Published for SISSA by Springer

Received: October 21, 2010
Revised: February 14, 2011
Accepted: March 8, 2011
Published: March 25, 2011

Exotic particles below the TeV from low scale flavour theories

Carlos A. Savoy and Marc Thormeier

Institut de Physique Théorique, CEA-Saclay,
91191 Gif-sur-Yvette, France[1]

E-mail: carlos.savoy@cea.fr, marc.thormeier@cea.fr

Abstract: A flavour gauge theory is observable only if the symmetry is broken at relatively low energies. The intrinsic parity-violation of the fermion representations in a flavour theory describing quark, lepton and higgsino masses and mixings generically requires anomaly cancellation by new fermions. Benchmark supersymmetric flavour models are built and studied to argue that: *i*) the flavour symmetry breaking should be about three orders of magnitude above the higgsino mass, enough also to efficiently suppress FCNC and CP violation coming from higher-dimensional operators; *ii*) new fermions with exotic decays into lighter particles are often predicted around the TeV region.

Keywords: Supersymmetry Phenomenology

[1] Laboratoire de la Direction des Sciences de la Matière du CEA et Unité de Recherche Associé au CNRS (URA2306).

doi:10.1007/JHEP03(2011)128
JHEP03(2011)128

Contents

1 Introduction

Notwithstanding our fair knowledge of quark masses, mixings and CP phases and strong constraints on neutrino ones, and the profusion of models in various frameworks, we have no cogent explanation for their origins. Even worse, most of the acceptable models are not directly testable as they do not predict any low energy energy effect but the fermion mass spectra they were designed for — some nice relations are encouraging but cannot quite prove a model. In this paper we focus on 4D perturbative supersymmetric gauged flavour theories — these five assumptions being relevant in our analysis — and claim that, under some circumstances, these models might predict new characteristic states within the reach of the LHC.[1] The Standard Model (no-suprrsymmetric) counter part is briefly commented on in the last section below.

Flavour symmetries are chiral, i.e., the parity conjugated states in the small mass operators of quarks, leptons, and higgsinos (μ-term) have different flavour charges so that the masses are controlled by the amount(s) of flavour symmetry breaking(s) and the charge

[1] Most of the results here were presented at the Planck2008 conference but were not published.

differences between parity conjugated states, which we call flavour-chiralities herein. These flavour-chiralities, as introduced to explain the fermion masses, would generate an anomalous coupling of flavour gauge bosons to photons and gluons. We argue that, in low energy abelian flavour models, anomaly cancellation generically requires a few extra charged and/or coloured particles whose flavour-chiralities are possibly close to the higgsino one, resulting into heavy states of mass $O(1\text{ TeV})$, in spite of a much higher flavour symmetry breaking scale. They would have peculiar decays into light states as they are required not to mix with light fermions to avoid, e.g., the destabilization of their mass matrices.

In a matter-of-fact approach, it is not necessary to impose anomaly compensation within the Standard Model (SM) or the Minimal Supersymmetric SM (MSSM) fermion field content. Just as some of their masses are reduced by the flavour symmetry, so could some states that are parity-symmetric with respect to the electroweak interactions, be flavour-chiral, get their masses suppressed with respect to the cutoff scale and contribute to anomaly-compensation below it. This is the generic case. If the cutoff is high enough, they can be integrated out together with the other flavour theory components, but it is not quite so when the cutoff occurs at relatively low energies.

Effective theories based on flavour symmetries are characterized by a cutoff scale Λ and the scales where the flavour symmetries are broken down, $\epsilon\Lambda$, $\epsilon'\Lambda$, $\ldots$. In the spirit of the Frogatt-Nielsen (FN) idea [1, 2], non-abelian flavour symmetries more naturally explain empirical relations between masses and mixings,[2] while abelian symmetries are suitable to deal with hierarchies. Here we consider gauged continuous symmetries — in particular to avoid Nambu Goldstone bosons — but also discrete symmetries that can result from symmetry breaking of the continuous flavour symmetry.

Some mostly dangerous baryon and lepton number violating operators can be eliminated by exact discrete symmetries like R or matter parity [16], baryon triality [17, 18] or proton hexality [19], that survive as relics of the flavour symmetry breaking. The last encompass the others and is implemented here, mainly to avoid proton decay dimension 5 operators. However, it is well-known that four-fermion operators associated to FCNC and CP violating processes must be suppressed by an effective cutoff at least $O(10^4\text{ TeV})$ to comply with the experimental limits [20]. This has been viewed as a generic lower limit on the cutoff Λ of low-energy effective flavour theories [21]. Therefore, when discussing flavour breaking at a low scale we mean a cutoff close to this bound, actually a bit lower due to flavour symmetries.

The MSSM Higgs sector being parity symmetrical, i.e., vector-like with respect to the electroweak symmetry, the action of the flavour symmetry on higgsinos must be such that their masses, the μ-term, is reduced by the FN mechanism from the natural value $O(\Lambda)$ to the effective supersymmetry breaking scale $O(\text{TeV})$. Hence, the higgsino flavour-chirality must be relatively large and give commensurate contributions to anomalies. Actually, this

[2] There is a large variety of those models in the literature; we can only quote a small part here [3–14]. They are not all consistent with the present data on fermion masses and mixings [15].

JHEP03(2011)128

is an example of a vector-like fermion that would be light enough to be detected as much as it gives a sizable contribution to the anomalies of the flavour symmetry! In this paper, we investigate whether anomaly cancellation might predict other coloured and/or charged states along the same lines. Notice that higgsinos do not mix to leptons because they are even under matter parity (odd under R-parity) and we shall generalize this property to other possible vector-like states by defining the discrete symmetry just mentioned.

We simplify our approach by picking a single $\mathrm{U}(1)_X$ flavour group broken at a scale $\epsilon\Lambda$ so that a coupling or a mass is reduced by a factor ϵ^n, where n is the flavour charge of the corresponding effective operator. We further require that a combination of the flavour and the weak hypercharge transformations contains the exact discrete symmetry that survive at low energies. If it is anomalous, one must rely on the Green-Schwarz cancellation mechanism [22], which assumes an underlying string theory. Then, the Dine-Seiberg-Wen-Witten mechanism [23–25] ensures the breaking of $\mathrm{U}(1)_X$ and defines the scale $\epsilon\Lambda$ a bit below the Planck scale [26–32]. However, this makes the search for direct signals of this $\mathrm{U}(1)_X$ moot.

Only if the $\mathrm{U}(1)_X$ is non-anomalous, one can adjust the flavour theory such that $\epsilon\Lambda$ is much lower than the Planck scale. It is known that anomaly cancellation within the MSSM field content in abelian flavour models is tightly constrained by the quark and lepton masses and mixings [28]. From the balance among the value for Λ, the types and the masses of the newly introduced heavy particles, we find, under (presumably) reasonable assumptions, that Λ should be at least $O(10^3\ )$TeV, while some new states could get much lower masse, plausibly within the LHC reach.

In order to avoid stable heavy "quarks" or "leptons", the models are also selected by the condition that heavy states decay into MSSM states, which is naturally implemented by the exact residual discrete symmetries.The new uncoloured weak doublets, are produced like heavy (s)leptons, but decay into three (s)quarks, one of each family! Actually, the "easier" signal at the LHC would be the production of a heavy coloured weak-isosinglet "squark" with more model dependent signatures: two quarks or one lepton plus one or two (s)quarks (the last possibility being favoured)!

Experiments on FCNC and CP violations impose severe suppressions of the coefficients of some dimension five operators in the effective superpotential and on dimension six operators in the effective superpotentials. The strongest bound comes from the latter once the flavour gauge boson is integrated out as already mentioned. The exchange of the new heavy quarks can also produce FCNC effects so excluding most of one of the three types of models.

In the next section, our requirements are stated and the effective theory is implemented in the framework of a single flavour charge. The realistic choices of flavour charges are selected from the fermion masses and the cancellation of anomalies via heavy fermions. The new exotic states are dealt with in section 3, where their masses are estimated and their decay modes established, for the abelian flavour benchmark model. Section 4 discusses FCNC constraints before and after integrating out the new heavy particles. The closing section presents a few conclusions, as well as comments on shortcomings and generalizations.

JHEP03(2011)128

2 An all-in-U(1) model

In this section, we construct and analyse a benchmark model based on the suggested scenario. Let us first recollect seven issues to be addressed by a supersymmetric flavour theory: **1)** the hierarchy among the SM fermion masses, the hierarchy among the entries of the CKM matrix and the value of the CP violation phase, δ; **2)** the contrasting pattern of the neutrino mass matrix, with (at least two) less hierarchical eigenvalues and two large mixing angles; **3)** the μ-problem: the higgsino mass must be suppressed from the cutoff scale down to the level of the supersymmetry breaking masses; **4)** renormalizable R-parity violating superpotential operators that cause the emergence of L and/or B violating terms and, in particular, those that destabilize the proton; **5)** non-renormalisable R-parity conserving superpotential operators (like $QQQL$) giving rise to L and/or B violations as well; **6)** non-renormalisable operators in the superpotential (like $UQDQ$) and in the Käler potential (like $Q^\dagger QD^\dagger D$) leading to FCNC and CP violations; **7)** flavour mixings and CP-violating phases in the supersymmetry breaking of the MSSM, some of them restricted by tight upper bounds from FCNCs and CP violation searches. The last, so-called supersymmetric flavour problem, is not addressed here since it strongly depends on the supersymmetry breaking and mediation mechanism, which is not specified here.[3] CP violations cannot be generated in the simple flavour sector discussed here and, in the absence of a CP theory, we consider only limits that would require a very small phase.

We try and choose the simplest flavour symmetry, consisting in a single abelian charge, denoted by X. It is hopeless to reduce proton decay to below the experimental bound, therefore we forbid it by assuming an exact Z_3 symmetry (baryon triality), that excludes supersymmetric operators like $QQQL$ or UDD. Lepton number conservation can be introduced through a Z_2 (matter parity), so to allow for neutrino masses. Their product is a Z_6 (proton hexality). This exact (gauged) discrete symmetries should result from the breaking of a continuous gauged anomaly-free symmetry and we make the economical and elegant choice that it coincides with $\mathrm{U}(1)_X$. More precisely, in general, it is a discrete subgroup of $\mathrm{U}(1)_X \otimes \mathrm{U}(1)_Y$ that leaves the Higgses invariant. This solution has a price: this Z_6 does not commute with SU(5), but, in practice, Abelian flavour models are only marginally consistent with grand-unification anyway.

In order to break the flavour symmetry we need flavoured SM singlets, or flavons, with both signs of X to allow for a symmetry breaking superpotential, and also for anomaly cancellation as discussed later on. We assume the anomaly-free $\mathrm{U}(1)_X$ flavour symmetry to be broken by a vector-like pair of flavon chiral superfields (A, B) with X-charges ∓ 1 into the residual discrete Z_6 symmetry. The breaking scale is given by the $v.e.v$'s

$$\epsilon \equiv \frac{\langle A \rangle}{\Lambda} = \frac{\langle B \rangle}{\Lambda}\,,$$

that result from a generic superpotential $W(A,B) = \Lambda AB(\epsilon + f(AB/\Lambda^2)$ where the small parameter ϵ will be fixed by the fermion mass matrices to be close to the Cabibbo angle.

[3] An inverted hierarchy in the squark and slepton mass differences could provide tests for the flavour model (see, e.g., [30]) but since they are already tightly constrained by FCNC experiments, they would be difficult to measure.

Within the Frogatt-Nielsen mechanism, the coefficient of an operator $\mathcal{O}$ in the effective Lagrangian below the flavour symmetry breaking is suppressed by a factor $\epsilon^{|\mathcal{X}_{\mathcal{O}}|}$, where the chirality $\mathcal{X}_{\mathcal{O}}$ is the sum of the X-charges of the fields in $\mathcal{O}$, since the lowest dimension corresponding invariant operator has $|\mathcal{X}_{\mathcal{O}}|$ additional A or B flavon fields. Hence the basic parameters in the Lagrangian are Λ, ϵ and, rather than the X-charges, the X-chirality matrices defined by the sum of the X eigenvalues of fermions with the same electric charge and colour, and their charge conjugated states, $\mathcal{X}_f = X(f) + X(f^c)$. Indeed, the observed flavour physics involve B and L conserving operators because of the exact discrete symmetry.

The first step is to define the action of the anomaly-free Z_6 symmetry on the MSSM fields and then write the Z_6-invariant MSSM effective model. The charges must be consistent with the presence of several operators in the superpotential, whose invariance under Z_6 means that the corresponding charge combinations must be integers. Of course, they must be family-independent to allow for family mixing. The appropriate choice of the charges can be written as:

$$\begin{aligned} &Z_Q = 0\,, && Z_U = Z_E = Z_{H_d} = 1/6\,, \\ &Z_L = -2/6 && Z_D = Z_{H_u} = -1/6\,. \end{aligned} \tag{2.1}$$

The X-charges are given by $X_i = \text{integer} + Z_i$. This Z_6 is broken by the Higgs $v.e.v$'s but the combination $X' = X + Y/3 = \text{integer} + Z'_i$ is such that $Z'_{H_i} = 0$ and so defines the exact abelian discrete symmetry that imposes the needed selection rules. The charges are simply $Z' = 1/18 = B/6$ for any quark, $Z' = 1/2 = L/2$ for any lepton, and the opposite ones for the C-conjugated states. For completeness, this is explained in the appendix.

This discrete symmetry dictates the selection rules that define the effective Lagrangian beneath the flavour symmetry breaking scale $\epsilon\Lambda$, including the terms containing the new fields to be added in the next sections. The general superpotential of the MSSM superfields with operators up to dimension five consistent with the Z_6 charges in (2.1) is:[4,5]

$$\begin{aligned} W = \mu\, H_d H_u \,+\, {Y^u}_{ij}\, Q^i H_u U^j \,+\, {Y^d}_{ij}\, Q^i H_d D^j \,+\, {Y^e}_{ij}\, L^i H_d E^j \\ +\, \frac{C^{qq}_{ijkl}}{\Lambda}\, U^i Q^j D^k Q^l \,+\, \frac{C^{qe}_{ijkl}}{\Lambda}\, U^i Q^j E^k L^l \,+\, \frac{C^h}{\Lambda}\,(H_d H_u)^2 \,+\, \frac{C^{ij}_{hl}}{\Lambda}\, L^i H_u L^j H_u. \end{aligned} \tag{2.2}$$

The orders of magnitude of the coefficients of the bilinear (μ-term), trilinear (Yukawa couplings to the Higgses) and quadrilinear couplings are given by powers of the parameter ϵ defined by the modulus of the sum of charges of the corresponding superfields (because of the symmetry $X \to -X$). These charge combinations are fixed by the phenomenology of the corresponding operators that we now turn to discuss.

[4] Notations are quite standard MSSM ones. As usual the X-charges are denoted by the same symbol as the left-handed fermions ($X(f) = f$) of the corresponding chiral multiplets, i, $j = 1, 2, 3$ are family indices.

[5] Possible dimension five operators (trilinear terms) in the Kähler potential can be transposed into the superpotential by an analytic field redefinition in the effective theory.

JHEP03(2011)128

2.1 SM fermion masses and mixings

The trilinear terms $Q^i H_d D^j$, $Q^i H_u U^j$ and $L^i H_d E^j$ yield the fermion mass and mixing hierarchies so that, with

$$q_i + h_u + \bar{u}_j = \mathcal{X}^u_{ij}, \quad q_i + h_d + \bar{d}_j = \mathcal{X}^d_{ij}, \quad l_i + h_d + \bar{e}_j = \mathcal{X}^e_{ij}.$$

then $\mathcal{X}^{u,d,e}{}_{ij} \in \mathbb{Z}$, and the Yukawa coupling matrices are

$$Y^u{}_{ij} \sim \epsilon^{|\mathcal{X}^u_{ij}|}, \qquad Y^d{}_{ij} \epsilon^{|\mathcal{X}^d_{ij}|}, \qquad Y^e{}_{ij} \epsilon^{|\mathcal{X}^e_{ij}|}. \tag{2.3}$$

Many of these $\mathcal{X}$'s can be specified from the known fermion masses and mixings. Because of the symmetry in the flavon sector, the results are invariant under $X \to -X$, so we choose the value of the $\mathcal{X}^u_{ij}$ and $\mathcal{X}^d_{ij}$ to be positive. The fact that all of them have the same sign comes from the strong hierarchies in quarks masses and mixings and the well-known strong correlations among them (thus only one flavon is relevant for their masses). Instead, for leptons, one must keep free the signs in the matrix elements of $\mathcal{X}^e$ as we shall prove later on. The dependence on $\tan\beta$ is taken into account by the parameter x, defined by $\tan\beta \sim \epsilon^{2-x}$. We also introduce two "fuzzy factors", y and z taking values 0 or 1, to account for some freedom in the relations. Then, with $\epsilon \sim \theta_C$, the Cabbibbo angle, the charged fermion masses lead to the following choices:

$$\mathcal{X}^u = \begin{pmatrix} 8 & 5+y & 3+y \\ 7-y & 4 & 2 \\ 5-y & 2 & 0 \end{pmatrix} \qquad \mathcal{X}^d = \begin{pmatrix} 4+x & 3+x+y & 3+x+y \\ 3+x-y & 2+x & 2+x \\ 1+x-y & x & x \end{pmatrix}. \tag{2.4}$$

We assume a hierarchical structure in Y^e that reproduces the charged lepton mass ratios,

$$\mathrm{diag}\,\mathcal{X}^e = \{\pm(4+x+z)\,,\ \pm(2+x)\,,\ \pm x\,,\}\,, \tag{2.5}$$

since the diagonal terms (or the trace) mostly appear in the relations below.

2.2 Effective neutrino masses and mixings

The lepton-higgsino X-chiralities, $l_i + h_u$, controlling both R-parity and neutrino masses, are defined similarly to the higgsino one that is in charge of the μ-term. The quadrilinear term $L^i H_u L^i H_u$ gives rise to the effective neutrino mass matrix,

$$\mathcal{M}_{\nu_{ij}} \sim \epsilon^{|\mathcal{X}^\nu_{ij}|}\,\frac{(174\ \mathrm{GeV})^2}{\Lambda}, \qquad \mathcal{X}^\nu_{ij} = l_i + h_u + l_j + h_u \tag{2.6}$$

The $\mathcal{X}^\nu_{ij} \in \mathbb{Z}$ must be odd by the Z_6 symmetry and large enough to suppress $(174\ \mathrm{GeV})^2/\Lambda$ down to the typical neutrino mass eigenvalues. Within the indeterminacy inherent to the model, we take a texture consistent with the small hierarchy and large mixings of the MNS matrix,

$$\mathrm{diag}\,\mathcal{X}^\nu = \pm\,(\mathcal{X}_\nu + 2v\,.\,\mathcal{X}_\nu\,,\ \mathcal{X}_\nu\,)\,, \qquad (v = 0, 1) \tag{2.7}$$

Hence, the mass parameter of atmospheric neutrino oscillations must satisfy

$$\epsilon^{\mathcal{X}_\nu} \sim m_{\mathrm{atm}}\Lambda/(174\ \mathrm{GeV})^2 \sim \epsilon^{13}\Lambda/(1000\ \mathrm{TeV}). \tag{2.8}$$

JHEP03(2011)128

2.3 μ-parameter

The bilinear term, $H_d H_u$, has a charge $h_d + h_u = \mathcal{X}_\mu \in \mathbb{Z}$, so the effective higgsino mass is naturally related to the cutoff by:

$$\mu \sim \epsilon^{|h_d+h_u|} \cdot \Lambda = \epsilon^{|\mathcal{X}_\mu|} \cdot \Lambda \tag{2.9}$$

and must be close to the MSSM scale, $O(\text{TeV})$, while Λ must be much larger to avoid FCNC and CP flavour problems and, anyway, for the superpotential in (2.2), to be meaningful. Therefore the higgsino X-chirality, $\mathcal{X}_\mu$ has to be large and contributes to the anomalies as displayed below. Its choice fixes the cutoff scale of the flavour model.

Of course, this is not quite a solution to the μ-problem since it does not relate the μ scale to the supersymmetry breaking one. Assuming another solution to the μ-problem, the contribution (2.9) must be subdominant. But, one cannot allow for a small contribution from (2.9) and invoke a standard Giudice-Masiero mechanism [33] because the flavour symmetry would imply a similar suppression factor with respect to the effective supersymmetric breaking scale.

Finally, note that since $H_d H_u$ exists, then so does $H_d H_u H_d H_u$, with $C_h \sim \mu^2/\Lambda^2$, which turns out to be very small and negligible to affect the electroweak symmetry breaking. And since QH_uD, QH_dD, LH_dE and H_dH_u must exist, neither $UQEL$ nor $UQDQ$ can be forbidden by flavour symmetries.

2.4 Anomaly cancellation

The next step is to fulfill the no-anomaly requirements

$$A_C = A_W = A_Y = A'_Y = 0, \tag{2.10}$$

corresponding to the vanishing of the strong, weak isospin, and the the two weak hypercharge anomalies, respectively. Since $Q_{\text{em}} = Y + T_3$, has vector-like representations, it is convenient to replace A_Y and A'_Y by the corresponding A_{em}, more directly related to the X-chiralities fitted to fermion masses, and A'_{em} (linear in Q_{em}). As already anticipated in (2.3), and as we generalize below, anomaly cancellation without extra-states is possible at the price of having lepton X-chiralities of both signs. This could lead to (very model dependent) patterns of lepton mixing different from quark mixings.

More generally, we must introduce X-chiral strongly and weakly interacting heavy matter to compensate the anomalies generated in the MSSM sector, which has to be vector-like under the SM symmetries, to lie above the weak scale. Our choice here is to preserve the nice MSSM gauge coupling unification and asymptotic freedom. Thus, we can only add SM vector-like matter associated to quarks and leptons filling one or two $\bar{\mathbf{5}} + \mathbf{5}$ representations of SU(5)): quarks, $(\mathfrak{D}_i, \bar{\mathfrak{D}}_i)$, and leptons $(\mathfrak{L}_i, \bar{\mathfrak{L}}_i)$, $i = 1, 2$ ($\mathfrak{D}_i$ and $\mathfrak{L}_i$ have the same SM charges as D's and L's, respectively). Their total X-chiralities, are the traces of the matrices (lowercase letters are the corresponding X-charges):

$$\mathcal{X}^{\mathfrak{d}}_{ij} = (\mathfrak{d}_i + \bar{\mathfrak{d}}_j) \qquad\qquad \mathcal{X}^{\mathfrak{l}}_{ij} = \left(\mathfrak{l}_i + \bar{\mathfrak{l}}_j\right) . \tag{2.11}$$

Correspondingly, their mass matrix elements are $m^{\mathfrak{D}}_{ij} \sim \epsilon^{|\mathcal{X}^{\mathfrak{d}}_{ij}|}\Lambda$ and $m^{\mathfrak{L}}_{ij} \sim \epsilon^{|\mathcal{X}^{\mathfrak{l}}_{ij}|}\Lambda$, respectively.

JHEP03(2011)128

Here we focus on anomalies quadratic in the SM vector-like charges, namely, colour and $Q_{\rm em}$, directly related to the Yukawa matrices through the X-chiralities defined in (2.3). Gathering the contributions from the MSSM states as well as the possible new heavy states, the anomalies to be cancelled are:

$$\begin{aligned} A_C &= \operatorname{Tr}\left[\mathcal{X}^u + \mathcal{X}^d + \mathcal{X}^{\mathfrak{d}}\right] - 3\mathcal{X}_\mu\,, \\ A_{\rm em} - \frac{4}{3}A_C &= \operatorname{Tr}\left[\mathcal{X}^e - \mathcal{X}^d - \mathcal{X}^{\mathfrak{d}} + \mathcal{X}^{\mathfrak{l}}\right] + \mathcal{X}_\mu\,. \end{aligned} \tag{2.12}$$

Hence anomaly cancellation means:

$$\begin{aligned} \operatorname{Tr}\mathcal{X}^{\mathfrak{d}} &= -\operatorname{Tr}\left[\mathcal{X}^u + \mathcal{X}^d\right] + 3\mathcal{X}_\mu\,, \\ \operatorname{Tr}\mathcal{X}^{\mathfrak{l}} &= \operatorname{Tr}\mathcal{X}^{\mathfrak{d}} + \operatorname{Tr}\left[\mathcal{X}^d - \mathcal{X}^e\right] - \mathcal{X}_\mu\,. \end{aligned} \tag{2.13}$$

Since $\mathcal{X}^u$ and $\mathcal{X}^d$ are non-negative matrices, we can replace (2.4) into (2.13) to get

$$\operatorname{Tr}\mathcal{X}^{\mathfrak{d}} = 3\left(\mathcal{X}_\mu - 6 - x\right)\,, \tag{2.14}$$

First note that (2.14) excludes $\mathcal{X}_\mu \leq 3$ which leads to $m_i^{\mathfrak{D}} \ll \mu$, and, anyhow, a cutoff too low to suppress rare processes . Without X-chiral heavy matter, $A_C = 0$ implies $\mathcal{X}_\mu = 6 + x$, hence a cutoff $\Lambda \gtrsim \epsilon^{-6}\mu \sim 2\times 10^4$ TeV. Any direct evidence for the model would show up far beyond the LHC reach, yet it provides an example of the need for a vector-like fermion, the higgsino which cancels the matter fermion anomalies as much as it is light. In order to have observable TeV-scale phenomena we need to introduce appropriate heavy states and $\mathcal{X}_\mu = 4$ or 5.

Now, let us define the difference $w = \operatorname{Tr}\mathcal{X}^{\mathfrak{d}} - \operatorname{Tr}\mathcal{X}^{\mathfrak{l}}$ and replace the fit to the fermions masses into the second relation in (2.13) to obtain,

$$\operatorname{Tr}\left[|\mathcal{X}^e| - \mathcal{X}^e\right] = \mathcal{X}_\mu + z - w\,. \tag{2.15}$$

The vanishing of the other two anomalies (as well as the pure $\mathrm{U}(1)_X$ anomalies) are not so simply related to the fermion mass eigenvalues and X-chiralities and will further constrain the charges. Since they can be fractional, we study in the appendix the cancellation of the fractional part of the anomalies. The weak anomaly, A_W, imposes the choice of the Z_6 as in (2.1), while $A'_{\rm em}$, involving X^2, just requires $w = 3n$. As discussed in the next section, $n \neq 0$ tend to spoil gauge coupling unification, and we keep only $w = 0$ hereafter. Notice that, from (2.15), one of the $\mathcal{X}^e_{ii}$ must always be negative for anomaly cancellation as stated before.

The integer part of the X-charges are not uniquely defined by the cancellations of A_W and $A'_{\rm em}$, the neutrino masses and some constraints from the other mass matrices. They are important for the decay properties of the heavy states, but this is not discussed in this paper to such a level.

Finally, for the relevant values, $\mathcal{X}_\mu = 4\,, 5\,, w = 0$, one gets the solutions in table 1, where the only negative $\mathcal{X}^e_i$ in each case is displayed and the associated values of the cutoff for a range of ϵ.

w	$\mathcal{X}_\mu$	z	x	$\mathcal{X}_i^e < 0$	Λ
0	4	0	0	$\mathcal{X}_\mu^e = -2$	$(350 - 1200)$ TeV
0	4	0	2	$\mathcal{X}_\tau^e = -2$	$(350 - 1200)$ TeV
0	5	1	1	$\mathcal{X}_\mu^e = -3$	$(1.5 - 7.0) \times 10^3$ TeV

Table 1. Solutions to the anomaly conditions, see text, and corresponding cutoff scale for $\epsilon = .20 \pm .03$.

3 Exotic matter below the Tev

Several properties of the new heavy states are fixed from the conditions and results stated in the previous section. We now turn to show how they their masses could be around the TeV and their couplings to the known quarks and leptons exotic. For this sake we impose approximate gauge coupling unification and ask the discrete symmetry to forbid the heavy states to mix to SM ones in the mass matrices but without making them stable. In this sense, the new matter hold exotic baryon and lepton numbers. We also simplify the analysis by considering more generic cases and skipping more peculiar issues since our aim is to define a robust benchmark model.

3.1 Masses

If one wants to preserve gauge coupling unification at a level close to that of the MSSM, the masses of the heavy leptons, $m_{\mathfrak{L}_i}$, and heavy quarks $m_{\mathfrak{D}_i}$ cannot differ too much. Indeed, their (one-loop) contribution to the difference between the strong and weak couplings at m_Z are given in terms of their mass matrices by

$$\Delta\left(\alpha_s^{-1} - \alpha_2^{-1}\right) = \frac{1}{2\pi} \ln\det \frac{m_{\mathfrak{L}}}{m_{\mathfrak{D}}} \tag{3.1}$$

The experimental uncertainties on this difference is $O(.12)$ and for the new contributions not to be larger than this uncertainty, we should impose $0.5 \lessapprox \det(m_{\mathfrak{L}}/m_{\mathfrak{D}}) \lesssim 2$. To translates it into a condition on charges, we have to fix the ambiguity in the pairing of the indices in the X-chiralities defined in (2.11).[6] We notice that, in the absence of fine-tuning, there is always a choice — not necessarily the one adopted later on - such that $\ln\det m_{\mathfrak{L}} \simeq Tr|\mathcal{X}^{\mathfrak{l}}| \ln\epsilon$, and similarly for $m_{\mathfrak{D}}$. With these choices we get

$$-0.5 \leq Tr|\mathcal{X}^{\mathfrak{l}}| - Tr|\mathcal{X}^{\mathfrak{d}}| \leq 0.5 \tag{3.2}$$

This is not enough to obtain a definite limit on the difference w defined above, but we find no solution with $w \neq 0$ to be consistent with (3.2) and (2.15).

Basically, the LHC could detect heavy quarks and, possibly, leptons whose masses are $O(\mu)$. To discuss this condition, it is convenient to redefine the indices in such a way that $|\mathfrak{d}_2 + \bar{\mathfrak{d}}_2| = \min|\mathfrak{d}_i + \bar{\mathfrak{d}}_j|$, so that the mass eigenvalues satisfy:

$$m_{\mathfrak{D}_2} \sim \epsilon^{|\mathfrak{d}_2 + \bar{\mathfrak{d}}_2|} \qquad\qquad m_{\mathfrak{D}_1} \lesssim \epsilon^{|\mathfrak{d}_1 + \bar{\mathfrak{d}}_1|}\,. \tag{3.3}$$

[6]Indeed, in general, the C-conjugated states defined by the mass eigenstates are not eigenstates of the broken charge X, unless these states differ by their transformation under the discrete symmetry.

JHEP03(2011)128

$\mathcal{X}_\mu$	4	4	4	5	5	5	5
x	0	0	0	1	1	1	1
$\mathrm{Tr}\mathcal{X}^{\mathfrak{d}}$	-6	-6	-6	-6	-6	-6	-6
$\mathfrak{d}_1 + \bar{\mathfrak{d}}_1$	-5	-4	-3	-6	-5	-4	-3
$\mathfrak{d}_2 + \bar{\mathfrak{d}}_2$	-1	-2	-3	0	-1	-2	-3
$m_{\mathfrak{D}_1}/\mu$	ϵ	ϵ^0	ϵ^{-1}	ϵ	ϵ^0	ϵ^{-1}	ϵ^{-2}
$m_{\mathfrak{D}_2}/\mu$	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{-5}	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}
N.B.	**!**	✓	?	**!**	✓	**?**	$\frown$

Table 2. Solutions to the anomaly conditions: $\mathcal{X}_\mu$ is the higgsino X-chirality, x is related to $\tan\beta$ as defined in the text, $\mathfrak{d}_i + \bar{\mathfrak{d}}_i$ are the X-chiralities of the heavy antiquarks, $\mathrm{Tr}\mathcal{X}^{\mathfrak{d}}$ is their contribution to the anomalies. The (orders of magnitude of the) masses of the heavy "quarks/antiquarks"corresponding to each solution are given in units of the higgsino mass as powers of the Cabbibbo angle, ϵ. The symbols in the last row denote one of the following situation with respect to the heavy quark range to be scanned at the LHC: within (✓), already excluded or within (**!**), above or within (**?**) and much above ($\frown$).

From the QCD anomaly condition (2.14), and the condition that the lightest heavy quark mass must be at least $O(\epsilon\mu)$, we have

$$\mathcal{X}_\mu + 1 \geq |\mathfrak{d}_1 + \bar{\mathfrak{d}}_1| \geq \frac{3}{2}(6 + x - \mathcal{X}_\mu) \tag{3.4}$$

This implies $\mathcal{X}_\mu > 3$ to avoid conflict with experimental limits on heavy quarks, leaving only two possibilities, $\mathcal{X}_\mu = 4\,, 5\,.$ The solutions to (3.4) are displayed in the table 2, where the masses are given by their ratios to μ in units of ϵ.

Therefore, after the Higgs $X-$chirality is chosen to allow for low energy flavour symmetry, and to fulfill the anomaly cancellation relations without states too light to have escaped observation, one ends with: $\mathcal{X}_\mu = 4$ or $5\,$, corresponding to a cutoff $\Lambda \sim 600\,\mu\,$, and $\Lambda \sim 3000\,\mu$ respectively; and several possibilities for the masses of heavy "quarks and leptons" . Notice that the heavy masses are independent of the higgsino X-chirality, hence of the cutoff.

Among the four solutions there are two with one of the states within the LHC reach, namely, those with masses $O(\mu) = O(\mathrm{TeV})$ or $O(.2\,\mathrm{TeV})$. The last case is more critical in many aspects. Notice that the masses are defined modulo $O(1)$ factors, renormalization from the cutoff and, for the scalars, supersymmetry breaking masses, that are supposed to be $O(\mathrm{TeV})$ as well. This is a serious obstacle for a generic discussion of the associated phenomenology at the LHC. Of course, the solutions can be different for the $\mathfrak{L}$'s and the $\mathfrak{D}$'s, corresponding to four different possibilities.

With regards to electroweak precision tests, the fact that there is no mixing to the light fermions and no large contribution to the heavy masses from Higgs couplings, preserve these states from these constraints which in other instances can be very strong (see, e.g., [34] and references therein).

JHEP03(2011)128

$Z_{\mathfrak{D}}$	0	1/3	1/2
$Z_{\mathfrak{L}}$	0		
$\mathfrak{D}$	UD	QL	UUE
$\bar{\mathfrak{D}}$	QQ	UE	$--$
$\mathfrak{L}$	QQQ		

Table 3. Main decays of exotic quarks and leptons.

3.2 Decays

Fields with the same SM and Z_6 quantum numbers can mix in the mass matrices. We do not want $\mathfrak{L}H_u/\bar{\mathfrak{L}}H_d$, $\bar{\mathfrak{L}}L$ and $\bar{\mathfrak{D}}D$ mass couplings that might destabilize the assumed light mass matrices (though this might be an interesting alternative in some cases) and we naturally implement it by the choice of the Z_6 charges, Z_i. From eq. (2.1), this amounts to choose: $Z_{\mathfrak{L}} \neq 1/6$, $-2/6$ and $Z_{\mathfrak{D}} \neq -1/6$.

For these states to be unstable and have at least one decay channel into MSSM states, we ask for such a coupling with dimension four or five (up to quadrilinear in the superpotential, trilinear in the Kähler potential). From eq. (2.1) one selects the $\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1) \otimes Z_6$ invariant operators according to $Z_{\mathfrak{L}}$ and $Z_{\mathfrak{D}}$. The solution $Z_{\mathfrak{L}} = 0$ is unique and leads to the operator $QQQ\mathfrak{L}$ while there are three solutions for $Z_{\mathfrak{D}}$ which we list below together with the respective allowed exotic superpotential operators:

- $Z_{\mathfrak{D}} = 3/6\,,\ Z_{\mathfrak{L}} = 0$: $QQQ\mathfrak{L}$, $UU\mathfrak{D}E$, $Q\bar{\mathfrak{D}}\bar{\mathfrak{D}}\bar{\mathfrak{L}}$, $LD\bar{\mathfrak{D}}\bar{\mathfrak{L}}$; the first two cause the decay of heavies into three MSSM particles.

- $Z_{\mathfrak{D}} = 2/6\,,\ Z_{\mathfrak{L}} = 0$: $QQQ\mathfrak{L}$, $QL\mathfrak{D}$, $EU\bar{\mathfrak{D}}$, $LDH_u\bar{\mathfrak{D}}$, $DD\bar{\mathfrak{D}}\bar{\mathfrak{D}}$; the first causes the decay of $\mathfrak{L}$ into three quarks; the decay of $\mathfrak{D}$ into a quark plus a lepton happens mainly due to the second and the third Yukawa couplings.

- $Z_{\mathfrak{D}} = 0\,,\ Z_{\mathfrak{L}} = 0$: $QQQ\mathfrak{L}$, $QQ\bar{\mathfrak{D}}$, $UD\mathfrak{D}$, $Q\mathfrak{L}\mathfrak{D}$, $Q\bar{\mathfrak{D}}\bar{\mathfrak{D}}\bar{\mathfrak{L}}$, $\bar{\mathfrak{D}}D\bar{\mathfrak{L}}H_d$, $\bar{\mathfrak{D}}U\bar{\mathfrak{L}}H_u$; the first causes the decay of $\mathfrak{L}$ into three (s)quarks; the decay of $\mathfrak{D}$ into a quark and a squark happens due to the second and the third term.

The two $\mathfrak{D}_i$'s may have different Z_6 charges and so may the two $\mathfrak{L}_i$'s. The corresponding decay modes are displayed in table 3. Some heavier states could also mostly decay by cascading. Lifetimes and flavour structures of the decay products are fixed by further defining the X-charges, consistently with the remaining anomalies, in particular. The variety of combinations of X-charges in the couplings introduce different patterns of suppression of the different decays. The phenomenology of these states has a strong model-dependence on the supersymmetry breaking terms that affect the spectrum, including the decay direction between fermions and scalars.

In spite of the exotic character and apparent distinguishing decay modes, they have not necessarily good signatures. Their masses are only predicted up to $O(1)$ factors. Here, we shall just discuss a few more or less generic features. We recall that only one of the $\mathfrak{D}_i$'s and/or one of the $\mathfrak{L}_i$'s would be present, and we skip the indices. The phenomenology of supersymmetric vector-like extra-matter that includes a $\mathfrak{D}$-like state (but with $B = 1/3$) has been recently analysed [34] by assuming very small mixings to the light quarks. Some results can be usefully adapted to the exotic states herein.

The $\mathfrak{L}$ decay into two squarks and one quark, one of each family. Roughly, it has[7] $c\tau \sim \epsilon^{2n} 10^{-6}$cm, where n is the absolute value of the total X-charge of the corresponding coupling. The reduction factor might increase $c\tau$ by several orders of magnitude, hardly enough to make it to cross the detector, perhaps a displaced vertex in some cases: the issue is very model dependent. The main problem is the weak production rate at the LHC.

Instead, the $\mathfrak{D}$ is strongly produced and more auspicious for LHC searches. We separate the three possible discrete symmetry charges (here, n is the smallest absolute value of the total X-charge of all flavour channels).

1. $Z_{\mathfrak{D}} = 1/2$: the decay is three-body, presumably decaying inside the detector, with a $c\tau$ analogous to $\mathfrak{L}$. The signature is the spectrum of the pair of prompt energetic leptons (if the squarks are not relatively too heavy, which they could be). Of course, the leptons can be neutrinos.

2. $Z_{\mathfrak{D}} = 1/3$ (lepto-quarks): certainly the easiest to see at the LHC, a pair decaying with $c\tau \sim \epsilon^{2n} 10^{-17}$cm and two very hard leptons — if the squark is not too heavy — and two jets, altogether. But n could be large.

3. $Z_{\mathfrak{D}} = 0$: (di-quarks)with a life-time analogous to the previous case, but a two-jet decay, more difficult to identify.

It could seem that for the last case the scalar $\mathfrak{D}$ could be produced as a resonance in quark-quark scattering. However, as discussed in the next section, this would be associated to strong FCNC violation and it seems difficult to choose the charges so to do that and still keep a reasonable cross section for the flavour conserving processes based only on the abelian symmetry. Idem for $Z_{\mathfrak{D}} = 1/3$ and lepton-quark scattering. In any instance, when allowed these lepto-quarks and di-quarks would presumably have their lifetime strongly increased by the flavour factor. Therefore only $Z_{\mathfrak{D}} = 1/2$ seems really generic.

Finally, it is worth noticing that the solutions displayed here are associated to a benchmark model with several optional assumptions. Other flavour models could have different spectra. Also, in the present model, there are other solutions where none of the heavy states is inside the LHC range. Still, it illustrates the fact that flavour theory could be observable in colliders through new heavy vector particles.

4 New sources of FCNC and CPV

The experiments are regularly tightening the already very restrictive bounds on new physics contributions to FCNC and CP violating processes. This has been translated in terms of effective operators into a cutoff $O(10^4\text{TeV})$ for several of them — specially if CP phases are larger — unless their coefficients could be suppressed, e.g., by the flavour symmetries.

[7] For $\Lambda \sim 5 \times 10^3$ TeV, since lower values are excluded by the FCNC measurements, as discussed below.

For details, see e.g., the recent review [20]. The well-known constraints on the MSSM can be avoided with supersymmetry breaking parameters above or close to the TeV scale, at the price of controlling some scalar mass differences and small CP phases. As already stated, we assume this to be the case. In this section we look for new effects, inherent to our model. We separate these contributions according to the dimension of the dangerous operators, formulated in the supersymmetric language.

4.1 Dimension five FCNC operators

The quadrilinear interactions in (2.2) are strongly bounded from the experimental limits on FCNC and CP violations so setting a lower limit on the flavour symmetry breaking scale. These bounds were numerically studied, e.g., in [36]. For our purposes here, we would rather present an analysis on an order of magnitude footing (that looks appropriate to models that only predict orders of magnitude!), which takes advantage of the direct relation between Λ, μ and $\mathcal{X}^\mu$. Their coefficients are,

$$C^{qe}_{ijkl} \sim \epsilon^{|\mathcal{X}^u_{ij}+\mathcal{X}^e_{kl}-\mathcal{X}^\mu|} \qquad\qquad C^{qq}_{ijkl} \sim \epsilon^{|\mathcal{X}^u_{ij}+\mathcal{X}^d_{kl}-\mathcal{X}^\mu|}\,. \tag{4.1}$$

and let us concentrate on the contributions from the operators $UQEL$ and $UQDQ$ to FCNC and CP violating electromagnetic transitions of leptons and quarks: $\ell_j \to \ell_k\,\gamma$ and $d_j \to d_k\,\gamma$ through the flavour changing magnetic moments μ^ℓ_{jk} and μ^d_{jk} and the electric dipole moments, d_{ℓ_j} and d_d. The two-loop diagrams are the supersymmetric analogous to the Barr-Zee one [35] — in the artificial limit where the higgsino mass is very large. Baring possible interferences between the different contributions, and for the sake of an order of magnitude estimate, we assume all the supersymmetry breaking parameters to be $O(\mu)$. Then, up to several $O(1)$ factors, the magnetic and electric dipole moments are roughly given by

$$(\mu\,+\,i\,d)_{jk}\;\sim\;\sum_i \frac{C^{qf}_{iijk}}{\Lambda}\frac{e\alpha_w}{8\pi^2}\frac{m_{u_i}}{\mu} \qquad\qquad (\,f=e\,,d\,) \tag{4.2}$$

where: the quark mass m_{u_i} keeps track of the chirality/isospin change. An estimate of the traditional (one-loop) supersymmetric contributions due to the textures in the A-terms to $(\mu\,+\,i\,d)_{jk}$ along the same lines gives $O(e\alpha_w m_{f_{jk}}\tan\beta\,/\,4\pi\,\mu)$, where the mass matrix elements represent the isospin, flavour and CP violations (of course this choice is only indicative).[8] Now, let us require that (4.2) are at most of the same order of magnitude as those traditional one-loop ones, namely,

$$C^{qf}_{iijk}\frac{\mu\,m_{u_i}}{\Lambda\,m_{f_{jk}}\tan\beta} \lesssim O(2\pi) \qquad\qquad (\,f=e\,,d\,) \tag{4.3}$$

and after replacing (2.3) and (4.1) we obtain the constraints,

$$\Delta\mathcal{X}^f_{jk} = |\mathcal{X}^u_{ii}+\mathcal{X}^f_{jk}-\mathcal{X}^\mu| + \mathcal{X}^\mu + \mathcal{X}^u_{ii} - |\mathcal{X}^f_{jk}| \geq -1, \tag{4.4}$$

[8] Actually we do not know the charged lepton mixing angles and CP phases, we are assuming they are similar in both scalar and fermion masses.

With the allowed values for the X-chiralities, this condition is always satisfied. The worst case is for $i = 3$ and $\mathcal{X}^f_{jk} \geq \mathcal{X}^\mu$ when $\Delta\mathcal{X}^f_{kl} = 0$ from the stop loops. Therefore, the only cases in the balance are $s \to d\,\gamma$, $b \to s\,\gamma$, possibly $\mu \to d_k\,\gamma$, as well as to d_e and d_d for some choices of X-chiralities.

One still has to check other processes, the most constraining coming from $K\bar{K}$-mixing. The corresponding operator has coefficient C^{qe}_{1212} and from the mass matrices and $\mathcal{X}^\mu$, $C^{qe}_{1212} \sim \epsilon^5$. Evaluating the one-loop diagram leading to the four-fermion interaction along the same lines as above, one obtains the effective cutoff:

$$\Lambda^2_{\text{eff}} \sim \frac{\alpha_{\text{w}}\mu\Lambda}{2\pi C^{qe}_{1212}} \sim \frac{\alpha_{\text{w}}}{2\pi}\Lambda^2 \tag{4.5}$$

which by comparison with the experimental bounds, puts a limit of about $O(10^3\text{TeV})$ on the cutoff Λ.

The conclusion is that the flavour/CP issues related to the $UQEL$ and $UQDQ$ terms in (2.2) are not worse than the standard MSSM A-term contributions. The explicit calculations of the bounds on Λ in [36], after the appropriate rescaling of μ, agree with our rough estimate within the many uncertainties. Therefore, the models discussed here will be typically as sensitive to the next round of FCNC/CP experiments as the renormalizable MSSM, even for unflavoured real soft terms.

Of course, by integrating out flavon fields one generates further contributions to C^{qe}_{ijkl}, C^{qq}_{ijkl}, as well as to C_h. The term generated from the later can be written as

$$\frac{1}{8\epsilon^2\Lambda^3}\left(\frac{\partial W_{\text{MSSM}}}{\partial\epsilon}\right)^2 \tag{4.6}$$

where W_{MSSM} is the superpotential (2.2) with the couplings replaced by the corresponding powes of ϵ. Because of a factor v^2/Λ^2 the contributions to C^{qe}_{ijkl} and C^{qq}_{ijkl} are sub-leading. Instead, the flavon exchange contribution is ϵ^{-2} larger than the original C_h, but still too small to be relevant .

For $Z_{\mathfrak{D}} \neq 1/2$ integrating out the heavy quark introduce new contributions to these dimension five operators. Because the $m_{\mathfrak{D}}$ is $O(\text{TeV})$, one needs a very large suppression, $O(\mu/\Lambda)$ to be compared with those in the discussion above. These new contributions depend on the largely arbitrary X-charges not the known X-chiralities. For a rough estimate, note that

$$\Delta C^{qq}_{ijkl}\frac{\Lambda}{\mu} \lesssim \epsilon^{|\mathcal{X}^u_{ij}+\mathcal{X}^d_{kl}-\mathcal{X}^\mu+\mathcal{X}^{\mathfrak{d}}|-|\mathcal{X}^{\mathfrak{d}}|-\mathcal{X}^\mu}\,, \tag{4.7}$$

where the r.h.s. is almost always very large. It is easy to check that for most choices of the charges the coefficients are not reduced enough, in particular for those related to $K\bar{K}$. Therefore these choices of $Z_{\mathfrak{D}}$ become more marginal while the safe case $Z_{\mathfrak{D}} = 1/2$ is preferred.

4.2 Dimension six FCNC operators

The relevant operators contributing to FCNC are those in the Kähler potential of the form $D^\dagger_i D_j Q^\dagger_k Q_l/\Lambda^2$ and analogs. The resulting limits from several measurements and without suppressions would be $\Lambda < O(10^3)$TeV). However their coefficients can be expressed in terms of the "known" X-chiralities, like in the previous example where the exponent of ϵ is $|\mathcal{X}^d_{jk} - \mathcal{X}^d_{ik}|$, hence as ratios of mass matrix elements. This reduces the coefficients for relevant cases. The only exception is for $\mu - e$ conversion since $\mathcal{X}^d_{22} < 0$, when the reduction is even more efficient.

Integrating out the gauge sector to define the supersymmetric Fermi approximation, one obtains quartic flavour diagonal corrections to the Kähler potential like those above but diagonal in the basis where X is diagonal, with a cutoff (equivalent to G_F) given by the flavour symmetry breaking scale, $(\epsilon\Lambda)^2$, hence ϵ^{-2} times larger than those discussed before. In the physical basis, FCNC interactions are introduced with coefficients given by the mixing angles that diagonalize the masses. For $K\bar{K}$-mixing this provides a factor ϵ^{-2} that compensates the same factor in the denominator and preserves the limit on Λ, for the others the reduction is even larger. It is important to note that these contributions are proportional to the charge-differences with a coefficient fixed by gauge universality and mixing angles, nothing else. Therefore the limit close to $10^{-}3$ on Λ is robust, just as stated in the literature.[9]

In summary, the solution to the anomaly cancellation problem with $\mathcal{X}^\mu = 4$ becomes somewhat marginal, $\mathcal{X}^\mu = 5$ ($\Lambda = O(3000\,\text{TeV})$ being more comfortable. But both are very close to be tested in rare process experiments perhaps before the new heavy particles could even be searched for at the LHC!

5 Conclusions

In this paper, we argue that gauged flavour theories generically require new states to compensate for anomalies from quarks and leptons in chiral representations of the gauged flavour group and that QCD freedom freedom may favour their masses being close to the higgsino mass, or μ-term, of supersymmetric theories. This has been explicitly shown in supersymmetric models with a single U(1) flavour group which, after its breaking, delivers discrete baryon and lepton symmetries that forbid dangerous processes such as proton decay as well as mixings between the MSSM states and the new ones.

As these new particles are often predicted to lie around the TeV scale, they provide a test for the flavour theories, which are hardly testable otherwise. They have exotic discrete baryon and lepton numbers, hence peculiar decay modes, although their signatures are model dependent and not necessarily distinguishing in the busy LHC environment. In most cases the heavy "quark" decays into a hard lepton plus jets, which could help in their searches. The heavy "lepton" goes into three quarks (one of each family) but is much less produced at the LHC.

The higher dimension dimension operators that are sources of FCNC/CPV supersymmetric operators cannot be all suppressed enough if the cutoff lies below 1000 TeV. This is due to the exchanges of the flavour gauge boson and supersymmetric partners. Remarkably, in the models studied here, where the small μ/Λ ratio is explained in terms of flavour symmetry: (i) there is a similar lower bound if asymptotic freedom is imposed to limit the

[9] For recent discussions see [21, 37].

number of heavy states; (ii) these theories are not testable for a cutoff well beyond 10^4 TeV. Of course these conclusions are stated within the limited framework of effective theories.

The charges in the models developed here are certainly quite confusing [10] although they are largely dictated by the known quark and lepton masses and mixings, and it seems difficult to conceive a UV completion yielding such a structure. These models are then consistent but not quite convincing at least for this reason. Also, they do not predict precise empirical properties of the mass matrices. These shortcomings could be remedied by introducing non-abelian flavour symmetries (or, at least, several abelian ones) and replace large charges by sequential and hierarchical symmetry breaking scales, should it seem more satisfactory. In principle, the arguments of this paper could be transposed to these cases: the lighter heavy states will be associated to the anomalies of the symmetries broken at the lowest scale, presumably in correspondence with lighter quark, neutrino or the higgsino masses. However, gauging these symmetries usually introduce FCNC because of the lighter flavour gauge bosons associated to the lower scales along the same lines also discussed above, and the lowest scale would still be quite high.[11]

Finally, let us comment on the non-supersymmetric counterpart of these flavour theories [40] with a cutoff lower bounded by neutrino masses and FCNC/CPV restrictions as above. In the simplest case, one needs only one Higgs doublet and one flavon field and, assuming that the Higgs mass can be fixed, the analysis is quite similar to the supersymmetric version, but for the absence of the μ-term and the corresponding higgsino chirality. This increases the SM anomalies to be compensated but one can take advantage of a larger number of new fermions consistent with asymptotic freedom. The most striking difference is that, because of the three-fermion decay of the new heavy fermions, the latter are long-lived and stable enough to leave nicer signatures at the LHC.

Acknowledgments

We thank Michele Frigerio, Stéphane Lavignac, Renata Funchal and Oscar Eboli for helpful conversations. The work of CAS was partially supported by the European Commission under the contracts PITN-GA-2009-237920 and MRTN-CT-2006-035863. M.T. greatly appreciates that he was funded by a Feodor Lynen fellowship of the Alexander-von-Humboldt-Foundation. C.A.S. acknowledges the nice atmosphere at the Galileo Galilei Institute for Theoretical Physics, during the workshop "Searching for New Physics at the LHC", where part of this work was carried on.

A Other anomalies

We study here the cancellation of the other two anomalies, A_W and $A'_{\rm em}$ and, in particular the vanishing of the fractional contributions related to the conserved discrete symmetry.

[10] But note that, in the simplest case,they can be all even and reduce to 4, 2, 1 or 0 by taking $\epsilon \sim \theta_C^2$.

[11] recently, it has been shown in [37] how to lower this scale while keeping FCNC under control (see also [38, 39]). However, the model discuted there is renormalizable and the mass parameters invariant under flavour symmetry are assumed to be much less than the non-invariant ones, which would be inconsistent with the effective theory formalism adopted here.

While the previously discussed anomalies involve only the X-chiralities, these two additional ones constrain the X-charges themselves. We shall just cancel the fractional part of the anomalies, $\mathrm{frac}(A_W)$ and $\mathrm{frac}(\,A'_{\mathrm{em}})$ since the integer part can be eliminated by two combinations of the various (integer parts of) the charges, $\mathrm{int}(X_i)$ or $\mathrm{int}(X'_i)$, with many solutions that we do not discuss here, although they are relevant for the properties of the heavy state decays.

For this purpose, notice that the conserved symmetry correspond to charges Z' such that: (i) they change sign under charge conjugation, hence all X-chiralities are integers, (ii) the experimental flavour mixing for quarks and leptons require the Z' to be generation independent. Therefore one has $Z'_Q = Z'_{Q_i} = -Z'_{U_i} = Z'_{D_i}$, $Z'_L = Z'_{L_i} = -Z'_{E_i}$, $Z'_{\mathfrak{D}_i} = -Z'_{\bar{\mathfrak{D}}_i}$, and $Z'_{\mathfrak{L}_i} = -Z'_{\bar{\mathfrak{L}}_i}$. Furthermore, the neutrino mass imposes $\mathrm{frac}(2Z'_L) = 0$. At the exotic side, the phenomenological constraints in section (3.2) gives $Z'_{\mathfrak{L}_i} = 0$ and $Z'_{\mathfrak{D}_i} = \delta/18$, with $\delta = 2\,, 8\,, -7$.

First consider the weak isospin anomaly, which we write for convenience in terms of X', as

$$A_W = \mathrm{Tr} X' T_3^2 = \mathcal{X}_\mu + \sum_{i=1}^{3} (3q_i + l_i) + \mathrm{Tr}\mathcal{X}^{\mathfrak{l}} = 0\,,$$
$$\mathrm{frac}(A_W) = \mathrm{frac}(9Z'_Q + 3Z'_L) = 0\,,$$

and notice, besides the well-known solution, $Z' \propto B - L$, which allows for the proton decay, the choice $Z' = (B - 3L)/6$, which forbids it and is chosen here, when applied to the MSSM states.

The $A'_{\mathrm{em}} = \mathrm{Tr} X' Q_{\mathrm{em}}^2$ anomaly reads,

$$A'_{\mathrm{em}} = {h_u}^2 - {h_d}^2 + \sum_i \left[2\left({q_i}^2 - {\bar{u}_i}^2\right) - \left({q_i}^2 - {\bar{d}_i}^2\right) - \left({l_i}^2 + {\bar{e}_i}^2\right) + \sum_i [\left(\mathfrak{d}_i^2 - \bar{\mathfrak{d}}_i^2\right) - \left(\mathfrak{l}_i^2 - \bar{\mathfrak{l}}_i^2\right) \right].$$

and its fractional part is then,

$$\mathrm{frac}\left[2\left(2\mathrm{Tr}\mathcal{X}^u - \mathrm{Tr}\mathcal{X}^d - 6h_u + 3h_d\right) Z'_Q - 2\left(\mathrm{Tr}\mathcal{X}^e - 3h_d\right) Z'_L + 2\mathrm{Tr}\mathcal{X}^{\mathfrak{d}} Z'_{\mathfrak{D}} - 2\mathrm{Tr}\mathcal{X}^{\mathfrak{l}} Z'_{\mathfrak{L}}\right]$$

Interestingly enough, when the Z'_i, the traces of the matrices given by (2.4), (2.5) in section (2.1) and the solutions to the anomaly cancelation conditions (2.13), (2.14) and (2.15) of section (2.4) are all replaced in this expression, we get a very simple result for its cancellation for any of the three values of δ, namely,

$$A'_{\mathrm{em}} = -\frac{w}{3} + \text{integer} = 0 \tag{A.1}$$

This requires $w = 0$ corresponding to the approximate equality between the products of masses of the exotic heavy quarks and of the exotic heavy leptons, otherwise the gauge coupling unification would be badly violated for $|w| = 3$ or larger, as previously discussed.

References

[1] C.D. Froggatt and H.B. Nielsen, *Hierarchy of quark masses, Cabibbo angles and CP-violation*, Nucl. Phys. **B 147** (1979) 277 [SPIRES].

[2] J. Bijnens and C. Wetterich, *Quark, lepton and neutrino masses in grand unified theories with local generation group*, Nucl. Phys. **B 292** (1987) 443 [SPIRES].

[3] Z.G. Berezhiani, *Horizontal symmetry and quark-lepton mass spectrum: the SU(5) x SU(3)-h model*, Phys. Lett. **B 150** (1985) 177 [SPIRES].

[4] A. Pomarol and D. Tommasini, *Horizontal symmetries for the supersymmetric flavor problem*, Nucl. Phys. **B 466** (1996) 3 [hep-ph/9507462] [SPIRES].

[5] R. Barbieri, L.J. Hall, S. Raby and A. Romanino, *Unified theories with U(2) flavor symmetry*, Nucl. Phys. **B 493** (1997) 3 [hep-ph/9610449] [SPIRES].

[6] R. Barbieri, L. Giusti, L.J. Hall and A. Romanino, *Fermion masses and symmetry breaking of a U(2) flavour symmetry*, Nucl. Phys. **B 550** (1999) 32 [hep-ph/9812239] [SPIRES].

[7] R. Barbieri, L.J. Hall and A. Romanino, *Precise tests of a quark mass texture*, Nucl. Phys. **B 551** (1999) 93 [hep-ph/9812384] [SPIRES].

[8] Z. Berezhiani and A. Rossi, *Grand unified textures for neutrino and quark mixings*, JHEP **03** (1999) 002 [hep-ph/9811447] [SPIRES].

[9] S.F. King and G.G. Ross, *Fermion masses and mixing angles from SU(3) family symmetry*, Phys. Lett. **B 520** (2001) 243 [hep-ph/0108112] [SPIRES].

[10] G.G. Ross, L. Velasco-Sevilla and O. Vives, *Spontaneous CP-violation and non-Abelian family symmetry in SUSY*, Nucl. Phys. **B 692** (2004) 50 [hep-ph/0401064] [SPIRES].

[11] I. Masina and C.A. Savoy, *Up quark masses from down quark masses*, Phys. Lett. **B 642** (2006) 472 [hep-ph/0606097] [SPIRES].

[12] Z. Berezhiani and F. Nesti, *Supersymmetric SO(10) for fermion masses and mixings: Rank-1 structures of flavour*, JHEP **03** (2006) 041 [hep-ph/0510011] [SPIRES].

[13] I. de Medeiros Varzielas, S.F. King and G.G. Ross, *Neutrino tri-bi-maximal mixing from a non-Abelian discrete family symmetry*, Phys. Lett. **B 648** (2007) 201 [hep-ph/0607045] [SPIRES].

[14] S. Antusch, S.F. King, M. Malinsky and G.G. Ross, *Solving the SUSY Flavour and CP Problems with Non-Abelian Family Symmetry and Supergravity*, Phys. Lett. **B 670** (2009) 383 [arXiv:0807.5047] [SPIRES];

[15] I. Masina and C.A. Savoy, *Real and imaginary elements of fermion mass matrices*, Nucl. Phys. **B 755** (2006) 1 [hep-ph/0603101] [SPIRES].

[16] M.C. Bento, L.J. Hall and G.G. Ross, *Generalized matter parities from the superstring*, Nucl. Phys. **B 292** (1987) 400 [SPIRES].

[17] L.E. Ibáñez and G.G. Ross, *Discrete gauge symmetry anomalies*, Phys. Lett. **B 260** (1991) 291 [SPIRES].

[18] L.E. Ibáñez and G.G. Ross, *Discrete gauge symmetries and the origin of baryon and lepton number conservation in supersymmetric versions of the standard model*, Nucl. Phys. **B 368** (1992) 3 [SPIRES].

[19] H.K. Dreiner, C. Luhn and M. Thormeier, *What is the discrete gauge symmetry of the MSSM?*, Phys. Rev. **D 73** (2006) 075007 [hep-ph/0512163] [SPIRES].

JHEP03(2011)128

[20] G. Isidori, Y. Nir and G. Perez, *Flavor physics constraints for physics beyond the standard model*, arXiv:1002.0900 [SPIRES].

[21] N. Arkani-Hamed, L.J. Hall, D. Tucker-Smith and N. Weiner, *Flavor at the TeV scale with extra dimensions*, *Phys. Rev.* **D 61** (2000) 116003 [hep-ph/9909326] [SPIRES].

[22] M.B. Green and J.H. Schwarz, *Anomaly cancellation in supersymmetric $D = 10$ gauge theory and superstring theory*, *Phys. Lett.* **B 149** (1984) 117 [SPIRES].

[23] M. Dine, N. Seiberg, X.G. Wen and E. Witten, *Nonperturbative effects on the string world sheet*, *Nucl. Phys.* **B 278** (1986) 769 [SPIRES].

[24] M. Dine, N. Seiberg, X.G. Wen and E. Witten, *Nonperturbative effects on the string world sheet. 2*, *Nucl. Phys.* **B 289** (1987) 319 [SPIRES].

[25] J.J. Atick, L.J. Dixon and A. Sen, *String calculation of Fayet-Iliopoulos d terms in arbitrary supersymmetric compactifications*, *Nucl. Phys.* **B 292** (1987) 109 [SPIRES].

[26] L.E. Ibáñez and G.G. Ross, *Fermion masses and mixing angles from gauge symmetries*, *Phys. Lett.* **B 332** (1994) 100 [hep-ph/9403338] [SPIRES].

[27] P. Binetruy and P. Ramond, *Yukawa textures and anomalies*, *Phys. Lett.* **B 350** (1995) 49 [hep-ph/9412385] [SPIRES].

[28] E. Dudas, S. Pokorski and C.A. Savoy, *Yukawa matrices from a spontaneously broken Abelian symmetry*, *Phys. Lett.* **B 356** (1995) 45 [hep-ph/9504292] [SPIRES].

[29] Y. Nir, *Gauge unification, Yukawa hierarchy and the mu problem*, *Phys. Lett.* **B 354** (1995) 107 [hep-ph/9504312] [SPIRES].

[30] E. Dudas, S. Pokorski and C.A. Savoy, *Soft scalar masses in supergravity with horizontal* $\mathrm{U}(1)_X$ *gauge symmetry*, *Phys. Lett.* **B 369** (1996) 255 [hep-ph/9509410] [SPIRES].

[31] P. Binetruy, S. Lavignac and P. Ramond, *Yukawa textures with an anomalous horizontal Abelian symmetry*, *Nucl. Phys.* **B 477** (1996) 353 [hep-ph/9601243] [SPIRES].

[32] P.H. Chankowski, K. Kowalska, S. Lavignac and S. Pokorski, *Update on fermion mass models with an anomalous horizontal U(1) symmetry*, *Phys. Rev.* **D 71** (2005) 055004 [hep-ph/0501071] [SPIRES].

[33] G.F. Giudice and A. Masiero, *A natural solution to the mu problem in supergravity theories*, *Phys. Lett.* **B 206** (1988) 480 [SPIRES].

[34] S.P. Martin, *Extra vector-like matter and the lightest Higgs scalar boson mass in low-energy supersymmetry*, *Phys. Rev.* **D 81** (2010) 035004 [arXiv:0910.2732] [SPIRES].

[35] S.M. Barr and A. Zee, *Electric dipole moment of the electron and of the neutron*, *Phys. Rev. Lett.* **65** (1990) 21 [Erratum-ibid. **65** (1990) 2920] [SPIRES].

[36] M. Pospelov, A. Ritz and Y. Santoso, *Flavor and CP-violating physics from new supersymmetric thresholds*, *Phys. Rev. Lett.* **96** (2006) 091801 [hep-ph/0510254] [SPIRES].

[37] B. Grinstein, M. Redi and G. Villadoro, *Low scale flavor gauge symmetries*, *JHEP* **11** (2010) 067 [arXiv:1009.2049] [SPIRES].

[38] Z.G. Berezhiani and J.L. Chkareuli, *Quark-leptonic families in a model with SU(5)* $\times$ *SU(3) symmetry.*, *Sov. J. Nucl. Phys.* **37** (1983) 618 [SPIRES].

[39] Z.G. Berezhiani, *The weak mixing angles in gauge models with horizontal symmetry: a new approach to quark and lepton masses*, *Phys. Lett.* **B 129** (1983) 99 [SPIRES].

[40] O.J.P. Éboli, C.A. Savoy and R. Zukanovich-Funchal, *A rationale for heavy metastable quarks and leptons at LHC: flavour theory, to appear.*

JHEP03(2011)128