Published for SISSA by Springer

Received: *August 14, 2014*
Accepted: *August 23, 2014*
Published: *September 17, 2014*

N = 1 supergravity and Maxwell superalgebras

P.K. Concha and E.K. Rodríguez

Departamento de Física, Universidad de Concepción,
Casilla 160-C, Concepción, Chile
Dipartimento di Scienza Applicata e Tecnologia (DISAT), Politecnico di Torino,
Corso Duca degli Abruzzi 24, Torino, I-10129 Italia
Istituto Nazionale di Fisica Nucleare (INFN) Sezione di Torino,
Via Pietro Giuria 1, Torino, 10125 Italia

E-mail: patrickconcha@udec.cl, everodriguez@udec.cl

Abstract: We present the construction of the $D = 4$ supergravity action from the minimal Maxwell superalgebra $s\mathcal{M}_4$, which can be derived from the $\mathfrak{osp}\,(4|1)$ superalgebra by applying the abelian semigroup expansion procedure. We show that $N = 1$, $D = 4$ pure supergravity can be obtained alternatively as the MacDowell-Mansouri like action built from the curvatures of the Maxwell superalgebra $s\mathcal{M}_4$. We extend this result to all minimal Maxwell superalgebras type $s\mathcal{M}_{m+2}$. The invariance under supersymmetry transformations is also analized.

Keywords: Supergravity Models, Gauge Symmetry

ArXiv ePrint: 1407.4635v2

doi:10.1007/JHEP09(2014)090

JHEP09(2014)090

Contents

1 Introduction

It is well known that the so-called Maxwell algebra $\mathcal{M}$ corresponds to a modification of the Poincaré symmetries, where a constant electromagnetic field background is added to the Minkowski space [1–6]. In $D = 4$ this algebra is obtained by adding to the Poincaré generators (J_{ab}, P_a) the tensorial central charges Z_{ab}, modifying the commutativity of the translation generators P_a as follows

$$[P_a, P_b] = Z_{ab}. \tag{1.1}$$

In this way, the Maxwell algebra is an enlargement of Poincaré algebra, i.e., if we consider $Z_{ab} = 0$ we recover the Poincaré algebra.

Recently, it was shown that the Maxwell algebra can be obtained as an expansion procedure of the AdS Lie algebra $\mathfrak{so}(3, 2)$ [7, 8]. In particular, in ref. [8] it was shown that the Maxwell algebra can be derived using the S-expansion procedure introduced in ref. [9], using $S_E^{(2)} = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3\}$ as the relevant semigroup. Furthermore, this result was extended to all Maxwell algebras type $\mathcal{M}_m$ which can be obtained as an S-expansion of the AdS algebra using $S_E^{(N)} = \{\lambda_\alpha\}_{\alpha=0}^{N+1}$ as an abelian semigroup [10]. The S-expansion procedure is not only an useful method to derive new Lie (super)algebras but it is a powerful tool in order to build new (super)gravity theories. For example, it was also shown that standard odd-dimensional General Relativity can be obtained from Chern-Simons gravity theory for these $\mathcal{M}_m$ algebras[1] and recently it was found that standard even-dimensional

[1] Also known as generalized Poincaré algebras $\mathfrak{B}_m$.

General Relativity emerges as a limit of a Born-Infeld like theory invariant under a certain subalgebra of the Lie algebra $\mathcal{M}_m$ [10–13].

In ref. [14], it was shown that the $N = 1$, $D = 4$ Maxwell superalgebra $s\mathcal{M}$ can be obtained as an enlargement of the Poincaré superalgebra. This is particularly interesting since it describes the supersymmetries of generalized $N = 1$, $D = 4$ superspace in the presence of a constant abelian supersymmetric field strength background. Very recently it was shown that minimal Maxwell superalgebra $s\mathcal{M}$ can be obtained using the Maurer Cartan expansion method [7]. Subsequently, this superalgebra and its generalization $s\mathcal{M}_{m+2}$ have been obtained as an S-expansion of the $\mathfrak{osp}(4|1)$ superalgebra [15]. This family of superalgebras which contain the Maxwell algebras type $\mathcal{M}_{m+2}$ as bosonic subalgebras may be viewed as a generalization of the D'Auria-Fré superalgebra [16] and Green algebras [17].

It is the purpose of this work to construct the minimal $D = 4$ supergravity action from the minimal Maxwell superalgebra $s\mathcal{M}_4$. To this aim, we apply the S-expansion procedure to the $\mathfrak{osp}(4|1)$ superalgebra and we build a Mac Dowell-Mansouri like action with the expanded 2-form curvatures. We show that $N = 1$, $D = 4$ pure supergravity can be derived alternatively as the MacDowell-Mansouri like action from the minimal Maxwell superalgebra $s\mathcal{M}_4$. This result corresponds to a supersymmetric extension of ref. [12] in which four-dimensional General Relativity is derived from Maxwell algebra as a Born-Infeld like action. We extend this result to all minimal Maxwell superalgebras type $s\mathcal{M}_{m+2}$ in $D = 4$. Interestingly, when the simplest Maxwell superalgebra is considered we obtain the action found in ref. [18].

This work is organized as follows: in section 2, we briefly review the principal aspects of the S-expansion procedure. In section 3, we review the $N = 1$, $D = 4$ supergravity with cosmological constant for the $\mathfrak{osp}(4|1)$ superalgebra. Section 4 and 5 contain our main results. In section 4, we obtain the supergravity action as a Mac Dowell-Mansouri like action from the Maxwell superalgebra $s\mathcal{M}_4$. We show that this action describes pure Supergravity. In section 5 we extend our results to all minimal Maxwell superalgebras type $s\mathcal{M}_{m+2}$ and we study the invariance under supersymmetry. Section 6 concludes the work with some comments.

2 S-expansion procedure

It is the purpose of this section to review the main properties of the S-expansion method introduced in ref. [9].

The S-expansion procedure consists in combining the inner multiplication law of a semigroup S with the structure constants of a Lie algebra $\mathfrak{g}$. Let $S = \{\lambda_\alpha\}$ be an abelian semigroup with 2-selector $K_{\alpha\beta}{}^{\gamma}$ defined by

$$K_{\alpha\beta}{}^{\gamma} = \begin{cases} 1 & \text{when } \lambda_\alpha\lambda_\beta = \lambda_\gamma, \\ 0 & \text{otherwise,} \end{cases} \tag{2.1}$$

and $\mathfrak{g}$ a Lie (super)algebra with basis $\{\mathbf{T}_A\}$ and structure constants $C_{AB}{}^{C}$,

$$[\mathbf{T}_A, \mathbf{T}_B] = C_{AB}{}^{C}\mathbf{T}_C. \tag{2.2}$$

JHEP09(2014)090

Then, the direct product $\mathfrak{G} = S \times \mathfrak{g}$ is also a Lie (super)algebra with structure constants $C_{(A,\alpha)(B,\beta)}{}^{(C,\gamma)} = K_{\alpha\beta}{}^{\gamma} C_{AB}{}^{C}$, given by

$$\left[\mathbf{T}_{(A,\alpha)}, \mathbf{T}_{(B,\beta)}\right] = C_{(A,\alpha)(B,\beta)}{}^{(C,\gamma)} \mathbf{T}_{(C,\gamma)}. \tag{2.3}$$

The Lie algebra $\mathfrak{G}$ defined by $\mathfrak{G} = S \times \mathfrak{g}$ is called S-expanded algebra of $\mathfrak{g}$.

When the semigroup has a zero element $0_S \in S$, it plays a somewhat peculiar role in the S-expanded algebra. The algebra obtained by imposing the condition $0_S \mathbf{T}_A = 0$ on $\mathfrak{G}$ is called 0_S-reduced algebra of $\mathfrak{G}$.

Interestingly, it is possible to extract smaller algebras from $S \times \mathfrak{g}$. However, before to extract smaller algebras it is necessary to apply a decomposition of the original algebra $\mathfrak{g}$. Let $\mathfrak{g} = \bigoplus_{p \in I} V_p$ be a decomposition of $\mathfrak{g}$ in subspaces V_p, where I is a set of indices. Then for each $p, q \in I$ it is always possible to define $i_{(p,q)} \subset I$ such that

$$[V_p, V_q] \subset \bigoplus_{r \in i_{(p,q)}} V_r. \tag{2.4}$$

Now, let $S = \bigcup_{p \in I} S_p$ be a subset decomposition of the abelian semigroup S such that

$$S_p \cdot S_q \subset \bigcup_{r \in i_{(p,q)}} S_p. \tag{2.5}$$

When such subset decomposition exists, then we say that this decomposition is in resonance with the subspace decomposition of $\mathfrak{g}$. Defining the subspaces of $\mathfrak{G} = S \times \mathfrak{g}$,

$$W_p = S_p \times V_p, \qquad p \in I \tag{2.6}$$

we have that

$$\mathfrak{G}_R = \bigoplus_{p \in I} W_p, \tag{2.7}$$

is a subalgebra of $\mathfrak{G} = S \times \mathfrak{g}$, and it is called a resonant subalgebra of the S-expanded algebra $\mathfrak{G}$.

Another case of smaller algebra can be derived when the semigroup has a zero element $0_S \in S$. The algebra obtained by imposing the condition $0_S \mathbf{T}_A = 0$ on $\mathfrak{G}$ is called 0_S-reduced algebra of $\mathfrak{G}$. Additionally a reduced algebra can be extracted from a resonant subalgebra.

A useful property of the S-expansion procedure is that it provides us with an invariant tensor for the S-expanded algebra $\mathfrak{G} = S \times \mathfrak{g}$ in terms of an invariant tensor for $\mathfrak{g}$. As was shown in ref. [9] the theorem VII.1 provide a general expression for an invariant tensor for an expanded algebra.

Theorem VII.1 *Let S be an abelian semigroup, $\mathfrak{g}$ a Lie (super)algebra of basis $\{\mathbf{T}_A\}$, and let $\langle \mathbf{T}_{A_n} \cdots \mathbf{T}_{A_n} \rangle$ be an invariant tensor for $\mathfrak{g}$. Then, the expression*

$$\langle \mathbf{T}_{(A_1,\alpha_1)} \cdots \mathbf{T}_{(A_n,\alpha_n)} \rangle = \alpha_\gamma K_{\alpha_1 \cdots \alpha_n}{}^{\gamma} \langle \mathbf{T}_{A_1} \cdots \mathbf{T}_{A_n} \rangle \tag{2.8}$$

where α_γ are arbitrary constants and $K_{\alpha_1 \cdots \alpha_n}{}^{\gamma}$ is the n-selector for S, corresponds to an invariant tensor for the S-expanded algebra $\mathfrak{G} = S \times \mathfrak{g}$.

The proofs of these definitions and theorem can be found in ref. [9].

3 $N = 1$, $D = 4$ AdS supergravity

In ref. [19] was presented a geometric formulation of $N = 1$ supergravity in four dimensions, where the relevant gauge fields of the theory are those corresponding to the $\mathfrak{osp}(4|1)$ supergroup. The resulting action, constructed only in terms of the gauge fields, leads to $N = 1$ supergravity plus cosmological and topological terms. In this section a brief review of this construction is considered.

The (anti)-commutation relations for the $\mathfrak{osp}\,(4|1)$ superalgebra are given by

$$\left[\tilde{J}_{ab}, \tilde{J}_{cd}\right] = \eta_{bc}\tilde{J}_{ad} - \eta_{ac}\tilde{J}_{bd} - \eta_{bd}\tilde{J}_{ac} + \eta_{ad}\tilde{J}_{bc}, \tag{3.1}$$

$$\left[\tilde{J}_{ab}, \tilde{P}_{c}\right] = \eta_{bc}\tilde{P}_{a} - \eta_{ac}\tilde{P}_{b}, \tag{3.2}$$

$$\left[\tilde{P}_{a}, \tilde{P}_{b}\right] = \tilde{J}_{ab}, \tag{3.3}$$

$$\left[\tilde{J}_{ab}, \tilde{Q}_{\alpha}\right] = -\frac{1}{2}\left(\gamma_{ab}\tilde{Q}\right)_{\alpha}, \qquad \left[\tilde{P}_{a}, \tilde{Q}_{\alpha}\right] = -\frac{1}{2}\left(\gamma_{a}\tilde{Q}\right)_{\alpha}, \tag{3.4}$$

$$\left\{\tilde{Q}_{\alpha}, \tilde{Q}_{\beta}\right\} = -\frac{1}{2}\left[\left(\gamma^{ab}C\right)_{\alpha\beta}\tilde{J}_{ab} - 2\left(\gamma^{a}C\right)_{\alpha\beta}\tilde{P}_{a}\right], \tag{3.5}$$

where $\tilde{J}_{ab}$, $\tilde{P}_{a}$ and $\tilde{Q}_{\alpha}$ correspond to the Lorentz generators, the AdS boost generators and the fermionic generators, respectively.

In order to write down a Lagrangian for this algebra, we start from the one-form gauge connection

$$A = \frac{1}{2}\omega^{ab}J_{ab} + \frac{1}{l}e^{a}P_{a} + \frac{1}{\sqrt{l}}\psi^{\alpha}Q_{\alpha}, \tag{3.6}$$

and the associated two-form curvature $F = dA + A \wedge A$

$$F = F^{A}T_{A} = \frac{1}{2}\mathcal{R}^{ab}J_{ab} + \frac{1}{l}R^{a}P_{a} + \frac{1}{\sqrt{l}}\rho^{\alpha}Q_{\alpha}, \tag{3.7}$$

where

$$\begin{aligned}
\mathcal{R}^{ab} &= d\omega^{ab} + \omega^{a}_{\ c}\omega^{cb} + \frac{1}{l^{2}}e^{a}e^{b} + \frac{1}{2l}\bar{\psi}\gamma^{ab}\psi, \\
R^{a} &= de^{a} + \omega^{a}_{\ b}e^{b} - \frac{1}{2}\bar{\psi}\gamma^{a}\psi, \\
\rho &= d\psi + \frac{1}{4}\omega_{ab}\gamma^{ab}\psi + \frac{1}{2l}e^{a}\gamma_{a}\psi = D\psi + \frac{1}{2l}e^{a}\gamma_{a}\psi.
\end{aligned}$$

The one-forms e^{a}, ω^{ab} and ψ are respectively the vierbein, the spin connection and the gravitino field (a Majorana spinor, i.e, $\bar{\psi} = \psi^{T}C$, where C is the charge conjugation matrix).

Here we have introduced a length scale l. This is done because we have chosen the Lie algebra generators $T_{A} = \{J_{ab}, P_{a}, Q_{\alpha}\}$ as dimensionless and thus the one form connection $A = A^{A}_{\mu}T_{A}dx^{\mu}$ must also be dimensionless. However, the vierbein $e^{a} = e^{a}_{\ \mu}dx^{\mu}$ must have dimensions of length if it is related to the spacetime metric $g_{\mu\nu}$ through the usual equation $g_{\mu\nu} = e^{a}_{\ \mu}e^{b}_{\ \nu}\eta_{ab}$. This means that the "true" gauge field must be considered as e^{a}/l, with

l a length parameter. In the same way, as the gravitino $\psi = \psi_\mu dx^\mu$ has dimensions of $(\text{length})^{1/2}$, we must consider that $\psi/\sqrt{l}$ is the gauge field of supersymmetry.

The general form of an action constructed with the 2-form curvature (3.7) is given by

$$S = 2\int \langle F \wedge F \rangle = 2\int F^A \wedge F^B \langle T_A T_B \rangle \,. \tag{3.8}$$

Let us note that if we choose $\langle T_A T_B \rangle$ as an invariant tensor (which satisfies the Bianchi identity) for the $Osp\,(4|1)$ supergroup, then the action (3.8) is a topological invariant and gives no equations of motion. Nevertheless, with the following choice of the invariant tensor

$$\langle T_A T_B \rangle = \begin{cases} \langle J_{ab} J_{cd} \rangle = \epsilon_{abcd} \\ \langle Q_\alpha Q_\beta \rangle = 2\,(\gamma_5)_{\alpha\beta} \end{cases} \tag{3.9}$$

the action (3.8) becomes

$$S = 2\int \frac{1}{4}\mathcal{R}^{ab}\mathcal{R}^{ab}\epsilon_{abcd} + \frac{2}{l}\bar{\rho}\gamma_5\rho \tag{3.10}$$

which corresponds to the Mac Dowell-Mansouri action [19]. This choice of the invariant tensor, which is necessary in order to reproduce a dynamical action, breaks the $Osp\,(4|1)$ supergroup to their Lorentz subgroup.

The explicit form of the action is given by,

$$\begin{aligned} S = \int \frac{1}{2}\epsilon_{abcd}\left(R^{ab}R^{cd} + \frac{2}{l^2}R^{ab}e^c e^d + \frac{1}{l^4}e^a e^b e^c e^d + \frac{2}{l^3}\bar{\psi}\gamma^{ab}\psi e^c e^d\right) \\ + \frac{4}{l^2}\bar{\psi}e^a\gamma_a\gamma_5 D\psi + \frac{4}{l}d\left(\bar{\psi}\gamma_5 D\psi\right) \end{aligned} \tag{3.11}$$

which can be written, modulo boundary terms, as follow

$$S = \int \frac{1}{l^2}\left(\epsilon_{abcd}R^{ab}e^c e^d + 4\bar{\psi}e^a\gamma_a\gamma_5 D\psi\right) + \frac{1}{2}\epsilon_{abcd}\left(\frac{1}{l^4}e^a e^b e^c e^d + \frac{2}{l^3}\bar{\psi}\gamma^{ab}\psi e^c e^d\right). \tag{3.12}$$

The action (3.12) corresponds to the Mac Dowell-Mansouri action for the $\mathfrak{osp}\,(4|1)$ superalgebra [19, 20]. This action, describing $N = 1$, $D = 4$ supergravity, is not invariant under the $\mathfrak{osp}\,(4|1)$ gauge transformations. However the invariance of the action under supersymmetry transformation can be obtained modifying the spin connection ω^{ab} supersymmetry transformation [21].

4 $D = 4$ supergravity from minimal Maxwell superalgebra $s\mathcal{M}_4$

It was shown in ref. [15] that the minimal Maxwell superalgebra $s\mathcal{M}_4$ in $D = 4$ can be found by an S-expansion of the $\mathfrak{osp}\,(4|1)$ superalgebra given by (3.1)–(3.5). In fact, following [15] let us consider the S-expansion of the Lie superalgebra $\mathfrak{osp}\,(4|1)$ using $S_E^{(4)} = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}$ as an abelian semigroup. The elements of the semigroup are dimensionless and obey the multiplication law

$$\lambda_\alpha\lambda_\beta = \begin{cases} \lambda_{\alpha+\beta}, & \text{when } \alpha+\beta \le \lambda_5, \\ \lambda_5, & \text{when } \alpha+\beta > \lambda_5, \end{cases} \tag{4.1}$$

where λ_5 plays the role of the zero element of the semigroup. After extracting a resonant subalgebra and considering a reduction, one finds a new algebra, whose generators $J_{ab} = \lambda_0\tilde{J}_{ab}$, $P_a = \lambda_2\tilde{P}_a$, $\tilde{Z}_{ab} = \lambda_2\tilde{J}_{ab}$, $Z_{ab} = \lambda_4\tilde{J}_{ab}$, $Q_\alpha = \lambda_1\tilde{Q}_\alpha$, $\Sigma_\alpha = \lambda_3\tilde{Q}_\alpha$ satisfy the following commutation relations

$$[J_{ab}, J_{cd}] = \eta_{bc}J_{ad} - \eta_{ac}J_{bd} - \eta_{bd}J_{ac} + \eta_{ad}J_{bc}, \tag{4.2}$$

$$[J_{ab}, P_c] = \eta_{bc}P_a - \eta_{ac}P_b, \qquad [P_a, P_b] = Z_{ab}, \tag{4.3}$$

$$[J_{ab}, Z_{cd}] = \eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac} + \eta_{ad}Z_{bc}, \tag{4.4}$$

$$[P_a, Q_\alpha] = -\frac{1}{2}\left(\gamma_a\Sigma\right)_\alpha, \tag{4.5}$$

$$[J_{ab}, Q_\alpha] = -\frac{1}{2}\left(\gamma_{ab}Q\right)_\alpha, \tag{4.6}$$

$$[J_{ab}, \Sigma_\alpha] = -\frac{1}{2}\left(\gamma_{ab}\Sigma\right)_\alpha, \tag{4.7}$$

$$\{Q_\alpha, Q_\beta\} = -\frac{1}{2}\left[\left(\gamma^{ab}C\right)_{\alpha\beta}\tilde{Z}_{ab} - 2\left(\gamma^a C\right)_{\alpha\beta}P_a\right], \tag{4.8}$$

$$\{Q_\alpha, \Sigma_\beta\} = -\frac{1}{2}\left(\gamma^{ab}C\right)_{\alpha\beta}Z_{ab}, \tag{4.9}$$

$$\left[J_{ab}, \tilde{Z}_{ab}\right] = \eta_{bc}\tilde{Z}_{ad} - \eta_{ac}\tilde{Z}_{bd} - \eta_{bd}\tilde{Z}_{ac} + \eta_{ad}\tilde{Z}_{bc}, \tag{4.10}$$

$$\left[\tilde{Z}_{ab}, \tilde{Z}_{cd}\right] = \eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac} + \eta_{ad}Z_{bc}, \tag{4.11}$$

$$\left[\tilde{Z}_{ab}, Q_\alpha\right] = -\frac{1}{2}\left(\gamma_{ab}\Sigma\right)_\alpha, \tag{4.12}$$

$$\text{others} = 0. \tag{4.13}$$

This new superalgebra obtained after a reduced resonant S-expansion of $\mathfrak{osp}\,(4|1)$ superalgebra corresponds to a minimal superMaxwell algebra $s\mathcal{M}_4$ in $D = 4$ which contains the Maxwell algebra $\mathcal{M}_4 = \{J_{ab}, P_a, Z_{ab}\}$ and the Lorentz type subalgebra $\mathcal{L}^{\mathcal{M}_4} = \{J_{ab}, Z_{ab}\}$ as subalgebras. One can see that setting $\tilde{Z}_{ab} = 0$ leads us to the minimal Maxwell superalgebra introduced in ref. [14]. This can be done since the Jacobi identities for spinors generators are satisfied due to the gamma matrix identity $(C\gamma^a)_{(\alpha\beta}\,(C\gamma_a)_{\gamma\delta)} = 0$ (cyclic permutations of α, β, γ). An alternative expansion method to obtain the minimal Maxwell superalgebra can be found in ref. [7].

In order to write down an action for $s\mathcal{M}_4$, we start from the one-form gauge connection

$$A = \frac{1}{2}\omega^{ab}J_{ab} + \frac{1}{2}\tilde{k}^{ab}\tilde{Z}_{ab} + \frac{1}{2}k^{ab}Z_{ab} + \frac{1}{l}e^a P_a + \frac{1}{\sqrt{l}}\psi^\alpha Q_\alpha + \frac{1}{\sqrt{l}}\xi^\alpha\Sigma_\alpha, \tag{4.14}$$

where the 1-form gauge fields are given by

$$\begin{aligned}
\omega^{ab} &= \omega^{(ab,0)} = \lambda_0\tilde{\omega}^{ab}, & e^a &= e^{(a,2)} = \lambda_2\tilde{e}^a,\\
\tilde{k}^{ab} &= \omega^{(ab,2)} = \lambda_2\tilde{\omega}^{ab}, & \psi^\alpha &= \psi^{(\alpha,1)} = \lambda_1\tilde{\psi}^\alpha,\\
k^{ab} &= \omega^{(ab,4)} = \lambda_4\tilde{\omega}^{ab}, & \xi^\alpha &= \psi^{(\alpha,3)} = \lambda_3\tilde{\psi}^\alpha,
\end{aligned}$$

in terms of $\tilde{e}^a, \tilde{\omega}^{ab}$ and $\tilde{\psi}$ which are the components of the $\mathfrak{osp}\,(4|1)$ connection.

JHEP09(2014)090

The associated two-form curvature $F = dA + A \wedge A$ is

$$F = F^A T_A = \frac{1}{2} R^{ab} J_{ab} + \frac{1}{l} R^a P_a + \frac{1}{2} \tilde{F}^{ab} \tilde{Z}_{ab} + \frac{1}{2} F^{ab} Z_{ab} + \frac{1}{\sqrt{l}} \Psi^\alpha Q_\alpha + \frac{1}{\sqrt{l}} \Xi^\alpha \Sigma_\alpha, \tag{4.15}$$

where

$$R^{ab} = d\omega^{ab} + \omega^a_{\ c} \omega^{cb}, \tag{4.16}$$

$$R^a = de^a + \omega^a_{\ b} e^b - \frac{1}{2} \bar{\psi} \gamma^a \psi, \tag{4.17}$$

$$\tilde{F}^{ab} = d\tilde{k}^{ab} + \omega^a_{\ c} \tilde{k}^{cb} - \omega^b_{\ c} \tilde{k}^{ca} + \frac{1}{2l} \bar{\psi} \gamma^{ab} \psi, \tag{4.18}$$

$$F^{ab} = dk^{ab} + \omega^a_{\ c} k^{cb} - \omega^b_{\ c} k^{ca} + \tilde{k}^a_{\ c} \tilde{k}^{cb} + \frac{1}{l^2} e^a e^b + \frac{1}{l} \bar{\xi} \gamma^{ab} \psi, \tag{4.19}$$

$$\Psi = d\psi + \frac{1}{4} \omega_{ab} \gamma^{ab} \psi = D\psi, \tag{4.20}$$

$$\begin{aligned} \Xi &= d\xi + \frac{1}{4} \omega_{ab} \gamma^{ab} \xi + \frac{1}{4} \tilde{k}_{ab} \gamma^{ab} \psi + \frac{1}{2l} e^a \gamma_a \psi \\ &= D\xi + \frac{1}{4} \tilde{k}_{ab} \gamma^{ab} \psi + \frac{1}{2l} e^a \gamma_a \psi. \end{aligned} \tag{4.21}$$

From the Bianchi identity $\nabla F = 0$, where $\nabla = d + [A, \cdot]$, it is possible to show that the Lorentz covariant exterior derivatives of the curvatures are given by,

$$DR^{ab} = 0, \tag{4.22}$$

$$DR^a = R^a_{\ b} e^b + \bar{\psi} \gamma^a \Psi, \tag{4.23}$$

$$D\tilde{F}^{ab} = R^a_{\ c} \tilde{k}^{cb} - R^b_{\ c} \tilde{k}^{ca} - \frac{1}{l} \bar{\psi} \gamma^{ab} \Psi \tag{4.24}$$

$$DF^{ab} = R^a_{\ c} k^{cb} - R^b_{\ c} k^{ca} + \tilde{F}^a_{\ c} \tilde{k}^{cb} - \tilde{F}^b_{\ c} \tilde{k}^{ca} + \frac{1}{l^2} R^a e^b - \frac{1}{l^2} e^a R^b \tag{4.25}$$

$$+ \frac{1}{l} \bar{\Xi} \gamma^{ab} \psi - \frac{1}{l} \bar{\xi} \gamma^{ab} \Psi, \tag{4.26}$$

$$D\Psi = \frac{1}{4} R_{ab} \gamma^{ab} \psi, \tag{4.27}$$

$$D\Xi = \frac{1}{4} R_{ab} \gamma^{ab} \xi - \frac{1}{4} \tilde{k}_{ab} \gamma^{ab} \Psi + \frac{1}{4} \tilde{F}_{ab} \gamma^{ab} \psi + \frac{1}{2l} R^a \gamma_a \psi - \frac{1}{2l} e^a \gamma_a \Psi. \tag{4.28}$$

Then, the action can be written as

$$S = 2 \int \langle F \wedge F \rangle = 2 \int F^A \wedge F^B \langle T_A T_B \rangle, \tag{4.29}$$

where $\langle T_A T_B \rangle$ corresponds to an S-expanded invariant tensor which is obtained from (3.9). Using theorem VII.1 of ref. [9] it is possible to show that these components are given by

$$\langle J_{ab} J_{cd} \rangle_{s\mathcal{M}_4} = \alpha_0 \langle J_{ab} J_{cd} \rangle, \tag{4.30}$$

$$\left\langle J_{ab} \tilde{Z}_{cd} \right\rangle_{s\mathcal{M}_4} = \alpha_2 \langle J_{ab} J_{cd} \rangle, \tag{4.31}$$

JHEP09(2014)090

$$\left\langle \tilde{Z}_{ab}\tilde{Z}_{cd}\right\rangle_{s\mathcal{M}_4} = \alpha_4 \left\langle J_{ab}J_{cd}\right\rangle, \tag{4.32}$$

$$\left\langle J_{ab}Z_{cd}\right\rangle_{s\mathcal{M}_4} = \alpha_4 \left\langle J_{ab}J_{cd}\right\rangle, \tag{4.33}$$

$$\left\langle Q_\alpha Q_\beta\right\rangle_{s\mathcal{M}_4} = \alpha_2 \left\langle Q_\alpha Q_\beta\right\rangle, \tag{4.34}$$

$$\left\langle Q_\alpha \Sigma_\beta\right\rangle_{s\mathcal{M}_4} = \alpha_4 \left\langle Q_\alpha Q_\beta\right\rangle, \tag{4.35}$$

where

$$\left\langle J_{ab}J_{cd}\right\rangle = \epsilon_{abcd} \tag{4.36}$$

$$\left\langle Q_\alpha Q_\beta\right\rangle = 2\left(\gamma_5\right)_{\alpha\beta} \tag{4.37}$$

and the α's are dimensionless arbitrary independent constants.

Considering the different components of the invariant tensor (4.30)–(4.35) and the two-form curvature (4.15), we found that the action can be written as

$$S = 2\int \left(\frac{1}{4}\alpha_0\epsilon_{abcd}R^{ab}R^{cd} + \frac{1}{2}\alpha_2\epsilon_{abcd}R^{ab}\tilde{F}^{cd} + \frac{1}{2}\alpha_4\epsilon_{abcd}R^{ab}F^{cd} + \frac{1}{4}\alpha_4\epsilon_{abcd}\tilde{F}^{ab}\tilde{F}^{cd} + \frac{2}{l}\alpha_2\bar{\Psi}\gamma_5\Psi + \frac{4}{l}\alpha_4\bar{\Psi}\gamma_5\Xi\right) \tag{4.38}$$

or explicitly,

$$\begin{aligned} S = \int & \frac{\alpha_0}{2}\epsilon_{abcd}R^{ab}R^{cd} + \alpha_2\epsilon_{abcd}\left(R^{ab}D\tilde{k}^{cd} + \frac{1}{2l}R^{ab}\bar{\psi}\gamma^{cd}\psi\right) \\ & + \frac{4}{l}\alpha_2 D\bar{\psi}\gamma_5 D\psi + \alpha_4\epsilon_{abcd}\left(R^{ab}Dk^{cd} + \frac{1}{2}D\tilde{k}^{ab}D\tilde{k}^{cd} + \frac{1}{l^2}R^{ab}e^c e^d\right. \\ & \left. + \frac{1}{2l}D\tilde{k}^{ab}\bar{\psi}\gamma^{cd}\psi + R^{ab}\tilde{k}^c{}_f\tilde{k}^{fd} + \frac{1}{l}R^{ab}\bar{\xi}\gamma^{cd}\psi\right) \\ & + \frac{8}{l}\alpha_4 D\bar{\psi}\gamma_5 D\xi + \frac{2}{l}\alpha_4 D\bar{\psi}\gamma_5\tilde{k}_{ab}\gamma^{ab}\psi + \frac{4}{l^2}\alpha_4\bar{\psi}e^a\gamma_a\gamma_5 D\psi \end{aligned} \tag{4.39}$$

where $D = d + [\omega, \cdot]$. Using the gravitino Bianchi identity

$$D\Psi = \frac{1}{4}R^{ab}\gamma_{ab}\psi, \tag{4.40}$$

and the gamma matrix identity

$$2\gamma_{ab}\gamma_5 = -\epsilon_{abcd}\gamma^{cd}, \tag{4.41}$$

it is possible to show that,

$$\begin{aligned} \frac{1}{2}\epsilon_{abcd}R^{ab}\bar{\psi}\gamma^{ab}\psi + 4D\bar{\psi}\gamma_5 D\psi &= d\left(4D\bar{\psi}\gamma_5\psi\right), \\ \epsilon_{abcd}R^{ab}\bar{\xi}\gamma^{cd}\psi + 8D\bar{\xi}\gamma_5 D\psi &= d\left(8D\bar{\xi}\gamma_5\psi\right), \\ \frac{1}{2}\epsilon_{abcd}D\tilde{k}^{ab}\bar{\psi}\gamma^{cd}\psi + 2\bar{\psi}\tilde{k}^{ab}\gamma_{ab}\gamma_5 D\psi &= d\left(\bar{\psi}\tilde{k}^{ab}\gamma_{ab}\gamma_5\psi\right). \end{aligned}$$

JHEP09(2014)090

Thus the Mac Dowell-Mansouri like action for the $s\mathcal{M}_4$ superalgebra is finally given by

$$
\begin{aligned}
S = \int & \frac{\alpha_0}{2}\epsilon_{abcd}R^{ab}R^{cd} + \alpha_2 d\left(\epsilon_{abcd}R^{ab}\tilde{k}^{cd} + \frac{4}{l}D\bar{\psi}\gamma_5\psi\right) \\
& + \alpha_4\left[\frac{1}{l^2}\epsilon_{abcd}R^{ab}e^c e^d + \frac{4}{l^2}\bar{\psi}e^a\gamma_a\gamma_5 D\psi\right. \\
& \left. + d\left(\epsilon_{abcd}\left(R^{ab}k^{cd} + \frac{1}{2}D\tilde{k}^{ab}\tilde{k}^{cd}\right) + \frac{8}{l}\bar{\xi}\gamma_5 D\psi + \frac{1}{l}\bar{\psi}\tilde{k}^{ab}\gamma_{ab}\gamma_5\psi\right)\right]. \qquad (4.42)
\end{aligned}
$$

Here we can see that the lagrangian is split into three independent pieces proportional to α_0, α_2 and α_4. The term proportional to α_0 corresponds to the Euler invariant. The piece proportional to α_2 is a boundary term. The term proportional to α_4 contains the Einstein-Hilbert term $\epsilon_{abcd}R^{ab}e^c e^d$ plus the Rarita-Schwinger lagrangian $4\bar{\psi}e^a\gamma_a\gamma_5 D\psi$, and a boundary term.

From (4.42) we can see that the minimal Maxwell superalgebra $s\mathcal{M}_4$ leads us to the pure supergravity action plus boundary terms. In this way the new Maxwell gauge fields do not contribute to the dynamics and enlarge only the boundary terms. Furthermore, as a consequence of the S-expansion procedure the supersymmetric cosmological term disappears completely from the action for $s\mathcal{M}_4$.

This result is particularly interesting since it corresponds to the supersymmetric case of refs. [10, 12], where Einstein-Hilbert action is obtained from Maxwell algebra[2] as a Born-Infeld like action.

Let us note that if we consider $\tilde{k}^{ab} = 0$, the term proportional to α_4 corresponds to the action found in [18], namely

$$
S|_{\tilde{k}^{ab}=0} = \alpha_4 \int \frac{1}{l^2}\left(\epsilon_{abcd}R^{ab}e^c e^d + 4\bar{\psi}e^a\gamma_a\gamma_5 D_\omega\psi\right) + d\left(\epsilon_{abcd}R^{ab}k^{cd} + \frac{8}{l}\bar{\xi}\gamma_5 D_\omega\psi\right) \qquad (4.43)
$$

which corresponds to four-dimensional pure supergravity plus a boundary term. This is not a surprise but something expected, because as we said before setting $\tilde{Z}_{ab} = 0$ in $s\mathcal{M}_4$ leads us to the simplest minimal Maxwell superalgebra [15], whose two-form curvature associated allows the construction of (4.43) as was shown in [18].

It would be interesting to study the possibility to obtain the action (4.42) from the approach considered in ref. [22] in which $N = 1$ and $N = 2$ supergravities are constructed in the presence of a non trivial boundary.

4.1 $s\mathcal{M}_4$ gauge transformations and supersymmetry

The gauge transformation of the connection A is

$$
\delta_\rho A = D\rho = d\rho + [A, \rho] \qquad (4.44)
$$

where ρ is the $s\mathcal{M}_4$ gauge parameter,

$$
\rho = \frac{1}{2}\rho^{ab}J_{ab} + \frac{1}{2}\tilde{\kappa}^{ab}\tilde{Z}_{ab} + \frac{1}{2}\kappa^{ab}Z_{ab} + \frac{1}{l}\rho^a P_a + \frac{1}{\sqrt{l}}\epsilon^\alpha Q_\alpha + \frac{1}{\sqrt{l}}\varrho^\alpha\Sigma_\alpha. \qquad (4.45)
$$

[2] Also known as $\mathfrak{B}_4$ algebra.

Then, using

$$\delta\left(A^A T_A\right) = d\rho + \left[A^B T_B, \rho^C T_C\right], \tag{4.46}$$

the $s\mathcal{M}_4$ gauge transformation are given by

$$\delta\omega^{ab} = D\rho^{ab}, \tag{4.47}$$

$$\delta\tilde{k}^{ab} = D\tilde{\kappa}^{ab} - \left(\tilde{k}^{a}_{\ c}\,\rho^{b}_{\ c} - \tilde{k}^{bc}\rho^{a}_{\ c}\right) - \frac{1}{l}\bar{\epsilon}\gamma^{ab}\psi, \tag{4.48}$$

$$\begin{aligned}\delta k^{ab} = {} & D\kappa^{ab} - \left(k^{ac}\rho^{b}_{\ c} - k^{bc}\rho^{a}_{\ c}\right) - \left(\tilde{k}^{ac}\tilde{\kappa}^{b}_{\ c} - \tilde{k}^{bc}\tilde{\kappa}^{a}_{\ c}\right)\\ & + \frac{2}{l^2}e^a\rho^b - \frac{1}{l}\bar{\varrho}\gamma^{ab}\psi - \frac{1}{l}\bar{\epsilon}\gamma^{ab}\xi,\end{aligned} \tag{4.49}$$

$$\delta e^a = D\rho^a + e^b\rho_b^{\ a} + \bar{\epsilon}\gamma^a\psi, \tag{4.50}$$

$$\delta\psi = d\epsilon + \frac{1}{4}\omega^{ab}\gamma_{ab}\epsilon - \frac{1}{4}\rho^{ab}\gamma_{ab}\psi, \tag{4.51}$$

$$\begin{aligned}\delta\xi = {} & d\varrho + \frac{1}{4}\omega^{ab}\gamma_{ab}\varrho + \frac{1}{2l}e^a\gamma_a\epsilon - \frac{1}{2l}\rho^a\gamma_a\psi - \frac{1}{4}\rho^{ab}\gamma_{ab}\xi\\ & + \frac{1}{4}\tilde{k}^{ab}\gamma_{ab}\epsilon - \frac{1}{4}\tilde{\kappa}^{ab}\gamma_{ab}\psi.\end{aligned} \tag{4.52}$$

In the same way, from the gauge variation of the curvature

$$\delta_\rho F = [F, \rho] \tag{4.53}$$

it is possible to show that the gauge transformations of the curvature F are given by

$$\delta R^{ab} = R^{ac}\rho_c^{\ b} - R^{cb}\rho^{a}_{\ c}, \tag{4.54}$$

$$\delta\tilde{F}^{ab} = \left(R^{ac}\tilde{\kappa}_c^{\ b} - R^{bc}\tilde{\kappa}^{a}_{\ c}\right) - \left(\tilde{F}^{ac}\rho^{b}_{\ c} - \tilde{F}^{bc}\rho^{a}_{\ c}\right) - \frac{1}{l}\bar{\epsilon}\gamma^{ab}\Psi, \tag{4.55}$$

$$\begin{aligned}\delta F^{ab} = {} & \left(R^{ac}\kappa_c^{\ b} - R^{bc}\kappa^{a}_{\ c}\right) - \left(F^{ac}\rho^{b}_{\ c} - F^{bc}\rho^{a}_{\ c}\right) - \left(\tilde{F}^{ac}\tilde{\kappa}^{b}_{\ c} - \tilde{F}^{ac}\tilde{\kappa}^{a}_{\ c}\right)\\ & + \frac{2}{l^2}R^a\rho^b - \frac{1}{l}\bar{\varrho}\gamma^{ab}\Psi - \frac{1}{l}\bar{\epsilon}\gamma^{ab}\Xi,\end{aligned} \tag{4.56}$$

$$\delta R^a = R^{a}_{\ b}\rho^b + R^b\rho_b^{\ a} + \bar{\epsilon}\gamma^a\Psi, \tag{4.57}$$

$$\delta\Psi = \frac{1}{4}R^{ab}\gamma_{ab}\epsilon - \frac{1}{4}\rho^{ab}\gamma_{ab}\Psi, \tag{4.58}$$

$$\delta\Xi = \frac{1}{4}R^{ab}\gamma_{ab}\varrho + \frac{1}{2l}R^a\gamma_a\epsilon - \frac{1}{2l}\rho^a\gamma_a\Psi - \frac{1}{4}\rho^{ab}\gamma_{ab}\Xi + \frac{1}{4}\tilde{F}^{ab}\gamma_{ab}\epsilon - \frac{1}{4}\tilde{\kappa}^{ab}\gamma_{ab}\Psi, \tag{4.59}$$

Although the Mac Dowell-Mansouri like action (4.42) is built from the $s\mathcal{M}_4$ curvature, it is *not* invariant under the $s\mathcal{M}_4$ gauge transformations. As we can see the action does not correspond to a Yang-Mills action, nor a topological invariant.

Furthermore, the action is not invariant under gauge supersymmetry. In fact, if we consider the variation of the action (4.42) under gauge supersymmetry, we find

$$\delta_{\text{susy}}S = -\frac{4}{l^2}\alpha_4\int R^a\bar{\Psi}\gamma_a\gamma_5\epsilon. \tag{4.60}$$

As in $\mathfrak{osp}\,(4|1)$ and super-Poincaré cases, the action is invariant under gauge supersymmetry imposing the super torsion constraint

$$R^a = 0. \tag{4.61}$$

This yields to express the spin connection ω^{ab} in terms of the vielbein and the gravitino fields. This leads to the supersymmetric action for $s\mathcal{M}_4$ superalgebra in second order formalism.

Alternatively, it is possible to have supersymmetry in first order formalism if we modify the supersymmetry transformation for the spin connection ω^{ab}. In fact, if we consider the variation of the action under an arbitrary $\delta\omega^{ab}$ we find

$$\delta_\omega S = \frac{2}{l^2}\alpha_4 \int \epsilon_{abcd} R^a e^b \delta\omega^{cd}, \tag{4.62}$$

thus the variation vanish for arbitrary $\delta\omega^{ab}$ if $R^a = 0$. Similarly to ref. [21], it is possible to modify $\delta\omega^{ab}$ adding an extra piece to the gauge transformation such that the variation of the action can be written as

$$\delta S = -\frac{4}{l^2}\alpha_4 \int R^a \left(\bar{\Psi}\gamma_a\gamma_5\epsilon - \frac{1}{2}\epsilon_{abcd} e^b \delta_{\text{extra}}\omega^{cd} \right). \tag{4.63}$$

In order to have an invariant action, $\delta_{\text{extra}}\omega^{ab}$ is given by

$$\delta_{\text{extra}}\omega^{ab} = 2\epsilon^{abcd} \left(\bar{\Psi}_{ec}\gamma_d\gamma_5\epsilon + \bar{\Psi}_{de}\gamma_c\gamma_5\epsilon - \bar{\Psi}_{cd}\gamma_e\gamma_5\epsilon \right) e^e, \tag{4.64}$$

with $\bar{\Psi} = \bar{\Psi}_{ab} e^a e^b$.

Then the action in the first order formalism is invariant under the following supersymmetry transformations

$$\delta\omega^{ab} = 2\epsilon^{abcd} \left(\bar{\Psi}_{ec}\gamma_d\gamma_5\epsilon + \bar{\Psi}_{de}\gamma_c\gamma_5\epsilon - \bar{\Psi}_{cd}\gamma_e\gamma_5\epsilon \right) e^e, \tag{4.65}$$

$$\delta\tilde{k}^{ab} = -\frac{1}{l}\bar{\epsilon}\gamma^{ab}\psi, \tag{4.66}$$

$$\delta k^{ab} = -\frac{1}{l}\bar{\epsilon}\gamma^{ab}\xi, \tag{4.67}$$

$$\delta e^a = \bar{\epsilon}\gamma^a\psi, \tag{4.68}$$

$$\delta\psi = d\epsilon + \frac{1}{4}\omega^{ab}\gamma_{ab}\epsilon = D\epsilon, \tag{4.69}$$

$$\delta\xi = \frac{1}{2l}e^a\gamma_a\epsilon + \frac{1}{4}\tilde{k}^{ab}\gamma_{ab}\epsilon. \tag{4.70}$$

On the other hand, it is important to note that there is a new supersymmetry related to the spinor charge Σ. The new supersymmetry transformations are given by

$$\delta\omega^{ab} = 0, \tag{4.71}$$

$$\delta\tilde{k}^{ab} = 0, \tag{4.72}$$

$$\delta k^{ab} = -\frac{1}{l}\bar{\varrho}\gamma^{ab}\psi, \tag{4.73}$$

$$\delta e^a = 0, \tag{4.74}$$

$$\delta\psi = 0, \tag{4.75}$$

$$\delta\xi = d\varrho + \frac{1}{4}\omega^{ab}\gamma_{ab}\varrho. \tag{4.76}$$

Considering the variation of the action (4.42) under the new gauge supersymmetry transformations, we find that the action is truly invariant

$$\delta S = 0. \tag{4.77}$$

Then one can see that the action is off-shell invariant under a subalgebra of $s\mathcal{M}_4$ given by $s\mathcal{L}_{\mathcal{M}_4} = \left\{ J_{ab}, \tilde{Z}_{ab}, Z_{ab}, \Sigma_\alpha \right\}$ which corresponds to a Lorentz type superalgebra. These results are interesting since we have shown that the Poincaré supersymmetry is not the only supersymmetry of $D = 4$ pure supergravity.

5 $D = 4$ supergravity from the minimal Maxwell superalgebra type $s\mathcal{M}_{m+2}$

It was shown in ref. [15] that the $D = 4$ minimal superMaxwell algebra type $s\mathcal{M}_{m+2}$ can be found by an S-expansion of the $\mathfrak{osp}\,(4|1)$ superalgebra given by (3.1)–(3.5). In fact, following [15] let us consider the S-expansion of the Lie superalgebra $\mathfrak{osp}\,(4|1)$ using $S_E^{(2m)} = \{\lambda_0, \lambda_1, \lambda_2, \cdots, \lambda_{2m+1}\}$ as the relevant finite abelian semigroup. The elements of the semigroup are dimensionless and obey the multiplication law

$$\lambda_\alpha \lambda_\beta = \begin{cases} \lambda_{\alpha+\beta}, & \text{when } \alpha+\beta \leq \lambda_{2m+1}, \\ \lambda_{2m+1}, & \text{when } \alpha+\beta > \lambda_{2m+1}, \end{cases} \tag{5.1}$$

where λ_{2m+1} plays the role of the zero element of the semigroup $S_E^{(2m)}$. After extracting a resonant subalgebra and considering a reduction, one finds the $D = 4$ minimal Maxwell superalgebra type $s\mathcal{M}_{m+2}$. The new superalgebra obtained by the S-expansion procedure is generated by

$$\left\{ J_{ab,(k)}, P_{a,(l)}, Q_{\alpha,(k)} \right\}, \tag{5.2}$$

where these new generators can be written as

$$J_{ab,(k)} = \lambda_{2k}\tilde{J}_{ab}, \tag{5.3}$$

$$P_{a,(l)} = \lambda_{2l}\tilde{P}_a, \tag{5.4}$$

$$Q_{\alpha,(p)} = \lambda_{2p-1}\tilde{Q}_\alpha, \tag{5.5}$$

with $k = 0, \ldots, m$; $l = p = 1, \ldots, m$. Here, the generators $\tilde{J}_{ab}, \tilde{P}_a$ and $\tilde{Q}_\alpha$ correspond to the $\mathfrak{osp}\,(4|1)$ generators. The new generators satisfy the commutation relations

$$\left[J_{ab,(k)}, J_{cd,(j)}\right] = \eta_{bc}J_{ad,(k+j)} - \eta_{ac}J_{bd,(k+j)} - \eta_{bd}J_{ac,(k+j)} + \eta_{ad}J_{bc,(k+j)}, \tag{5.6}$$

$$\left[J_{ab,(k)}, P_{a,(l)}\right] = \eta_{bc}P_{a,(k+l)} - \eta_{ac}P_{b,(k+l)}, \tag{5.7}$$

$$\left[P_{a,(l)}, P_{b,(n)}\right] = J_{ab,(l+n)}, \tag{5.8}$$

JHEP09(2014)090

$$\left[J_{ab,(k)},Q_{\alpha,(p)}\right]=-\frac{1}{2}\left(\gamma_{ab}Q\right)_{\alpha,(k+p)}, \tag{5.9}$$

$$\left[P_{a,(l)},Q_{\alpha,(p)}\right]=-\frac{1}{2}\left(\gamma_{a}Q\right)_{\alpha,(l+p)}, \tag{5.10}$$

$$\left\{Q_{\alpha,(p)},Q_{\beta,(q)}\right\}=-\frac{1}{2}\left[\left(\gamma^{ab}C\right)_{\alpha\beta}J_{ab,(p+q)}-2\left(\gamma^{a}C\right)_{\alpha\beta}P_{a,(p+q)}\right]. \tag{5.11}$$

Naturally, when $k+j>m$ the generators $T_A^{(k)}$ and $T_B^{(j)}$ are abelian. One sees that if we redefine the generators as

$$\begin{aligned}
J_{ab}&=J_{ab,0}=\lambda_0\tilde{J}_{ab}, & P_a&=P_{a,2}=\lambda_2\tilde{P}_a,\\
Z_{ab}^{(k)}&=J_{ab,4k}=\lambda_{4k}\tilde{J}_{ab}, & Z_a^{(l)}&=P_{a,4l+2}=\lambda_{4l+2}\tilde{P}_a,\\
\tilde{Z}_{ab}^{(k)}&=J_{ab,4k-2}=\lambda_{4k-2}\tilde{J}_{ab}, & \tilde{Z}_a^{(l)}&=P_{a,4l}=\lambda_{4l}\tilde{P}_a,\\
Q_\alpha&=Q_{\alpha,1}=\lambda_1\tilde{Q}_\alpha, & \Sigma_\alpha^{(k)}&=Q_{\alpha,4k-1}=\lambda_{4k-1}\tilde{Q}_\alpha,\\
\Phi_\alpha^{(l)}&=Q_{\alpha,4k+1}=\lambda_{4k+1}\tilde{Q}_\alpha,
\end{aligned}$$

we obtain the commutation relations for $s\mathcal{M}_{m+2}$ introduced in [15]. However, if we want to build an action and avoid extensive terms we shall use (5.3)–(5.5). In order to write down a Lagrangian for $s\mathcal{M}_{m+2}$, we start from the one-form gauge connection

$$A=\frac{1}{2}\sum_k\omega^{ab,(k)}J_{ab,(k)}+\frac{1}{l}\sum_l e^{a,(l)}P_{a,(l)}+\frac{1}{\sqrt{l}}\sum_p\psi^{\alpha,(p)}Q_{\alpha,(p)}, \tag{5.12}$$

where the different components are given by

$$\omega^{ab,(k)}=\lambda_{2k}\tilde{\omega}^{ab}, \tag{5.13}$$

$$e^{a,(l)}=\lambda_{2l}\tilde{e}^{a}, \tag{5.14}$$

$$\psi^{\alpha,(p)}=\lambda_{2p-1}\tilde{\psi}^{\alpha}, \tag{5.15}$$

in terms of $\tilde{e}^a,\tilde{\omega}^{ab}$ and $\tilde{\psi}$ which are the components of the $\mathfrak{osp}\,(4|1)$ connection.

The associated two-form curvature $F=dA+A\wedge A$ is

$$F=F^AT_A=\frac{1}{2}\sum_k\mathcal{R}^{ab,(k)}J_{ab,(k)}+\frac{1}{l}\sum_l R^{a,(l)}P_{a,(l)}+\frac{1}{\sqrt{l}}\sum_p\Psi^{\alpha,(p)}Q_{\alpha,(p)}, \tag{5.16}$$

where

$$\begin{aligned}
\mathcal{R}^{ab,(k)}&=d\omega^{ab,(k)}+\omega^{a}{}_{c}{}^{(i)}\wedge\omega^{cb,(j)}\delta_{i+j}^{k}+\frac{1}{l^2}e^{a,(l)}e^{b,(n)}\delta_{l+n}^{k}\\
&\quad+\frac{1}{2l}\bar{\psi}^{(p)}\gamma^{ab}\wedge\psi^{(q)}\delta_{p+q}^{2k},
\end{aligned} \tag{5.17}$$

$$R^{a,(l)}=de^{a,(l)}+\omega^{a}{}_{b}{}^{(k)}\wedge e^{b,(n)}\delta_{k+n}^{l}-\frac{1}{2}\bar{\psi}^{(p)}\gamma^{a}\wedge\psi^{(q)}\delta_{p+q}^{2l}, \tag{5.18}$$

$$\Psi^{(p)}=d\psi^{(p)}+\frac{1}{4}\omega_{ab}{}^{(k)}\gamma^{ab}\wedge\psi^{(q)}\delta_{k+q}^{p}+\frac{1}{2l}e^{a,(l)}\gamma_a\wedge\psi^{(q)}\delta_{l+q}^{p}, \tag{5.19}$$

with $k = 0, \ldots, m$; $l = p = 1, \ldots, m$. The Bianchi identities can be obtained by considering $\nabla F = 0$, where $\nabla = d + [A, \cdot]$,

$$D\mathcal{R}^{ab,(k)} = \left(\mathcal{R}^{ac,(i)}\omega_c{}^{b,(j+1)} - \mathcal{R}^{bc,(i)}\omega_c{}^{a,(j+1)}\right)\delta^k_{i+j+1} + \frac{1}{l}\left(R^{a,(l)}e^{b,(n)} - e^{a,(n)}R^{b,(l)}\right)\delta^k_{l+n} - \frac{1}{l}\bar{\psi}^{(p)}\gamma^{ab}\Psi^{(q)}\delta^{2k}_{p+q}, \tag{5.20}$$

$$DR^{a,(l)} = \mathcal{R}^{ab,(i)}e_b{}^{,(j)}\delta^l_{i+j} + R^{c,(n)}\omega_c{}^{a,(j+1)}\delta^l_{n+j+1} + \bar{\psi}^{(p)}\gamma^a\Psi^{(q)}\delta^{2l}_{p+q}, \tag{5.21}$$

$$D\Psi^{(p)} = \frac{1}{4}\left(\mathcal{R}^{ab,(i)}\gamma_{ab}\psi^{(q)}\right)\delta^p_{i+q} - \frac{1}{4}\left(\omega^{ab,(i+1)}\gamma_{ab}\Psi^{(q)}\right)\delta^p_{i+1+q} + \frac{1}{2l}\left(T^{a,(l)}\gamma_a\psi^{(q)}\right)\delta^p_{l+q} - \frac{1}{2l}\left(e^{a,(l)}\gamma_a\Psi^{(q)}\right)\delta^p_{l+q}, \tag{5.22}$$

where D corresponds to the Lorentz covariant exterior derivative $D = d + [\omega, \cdot]$.

Then the action can be written as

$$S = 2\int \langle F \wedge F\rangle = 2\int F^A \wedge F^B \langle T_A T_B\rangle, \tag{5.23}$$

where $\langle T_A T_B\rangle$ corresponds to an S-expanded invariant tensor which is obtained from (3.9). Using theorem VII.1 of ref. [9] it is possible to show that these components are given by

$$\left\langle J_{ab,(k)}J_{cd,(j)}\right\rangle_{s\mathcal{M}_{m+2}} = \alpha_{2(k+j)}\left\langle J_{ab}J_{cd}\right\rangle, \tag{5.24}$$

$$\left\langle Q_{\alpha,(p)}Q_{\beta,(q)}\right\rangle_{s\mathcal{M}_{m+2}} = \alpha_{2(p+q-1)}\left\langle Q_\alpha Q_\beta\right\rangle, \tag{5.25}$$

which can be written as

$$\left\langle J_{ab,(k)}J_{cd,(j)}\right\rangle_{s\mathcal{M}_{m+2}} = \alpha_{2(k+j)}\epsilon_{abcd}, \tag{5.26}$$

$$\left\langle Q_{\alpha,(p)}Q_{\beta,(q)}\right\rangle_{s\mathcal{M}_{m+2}} = 2\alpha_{2(p+q-1)}\left(\gamma_5\right)_{\alpha\beta}, \tag{5.27}$$

where the α's are arbitrary independent constants and $J_{ab,(k)}$, $Q_{\alpha,(p)}$ are given by (5.3), (5.5), respectively. Using the different components of the invariant tensor (5.26)–(5.27) and the two-form curvature (5.16), we found that the action is given by

$$S = 2\int \sum_{k,j}\frac{\alpha_{2(k+j)}}{2}\epsilon_{abcd}\mathcal{R}^{ab,(k)}\mathcal{R}^{cd,(j)} + \sum_{p,q}\alpha_{2(p+q-1)}\frac{4}{l}\bar{\Psi}^{(p)} \wedge \gamma_5\Psi^{(q)}, \tag{5.28}$$

with $k, j = 0, \ldots, m$; $p, q = 1, \ldots, m$.

5.1 $s\mathcal{M}_{m+2}$ gauge transformations and supersymmetry

The gauge transformation of the connection A is

$$\delta_\rho A = D\rho = d\rho + [A, \rho] \tag{5.29}$$

where ρ is the $s\mathcal{M}_{m+2}$ gauge parameter:

$$\rho = \frac{1}{2}\sum_k \rho^{ab,(k)}J_{ab,(k)} + \frac{1}{l}\sum_l \rho^{a,(l)}P_{a,(l)} + \frac{1}{\sqrt{l}}\sum_p \epsilon^{\alpha,(p)}Q_{\alpha,(p)}. \tag{5.30}$$

Here we have written the components of the gauge parameter as an S-expansion of the component of the $\mathfrak{osp}(4|1)$ gauge parameter,

$$
\begin{aligned}
\rho^{ab,(k)} &= \lambda_{2k}\tilde{\rho}^{ab}, \\
\rho^{a,(l)} &= \lambda_{2l}\tilde{\rho}^{a}, \\
\epsilon^{\alpha,(p)} &= \lambda_{2p-1}\tilde{\epsilon}^{\alpha},
\end{aligned}
$$

with $k = 0, \ldots, m$; $l = p = 1, \ldots, m$ and $\lambda_i \in S_E^{(2m)} = \{\lambda_0, \lambda_1, \lambda_2, \cdots, \lambda_{2m+1}\}$. Then, using the multiplication law of the semigroup (5.1) and

$$
\delta\left(A^A T_A\right) = d\rho + \left[A^B T_B, \rho^C T_C\right], \tag{5.31}
$$

it is possible to show that the gauge transformations are given by

$$
\begin{aligned}
\delta\omega^{ab,(k)} &= D\rho^{ab,(k)} - \left(\omega^{ac,(i+1)}\rho_{c}^{\ b,(j)} - \omega^{bc,(i+1)}\rho_{c}^{\ a,(j)}\right)\delta_{i+j+1}^{k} \\
&\quad + \frac{2}{l^2}e^{a,(l)}\rho^{b,(n)}\delta_{l+n}^{k} - \frac{1}{l}\bar{\epsilon}^{(p)}\gamma^{ab}\psi^{(q)}\delta_{p+q}^{2k}, &(5.32)\\
\delta e^{a,(l)} &= D\rho^{a,(l)} + \omega_{\ b}^{a\ ,(k+1)}\rho^{b,(n)}\delta_{k+n+1}^{l} + e^{b,(n)}\rho_{b}^{\ a,(k)}\delta_{n+k}^{l} + \bar{\epsilon}^{(p)}\gamma^{a}\psi^{(q)}\delta_{p+q}^{2l}, &(5.33)\\
\delta\psi^{(p)} &= d\epsilon^{(p)} + \frac{1}{4}\omega^{ab,(k)}\gamma_{ab}\epsilon^{(q)}\delta_{k+q}^{p} + \frac{1}{2l}e^{a,(l)}\gamma_a\epsilon^{(q)}\delta_{l+q}^{p} \\
&\quad - \frac{1}{4}\rho^{ab,(k)}\gamma_{ab}\psi^{(q)}\delta_{k+q}^{p} - \frac{1}{2l}\rho^{a,(l)}\gamma_a\psi^{(q)}\delta_{l+q}^{p}. &(5.34)
\end{aligned}
$$

In the same way, from the gauge variation of the curvature

$$
\delta_\lambda F = [F, \lambda] \tag{5.35}
$$

it is possible to show that the gauge transformations of the curvature F are given by

$$
\begin{aligned}
\delta\mathcal{R}^{ab,(k)} &= \left(\mathcal{R}^{ac,(i)}\rho_{c}^{\ b,(j)} - \mathcal{R}^{cb,(i)}\rho_{\ c}^{a\ ,(j)}\right)\delta_{i+j}^{k} + \frac{2}{l^2}R^{a,(l)}\rho^{b,(n)}\delta_{l+n}^{k} \\
&\quad - \frac{1}{l}\bar{\epsilon}^{(p)}\gamma^{ab}\Psi^{(q)}\delta_{p+q}^{2k}, &(5.36)\\
\delta R^{a,(l)} &= \mathcal{R}_{\ b}^{a\ ,(k)}\rho^{b,(n)}\delta_{k+n}^{l} + R^{b,(n)}\rho_{b}^{\ a,(k)}\delta_{k+n}^{l} + \bar{\epsilon}^{(p)}\gamma^{a}\Psi^{(q)}\delta_{p+q}^{2l}, &(5.37)\\
\delta\Psi^{(p)} &= \frac{1}{4}\mathcal{R}^{ab,(k)}\gamma_{ab}\epsilon^{(q)}\delta_{k+q}^{p} + \frac{1}{2l}R^{a,(l)}\gamma_a\epsilon^{(q)}\delta_{l+q}^{p} - \frac{1}{4}\rho^{ab,(k)}\gamma_{ab}\Psi^{(q)}\delta_{k+q}^{p} \\
&\quad - \frac{1}{2l}\rho^{a,(l)}\gamma_a\Psi^{(q)}\delta_{l+q}^{p}, &(5.38)
\end{aligned}
$$

with $k = i = j = 0, \ldots, m$; $l = n = p = q = 1, \ldots, m$.

Although the Mac Dowell-Mansouri like action (5.28) is built from the $s\mathcal{M}_{m+2}$ curvature, it is *not* invariant under the $s\mathcal{M}_{m+2}$ gauge transformations.

Besides the action is not invariant under gauge supersymmetry. In fact, if we consider the variation of the action (5.28) under gauge supersymmetry related to $Q_{(1)}$, we find

$$
\delta_{\text{susy}}S = -\frac{4}{l^2}\int\sum_k \alpha_{2k}R^{a,(l)}\bar{\Psi}^{(p)}\gamma_a\gamma_5\epsilon\delta_{l+p}^{k}, \tag{5.39}
$$

with $k = 2, \ldots, m$; $l, p \geq 1$ and where ϵ is the gauge parameter associated to the spinor charge $Q_{(1)}$.

As in the previous case the action is invariant for every value of k under gauge supersymmetry imposing the expanded super torsion constraint

$$R^{a,(l)} = 0. \tag{5.40}$$

This yields to express the expanded spin connection $\omega^{ab,(k)}$ in terms of the expanded fields as we can see in (5.18). This leads to the supersymmetric action for the $s\mathcal{M}_{m+2}$ superalgebra in the second order formalism.

Alternatively, since the α's are arbitrary and independent we can study the supersymmetry in each term separately. Then if we consider the variation of the action proportional to α_{2k} under gauge supersymmetry transformations asociated to $Q_{(k-1)}$, we find

$$\delta_{\text{susy}} S = -\frac{4}{l^2}\alpha_{2k} \int R^a \bar{\Psi}\gamma_a\gamma_5\epsilon^{(k-1)}, \tag{5.41}$$

with $k = 2, \ldots, m$ and where $\epsilon^{(k-1)}$ is the gauge parameter associated to the spinor charge $Q_{(k-1)}$. Here R^a and Ψ correspond to $R^{a,(1)}$ and $\Psi^{(1)}$ respectively.

It is possible to have invariance under supersymmetry in first order formalism in every term if we modify the supersymmetry transformation for every expanded spin connection. In fact, if we consider the variation of the action under an arbitrary $\delta\omega^{ab,(k-2)}$ we find

$$\delta_\omega S = \frac{2}{l^2}\alpha_{2k} \int \epsilon_{abcd} R^a e^b \delta\omega^{cd,(k-2)}, \tag{5.42}$$

with $k = 2, \ldots, m$; $R^a = R^{a,(1)}$ and $e^a = e^{a,(1)}$. One can see that the variation vanishes for arbitrary $\delta\omega^{ab,(k-2)}$ if $R^a = 0$.

Nevertheless it is possible to modify $\delta\omega^{ab,(k-2)}$ by adding an extra piece such that the variation of the action ($\sim \alpha_{2k}$) can be written as

$$\delta S = -\frac{4}{l^2}\alpha_{2k} \int R^a \left(\bar{\Psi}\gamma_a\gamma_5\epsilon^{(k-1)} - \frac{1}{2}\epsilon_{abcd} e^b \delta_{\text{extra}}\omega^{cd,(k-2)} \right). \tag{5.43}$$

Thus the transformation of the $\omega^{ab,(k-2)}$ field leaving the term proportional to α_{2k} invariant is

$$\delta_{\text{extra}}\omega^{ab,(k-2)} = 2\epsilon^{abcd} \left(\bar{\Psi}_{ec}\gamma_d\gamma_5\epsilon^{(k-1)} + \bar{\Psi}_{de}\gamma_c\gamma_5\epsilon^{(k-1)} - \bar{\Psi}_{cd}\gamma_e\gamma_5\epsilon^{(k-1)} \right) e^e,$$

with $\bar{\Psi} = \bar{\Psi}_{ab} e^a e^b$.

On the other hand, it is important to note that the term proportional to α_{2k} is truly invariant under gauge supersymmetry transformations associated to $Q_{(q)}$, with $q \geq k$.

Clearly for $m = 2$ in $s\mathcal{M}_{m+2}$ we recover the results presented in the previous section.

JHEP09(2014)090

5.2 Pure supergravity from the minimal Maxwell algebra type $s\mathcal{M}_{m+2}$

Since we are interested in obtaining the Einstein-Hilbert and the Rarita-Schwinger terms, we shall consider only the piece proportional to α_4. Then the action is written with the following choice for the non-vanishing components of an invariant tensor

$$\left\langle J_{ab,(0)} J_{cd,(4)} \right\rangle_{s\mathcal{M}_{m+2}} = \alpha_4 \left\langle J_{ab} J_{cd} \right\rangle, \tag{5.44}$$

$$\left\langle J_{ab,(2)} J_{cd,(2)} \right\rangle_{s\mathcal{M}_{m+2}} = \alpha_4 \left\langle J_{ab} J_{cd} \right\rangle, \tag{5.45}$$

$$\left\langle Q_{\alpha,(1)} Q_{\beta,(3)} \right\rangle_{s\mathcal{M}_{m+2}} = \alpha_4 \left\langle Q_{\alpha} Q_{\beta} \right\rangle, \tag{5.46}$$

which can be written as

$$\left\langle J_{ab,(0)} J_{cd,(4)} \right\rangle_{s\mathcal{M}_{m+2}} = \alpha_4 \epsilon_{abcd}, \tag{5.47}$$

$$\left\langle J_{ab,(2)} J_{cd,(2)} \right\rangle_{s\mathcal{M}_{m+2}} = \alpha_4 \epsilon_{abcd}, \tag{5.48}$$

$$\left\langle Q_{\alpha,(1)} Q_{\beta,(3)} \right\rangle_{s\mathcal{M}_{m+2}} = 2\alpha_4 \left(\gamma_5\right)_{\alpha\beta}. \tag{5.49}$$

Then we only need the two-form curvatures asociated to $J_{ab,(0)}$, $J_{ab,(2)}$, $J_{(ab,4)}$, $Q_{\alpha,(1)}$ and $Q_{\alpha,(3)}$ which can be derived from (5.17)–(5.19).

Considering the different non-vanishing components of the invariant tensor and the respective two-form curvatures we obtain the following action for the S-expanded superalgebra

$$S = 2\alpha_4 \int \left(\frac{1}{2} \epsilon_{abcd} \mathcal{R}^{ab,(0)} \mathcal{R}^{cd,(4)} + \frac{1}{4} \epsilon_{abcd} \mathcal{R}^{ab,(2)} \mathcal{R}^{cd,(2)} + \frac{4}{l} \bar{\Psi}^{(3)} \wedge \gamma_5 \Psi^{(1)} \right), \tag{5.50}$$

which can be written explicitly as follows

$$\begin{aligned} S = \alpha_4 \int & \epsilon_{abcd} \frac{1}{l^2} \left(\mathcal{R}^{ab,(0)} e^{c,(2)} e^{d,(2)} + 4 \bar{\psi}^{(1)} e^{a,(2)} \gamma_a \gamma_5 D_\omega \psi^{(1)} \right) \\ & + d \left(\epsilon_{abcd} \left(\mathcal{R}^{ab,(0)} \omega^{ab,(4)} + \frac{1}{2} D_\omega \omega^{ab,(2)} \omega^{cd,(2)} \right) \right. \\ & \left. + \frac{8}{l} D_\omega \bar{\psi}^{(1)} \gamma_5 \psi^{(3)} + \frac{1}{l} \bar{\psi}^{(1)} \omega^{ab,(2)} \gamma_{ab} \gamma_5 \psi^{(1)} \right). \end{aligned} \tag{5.51}$$

Here we have used the gravitino Bianchi identity $D\Psi^{(1)} = \frac{1}{4} R^{ab} \gamma_{ab} \Psi^{(1)}$ and the matrix gamma identity (4.41) to show that

$$\epsilon_{abcd} \mathcal{R}^{ab,(0)} \bar{\psi}^{(3)} \gamma^{cd} \psi^{(1)} + 8 D \bar{\psi}^{(1)} \gamma_5 D_\omega \psi^{(3)} = D \left(8 D_\omega \bar{\psi}^{(1)} \gamma_5 \psi^{(3)} \right)$$

$$\frac{1}{2} \epsilon_{abcd} D \omega^{ab,(2)} \bar{\psi}^{(1)} \gamma^{cd} \psi^{(1)} + 2 \bar{\psi}^{(1)} \omega^{ab,(2)} \gamma_{ab} \gamma_5 D \psi^{(1)} = D \left(\bar{\psi}^{(1)} \omega^{ab,(2)} \gamma_{ab} \gamma_5 \psi^{(1)} \right).$$

Then using the following identification

$$\begin{aligned} \omega^{ab,(0)} &= \omega^{ab}, & \omega^{ab,(2)} &= \tilde{k}^{ab}, \\ \omega^{ab,(4)} &= k^{ab}, & e^{a,(2)} &= e^{a}, \\ \mathcal{R}^{ab,(0)} &= R^{ab}, & \psi^{(1)} &= \psi, \\ \psi^{(3)} &= \xi, \end{aligned}$$

the action is given by

$$S = \alpha_4 \int \epsilon_{abcd} \frac{1}{l^2} \left(R^{ab} e^c e^d + 4\bar{\psi} e^a \gamma_a \gamma_5 D_\omega \psi \right) + d\left(\epsilon_{abcd} \left(R^{ab} k^{cd} + \frac{1}{2} D_\omega \tilde{k}^{ab} \tilde{k}^{cd} \right) + \frac{8}{l} \bar{\xi} \gamma_5 D_\omega \psi + \frac{1}{l} \bar{\psi} \tilde{k}^{ab} \gamma_{ab} \gamma_5 \psi \right). \tag{5.52}$$

Here we can see that the action proportional to α_4 contains the Einstein-Hilbert term $\epsilon_{abcd} R^{ab} e^c e^d$, the Rarita-Schwinger lagrangian $4\bar{\psi} e^a \gamma_a \gamma_5 D\psi$ and a boundary term involving the new fields k_{ab}, $\tilde{k}_{ab}$, ξ and the original ones.

Unlike the Mac Dowell-Mansouri lagrangian for $\mathfrak{osp}\,(4|1)$ superalgebra the supersymmetric cosmological constant does not appear explicitly in this action. This is due to the S-expansion procedure since if we want to obtain the supersymmetric cosmological constant

$$\frac{1}{2l^4} e^a e^b e^c e^d + \frac{1}{l^3} \bar{\psi} \gamma^{ab} \psi e^c e^d$$

in the action, it should be necessary to consider the components $\langle J_{ab,(4)} J_{cd,(4)} \rangle$ and $\langle J_{ab,(2)} J_{cd,(4)} \rangle$ which are proportional to α_8 and α_6, respectively.

Independently of the numbers of new generators of the Maxwell superalgebra, the new Maxwell fields do not contribute to the dynamics of the term proportional to α_4. In this way, we have shown that $N = 1$, $D = 4$ pure supergravity can be obtained as a Mac Dowell-Mansouri like action for the minimal Maxwell superalgebras $s\mathcal{M}_{m+2}$ (with $m > 1$).

$$S = \alpha_4 \int \frac{1}{l^2} \left[\epsilon_{abcd} R^{ab} e^c e^d + 4\bar{\psi} e^a \gamma_a \gamma_5 D\psi \right] + \text{boundary terms}. \tag{5.53}$$

It is important to note that the case $m = 1$ corresponds to $D = 4$ Poincaré superalgebra $s\mathcal{P} = \{J_{ab}, P_a, Q_\alpha\}$ as mencioned in ref. [15]. However it is not possible to derive the pure supergravity action from a Mac Dowell-Mansouri like action for this superalgebra since it is not possible to obtain the Eintein-Hilbert term from $\langle J_{ab} J_{cd} \rangle$ for $s\mathcal{P}$.

6 Comments and possible developments

In the present work we have derived the minimal $D = 4$ supergravity action from the minimal Maxwell superalgebra $s\mathcal{M}_4$. For this purpose we have applied the abelian semigroup expansion procedure to the $\mathfrak{osp}\,(4|1)$ superalgebra allowing us to build a Mac Dowell-Mansouri like action. Interestingly, the action obtained describes pure supergravity in four dimensions. This result can be seen as a supersymmetric generalization of ref. [12] in which four-dimensional General Relativity is derived from Maxwell algebra as a Born-Infeld like action.

We have also obtained the $D = 4$ supergravity action from the minimal Maxwell superalgebra type $s\mathcal{M}_{m+2}$ using bigger semigroups. These superalgebras enlarge the pure supergravity action adding new terms containing new Maxwell symmetries. In particular, the action found in ref. [18] can be obtained when the simplest Maxwell superalgebra is considered. The invariance of the actions under the new supersymmetry transformations has been analized in detail.

JHEP09(2014)090

Our results provide another example showing that the S-expansion procedure is not only a useful method to derive new lie superalgebras but it is a powerful and simple tool in order to construct a supergravity action for an S-expanded superalgebra. Moreover the invariance of the pure supergravity action under new supersymmetry transformations could not be guessed trivially.

A future work could be consider the N-extended Maxwell superalgebras and the construction of N-extended supergravities in a very similar way to the one shown here. It would be also interesting to build lagrangians in odd dimensions using the Chern-Simons formalism and the S-expansion method. It seems that it should be possible to recover standard odd-dimensional supergravity from the Maxwell superalgebras [work in progress].

Acknowledgments

The authors wish to thank L. Andrianopoli, R. D'Auria and M. Trigiante for enlightening discussions and their kind hospitality at Dipartimento Scienza Applicata e Tecnologia of Politecnico di Torino, where this work was done. We are also grateful to P. Salgado for introducing the topics covered in the present work. The autors were supported by grants from the Comisión Nacional de Investigación Científica y Tecnológica (CONICYT) and from the Universidad de Concepción, Chile. This work was supported in part by FONDECYT Grants N° 1130653.

References

[1] H. Bacry, P. Combe and J.L. Richard, *Group-theoretical analysis of elementary particles in an external electromagnetic field. 1. The relativistic particle in a constant and uniform field*, *Nuovo Cim.* **A 67** (1970) 267 [INSPIRE].

[2] R. Schrader, *The Maxwell group and the quantum theory of particles in classical homogeneous electromagnetic fields*, *Fortsch. Phys.* **20** (1972) 701 [INSPIRE].

[3] D.V. Soroka and V.A. Soroka, *Tensor extension of the Poincaré algebra*, *Phys. Lett.* **B 607** (2005) 302 [hep-th/0410012] [INSPIRE].

[4] J. Gomis, K. Kamimura and J. Lukierski, *Deformations of Maxwell algebra and their Dynamical Realizations*, *JHEP* **08** (2009) 039 [arXiv:0906.4464] [INSPIRE].

[5] S. Bonanos and J. Gomis, *A Note on the Chevalley-Eilenberg Cohomology for the Galilei and Poincaré Algebras*, *J. Phys.* **A 42** (2009) 145206 [arXiv:0808.2243] [INSPIRE].

[6] G.W. Gibbons, J. Gomis and C.N. Pope, *Deforming the Maxwell-Sim Algebra*, *Phys. Rev.* **D 82** (2010) 065002 [arXiv:0910.3220] [INSPIRE].

[7] J.A. de Azcarraga, J.M. Izquierdo, J. Lukierski and M. Woronowicz, *Generalizations of Maxwell (super)algebras by the expansion method*, *Nucl. Phys.* **B 869** (2013) 303 [arXiv:1210.1117] [INSPIRE].

[8] P. Salgado and S. Salgado, $\mathfrak{so}(D-1,1)\otimes\mathfrak{so}(D-1,2)$ *algebras and gravity*, *Phys. Lett.* **B 728** (2014) 5 [INSPIRE].

[9] F. Izaurieta, E. Rodríguez and P. Salgado, *Expanding Lie (super)algebras through Abelian semigroups*, *J. Math. Phys.* **47** (2006) 123512 [hep-th/0606215] [INSPIRE].

[10] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez and P. Salgado, *Chern-Simons and Born-Infeld gravity theories and Maxwell algebras type*, *Eur. Phys. J.* **C 74** (2014) 2741 [arXiv:1402.0023] [INSPIRE].

[11] F. Izaurieta, P. Minning, A. Perez, E. Rodríguez and P. Salgado, *Standard General Relativity from Chern-Simons Gravity*, *Phys. Lett.* **B 678** (2009) 213 [arXiv:0905.2187] [INSPIRE].

[12] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez and P. Salgado, *Even-dimensional General Relativity from Born-Infeld gravity*, *Phys. Lett.* **B 725** (2013) 419 [arXiv:1309.0062] [INSPIRE].

[13] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez and P. Salgado, *Generalized Poincaré algebras and Lovelock gravity theory*, arXiv:1405.7078 [INSPIRE].

[14] S. Bonanos, J. Gomis, K. Kamimura and J. Lukierski, *Maxwell Superalgebra and Superparticle in Constant Gauge Badkgrounds*, *Phys. Rev. Lett.* **104** (2010) 090401 [arXiv:0911.5072] [INSPIRE].

[15] P.K. Concha and E.K. Rodríguez, *Maxwell Superalgebras and Abelian Semigroup Expansion*, *Nucl. Phys.* **B 886** (2014) 1128 [arXiv:1405.1334] [INSPIRE].

[16] R. D'Auria and P. Fré, *Geometric Supergravity in $D=11$ and Its Hidden Supergroup*, *Nucl. Phys.* **B 201** (1982) 101 [*Erratum ibid.* **B 206** (1982) 496] [INSPIRE].

[17] M.B. Green, *Supertranslations, Superstrings and Chern-Simons Forms*, *Phys. Lett.* **B 223** (1989) 157 [INSPIRE].

[18] J.A. de Azcarraga and J.M. Izquierdo, *Minimal $D=4$ supergravity from the superMaxwell algebra*, *Nucl. Phys.* **B 885** (2014) 34 [arXiv:1403.4128] [INSPIRE].

[19] S.W. MacDowell and F. Mansouri, *Unified Geometric Theory of Gravity and Supergravity*, *Phys. Rev. Lett.* **38** (1977) 739 [*Erratum ibid.* **38** (1977) 1376] [INSPIRE].

[20] L. Castellani, $OSp(1|4)$ *supergravity and its noncommutative extension*, *Phys. Rev.* **D 88** (2013) 025022 [arXiv:1301.1642] [INSPIRE].

[21] P. van Nieuwenhuizen, *Supergravity as a Yang-Mills theory*, in *50 years of Yang-Mills theory*, G. 't Hooft ed., World Pub. Co. (2004), pg. 433 [hep-th/0408137] [INSPIRE].

[22] L. Andrianopoli and R. D'Auria, *$N=1$ and $N=2$ pure supergravities on a manifold with boundary*, *JHEP* **08** (2014) 012 [arXiv:1405.2010] [INSPIRE].