Natural inflation with natural trans-planckian axion decay constant from anomalous $\mathrm{U}(1)_X$

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ABSTRACT: We propose a natural inflation model driven by an imaginary or axionic component of a Kähler modulus in string-inspired supergravity. The shift symmetry of the axion is gauged under an anomalous $\mathrm{U}(1)_X$ symmetry, which leads to a modulus-dependent Fayet-Iliopoulos (FI) term. The matter fields are stabilized by F-terms, and the real component of the modulus is stabilized by the $\mathrm{U}(1)_X$ D-term, while its axion remains light. Therefore, the masses of real and imaginary components of the modulus are separated at different scales. The scalar potential for natural inflation is realized by the superpotential from the non-perturbative effects. The trans-Planckian axion decay constant, which is needed to fit with BICEP2 observations, can be obtained naturally in this model.

Contents

1 Introduction

The recent discovery of the B-mode polarization by the BICEP2 Collaboration, if confirmed, provides further strong evidence on the inflationary paradigm for the early epoch of the Universe [1]. The observed tensor-to-scalar ratio r is $r = 0.20^{+0.07}_{-0.05}$, or $r = 0.16^{+0.06}_{-0.05}$ without the dust contributions. One of the inflation models, which agrees with the BICEP2 results, is the well-known natural inflation [2, 3].

The motivation of natural inflation [2, 3] is to solve the flatness problem of inflation potential at tree level, and remains flat against radiative corrections. The continuous shift symmetry protects the flatness of inflation potential. To realize inflation, the continuous symmetry should be broken to a discrete shift symmetry $\phi \to \phi + 2\pi f$ with f the axion (or inflaton) decay constant, and the potential for natural inflation is

$$V(\phi) = \Lambda^4 \left(1 \pm \cos\left(\frac{\phi}{f}\right) \right), \tag{1.1}$$

where Λ is the inflation energy scale around $2 \times 10^{16}\,\mathrm{GeV}$ or $10^{-2}\,\mathrm{M_{Pl}}$ for $r = 0.16/0.20$ with $\mathrm{M_{Pl}}$ the reduced Planck mass ($\mathrm{M_{Pl}} = 2.4 \times 10^{18}\,\mathrm{GeV}$).

Axion is a "natural" inflaton candidate since it preserves the exact continuous shift symmetry at perturbative level. Axions can be obtained from antisymmetric tensor fields in string theory through spacetime compactification [4]. Considering the non-perturbative effects, such as gaugino condensation or instanton effect, one can break such continuous shift symmetry of axion to the discrete symmetry and then realize natural inflation in string theory.

The challenge to natural inflation is the large decay constant f. To generate sufficient large tensor fluctuations that are consistent with BICEP2 observations, the decay constant

f should be trans-Planckian $f \sim O(10)$ in the Planck units [5]. However, in string theory the axion decay constant cannot be larger than the string scale M_{String} [4, 6, 7], which is about one order below the reduced Planck scale as required by the weak interaction assumption. An effective large axion decay constant was realized by the N-flation [8, 9] or the aligned axion mechanism [10]. In the aligned natural inflation [10], two axions with small decay constants are carefully adjusted to form a flat direction for inflation, and then one-linear combination of two axions can have the effective trans-Planckian decay constant. Recently, there are many works proposed to realize large decay constant with multi-aligned axions [11–14] or realize N-flation and natural inflation in string theory [15–21].

To obtain inflation in string theory or its low energy approximation-supergravity (SUGRA), there is a general problem on the moduli stabilization. For single field inflation, all the scalars except the inflaton should be fixed during inflation. The well-known KKLT mechanism based on F-term was proposed in [22], where the complex-structure moduli are stabilized by the fluxes [23] while the Kähler modulus is stabilized through the non-perturbative effects. The difficulty to realize axion inflation based on the KKLT mechanism is: once the real component of the Kähler modulus is fixed, its axionic component obtains large mass as well and then destroys inflation [24, 25]. This problem can be solved by considering the modulus-dependent FI term associated with anomalous $U(1)_X$ [26]. The FI term only depends on the real component of modulus, which can obtain large mass from D-term flatness. The axion is still light as it decouples from D-term.

In this work, we construct a natural inflation model in string-inspired SUGRA where only one modulus couples to the matter fields. The shift symmetry of axion is gauged to obtain anomalous $U(1)_X$. The gauge invariant superpotential consists of the non-perturbative term of modulus and various couplings among matter fields. All the matter fields are stabilized by F-terms, and the vanishing D-term gives large mass to the real component of the modulus. The axion is still light after modulus stabilization and its potential is given exactly by eq. (1.1). Besides, the trans-Planckian axion decay constant can be obtained naturally by taking the large condensation gauge group and numbers of $U(1)_X$ charged matter fields (around 20).

This paper is organized as follows. In section 2 we provide the string-inspired SUGRA structure for model building and show the anomaly cancellation of $U(1)_X$. In section 3 the matter fields/modulus stabilizations based on the F-terms/D-term are discussed. In section 4 we obtain the natural inflation potential after stabilizations, and the trans-Planckian axion decay constant is realized from the non-perturbative effects with suitable condensation gauge group. We conclude in section 5.

2 Natural inflation model building

We consider the following Kähler potential

$$K = -\ln(T + \bar{T}) + \phi_i \bar{\phi}_i + \chi_j \bar{\chi}_j + \varphi_k \bar{\varphi}_k + Q\bar{Q} + X_l \bar{X}_l, \qquad (2.1)$$

in which the indexes are $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, m-2$, $k = 1, 2, 3, 4$ and $l = 0, 1, 2, \cdots, m$. The modulus T can be dilaton superfield or one of the Kähler moduli from

string compactification. The $U(1)_X$ charged matter fields $(\phi_i, \chi_j, \varphi_k, Q)$, which will be generically denoted as z_h, are introduced to construct gauge invariant superpotential and cancel the gauge anomalies. The modulus T and matter fields z_h transform under $U(1)_X$ as follows

$$
\begin{aligned}
T &\to T + i\delta\epsilon \\
z_h &\to z_h e^{i\epsilon q_{z_h}} \,,
\end{aligned}
\tag{2.2}
$$

in which the $U(1)_X$ charges q_z are $q_{\phi_i} = -q_{\chi_j} = q$, $q_{\varphi_k} = (-1)^k q$, $q_Q = -2q$. X_l are neutral under $U(1)_X$ and vanish during inflation, their F-terms are used to stabilize ϕ_i and $\varphi_{1,2}$ with non-zero vacuum expectation values (VEV).

The superpotential is

$$
\begin{aligned}
W = {}& w_0 + a\phi_1^{\frac{1}{n}} e^{-bT} + X_i(\phi_i\varphi_1 - \lambda_1) + X_0(\varphi_1\varphi_2 - \lambda_2) \\
& + c_1(\varphi_1\varphi_4 + \varphi_2\varphi_3) + m_j\varphi_4\chi_j + \varphi_4^2 Q.
\end{aligned}
\tag{2.3}
$$

Given $q = nb\delta$, above superpotential is obviously $U(1)_X$ invariant. The first two terms in (2.3) is similar with the KKLT scenario in a gauge invariant form [22]. The constant term w_0 is from the complex-structure moduli stabilization. Different from the KKLT scenario, the non-perturbative term $a\phi_1^{\frac{1}{n}} e^{-bT}$ is $U(1)_X$ invariant. The $U(1)_X$ transformation of matter field $\phi_1^{1/n} \to \phi_1^{1/n} e^{iq/n}$ cancels the phase factor $e^{-ib\delta}$ from shift of the modulus $T \to T + i\epsilon\delta$ under $U(1)_X$. Such kind of non-perturbative superpotential can be obtained from the gaugino condensation with massive chiral superfields, which form representation of the condensation group, such as $SU(n)$ and can be integrated out in effective field theory. The effective superpotential is guaranteed to be $U(1)_X$ invariant [27–29, 32]. Effects of the gauge invariant non-perturbative term on moduli stabilization and inflation have been studied in [30–35]. Furthermore, in this form the matter field ϕ_1 has positive exponent, and gives an analytic coefficient for the non-perturbative term, which makes the anomalous $U(1)_X$ D-term cancellable. The D-term flatness is needed for modulus stabilization.

In eq. (2.3), we will have many other superpotential terms, for example, $\phi_i\chi_j$, etc, which are allowed by the $U(1)_X$ symmetry but neglected. To solve this problem, we can introduce a Z_m discrete symmetry under which φ_3, φ_4, χ_j, and Q transform as follows

$$
\varphi_3 \to \omega^k \varphi_3, \ \varphi_4 \to \omega^{-k}\varphi_4, \ \chi_j \to \omega^k\chi_j, \ Q \to \omega^{2k}Q,
\tag{2.4}
$$

with $\omega^m = 1$, while all the other fields are neutral under Z_m. Thus, superpotential will be

$$
\begin{aligned}
W = {}& w_0 + a\phi_1^{\frac{1}{n}} e^{-bT} + X_i(\phi_i\varphi_1 - \lambda_1) + X_0(\varphi_1\varphi_2 - \lambda_2) \\
& + c_1\varphi_1\varphi_2 + m_0\varphi_3\varphi_4 + m_j\varphi_4\chi_j + \varphi_4^2 Q \,.
\end{aligned}
\tag{2.5}
$$

With our numerical study assumption, we will point out that the superpotentials in both eq. (2.3) and eq. (2.5) will give the similar inflaton potential. In general, we can have the superpotential terms $c_i'\phi_i\varphi_1 + c_1\varphi_1\varphi_2$. Without loss of generality, we can make a transformation for ϕ_i/φ_2, and X_i, and obtain the above superpotential since φ_1 is only coupled to one linear combination of ϕ_i and φ_2.

Gauge anomaly cancellation. The anomalous $U(1)_X$ plays a special role in the quantum anomaly cancellation through the Green-Schwarz mechanism in four dimensional spacetime [36]. The gauge kinetic term of $U(1)_X$ is

$$\int d^2\theta f W_\alpha^2, \tag{2.6}$$

in which W_α is $U(1)_X$ gauge field strength. Here we take the gauge kinetic function $f = k_X T$. The gauge kinetic term contains two parts $Re(f)F^2$ and $Im(f)F\tilde{F}$. The first term is $U(1)_X$ invariant, while the second term transforms non-trivially under $U(1)_X$. The shift of modulus T introduces an extra term $i\delta k_X \int d^2\theta W_\alpha^2$, which cancels the anomaly from charged fermionic fields and keep the theory anomaly free.

Ignoring the anomaly of condensation gauge group $SU(n)$, we need to consider two kinds of anomalies: the gravitational anomaly $U(1)_X$ and the cubic anomaly $U(1)_X^3$. The fermionic contributions are:

$$\begin{aligned}
\text{Tr } q_z &= \sum_z q_z = 0\,, \\
\text{Tr } q_z^3 &= \sum_z q_z^3 = -6q^3.
\end{aligned} \tag{2.7}$$

The gravitational anomaly is canceled without higher derivative terms R^2. Anomaly cancelation of cubic term $U(1)_X^3$ requires

$$k_X\delta = -\frac{1}{48\pi^2}\sum_z q_z^3 = \frac{1}{8\pi^2}q^3. \tag{2.8}$$

As $q = nb\delta$, we have $k_X = nbq^2/8\pi^2$.

3 Matter fields and modulus stabilization

In this model, the matter fields are stabilized by F-terms. Even though there is the KKLT type superpotential in (2.3), it has nothing to do with modulus stabilization but provides potential for natural inflation, actually, the real component of the modulus is stabilized by $U(1)_X$ D-term.

The F-term scalar potential is given by

$$V_F = e^K(K^{i\bar{j}}D_i W D_{\bar{j}}\bar{W} - 3W\bar{W})\,, \tag{3.1}$$

in which $K^{i\bar{j}}$ is the inverse of the Kähler metric $K_{i\bar{j}} = \partial_i\partial_{\bar{j}}K$ and $D_i W = W_i + K_i W$. The complete expression of V_F is rather tedious, nevertheless, it can be remarkably simplified after field stabilization.

3.1 Matter fields stabilization

For matter fields stabilization, we ignore the constant term and the non-perturbative term, as they provide inflationary potential which is significantly lower than the matter fields stabilization scale. Their effects will be estimated later.

Clearly the matter fields $\chi_j, \varphi_{3,4}, Q$ and X_l have global minimum at origin while extra matter fields ϕ_i and $\varphi_{1,2}$ will get non-trivial VEVs. During inflation they will evolve to the minimum rapidly driven by the exponential factor e^K of F-term scalar potential and the large masses obtained from the matter couplings in eq. (2.3). Therefore the value of superpotential during inflation is simplified as $\langle W \rangle \equiv W_0 = w_0 + a\phi_1^{\frac{1}{n}}e^{-bT}$. In eq. (3.1), only these terms independent with $\chi_j, \varphi_{3,4}, Q$ and X_l are non-vanishing. The non-vanishing F-terms V_z introduced by fields z are (multiplied by an overall factor e^K):

- for T:

$$V_T = (T + \bar{T})^2 \left[a^2 b^2 (\phi_1\bar{\phi}_1)^{\frac{1}{n}} e^{-b(T+\bar{T})} + 2a^2 b \frac{1}{T+\bar{T}} (\phi_1\bar{\phi}_1)^{\frac{1}{n}} e^{-b(T+\bar{T})} \right.$$
$$\left. + abw_0 \frac{1}{T+\bar{T}} (\phi^{\frac{1}{n}} e^{-bT} + \bar{\phi}^{\frac{1}{n}} e^{-b\bar{T}}) \right] + W_0\bar{W}_0, \tag{3.2}$$

- for ϕ_1:

$$V_{\phi_1} = \frac{a^2}{n^2} (\phi_1\bar{\phi}_1)^{\frac{1}{n}-1} e^{-b(T+\bar{T})} + \frac{2a^2}{n} (\phi_1\bar{\phi}_1)^{\frac{1}{n}} e^{-b(T+\bar{T})}$$
$$+ \frac{a}{n} w_0 (\phi^{\frac{1}{n}} e^{-bT} + \bar{\phi}^{\frac{1}{n}} e^{-b\bar{T}}) + \phi_1\bar{\phi}_1 W_0\bar{W}_0, \tag{3.3}$$

- for $\phi_{i>1}$:

$$V_{\phi_{i>1}} = \phi_{i>1}\bar{\phi}_{i>1} W_0\bar{W}_0, \tag{3.4}$$

- and similarly, for φ_1 and φ_2:

$$V_{\varphi_{1,2}} = (\varphi\bar{\varphi})_{1,2} W_0\bar{W}_0. \tag{3.5}$$

Above terms are proportional to a^2, aw_0 or w_0^2. In this model, the parameters have orders of $a \sim w_0 \sim 10^{-5}$, while $c_1 \sim 10^{-3} - 10^{-2}$, therefore above terms are extremely small comparing with the following F-terms.

For the superpotential in eq. (2.5), V_T, V_{ϕ_1}, $V_{\phi_{i>1}}$ are the same, while $V_{\varphi_{1,2}}$ become

$$V_{\varphi_{1,2}} = |\bar{\varphi} W_0 + c_1 \varphi_{2,1}|^2 . \tag{3.6}$$

Note that in the above equation only the terms $|c_1\varphi_{1,2}|^2$ are relevant while all the other terms are small and negligible, thus, all the rest discussions will be the same and we will not repeat here.

The vanishing neutral matter fields X_l provide F-terms $|W_{X_l}|^2$ in the scalar potential, besides, φ_1 and φ_2 get mass terms from $|W_{\varphi_4}|^2$ and $|W_{\varphi_3}|^2$, respectively. Combining these F-terms together we have

$$V_{F_1} = e^K |W_{X_l}|^2 = e^K \left(\sum_i |\phi_i\varphi_1 - \lambda_1|^2 + |\varphi_1\varphi_2 - \lambda_2|^2 + c_1^2(|\varphi_1|^2 + |\varphi_2|^2) \right). \tag{3.7}$$

Of course there are numerous corrections containing ϕ_i and $\varphi_{1,2}$ in (3.2), (3.3), (3.4), (3.5), however, they are either significantly smaller than (3.7) or can be canceled with each other. The VEVs of ϕ_i are mainly dominated by the results obtained from (3.7).

Taking $\varphi_i = r_i e^{i\theta_i}$, the potential (3.7) becomes

$$
\begin{aligned}
V_{F_1} = e^K |W_{X_l}|^2 &= e^K \bigg\{ \sum_i |\phi_i \varphi_1 - \lambda_1|^2 + r_1^2 r_2^2 - 2\lambda_2 r_1 r_2 \cos(\theta_1 + \theta_2) + \lambda_2^2 + c_1^2(r_1^2 + r_2^2) \bigg\} \\
&\geqslant e^K \bigg\{ \sum_i |\phi_i \varphi_1 - \lambda_1|^2 + r_1^2 r_2^2 - 2\lambda_2 r_1 r_2 + \lambda_2^2 + c_1^2(r_1^2 + r_2^2) \bigg\} \\
&= e^K \bigg\{ \sum_i |\phi_i \varphi_1 - \lambda_1|^2 + (r_1 r_2 - (\lambda_2 - c_1^2))^2 + c_1^2(r_1 - r_2)^2 + c_1^2(2\lambda_2 - c_1^2) \bigg\}.
\end{aligned}
\tag{3.8}
$$

The vacuum takes place at $\theta_1 + \theta_2 = 2n\pi$, while the direction $\theta_1 - \theta_2$ is flat. The minimization of the potential in the bracket gives

$$
\begin{aligned}
r_1 = r_2 = r_0 &= \sqrt{\lambda_2 - c_1^2}, \\
|\phi_i| \equiv r &= \frac{\lambda_1}{r_0}.
\end{aligned}
\tag{3.9}
$$

The orders of parameters are simply taken as $r_0 \sim 10^{-2}$ and $r^2 = 0.1$ in following estimations. Eq. (3.8) provides strong stabilization on the matter fields. For the condition $r_1 = r_2$, the mass of direction along $(r_1 - r_2)/\sqrt{2}$ is $2c_1 \sim 10^{-3} - 10^{-2} \gg H$, where H is Hubble constant during inflation, therefore it is strongly stabilized as well. Besides, we get a cosmology constant term $V_0 = c_1^2(2\lambda_2 - c_1^2)$.

The anomalous $U(1)_X$ is spontaneously broken by VEVs of charged matter fields. The phases of $\phi_i = r e^{i\alpha_i}$ satisfy $\alpha_i = \theta_2$, we take the $U(1)_X$ gauge $\theta_2 = 0$ for simplicity. The flat direction is absorbed by $U(1)_X$ massive vector field through the Higgs mechanism after spontaneous symmetry broken.

It should be careful to consider above values as VEVs of these fields. Actually the cosmology constant term V_0, together with the overall factor e^K, can shift the non-zero VEVs. Specifically, the first condition $r_1 = r_2$ remains the same, while the results of ϕ_i and $r_0 = \sqrt{\lambda_2 - c_1^2}$ will be slightly modified.

In general, considering a field ϕ with mass m_0 and non-zero VEV ϕ_0, assuming its Kähler potential is minimal, and the overall scalar potential $V = e^{\phi^2}(\frac{1}{2}m_0^2(\phi - \phi_0)^2 + V_0)$, where V_0 is the residual cosmology constant term after field stabilization or during inflation, the vacuum is determined by

$$
\frac{dV}{d\phi} = e^{\phi^2}(m_0^2(\phi - \phi_0) + m_0^2 \phi(\phi - \phi_0)^2 + 2V_0 \phi) = 0,
\tag{3.10}
$$

which gives a new VEV $\phi_0' \simeq \phi_0 - 2V_0 \phi_0/m_0^2$. During inflation, the quasi-cosmology constant provided by inflaton is $V_0 \sim 10^{-8}$ in the Planck units, m_0 is of order $O(10^{-2})$ in our model, therefore the shift of matter fields is about $\Delta\phi/\phi_0 \simeq 10^{-4}$. The vacuum energy is reduced by $2V_0^2 \phi_0^2/m_0^2 \sim 10^{-13}$, which is completely ignorable during inflation. Therefore we can safely consider the values obtained in (3.9) are VEVs of these fields. Besides, there is a coupling between ϕ_1 and modulus T through $a\phi_1^{\frac{1}{n}} e^{-bT}$, however, we will show that the interaction is seriously suppressed by $aw_0 \sim 10^{-9}$ and has ignorable effect on ϕ_1 stabilization as well.

3.2 F-term potential after stabilization

After field stabilization, the F-term potential is simplified. Besides an overall factor e^K, it is

$$
\begin{aligned}
V = (T + \bar{T})^2 &\left[a^2 b^2 (\phi_1 \bar{\phi}_1)^{\frac{1}{n}} e^{-b(T+\bar{T})} + 2a^2 b \frac{1}{T+\bar{T}} (\phi_1 \bar{\phi}_1)^{\frac{1}{n}} e^{-b(T+\bar{T})} \right. \\
&\left. + abw_0 \frac{1}{T+\bar{T}} \left(\phi_1^{\frac{1}{n}} e^{-bT} + \bar{\phi}_1^{\frac{1}{n}} e^{-b\bar{T}} \right) \right] + \frac{a^2}{n^2} (\phi_1 \bar{\phi}_1)^{\frac{1}{n}-1} e^{-b(T+\bar{T})} \\
&+ \frac{2a^2}{n} (\phi_1 \bar{\phi}_1)^{\frac{1}{n}} e^{-b(T+\bar{T})} + \frac{a}{n} w_0 \left(\phi_1^{\frac{1}{n}} e^{-bT} + \bar{\phi}_1^{\frac{1}{n}} e^{-b\bar{T}} \right) \\
&+ c_1^2 (2\lambda_2 - c_1^2) + \left(\sum_i |\phi_i|^2 + |\varphi_1|^2 + |\varphi_2|^2 - 2 \right) W_0 \bar{W}_0.
\end{aligned}
\tag{3.11}
$$

Taking $r_0^2 \ll r^2$, we ignore the contributions from $|\varphi_{1,2}|^2 W_0 \bar{W}_0$. The parameters can be simply taking as $\sum_i |\phi_i|^2 = mr^2 = 2$ so that the last term in eq. (3.11) vanishes.

Couplings between modulus T and ϕ_1 are shown in eq. (3.11), the dominant term is $g(T)\phi_1^{\frac{1}{n}}$ with $g(T) \sim 10^{-8}$. Ignoring the factor e^K, the potential of ϕ_1 is approximate to

$$
V_{\phi_1} = \frac{1}{2} m_1^2 (\phi_1 - r)^2 + g(T) \phi_1^{\frac{1}{n}},
\tag{3.12}
$$

in which $m_1^2 \sim 10^{-4}$. VEV of ϕ_1 is shifted about $\Delta \langle \phi_1 \rangle \simeq g(T) r^{\frac{1}{n}} / n m_1^2 r \sim 10^{-4}$, and the energy is reduced by $\frac{1}{2} \frac{g(T)^2 r^{2/n}}{n^2 m_1^2 r^2} \sim 10^{-12}$, which confirms that the non-perturbative effect is ignorable for matter field stabilization.

3.3 Anomalous U(1)$_X$ D-term and modulus stabilization

The anomalous U(1)$_X$ D-term scalar potential is given by

$$
V_D = \frac{1}{2\mathrm{Re}(f)} D^2,
\tag{3.13}
$$

where $D = iK_z X^z + i\frac{W_z}{W} X^z$. X^z are the components of Killing vector corresponding to U(1)$_X$ isometries of the Kähler manifold, which are

$$
(X^T, X^{\phi_i}, X^{\chi_j}, X^{\varphi_k}, X^Q) = (i\delta, iq\phi_i, -iq\chi_j, (-1)^k iq\varphi_k, -2iqQ).
\tag{3.14}
$$

In this model the superpotential is gauge invariant, the D-term is simplified as $D = iK_z X^z$ and it reads

$$
D = \frac{\delta}{T + \bar{T}} - q \sum_i |\phi_i|^2 + q \sum_j |\chi_j|^2 - (-1)^k q \sum_k |\varphi_k|^2 + 2q|Q|^2.
\tag{3.15}
$$

After field stabilization, $\chi_j, \varphi_{3,4}$ and Q have vanshing VEVs, the φ_1 and φ_2 D-terms cancel each other due to $|\varphi_1| = |\varphi_2|$, and then the D-term is reduced to

$$
D = \frac{\delta}{T + \bar{T}} - q \sum_i |\phi_i|^2 = \frac{\delta}{T + \bar{T}} - mqr^2 .
\tag{3.16}
$$

The D-term potential is

$$V_D = \frac{1}{2k_X T_R}\left(\frac{\delta}{T+\bar{T}} - mqr^2\right)^2$$

$$= \frac{4\pi^2}{(nb)^3 T_R}\left(\frac{1}{2T_R} - mnbr^2\right)^2,$$

(3.17)

in which $T_R \equiv \operatorname{Re} T$ and the gauge invariant condition $q = nb\delta$ and cubic anomaly $\mathrm{U}(1)_X^3$ cancelation $k_X = nbq^2/8\pi^2$ are used. The D-term vanishing condition gives $\langle T_R\rangle \equiv T_0 = 1/2mnbr^2$. In the simplified case with $mr^2 = 2$, the real component of modulus is $T_0 = 1/4nb$.

4   Natural inflation potential

We have stabilized all the fields except the imaginary component of T. Now the scalar potential (3.11) becomes

$$V = 2e^2 b\left[a^2 r^{\frac{2}{n}} e^{-\frac{1}{2n}}\left(\frac{1}{4n} + 3 + \frac{1}{nr^2}\right) \right.$$

$$\left. + nc_1^2(2\lambda_2 - c_1^2) + 3aw_0 r^{\frac{1}{n}} e^{-\frac{1}{4n}}\cos(b\theta)\right],$$

(4.1)

where θ is the imaginary component of T. To get global Minkowski vacuum the parameters need to be adjusted so that

$$nc_1^2(2\lambda_2 - c_1^2) + a^2 r^{\frac{2}{n}} e^{-\frac{1}{2n}}\left(\frac{1}{4n} + 3 + \frac{1}{nr^2}\right) = 3aw_0 r^{\frac{1}{n}} e^{-\frac{1}{4n}},$$

(4.2)

and we get the scalar potential

$$V = 6abw_0 e^2 r^{\frac{1}{n}} e^{-\frac{1}{4n}}\left(1 + \cos(b\theta)\right),$$

(4.3)

which is of the same form in eq. (1.1) with $\Lambda^4 = 6abw_0 e^2 r^{1/n} e^{-1/4n}$. Taking $a \simeq 5\times 10^{-5}$, $w_0 \sim 6\times 10^{-5}$, $r^2 \sim 0.1$, $b = 0.1$ and $n \geqslant 6$, it gives the inflation scale $\Lambda \sim 10^{-2}$ in the Planck units.

Before we consider θ as our inflaton, a field re-scale is needed to get canonical kinetic term, and this will affect the decay constant of θ.

The kinetic term of T is

$$L_K = \frac{1}{(T+\bar{T})^2}\partial_\mu T \partial^\mu T = \frac{1}{4T_0^2}(\partial_\mu T_R \partial^\mu T_R + \partial_\mu \theta \partial^\mu \theta).$$

(4.4)

Defining $\theta = \sqrt{2}T_0\rho$, we get the action of inflaton

$$L = \frac{1}{2}\partial_\mu \rho \partial^\mu \rho + \Lambda^4\left(1 + \cos\left(\frac{\rho}{\sqrt{2}mnr^2}\right)\right),$$

(4.5)

in which the D-term stabilization condition $bT_0 = 1/2mnr^2$ has been used. The axion decay constant in this model is $f = \sqrt{2}mnr^2$. The VEV of matter fields r is smaller than

the Planck mass. Without uplift from parameters m, n, the axion decay constant f cannot be super-Planckian. Nevertheless, f is proportional to the product of charged field number m and the degree of condensation gauge group n, it is very easy to get super-Plankian f by taking large m or n. We used $r^2 = 0.1$ and $mr^2 = 2$ before, in such case we have $m = 20$, then $f = 2\sqrt{2}n$ and it is of order $O(10)$ with $n = 4$. By using larger n the axion decay constant f increases linearly, the potential gets close to the type $\frac{1}{2}m^2\rho^2$, and we get chaotic inflation.

Gravitino mass.   Even though the matter fields are stabilized at the scales much higher than inflation scale, they do not introduce too heavy gravitino mass. After field stabilization, the pure matter couplings in the superpotential in eq. (2.3) vanish, the VEV of superpotential $\langle W \rangle$ during inflation is

$$\langle W \rangle \equiv W_0 = w_0 + a\langle \phi_1^{\frac{1}{n}} e^{-bT}\rangle = w_0 - ar^{\frac{1}{n}}e^{-\frac{1}{4n}}. \tag{4.6}$$

Besides, we also have the VEV of e^K

$$\langle e^K \rangle = \frac{e^2}{2T_0}, \tag{4.7}$$

in which we have used $mr^2 = 2$ and the small term $2r_1^2$ from VEVs of $\varphi_{1,2}$ is ignored. The gravitino mass is

$$\begin{aligned}
M_{\frac{3}{2}} \equiv \exp\left(\frac{\langle G \rangle}{2}\right) &= \langle e^{\frac{K}{2}}(W_0\bar{W}_0)^{\frac{1}{2}}\rangle \\
&= e\sqrt{2nb}\left(w_0 - ar^{\frac{1}{n}}e^{-\frac{1}{4n}}\right).
\end{aligned} \tag{4.8}$$

The gravitino mass relates to the inflation energy scale through

$$(e\sqrt{2nb})^2 w_0 ar^{\frac{1}{n}}e^{-\frac{1}{4n}} = \frac{n}{3}\Lambda^4. \tag{4.9}$$

Small gravitino mass can be obtained by taking $w_0 \to ar^{\frac{1}{n}}e^{-\frac{1}{4n}}$, however, eq. (4.2) provides a lower bound on it

$$\begin{aligned}
w_0 - ar^{\frac{1}{n}}e^{-\frac{1}{4n}} &= nc_1^2(2\lambda_2 - c_1^2)\left(3ar^{\frac{1}{n}}e^{-\frac{1}{4n}}\right)^{-1} + \frac{1}{3n}ar^{\frac{1}{n}}e^{-\frac{1}{4n}}\left(\frac{1}{4} + \frac{1}{r^2}\right) \\
&\geqslant \frac{2}{3}c_1(2\lambda_2 - c_1^2)^{\frac{1}{2}}\left(\frac{1}{4} + \frac{1}{r^2}\right)^{\frac{1}{2}}.
\end{aligned} \tag{4.10}$$

The minimum locates at $c_1^2(2\lambda_2 - c_1^2) \simeq (\frac{1}{4} + \frac{1}{r^2})w_0^2/n^2$. For $c_1 \sim 10^{-3}$, $\lambda_2 \sim 10^{-4}$, the gravitino mass can be reduced to the order of 10^{-5}, while it cannot get significantly smaller otherwise the field stabilization is not strong enough for inflation. In short, the gravitino mass will not affect inflation in our model.

5 Conclusion

We have proposed a natural inflation model based on string inspired SUGRA with gauged shift symmetry $U(1)_X$. The matter fields are stabilized by F-terms, part of them obtain non-zero VEVs which break the anomalous $U(1)_X$ spontaneously. They obtain masses several orders larger than the mass of inflaton, consequently the effect of vacuum energy from inflaton on field stabilization is seriously suppressed and ignorable. The modulus-dependent FI term of $U(1)_X$ plays a critical role in modulus stabilization. As the coupling between matter field and modulus is analytic, the D-term can be vanished in our model. Once the matter fields obtain non-zero VEVs, the real component of modulus T is fixed by the D-term flatness. While the D-term is independent with the axionic component of T, therefore its cancellation has no effect on axion which remains light after modulus stabilization. The anomalous $U(1)_X$ splits the masses of real and imaginary components of T. Such role of anomalous $U(1)_X$ D-term on modulus stabilization has been studied in [26]. In the F-term moduli stabilization, usually the imaginary component of modulus obtains mass as large as the real component [24], the axion inflation cannot be realized. The reason is in F-term potential, the real and imaginary components of moduli couple with each other, it is highly non-trivial to split their masses at different scale so that one of them can play the role of inflaton while the other is frozen during inflation.

Potential for natural inflation is obtained from non-perturbative effect. Generally it is very difficult to get trans-Planckian axion decay constant, which is needed to fit with BICEP2 results. In our model, the axion decay constant linearly depends on the degree of condensation gauge group $SU(n)$ and number of $U(1)_X$ charged matter fields, therefore, the super-Planckian axion decay constant can be easily fulfilled by using large condensation gauge group and more $U(1)_X$ charged fields.

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