a_3 = -1,\ -100 \lesssim a_2 \lesssim -10, \text{ and } B_3 = 500\ \mathrm{GeV}. \tag{4.37}$$

Our results show that a certain ratio between the gluino mass and wino mass is favorable. Also, the tachyonic initial condition for stop masses at the messenger scale would be favorable, in particular in the low messenger scale scenario. For $M < 10^6\,\mathrm{GeV}$, the favorable region corresponds to only negative values of both a_2 and a_3. The A-term A_t plays a role in this result. Its initial value vanishes at M, i.e. $A_t(M) = 0$, and its value at M_Z is generated by RG effect as eqs. (4.8),(4.13),(4.18), which are determined mainly by

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B_3 [GeV]	b_2	a_2	a_3	Figure
500	4.19	$-45 \lesssim a_2 \lesssim 5$	$-10 \lesssim a_3 \lesssim 2$	2 (a)
300	4.01	$-40 \lesssim a_2 \lesssim 10$	$-9 \lesssim a_3 \lesssim 5$	2 (b)
500	1	$-10 \lesssim a_2 \lesssim 50$	$-15 \lesssim a_3 \lesssim 0$	2 (c)
300	1	$-10 \lesssim a_2 \lesssim 50$	$-12 \lesssim a_3 \lesssim 3$	2 (d)

Table 1. Favorable parameter regions for (i) $M = \Lambda_{\rm GUT}$.

B_3 [GeV]	b_2	a_2	a_3	Figure
500	4.12	$-35 \lesssim a_2 \lesssim 15$	$-5 \lesssim a_3 \lesssim 0$	3 (a)
300	3.86	$-35 \lesssim a_2 \lesssim -10$	$-5 \lesssim a_3 \lesssim -1$	3 (b)
500	1	$-10 \lesssim a_2 \lesssim 10$	$-6 \lesssim a_3 \lesssim -3$	3 (c)
300	1	$-12 \lesssim a_2 \lesssim -2$	$-5 \lesssim a_3 \lesssim -4$	3 (d)

Table 2. Favorable parameter regions for (ii) $M = 10^{10}\,\text{GeV}$.

B_3 [GeV]	b_2	a_2	a_3	Figure
1000	4.10	$-20 \lesssim a_2 \lesssim -5$	$-2 \lesssim a_3 \lesssim 0$	4 (a)
500	3.93	$-20 \lesssim a_2 \lesssim -10$	$-2 \lesssim a_3 \lesssim -1$	4 (b)
1000	1	$-5 \lesssim a_2 \lesssim 5$	$-2 \lesssim a_3 \lesssim -1$	4 (c)
500	1	$-5 \lesssim a_2 \lesssim -3$	$-3 \lesssim a_3 \lesssim -2$	4 (d)

Table 3. Favorable parameter regions for (ii) $M = 10^6\,\text{GeV}$.

B_3 and B_2. However, a value of $|A_t(M_Z)|$ at M_Z is smaller as the messenger scale becomes lower, because the RG effects become smaller. On the other hand, a large value of the stop mixing $|A_t/m_{\tilde{t}}|$ is favorable to increase the Higgs mass, m_h. Thus, if a value of $|A_t(M_Z)|$ is small, we have to decrease a value $m_{\tilde{t}}$ to obtain a large stop mixing $|A_t/m_{\tilde{t}}|$. That can be realized by imposing the tachyonic initial condition of the stop mass at M.

We also give the mass spectra of gluino, wino, and stop for typical parameters of the favorable regions in table 4. We find that the smallest masses of wino and stop are realized in the case (i) with $B_3 = 300\,\text{GeV}$, $a_3 = -1$, and $a_2 = 30$ as $M_2 \simeq 517$ GeV and $m_{\tilde{t}} \simeq 555\,\text{GeV}$. On the other hand, the largest masses of wino and stop are given in the case (iii) with $B_3 = 10^3\,\text{GeV}$, $a_3 = 1$ and $a_2 = -50$ as $M_2 \simeq 7150\,\text{GeV}$ and $m_{\tilde{t}} \simeq 2420\,\text{GeV}$.

5 $\mu - B$ problem

Here, we comment on the μ-term and B-term. How to generate the μ-term and B-term is another important issue. Within the framework of the gauge mediation, a simple mechanism to generate the μ-term would lead to

$$\frac{\mu B}{\mu^2} = \mathcal{O}(16\pi^2). \tag{5.1}$$

M [GeV]	B_3 [GeV]	a_3	a_2	b_2	M_3 [GeV]	M_2 [GeV]	$m_{\tilde{t}}$ [GeV]
2×10^{16}	500	1	-50	5.71	1500	2450	1180
2×10^{16}	500	-1	-50	4.22	1500	1810	863
2×10^{16}	500	-1	-1	1.31	1500	563	865
2×10^{16}	500	-1	40	1.42	1500	609	803
2×10^{16}	300	1	-50	5.59	900	1440	722
2×10^{16}	300	1	1	4.01	900	1030	616
2×10^{16}	300	1	30	2.63	900	677	522
2×10^{16}	300	-1	-50	5.36	900	1380	704
2×10^{16}	300	-1	1	3.67	900	947	615
2×10^{16}	300	-1	30	2.01	900	517	555
10^{10}	500	1	-50	6.48	1500	2790	1280
10^{10}	500	1	1	4.12	1500	1770	1230
10^{10}	500	1	10	3.58	1500	1520	1210
10^{10}	500	-1	-50	6.04	1500	2590	1150
10^{10}	500	-1	1	3.34	1500	1440	1110
10^{10}	500	-1	10	2.55	1500	1100	1100
10^{10}	300	1	-30	5.50	900	1420	755
10^{10}	300	-1	-50	5.89	900	1510	682
10^{10}	300	-1	-10	3.83	900	987	669
10^{6}	1000	1	-50	8.32	3000	7150	2420
10^{6}	1000	-1	-50	7.59	3000	6520	1800
10^{6}	1000	-1	1	2.21	3000	1900	1800
10^{6}	500	-1	-30	5.97	1500	2570	897

Table 4. Mass spectra of gluino, wino, and stop in typical parameter space.

This ratio would cause a problem if

$$\mu^2 \sim m_{H_u}^2(M_Z),\ m_{H_d}^2(M_Z). \tag{5.2}$$

When both (5.1) and (5.2) hold, we could not realize the successful EW symmetry breaking. That is often called the $\mu - B$ problem of the gauge mediation.

However, in the previous section, we have studied models with spectra different from eq. (5.2). From the viewpoint of fine-tuning between μ^2 and $m_{H_u}^2(M_Z)$, the favorable spectrum is that $\mu, |m_{H_u}(M_Z)| = \mathcal{O}(100)$GeV and other SUSY breaking masses are of order of a few TeV. Indeed, if we can obtain the following hierarchy,

$$\mu^2 \sim m_{H_u}^2 \ll \mu B \ll m_{H_d}^2, \tag{5.3}$$

we can realize the successful EW symmetry breaking. It has been already pointed out in [79] that the above hierarchy would be favorable in the gauge mediation. Also, such a pattern has been studied within the framework of the TeV scale mirage scenario [34], i.e. the mass pattern II.

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This pattern of hierarchy can be realized in our analyses. A relatively large B_2 is favorable to obtain a large m_{H_d} seen as in (4.7), (4.12), and (4.17). For example, if we take $M = 10^6\,\mathrm{GeV}$, $B_3 = 1\,\mathrm{TeV}$, $a_3 = 1$, $a_2 = -50$, and $b_2 \simeq 8.32$, which lead to $\Delta_{B_3} = 10$, $M_3 \simeq 3\,\mathrm{TeV}$, and $M_2 \simeq 7.15\,\mathrm{TeV}$, and $m_{\tilde{t}} = 2.42\,\mathrm{TeV}$, we obtain

$$m_{H_d}^2(M_Z) \simeq 2.89^2\ \mathrm{TeV}^2. \tag{5.4}$$

By using

$$\sin 2\beta = \frac{2\mu B}{2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2}, \tag{5.5}$$

with $\tan\beta = 10$, the above value of $m_{H_d}^2(M_Z) \simeq 2.89^2\mathrm{TeV}^2$ determines the value of μB as

$$\mu B \simeq 911^2\ \mathrm{GeV}^2. \tag{5.6}$$

That is, we have $\mu B/\mu^2 = \mathcal{O}(100)$ for $\mu \sim 100\,\mathrm{GeV}$. Such a ratio $\mu B/\mu^2$ could be realized by a simple mechanism to generate the μ-term and B-term (5.1).[5] Therefore, this parameter set, which relaxes the fine-tuning problem, would also be favorable from the viewpoint of the $\mu - B$ problem.

6 Summary

We have studied the fine-tuning problem in the context of general gauge mediation. Numerical analyses toward for relaxing the fine-tuning in the problem have been presented. We analysed the problem in typical three cases of the messenger scale, that is, GUT ($2 \times 10^{16}\,\mathrm{GeV}$), intermediate ($10^{10}\,\mathrm{GeV}$), and relatively low energy ($10^6\,\mathrm{GeV}$) scales. In each case, the parameter space with less fine-tuning such as 10% has been found. It has also been shown that the favorable region becomes narrow as the messenger scale becomes lower, especially, $-10 \lesssim a_2 \lesssim 50$ and $-15 \lesssim a_3 \lesssim 0$ are allowed for $B_3 = 500\,\mathrm{GeV}$ and $b_1 = b_2 = a_1 = 1$ in the case (i), $-10 \lesssim a_2 \lesssim 10$ and $-6 \lesssim a_3 \lesssim -3$ for $B_3 = 500\,\mathrm{GeV}$ and $b_1 = b_2 = a_1 = 1$ in the case (ii), and $-5 \lesssim a_2 \lesssim -3$ and $-3 \lesssim a_3 \lesssim -2$ for $B_3 = 500\,\mathrm{GeV}$ and $b_1 = b_2 = a_1 = 1$ in the case (iii). Our results imply that certain ratios between the gluino and wino masses as well as scalar masses are favorable to relax the fine-tuning problem. Also, tachyonic initial conditions of scalar masses are favored, in particular in the relatively low messenger scale scenario. Furthermore, the type of spectra with $\mu \approx 100\,\mathrm{GeV}$ and a few TeV of other SUSY breaking masses is also favorable from the viewpoint of the $\mu - B$ problem. Thus, it would be important to construct explicit models, which realize certain ratios among gaugino and scalar masses.

Note to be added. While this paper was being completed, ref. [80] appeared, where also fine tuning in the GGM was studied.

[5] In this example, we use the large ratio of $|a_2/a_3|$, i.e. $a_3 = 1$ and $a_2 = -50$, but realization of such a ratio may not be straightforward in explict model building. As another example, we take $M = 10^{10}\,\mathrm{GeV}$, $B_3 = 500\,\mathrm{GeV}$, $a_3 = 1$, $a_2 = 1$, and $b_2 \simeq 4.12$. This example leads to $m_{H_d}(M_Z) \simeq 1.06\,\mathrm{TeV}$ and $\mu B/\mu^2 = \mathcal{O}(10)$. That would lead to the above hierarchy (5.3) although the gap of hierarchy would be smaller than the first example.

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Acknowledgments

T. K. is supported in part by the Grant-in-Aid for Scientific Research No. 20540266 from the Ministry of Education, Culture, Sports, Science and Technology of Japan. T. K. and R. T. are also supported in part by the Grant-in-Aid for the Global COE Program "The Next Generation of Physics, Spun from Universality and Emergence" from the Ministry of Education, Culture,Sports, Science and Technology of Japan.

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