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k -string tension from Eguchi-Kawai reduction

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ABSTRACT: We consider three-dimensional $SU(N)$ gauge theories with massless Dirac or Majorana fermions in the adjoint representation. We use the Eguchi-Kawai volume reduction to two-dimensions to calculate the tension σ_k of the k -string in such theories. Under some assumptions whose validity we discuss, we derive the previously conjectured sine formula, $\sigma_k \sim N \sin\left(\pi \frac{k}{N}\right)$.

KEYWORDS: Confinement, $1/N$ Expansion, QCD

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1 Introduction

The dynamics of QCD in the non-perturbative regime is extremely complex and difficult. In order to simplify the problem 't Hooft suggested a long time ago to replace the gauge group $SU(3)$ of QCD by an $SU(N)$ group and to take the large- N limit, while keeping $g^2 N$ fixed [1]. In this limit the theory is expected to be controlled by a classical master field. Yet, despite the expectation of a huge simplification, very few analytical results were achieved by using the large- N approximation.

An important example of these is the Eguchi-Kawai volume reduction [2]. It was argued that when a planar theory is compactified on a circle, quantities that are neutral under the center of the gauge group should not depend on the radius of the circle and hence calculations of neutral quantities by a reduction to an arbitrarily small volume should be possible. Shortly after the seminal paper by Eguchi and Kawai, it was shown that a necessary condition for the reduction is that the Z_N center of the gauge group is not spontaneously broken, namely that the theory is confining at arbitrarily small volumes [3]. Unfortunately the pure $SU(N)$ Yang-Mills theory undergoes a confinement/deconfinement phase transition at a certain critical radius and hence a reduction to a radius below the critical radius is not allowed.

The idea of volume reduction was resurrected recently by Kovtun, Unsal and Yaffe [4]. They argued that the Eguchi-Kawai reduction can be used for QCD-like theories with fermions that transform in the *adjoint* representation of the gauge group. They showed that if the adjoint fermions obey periodic boundary conditions (hence opposite boundary conditions w.r.t. the finite temperature case), the theory is confining at an arbitrarily small radius and the condition for volume reduction is satisfied. Following their work several papers [5–17] supporting the validity of volume reduction for theories with adjoint fermions were published. This observation becomes particularly interesting when combined with

another intriguing outcome of the large- N limit: a set of equivalences amongst seemingly distinct gauge theories [18, 19]. In particular, a sector of QCD observables has a counterpart in the theory with adjoint fermions. In principle this could open the way to computing physical QCD quantities in the large- N limit through small volume calculations.

In this paper we consider three-dimensional gauge theories with adjoint fermions and use volume-reduction to two-dimensions in order to calculate the k -string tension. The k -string can be thought of as a bound state of k fundamental QCD-strings. Its tension has been vastly discussed in the literature [20–24]. In particular, most analytic approaches in the case of theories with adjoint fermions suggest that the k -string tension is proportional to $\sin(\pi \frac{k}{N})$ [25–28].

Our derivation of the string tension is similar to the corresponding one in the 2d Abelian (massive Schwinger model) [29] and the non-Abelian [30, 31] cases. We rely on the fact that in two dimensions an external charge at spatial infinity is equivalent to a θ -term [32]. The main result of this paper is

$$\sigma_k \sim N \sin\left(\pi \frac{k}{N}\right). \tag{1.1}$$

The paper is organized as follows: in section 2 we review the derivation of the string tension in the Abelian case. In section 3 we derive the k -string tension for the dimensionally reduced theory. In section 4 we discuss the validity of our derivation. Section 5 is devoted to a summary. In the appendix we discuss how the derivation of section 3 can be repeated in the case of a 3d theory with a single Majorana adjoint fermion.

2 A reminder of the string tension in the massive Schwinger model

Consider two-dimensional QED with one massive Dirac fermion whose electric charge (in units of the elementary charge) is Q . The action is

$$S = \int d^2x \left(-\frac{1}{4e^2} F_{\mu\nu}^2 + \bar{\Psi} i \not{\partial} \Psi - m \bar{\Psi} \Psi + Q A_\mu \bar{\Psi} \gamma^\mu \Psi \right), \tag{2.1}$$

where e is the gauge coupling. It is convenient to bosonize the fermion and to use the gauge $A_1 = 0$. Denoting the boson by ψ the resulting action takes the form

$$S = \int d^2x \left(\frac{1}{2e^2} (\partial_1 A_0)^2 + \frac{1}{2} (\partial_\mu \psi)^2 + m\mu \cos(2\sqrt{\pi}\psi) + \frac{Q}{\sqrt{\pi}} A_0 \partial_1 \psi \right), \tag{2.2}$$

where $\mu = e \frac{\exp \gamma}{2\pi^{3/2}}$. In the massless limit diagonalization of the quadratic terms reveals the existence of a free massive boson with $m^2 = \frac{e^2 Q^2}{\pi}$. When $m \ll e$, namely when the mass term is considered as a small perturbation, the vacuum configuration corresponds to $\psi = 0$ for $m > 0$ and to $2\sqrt{\pi}\psi = \pi$ for $m < 0$. The vacuum energy is negative and given by $-|m|\mu$.

Let us add a heavy electron-positron pair of charge $\pm Q'$ at $x = \pm L$ having in mind to take the limit $L \rightarrow \infty$ at the end. This means adding the term

$$S' = \int d^2x Q' A_0 (\delta(x - L) - \delta(x + L)) \tag{2.3}$$

to the previous action. We can integrate out the non-dynamical field A_0 ,

$$\partial_1 A_0 = e^2 \left(\frac{Q}{\sqrt{\pi}} \psi + Q'(\theta(x-L) - \theta(x+L)) \right), \quad (2.4)$$

to arrive at the following action:

$$S = \int d^2x \left(\frac{1}{2} (\partial_\mu \psi)^2 + m\mu \cos(2\sqrt{\pi}\psi) - \frac{e^2}{2} \left(\frac{Q}{\sqrt{\pi}} \psi + Q'\Theta \right)^2 \right), \quad (2.5)$$

where $\Theta \equiv \theta(x-L) - \theta(x+L)$.

The corresponding Hamiltonian of the theory is given by

$$H = \int dx \left(\frac{1}{2} \Pi_\psi^2 + \frac{1}{2} (\partial_1 \psi)^2 - m\mu \cos(2\sqrt{\pi}\psi) + \frac{e^2}{2} \left(\frac{Q}{\sqrt{\pi}} \psi + Q'\Theta \right)^2 \right). \quad (2.6)$$

Let us restrict ourselves to the limit $0 < m\mu/e^2 \ll 1$. In this limit the solution of the equation of motion for ψ is

$$\psi = -\sqrt{\pi} \frac{Q'\Theta}{Q} + \mathcal{O}\left(\frac{m\mu}{e^2}\right). \quad (2.7)$$

The vacuum energy is given by substituting (2.7) in (2.6):

$$E = H - H_0 = m\mu \left(1 - \cos\left(2\pi \frac{Q'}{Q}\right) \right) 2L, \quad (2.8)$$

where H_0 is the Hamiltonian of the theory without the external charge. After dealing similarly with the case of a small negative mass we find that the string tension is

$$\sigma = |m|\mu \left(1 - \cos\left(2\pi \frac{Q'}{Q}\right) \right). \quad (2.9)$$

The tension (2.9) vanishes if Q' is a multiple of Q and, interestingly, also when the quark mass is zero (corresponding to total charge screening).

3 The string tension in three-dimensional adjoint QCD

Consider three-dimensional adjoint QCD with one Dirac fermion, based on a gauge group $U(N)$:

$$\mathcal{L} = \text{Tr} \left(-\frac{1}{2g_3^2} F^{mn} F_{mn} + i\bar{\psi} \not{D}\psi \right) \quad ; \quad m, n = 0, 1, 2, \quad (3.1)$$

and compactified on $\mathbb{R}^2 \times S^1$. By imposing periodic boundary conditions on both the gauge and fermionic fields, the full action, including the Kaluza-Klein modes, is given by:

$$\begin{aligned} \sum_{q \in \mathbb{Z}} \text{Tr} \left(-\frac{1}{2g^2} F_{-q}^{\mu\nu} F_{q\mu\nu} + (D^\mu \phi)_{-q} (D_\mu \phi)_q - \frac{2}{gR} A_{-q}^\mu (D_\mu \phi)_q + \frac{q^2}{g^2 R^2} A_{-q}^\mu A_{q\mu} + \right. \\ \left. + i\bar{\psi}_{-q} \gamma^\mu (D_\mu \psi)_q - \frac{q}{R} \bar{\psi}_{-q} \gamma^2 \psi_q - g \sum_{q_1} \bar{\psi}_{-q} \gamma^2 [\phi_{q_1} \cdot \psi_{q-q_1}] \right), \quad (3.2) \end{aligned}$$

where $g = g_3/\sqrt{R}$ is the 2-dimensional gauge coupling, the “covariant” derivative on a generic field φ is given by $(D^\mu\varphi)_q = \partial^\mu\varphi_q + i\sum_{q_1}[A_{q_1}^\mu, \varphi_{q-q_1}]$ with $\mu, \nu = 0, 1$, and we introduced the tower of scalars $A_2/g = \sum_{q \in \mathbb{Z}} \phi_q e^{iqy/R}$.

Usual Kaluza-Klein reduction gives a tower of states whose only massless fields are those related to the “zero” modes with $q = 0$. All the others acquire a mass of order q/R , where R is the radius of S^1 . This is no longer true when the scalars of the theory acquire v.e.v.s, since these modify the mass spectra through Yukawa and gauge interactions and, as a result, the different mass levels get shifted and split.

V.e.v.s for the scalars arise quite naturally. Let us denote the $q = 0$ component of the gauge field along the circle by ϕ and study its vev. The vacuum configuration, obtained by a minimization of the effective action for the Polyakov loop $\text{Tr} \exp(i \int_0^{2\pi} \phi)$, yields [19]:

$$\langle \phi \rangle_{nm} = v_n \delta_{nm} \equiv \frac{(n - (N + 1)/2)}{N} \delta_{nm} \tag{3.3}$$

and hence the center-symmetry is not spontaneously broken.

Let us focus for the moment on the zero modes only, namely let us carry out a dimensional reduction down to two dimensions. As just mentioned, actually this is not necessarily justified. The correct procedure is to keep the lowest mass modes, which are not necessarily the $q = 0$ modes: the lightest modes may include KK momentum or winding. We will discuss this important issue in section 4.

The reduced action is

$$S = \int d^2x \text{Tr} \left(-\frac{1}{2g^2} F_{\mu\nu}^2 + (D_\mu\phi)^2 + \bar{\Psi} i \not{D} \Psi + ig[\phi, \bar{\Psi}] \gamma^3 \Psi \right), \tag{3.4}$$

where the matrix $\gamma_3 = \text{diag}(1, -1)$ is the chiral matrix in $2d$ which is equal to $-i\gamma^2$ in $3d$ and the Yukawa coupling is $g \sim \frac{g_3}{\sqrt{R}}$. Let us choose again the gauge $A_1 = 0$. In the presence of the v.e.v. (3.3) the gauge symmetry is broken to $U(1)^N$, with the massless gauge fields given by

$$(A_0)_{nm} \equiv A_n \delta_{nm}. \tag{3.5}$$

Let us focus on the gauge and Yukawa interactions, since these terms in the action will determine the 2d string tension. In the presence of the v.e.v. the terms

$$\text{Tr} i[\phi, \bar{\Psi}] \gamma^3 \Psi \quad ; \quad \text{Tr} [A_0, \bar{\Psi}] \gamma^0 \Psi \tag{3.6}$$

become, respectively:

$$i(v_n - v_m) \bar{\Psi}_{nm} \gamma^3 \Psi_{mn} \quad ; \quad (A_n - A_m) \bar{\Psi}_{nm} \gamma^0 \Psi_{mn}. \tag{3.7}$$

The action for the $U(1)^N$ gauge theory, excluding the kinetic terms for the scalar and fermi fields –which do not play any role in the calculation of the string tension– is:

$$S = \int d^2x \left\{ \sum_n \frac{1}{2g^2} (\partial_1 A_n)^2 + m \sum_{mn} i(v_n - v_m) \bar{\Psi}_{nm} \gamma^3 \Psi_{mn} + \sum_{mn} (A_n - A_m) \bar{\Psi}_{nm} \gamma^0 \Psi_{mn} \right\}. \tag{3.8}$$

As the derivation of the string tension involves terms which are not invariant under chiral rotation, we should also incorporate the chiral anomaly in the action. In analogy with the well-known case of QCD [33–35], the effective action can be written in the form:

$$S_{\text{eff}} = \int d^2x \left\{ \sum_n \frac{1}{2g^2} F_n^2 + m \sum_{mn} i(v_n - v_m) \bar{\Psi}_{nm} \gamma^3 \Psi_{mn} \right. \quad (3.9)$$

$$\left. + \sum_{mn} (A_n - A_m) \bar{\Psi}_{nm} \gamma^0 \Psi_{mn} + \sum_n F_n \left[\frac{-i}{8\pi} \sum_m \ln \left(\frac{\Psi_{Lnm}^* \Psi_{Rmn} \Psi_{Rmn}^* \Psi_{Lnm}}{\Psi_{Lmn}^* \Psi_{Rnm} \Psi_{Rnm}^* \Psi_{Lmn}} \right) \right] \right\},$$

where $F_n = \partial_1 A_n$.

One can easily check that, under a chiral transformation in the a^{th} direction of the Cartan subalgebra,

$$\delta \Psi_{Lij} = i\alpha \delta_{ia} \Psi_{Laj} - i\alpha \Psi_{Lia} \delta_{aj} + O(\alpha^2) \quad (3.10)$$

$$\delta \Psi_{Lij}^* = i\alpha \delta_{ia} \Psi_{Laj}^* - i\alpha \Psi_{Lia}^* \delta_{aj} + O(\alpha^2) \quad (3.11)$$

$$\delta \Psi_{Rij} = \delta \Psi_{Rij}^* = 0, \quad (3.12)$$

the effective action transforms as it should:

$$\delta_a \mathcal{L}_{\text{eff}} = \frac{1}{2\pi} \left(\alpha N F^a - \alpha \sum_{i=1}^N F^i \right).$$

Clearly the overall U(1) is not anomalous since $\sum_{a=1}^N \delta_a \mathcal{L} = 0$. Consequently, we have selected indeed an SU(N), rather than a U(N), anomaly.

Let us add now a source of the form

$$S' = \int d^2x \left(\frac{k}{2} A^a - \frac{k}{2} A^b \right) (\delta(x-L) - \delta(x+L)) \quad (3.13)$$

$$= - \int d^2x \left(\frac{k}{2} F_a - \frac{k}{2} F_b \right) \Theta, \quad (3.14)$$

that corresponds to k units of fundamental charge placed at the end of the interval and points along the SU(N) Cartan subalgebra, namely $\vec{k} = (0, \dots, 0, k, 0, \dots, 0, -k, 0, \dots)$.

In the presence of the external charge the relevant part of the effective action is

$$S_{\text{eff}} = \int d^2x \left\{ \sum_n \frac{1}{2g^2} F_n^2 + m \sum_{mn} i(v_n - v_m) (\Psi_{Lnm}^* \Psi_{Rmn} - \Psi_{Rnm}^* \Psi_{Lmn}) \right. \quad (3.15)$$

$$\left. + \sum_n F_n \left[\frac{-i}{8\pi} \sum_m \ln \left(\frac{\Psi_{Lnm}^* \Psi_{Rmn} \Psi_{Rmn}^* \Psi_{Lnm}}{\Psi_{Lmn}^* \Psi_{Rnm} \Psi_{Rnm}^* \Psi_{Lmn}} \right) - \frac{k}{2} \delta_n^a + \frac{k}{2} \delta_n^b \right] \right\}.$$

In order to write the effective action in terms of bosonic fields let us introduce

$$\Psi_{Lnm}^* \Psi_{Rmn} = \mu \exp(i\psi_{mn}) \quad ; \quad \Psi_{Rmn}^* \Psi_{Lnm} = \mu \exp(-i\psi_{nm}). \quad (3.16)$$

The bosonized action takes the form

$$S_{\text{eff}} = \int d^2x \left\{ \sum_n \frac{1}{2g^2} F_n^2 - 2m\mu \sum_{mn} (v_n - v_m) \sin \psi_{mn} + \sum_n F_n \left[\frac{-i}{8\pi} \sum_m \ln \exp(2i(\psi_{mn} - \psi_{nm})) - \frac{k}{2} \delta_n^a + \frac{k}{2} \delta_n^b \right] \right\}. \quad (3.17)$$

Integrating out F_n the Hamiltonian density becomes:

$$H = 2m\mu \sum_{mn} (v_n - v_m) \sin \psi_{mn} + g^2 \sum_n \left[\frac{1}{4\pi} \sum_m (\psi_{mn} - \psi_{nm}) + \frac{2\pi q_n}{8\pi} - \frac{k}{2} \delta_n^a + \frac{k}{2} \delta_n^b \right]^2, \quad (3.18)$$

where the multivaluedness of the logarithm is encoded into $q_n \in \mathbb{Z}$.

As for the Schwinger model we should now solve the equations of motion and evaluate H on-shell. The problem at hand is greatly simplified in the limit $m/gN \ll 1$. The full set of equations of motion, which also includes the equation of motion for the scalar,

$$(D_\mu D^\mu \phi)_n = 2m \sum_{l \neq n} \sin \psi_{nl}, \quad (3.19)$$

is solved by:

$$\psi_{am} = \pi + \frac{\pi k}{N}, \quad M_{ma} < 0, \quad (3.20)$$

$$\psi_{am} = \frac{\pi k}{N}, \quad M_{ma} > 0, \quad (3.21)$$

$$\psi_{bm} = \pi - \frac{\pi k}{N}, \quad M_{mb} > 0, \quad (3.22)$$

$$\psi_{bm} = -\frac{\pi k}{N}, \quad M_{mb} < 0, \quad (3.23)$$

$$\psi_{mn} = 0, \quad m, n \neq a, b, \quad (3.24)$$

$$\psi_{mn} = -\psi_{nm}, \quad (3.25)$$

where M_{mn} is the mass coefficient multiplying $\sin \psi_{nm}$ (we will discuss in the next section the importance of KK modes and how they modify this mass matrix), while the scalar fluctuations are set to zero: $\phi = 0$.

The integers q_i are chosen in such a way that the quadratic term, which is the dominant contribution to the energy, vanishes on-shell and the only non-zero piece comes from the sum of the mass terms.

Note that an exact solution for the equation of motion for the scalar (3.19) requires $b - a = N/2$. Substituting the solution in the Hamiltonian we find the energy density

$$\langle H \rangle = 2m\mu \sum_{mn} (v_n - v_m) \sin \psi_{mn} = \frac{8m\mu}{N} \sum_{n=0}^N \left| n - \frac{N}{2} \right| \sin \left(\pi \frac{k}{N} \right). \quad (3.26)$$

Hence

$$\sigma_k \sim m\mu N \sin\left(\pi \frac{k}{N}\right). \tag{3.27}$$

Had we started directly with a two-dimensional theory, m would have been an arbitrary mass parameter, thus depending on its sign we should have found either the solution just proposed (when $m > 0$) or another solution shifted by π when $m < 0$, precisely as it happens in the Abelian case. Independently on the sign of m the string tension will always be $\sigma_k \sim |m|\mu N \sin\left(\pi \frac{k}{N}\right) > 0$.

4 Comments on the validity of the derivation

The main result of our paper is the string tension (3.27). In this section we discuss the assumptions that were made in our derivation and the validity of the result.

Similarly to the Abelian case (the massive Schwinger model that we reviewed in section 2), one needs to assume that the mass term is smaller than the gauge-interaction term. Since the mass term is $\mathcal{O}(N^2)$, while the gauge-interaction term is $\mathcal{O}(N^3)$, the condition is

$$m\mu N^2 \ll g^2 N^3. \tag{4.1}$$

By using $m \sim \frac{1}{R}$, $\mu \sim g$ and the relation between the 3d gauge coupling and the 2d gauge coupling $g^2 R = g_3^2$, we can rewrite the above condition as

$$\lambda_3 RN \gg 1, \tag{4.2}$$

where λ_3 is the 3d 't Hooft coupling. We now encounter [36] the following difficulty: the masses of the KK modes and the lightest W-bosons are both

$$M_{KK} \sim M_W \sim \frac{1}{RN}, \tag{4.3}$$

hence the condition that the mass term is a small perturbation implies that the W-bosons mass (in units of the 3d 't Hooft coupling) goes to zero, namely

$$\frac{M_W}{\lambda_3} \sim \frac{1}{\lambda_3 RN} \ll 1. \tag{4.4}$$

If the W-bosons become massless, our assumption that the dynamics is controlled by the Cartan sub-algebra degrees of freedom may be invalid. Note that (4.4) does not invalidate our derivation of the string tension within the 2d framework: within 2d we can always assume a small mass term. The problem is that the Eguchi-Kawai procedure requires a full non-Abelian dynamics and that an ‘‘Abelianization’’ of the problem may not be trusted. We will return to this issue in the summary section.

Another important issue is the inclusion of KK modes. The mass of a generic mode that couples to the fermion bi-linear is

$$M_{mn}^l = \frac{1}{R} \left(l + w_m - w_n + \frac{m - n}{N} \right) \tag{4.5}$$

where l is the KK momentum and w 's are integers that correspond to winding (the Polyakov loop is defined only mod N). It means that when $(m - n) \sim \mathcal{O}(N)$ there is no separation between the lowest KK modes and higher modes. For this reason we should, in principle, consider all the modes and dimensional reduction is invalid. In particular the lowest N fermionic masses are in the range $M_{mn}^l RN \in (-N/2, \dots, N/2)$ and they include modes with $w = 0, \pm 1$. These are the ψ_{ij} modes that we used in our derivation, *not* the zero modes. The rest of the modes were set to zero.

Returning to the issue of the W's, we can require that their mass $M_W \sim \frac{1}{RN}$ be fixed in terms of λ_3 which means $1/R = r\lambda_3 N$ with r a dimensionless number. The expansion parameter for our previous solution $\epsilon = \frac{m\mu}{g^2 N}$ can be rewritten using all the above formulas and it is precisely $\epsilon = r$. The proposed solution admits a series expansion in power of ϵ and at first order we obtained eq. (3.27) which can be rewritten in the more suggestive form:

$$\sigma_k \sim \lambda_3^2 N^2 \sin\left(\pi \frac{k}{N}\right) \sim \lambda_2 N \sin\left(\pi \frac{k}{N}\right). \quad (4.6)$$

5 Summary and discussion

In this paper we used the Eguchi-Kawai large- N volume reduction to calculate the k -string tension in 3d gauge theories with adjoint fermions. As we have elaborated in the previous section, our derivation relies on the limit $\lambda_3 RN \gg 1$. In this limit the W-bosons masses go to zero and hence our assumption that the dynamics is governed by a $U(1)^N$ theory may not be justified. On the other hand, we cannot exclude the option that for dynamical reasons the 3d string tension *is* dominated by the contribution of the Cartan sub-algebra, as in various other models [37].

Our work can be extended in various directions: it is natural to start from a 4d theory with adjoint fermions and to consider it on $R^2 \times T^2$ and to calculate the string tension in 4d by using the reduced 2d theory. Other directions would be to calculate the quark condensate and the glueball masses by the reduced theory.

The reduction from 4d to 2d raises several conceptual problems. In particular the understanding of the 4d running of the gauge coupling from the 2d theory. Once the running of the coupling is understood, it will be possible to discuss issues such as asymptotic freedom and the range of the conformal window in 4d from 2d. We hope to return to these problems in a future work.

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A Majorana fermions

In this appendix we will briefly discuss the case in which the starting 3 dimensional theory contains Majorana rather than Dirac fermions. This discussion makes possible the analysis of the $\mathcal{N} = 1$ supersymmetric case which after reduction will give us a $\mathcal{N} = (1, 1)$ SUSY theory in 2 dimensions.

The Dirac matrices are constructed using Pauli σ matrices: $\gamma^0 = \sigma_1$, $\gamma^1 = i\sigma_2$, $\gamma^2 = i\sigma_3$ and satisfy the usual Clifford algebra with Minkowski signature. The Majorana condition reads $\Psi^* = \sigma_3\Psi$, so denoting the two components of Ψ as Ψ_R and Ψ_L (with a slight abuse of notation since clearly in $3d$ there is no chirality) we have simply $\Psi_R = \Psi_R^*$ while $\Psi_L = -\Psi_L^*$. Since our fermions are in the adjoint representation of the gauge group it is consistent to impose a Majorana constraint on them and after the reduction to 2 dimensions for every $3d$ Majorana we get 2 Majorana-Weyl fermions.

The reduction of the action is straightforward and using the above conditions it is easy to obtain:

$$\mu \exp(i\psi_{mn}) = \Psi_{Lnm}^* \Psi_{Rmn} = \Psi_{Rmn}^* \Psi_{Lnm} = \mu \exp(-i\psi_{nm}), \quad (\text{A.1})$$

which gives us directly the antisymmetry of the phases ψ_{mn} (modulo 2π factors).

As a consequence of eq. (A.1) the effective term for the anomaly can be simplified to

$$\delta\mathcal{L}_{\text{eff}} = \sum_n F_n \left[\frac{-i}{4\pi} \sum_m \ln \left(\frac{\Psi_{Lnm}^* \Psi_{Rmn}}{\Psi_{Lmn}^* \Psi_{Rnm}} \right) \right].$$

Using further the Majorana condition we can rearrange the Yukawa term in the form

$$ig\text{Tr}([\phi, \bar{\Psi}]\gamma^3\Psi) = 2g\text{Tr}([\phi, \Psi_L]\Psi_R),$$

and after rewriting this term together with the gauge interactions and the effective anomaly in terms of the phases ψ_{mn} we can reproduce precisely eq. (3.17) in the Majorana case as well.

Nothing new happens when we try to solve the equation of motions, we can directly substitute the solution presented in eq. (3.20)–(3.24) obtaining for the string tension:

$$\sigma_k \sim \frac{m\mu}{2} N \sin\left(\pi \frac{k}{N}\right), \quad (\text{A.2})$$

with an the extra factor $1/2$ with respect to the Dirac case.

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