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Retailer's inventory system in a two-level trade credit financing with selling price discount and partial order cancellations

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Abstract

In today's fast marketing over the Internet or online, many retailers want to trade at the same time and change their marketing strategy to attract more customers. Some of the customers may decide to cancel their orders partially with a retailer due to various reasons such as increase in customer's waiting time, loss of customer's goodwill on retailer's business, and attractive promotional schemes offered by other retailers. Even though there is a lag in trading and order cancellation, this paper attempts to develop the retailer's inventory model with the effect of order cancellations during advance sales period. The retailer announces a price discount program during advance sales period to promote his sales and also offers trade credit financing during the sales periods. The retailer availing trade credit period from his supplier offers a permissible delay period to his customers. The customer who gets an item is allowed to pay on or before the permissible delay period which is accounted from the buying time rather than from the start period of inventory sales. This accounts for significant changes in the calculations of interest payable and interest earned by the retailer. The retailer's total cost is minimized so as to find out the optimal replenishment cycle time and price discount policies through a solution procedure. The results derived in mathematical theorems are implemented in numerical examples, and sensitivity analyses on several inventory parameters are obtained.

Keywords: Inventory; Advance sales; Price discount; Two-echelon trade credit

Introduction

In today's business era, retailers have the dominant power of controlling or affecting another member's decision in a supply chain. A retailer has the ability to offer an effective promotional effect such as price discount and credit period. The retailers often try to stimulate the demand by offering price discounts. Price discounts could improve economic benefits to consumers and influence consumers' beliefs about the brand which will increase consumers' purchase intentions. In real life, there are situations in which the retailer announces price discount offers to the customers who can commit their orders before the selling period. Due to the booming in IT, customers can easily commit their orders prior to selling period and the estimation error in demand can also be reduced. This situation is adopted more in the

selling of musical disks, apparel, video games, or books. In dairy-product manufacturing scheme prior to selling season, the retailer offers price discount to the customers who register their orders via email or phone call.

Certainly, the credit period facility would promote the purchases, and it attracts new customers who consider trade credit policy as a type of price reduction. To handle the risks of trade credit situations, retailer collects a higher interest from his customers when they did not settle the payment within the credit period time. In this paper, supplier offers the retailer a trade credit period t_1. The retailer offers his customers a credit period t_2, and he receives the revenue from t_2 to $T + t_2$, where T is the cycle time at the retailer. Under this situation, three cases such as $t_1 \le T$, $T \le t_1 \le T + t_2$, and $T + t_2 \le t_1$ are to be considered. Customers under advance sales booking system may cancel their reservations due to various reasons such as increase in customer's waiting time, loss of customer's goodwill on retailer's business, and attractive

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promotional schemes offered by other retailers. Here, we consider the partial order cancellations during advance sales period.

This paper investigates retailer inventory system in which customers are partially canceling their orders during advance sales period. The customers commit their orders before the selling period. Among the committed orders, a fraction of the orders are cancelled. The customer who receives an item at time 't' will remit at time $t + t_2$ due to his availability of trade credit period t_2. So, the retailer gets revenue from earning the interest on customer's payment during the period from t_2 to t_1, and he pays interest during the period t_2 to $T + t_2$. So, the cases in Tsao (2009) are to be reconsidered as $t_1 \leq T$, $T \leq t_1 \leq T + t_2$, and $T + t_2 \leq t_1$. The retailer also earns interest from standby orders during the advance sales period. During the normal sales period, all customers receive their orders at the time of their purchase. With the help of derived mathematical theorems in the model, a simple solution procedure is provided to find the optimal solution. Numerical examples are given to illustrate the solution process, and sensitivity analyses are performed for various inventory key parameters.

Literature review

During the past few years, many researchers have studied inventory models for permissible delay in payments. Goyal (1985) was the first proponent for developing an economic order quantity (EOQ) model under the conditions of permissible delay in payments. Shah (1993) considered a stochastic inventory model when items in the inventory deteriorate and delays in payments are permissible. Aggarwal and Jaggi (1995) extended Goyal's model (1985) to allow the inventory to have deteriorating items. Jamal et al. (1997) further generalized Aggarwal and Jaggi's model (1995) to allow for shortages. Hwang and Shinn (1997) developed a model considering exponentially deteriorating items and found decision policy for selling price and lot size. Teng (2002) amended Goyal's model (1985) by considering the difference between unit price and unit cost and established an easy analytical closed-form solution to the problem. Chang et al. (2003) constructed a mathematical model for an EOQ inventory with deteriorating items and supplier credits are linked to ordering quantity. Chung and Huang (2003) generalized Goyal's EOQ model (1985) to an economic production quantity (EPQ) model in which the selling price is the same as the purchase cost. Huang (2003) extended Goyal's model (1985) to the case in which the supplier offers the retailer the permissible delay period M (i.e., the upstream trade credit), and the retailer in turn provides the trade credit period N (with $N < M$) to his customers (i.e., the downstream trade credit). Ouyang et al. (2006) developed an EOQ model for

deteriorating items under trade credits. Teng and Goyal (2007) amended Huang's model (2003) by complementing his shortcomings. Liao (2007) established an EPQ model for deteriorating items under permissible delay in payments. Chang et al. (2008) reviewed the contributions on the literature in modeling of inventory lot sizing under trade credits. Ho et al. (2008) developed an integrated supplier–buyer inventory model with the assumption that demand is sensitive to retail price and the supplier adopts a two-part trade credit policy. Huang and Hsu (2008) have developed an inventory model under two-level trade credit policy by incorporating partial trade credit option at the customers of the retailer. Liao (2008) developed an EOQ model with non-instantaneous receipt and exponentially deteriorating items under two-level trade credit financing. Teng and Chang (2009) extended Huang's model (2007) by relaxing the assumption $N < M$. Jaggi et al. (2008) developed a simple EOQ model in which the retailer's demand is linked to credit period. Thangam and Uthayakumar (2009) developed an EPQ model for perishable items under two-level trade credit policy when demand depends on selling price and credit period. Teng (2009) developed an EOQ model for a retailer who receives a full trade credit from its supplier and offers a partial trade credit to its bad credit customers or a full trade credit to its good credit customers. Teng et al. (2009) developed a mathematical model for an EOQ inventory with two warehouses and solved the problem by an arithmetic–geometric inequality method. Tsao (2009) developed a model by considering advance sales discount and trade credits. In the paper of Tsao (2009), he considered a strategy, namely advance sales discount (ASD) program, that the customers can commit their orders at a discount price prior to the selling season. He considers the cases such as $T \geq t_1$, $t_2 \leq T \leq t_1$, and $T \leq t_2$. Chen and Kang (2010a) considered trade credit and imperfect quality in an integrated vendor-buyer supply chain model. Concurrently, Chen and Kang (2010b) developed an integrated vendor-buyer inventory model with two-level trade credits and price negotiation scheme. Chang et al. (2010) have extended Liao's model (2008) by considering the case $M < N$ also. Hu and Liu (2010) established an EPQ model with permissible delay in payments and allowable shortages. Cárdenas-Barrón et al. (2010) developed a model which considers the advantage of a one-time discount offer with allowed backorders. Balkhi (2011) has developed a finite horizon inventory model with deteriorating items under inflation and time value of money when shortages are not allowed. Thangam and Uthayakumar (2011) have built a mathematical model for a retailer under two-level trade credit and two-payment methods. Tsao (2011) developed an EOQ model by considering trade credit and logistics risk. Teng

et al. (2011) extended an EOQ model for stock-dependent demand to supplier's trade credit with a progressive payment scheme. Skouri et al. (2011) studied supply chain models for deteriorating items with ramp-type demand rate under permissible delay in payments. Jaggi et al. (2012) developed an EOQ model under two-levels of trade credit policy when demand is influenced by credit period. Liao et al. (2012) have developed a two-warehouse lot-sizing model with order-dependent trade credit period. Tsao and Sheen (2012) have developed a multi-item supply chain model with trade credit periods and weight freight cost. Thangam (2012) developed a two-level trade credit financing model for a supply chain with deteriorating items and advance payment scheme. Teng et al. (2012a) developed vendor-buyer inventory models with trade credit financing under a non-cooperative and integrated environments. Concurrently, Teng et al. (2012b) proposed an EOQ model with trade credit financing for increasing demand. Min et al. (2012) established an EPQ model with inventory level-dependent demand and permissible delay in payments. Tsao (2012) considered manufacturer's production and warranty decisions for an imperfect production system under system maintenance and trade credit. Teng et al. (2013) have developed a two-level trade credit financing model with timely increasing demand at the retailer. Feng et al. (2013) have developed an EPQ inventory model with supplier's cash discount and two-level trade credit financing. Ouyang et al. (2013) have developed a mathematical model with two-level trade credit financing in which trade credit offer depends on the amount of ordering quantity. Taleizadeh et al. (2013) have developed an EOQ model with perishable items, special sale offers, and shortages. Chung and Cárdenas-Barrón (2013) presented a simplified solution procedure to an EOQ model for deteriorating items by Min et al. (2010) with stock-dependent demand and two-level trade credit. Chern et al. (2013) established Stackelberg solution in a vendor-buyer supply chain model with permissible delay in payments. Ouyang and Chang (2013) proposed an optimal production lot with imperfect production process under permissible delay in payments and complete backlogging. Chen et al. (2013a) established the retailer's optimal EOQ when the supplier offers conditionally permissible delay in payments linked to order quantity. Concurrently, Chen et al. (2013b) attempted to overcome some shortcomings of mathematical model and expressions in Liao et al. (2012). Jaggi et al. (2013) established an EOQ inventory model with defective items under allowable shortages and trade credit.

Mathematical model formulation

We follow the same notations as in Tsao (2009) and introduce a new notation, namely δ (fixed) referring the rate at which the orders are cancelled during advance sales period:

p unit retailer price,
c unit purchase cost,
A ordering cost per order,
H unit inventory holding cost,
t_1 retailer's credit period provided by supplier,
t_2 customer's credit period provided by retailer,
r price discount,
δ the rate at which the orders are cancelled during advance sales period,
I_p the interest paid per dollar per unit time,
I_e the interest earned per dollar per unit time,
T replenishment cycle time,
D_1 annual demand rate for the retailer, say retailer 1, to whom optimum decision policy is considered,
D_2 annual demand rate for other retailers,
Y_1 fraction of retailer 1's customers who use advance sales discount program,
Y_2 fraction of other retailers' customers who use advance sales discount program,
$(1 - Y_1)D_1$ annual demand of customers who are not using advance sales discount program.

We follow the assumptions as in Tsao (2009) and include other assumptions that

1. The orders during advance sales period are partially cancelled.
2. The customers who gets an item at time 't' pays at time $t + t_2$ and so the retailer earns interest from the revenue obtained during the time t_2 to t_1 instead of time $t = 0$ to $t = t_1$. Retailer starts paying interest for the items in stock at the rate I_p.

Assumptions as in Tsao (2009) are as follow:

1. The problem considers an inventory system with single item.
2. The retailer offers price discount r to his customers if they can commit their orders prior to the sales period.
3. Y_1 percentage of the retailer 1's customers use advance sales discount program and Y_2 percentage of other retailer's customers use advance sales discount program.

The objective is to minimize the annual total cost incurred at the retailer, TC(T) = Annual ordering cost + Annual stock holding cost + Annual interest payable – Annual interest earned:

1. Annual ordering cost = A/T,
2. Annual holding cost = $\frac{(1-Y_1)D_1TH}{2}$,
3. Annual interest earned by the retailer.

Case 1: when $t_1 \leq T$

For this case, please see Figure 1.

Since $(1-\delta)(Y_1 D_1 + Y_2 D_2)$ number of orders are stand on, the interest earned during the period from t_2 to t_1 due to advance sales discount (ASD) program is $(1-\delta)(Y_1 D_1 + Y_2 D_2) \, p(1-r) \, I_e \, (t_1 - t_2)T$. The interest earned during the normal sales period is $pI_e(1-Y_1)D_1\left[\frac{(t_1-t_2)^2}{2}\right]$.

The annual interest earned is $(1-\delta)(Y_1 D_1 + Y_2 D_2) p(1-r)I_e(t_1-t_2) + \frac{pI_e(1-Y_1)D_1}{2T}(t_1-t_2)^2$.

Case 2: when $T \leq t_1 \leq T + t_2$

For this case, see Figure 2.

Interest earned due to ASD program is $(1-\delta)(Y_1 D_1 + Y_2 D_2) \, p(1-r) \, I_e \, (t_1 - t_2)T$. The interest earned during the normal sales period is $pI_e(1-Y_1)D_1\left[\frac{(t_1-t_2)^2}{2}\right]$. The annual interest earned is $(1-\delta)(Y_1 D_1 + Y_2 D_2)p(1-r)I_e(t_1-t_2)+\frac{pI_e(1-Y_1)D_1}{2T}(t_1-t_2)^2$.

Case 3: when $T + t_2 \leq t_1$ (see Figure 3)

The interest earned due to ASD program is $(1-\delta)(Y_1 D_1 + Y_2 D_2)p(1-r)I_e(t_1-t_2)T$. The interest earned during the normal sales period is $pI_e(1-Y_1)D_1\left[\frac{T^2}{2} + T(t_1-T-t_2)\right]$. The annual interest earned by the retailer is $(1-\delta)(Y_1 D_1 + Y_2 D_2)p(1-r)I_e(t_1-t_2) +pI_e(1-Y_1)D_1\left[t_1-t_2-\frac{T}{2}\right]$.

Annual interest payable by the retailer

Case 1: when $t_1 \leq T$

See Figure 1 for this case. The interest payable for the items in stock is $cI_p(1-Y_1)D_1\left[\frac{(T-t_1)^2}{2}\right]$. The interest payable for the items, which are sold but not paid yet, is $pI_p(1-Y_1)D_1t_2\left[T-t_1+\frac{t_2}{2}\right]$. Therefore, the annual interest payable is $\frac{cI_p(1-Y_1)D_1}{2T}(T-t_1)^2 + \frac{pI_p(1-Y_1)D_1t_2}{T}\left[T-t_1+\frac{t_2}{2}\right]$.

Case 2: when $T \leq t_1 \leq T + t_2$

Please see Figure 2 for this case. Since there is no stock on hand, the retailer does not need to pay interest for the items in stock. However, he pays interest for the items which are sold, but not paid yet. Therefore, the annual interest payable is $\frac{pI_p(1-Y_1)D_1}{2T}\left[T + t_2-t_1\right]^2$.
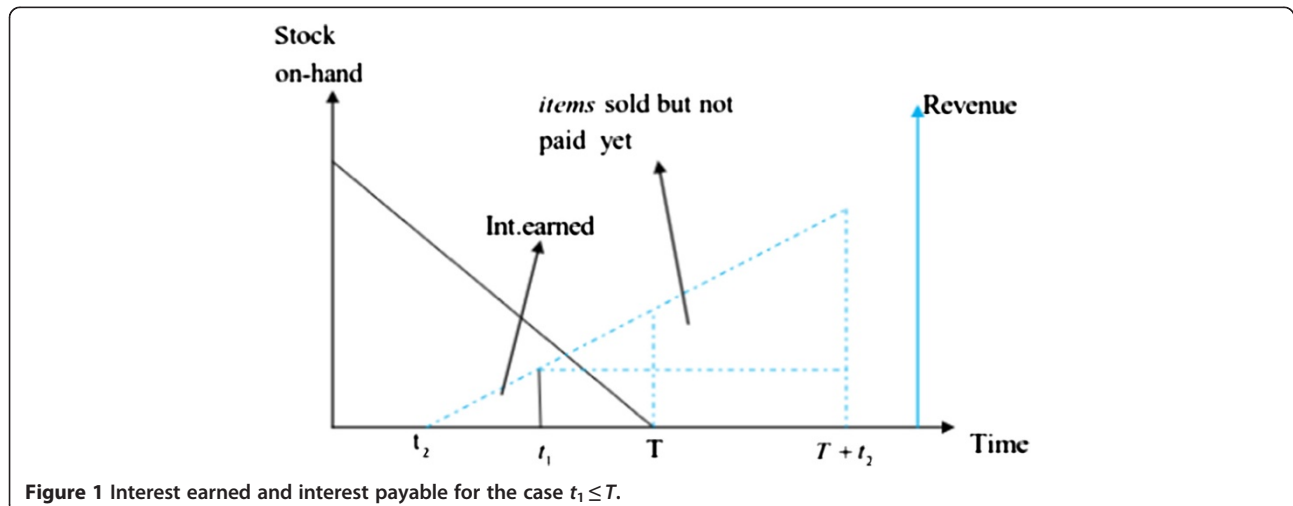
Case 3: when $T + t_2 \leq t_1$

Since the retailer pays off his items at time t_1, which is later than the time $T + t_2$ at which he receives all payment from his customers, there is no interest payable by the retailer. Therefore, the total cost TC(T) incurred at the retailer is

$$\mathrm{TC}(T) = \begin{cases} \mathrm{TC}_1(T) & \text{if} & t_1 \leq T \\ \mathrm{TC}_2(T) & \text{if} & T \leq t_1 \leq T + t_2 , \\ \mathrm{TC}_3(T) & \text{if} & T + t_2 \leq t_1 \end{cases}$$

where

$$\mathrm{TC}_1(T) = \frac{1}{2T}\left[2A + (1-Y_1)D_1\left[cI_p t_1{}^2 - pI_e(t_1-t_2)^2 - pI_p t_2(2t_1-t_2)\right]\right]$$
$$+ \frac{T}{2}\left[(1-Y_1)D_1\left(cI_p + H\right)\right] + (1-Y_1)D_1\left[pI_p t_2 - cI_p t_1\right]$$
$$-(1-\delta)(Y_1 D_1 + Y_2 D_2)P(1-r)I_e(t_1-t_2),$$

$$(1)$$

$$\mathrm{TC}_2(T) = \frac{1}{2T}\left[2A - (1-Y_1)D_1 p\left(I_e - I_p\right)(t_1-t_2)^2\right]$$
$$+ \frac{T}{2}\left[(1-Y_1)D_1\left(H + pI_p\right)\right] - pI_p(1-Y_1)D_1[t_1-t_2]$$
$$-(1-\delta)(Y_1 D_1 + Y_2 D_2)P(1-r)I_e(t_1-t_2),$$

$$(2)$$

Figure 1 Interest earned and interest payable for the case $t_1 \leq T$.
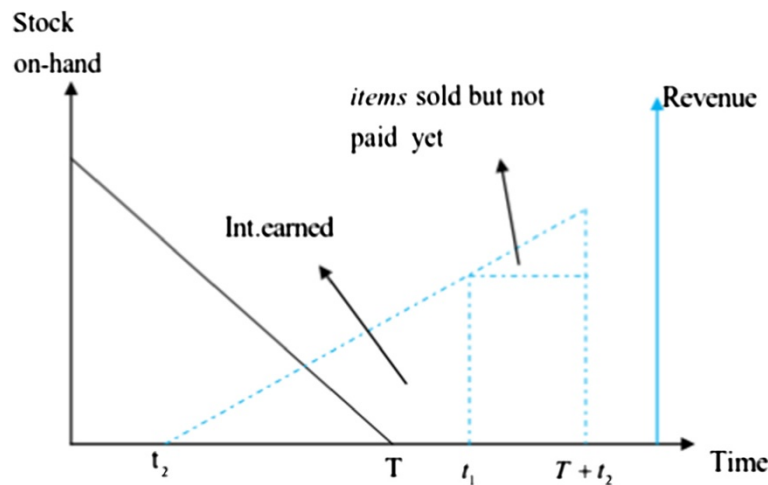
Figure 2 Interest earned and interest payable for the case $T \le t_1 \le T + t_2$.

$$\mathrm{TC}_3(T) = \frac{A}{T} + \frac{T}{2}\big[(1-Y_1)D_1(H+pI_e)\big]$$
$$-pI_e(1-Y_1)D_1[t_1-t_2]$$
$$-(1-\delta)(Y_1D_1 + Y_2D_2)P(1-r)I_e(t_1-t_2). \tag{3}$$

Optimal solutions
When retail price discount rate r is fixed
The first-order and second-order derivatives of $\mathrm{TC}_i(T)$, $i = 1,2,3$, are as follows:

$$\frac{d\mathrm{TC}_1(T)}{dT} = \frac{-1}{2T^2}\big[2A + (1-Y_1)D_1(cI_pt_1{}^2 - pI_e(t_1-t_2)^2$$
$$-pI_pt_2(2t_1-t_2))\big]$$
$$+\frac{1}{2}\big[(1-Y_1)D_1(H+cI_p)\big]$$

$$\frac{d^2\mathrm{TC}_1(T)}{dT^2} = \frac{1}{T^3}\big[2A + (1-Y_1)D_1(cI_pt_1{}^2 - pI_e(t_1-t_2)^2$$
$$-pI_pt_2(2t_1-t_2))\big]$$

$$\frac{d\mathrm{TC}_2(T)}{dT} = \frac{-1}{2T^2}\big[2A - p(1-Y_1)D_1(t_1-t_2)^2(I_e-I_p)\big]$$
$$+\frac{1}{2}\big[(1-Y_1)D_1(H+pI_p)\big]$$

$$\frac{d^2\mathrm{TC}_2(T)}{dT^2} = \frac{1}{T^3}\big[2A - p(I_e-I_p)(1-Y_1)D_1(t_1-t_2)^2\big]$$

$$\frac{d\mathrm{TC}_3(T)}{dT} = \frac{-A}{T^2} + \frac{1}{2}\big[(1-Y_1)D_1(H+pI_e)\big]$$

$$\frac{d^2\mathrm{TC}_3(T)}{dT^2} = \frac{2A}{T^3} > 0.$$

From the above, we observe that $\mathrm{TC}_1(T)$ is a convex function on T if $[2A + (1 - Y_1)D_1(cI_pt_1^2 - pI_e(t_1 - t_2)^2 - pI_pt_2(2t_1 - t_2))] > 0$. If $[2A + (1 - Y_1)D_1(cI_pt_1^2 - pI_e(t_1 - t_2)^2 - pI_pt_2(2t_1 - t_2))] < 0$, then $\mathrm{TC}_1(T)$ is a concave function on T

and $\frac{d\mathrm{TC}_1(T)}{dT}$ is an increasing function on $[t_1, \infty)$. Therefore, minimum $\mathrm{TC}_1(T)$ is attained at $T_1{}^* = t_1$ when $\mathrm{TC}_1(T)$ is a concave function of T.

$\mathrm{TC}_2(T)$ is a convex function on T if $[2A - p(I_e - I_p)(1 - Y_1) D_1(t_1 - t_2)^2] > 0$. If $[2A - p(I_e - I_p)(1 - Y_1)D_1(t_1 - t_2)^2] < 0$, then $\mathrm{TC}_2(T)$ is a concave function on T and $\frac{d\mathrm{TC}_2(T)}{dT}$ is an increasing function on $[t_1 - t_2, t_1]$. Therefore, minimum $\mathrm{TC}_2(T)$ is attained at $T_2{}^* = t_1 - t_2$ when $\mathrm{TC}_1(T)$ is a concave function of T.

The optimal cycle times $T_i{}^*$ ($i = 1,2,3$) are obtained by solving $\frac{d\mathrm{TC}_i(T)}{dT} = 0$ ($i = 1,2,3$) respectively.

$$T_1{}^* = \left[\frac{2A + (1-Y_1)D_1\left(cI_pt_1{}^2 - pI_e(t_1-t_2)^2 - pI_pt_2(2t_1-t_2)\right)}{(1-Y_1)D_1(H+cI_p)}\right]^{\frac{1}{2}} \tag{4}$$

$$T_2{}^* = \left[\frac{2A - p(1-Y_1)D_1(t_1-t_2)^2\left(I_e-I_p\right)}{(1-Y_1)D_1\left(H+pI_p\right)}\right]^{\frac{1}{2}} \tag{5}$$

$$T_3{}^* = \left[\frac{2A}{(1-Y_1)D_1(H+pI_e)}\right]^{\frac{1}{2}}. \tag{6}$$

Ensuring the condition that $T_1{}^* \ge t_1$, we have $2A \ge (1 - Y_1) D_1(Ht_1^2 + pI_e(t_1 - t_2)^2 + pI_pt_2(2t_1 - t_2))$ if and only if $T^* = T_1{}^*$. Ensuring the condition that $T_2{}^* \le t_1 \le T_2{}^* + t_2$, we have $T^* = T_2{}^*$ if and only if $2A \ge (1 - Y_1)D_1(t_1 - t_2)^2(H + pI_e)$ and $2A \le (1 - Y_1)D_1(Ht_1^2 + pI_e(t_1 - t_2)^2 + pI_pt_2(2t_1 - t_2))$. Ensuring the condition that $T_3{}^* + t_2 \le t_1$, we have $T^* = T_3{}^*$ if and only if $2A \le (1 - Y_1)D_1(t_1 - t_2)^2(H + pI_e)$.

Let

$$\Delta_1 = (1-Y_1)D_1\big(Ht_1{}^2 + pI_e(t_1-t_2)^2 + pI_pt_2(2t_1-t_2)\big),$$
$$\Delta_2 = (1-Y_1)D_1(t_1-t_2)^2(H+pI_e).$$

It is to observe that $\Delta_1 - \Delta_2 = (1 - Y_1)D_1[H(t_1^2 - (t_1 - t_2)^2) + pI_pt_2(2t_1 - t_2)] \ge 0$ and so $\Delta_1 \ge \Delta_2$.
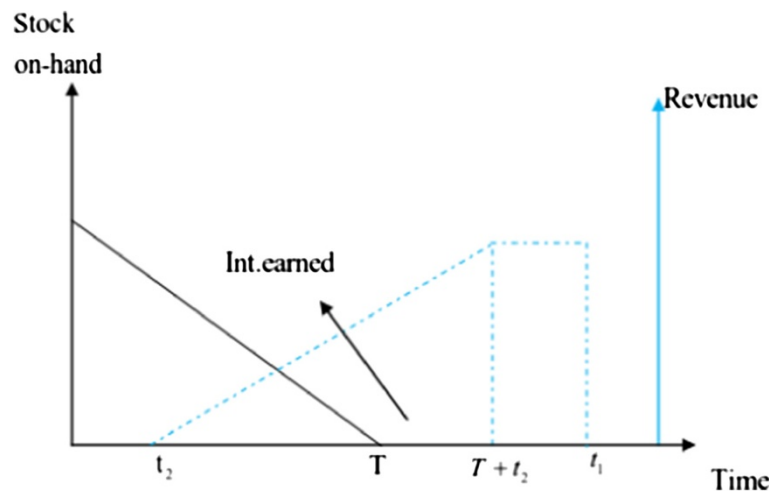
Figure 3 Interest earned and interest payable for the case $T + t_2 \le t_1$.

Theorem 1

1. If $2A \ge \Delta_1$, then $T^* = T_1^*$.
2. If $2A \le \Delta_1$ and $2A \ge \Delta_2$, then $T^* = T_2^*$.
3. If $2A \le \Delta_2$, then $T^* = T_3^*$.

Proof

(1) $2A \ge \Delta_1$ is implied from $t_1 \le T_1^*$. Since $TC_1'(T)$ is an increasing function on $[t_1, \infty)$, $TC_1'(t_1) \le TC_1'(T_1^*)$. So, we have $TC_1'(t_1) \le 0$, $TC_1'(T_1^*) = 0$, and $TC_1'(T) > 0$ for $T \in [T_1^*, \infty]$. Thus, $TC_1(T)$ is decreasing on $[t_1, T_1^*]$ and increasing on $[T_1^*, \infty)$.

Since $2A \ge \Delta_1$ can also be implied from $T_2^* \ge t_1$. Since $TC_2'(T)$ is an increasing function, $TC_2'(T_2^*) \ge TC_2'(t_1)$ which implies that $TC_2'(t_1) \le 0$. $t_1 - t_2 \le t_1$ implies that $TC_2'(t_1 - t_2) \le TC_2'(t_1)$. Hence, $TC_2(T)$ is decreasing on $[t_1 - t_2, t_1]$.

$2A \ge \Delta_1$ implies $2A \ge \Delta_2$. $2A \ge \Delta_2$ is implied from $T_3^* \ge t_1 - t_2$. Since $TC_3'(T)$ is an increasing function, $TC_3'(t_1 - t_2) \le TC_3'(T_3^*)$ which implies that $TC_3'(t_1 - t_2) \le 0$. Since $TC_3'(0) \le 0$, $TC_3(T)$ is decreasing on $[0, t_1 - t_2]$. From the discussions, we have

(1) $TC_1(T)$ is decreasing on $[t_1, T_1^*]$ and increasing on $[T_1^*, \infty)$.
(2) $TC_2(T)$ is decreasing on $[t_1 - t_2, t_1]$.
(3) $TC_3(T)$ is decreasing on $[0, t_1 - t_2]$.

Therefore, TC(T) attains minimum at $T^* = T_1^*$ and $TC^*(T) = TC_1(T_1^*)$.

(2) Let $2A \ge \Delta_1$ and $2A \ge \Delta_2$. $2A \ge \Delta_1$ is implied from $t_1 \ge T_1^*$. So, $TC_1'(t_1) \ge 0$. Since $TC_1'(T)$ is an increasing function on $[t_1, \infty]$, $TC_1'(T) \ge 0$ for $T \in [t_1, \infty)$. Therefore $TC_1(T)$ is increasing on $[t_1, \infty]$. $2A \le \Delta_1$ can also be implied from $t_1 \ge T_2^*$. $2A \ge \Delta_2$ is implied from $t_1 - t_2 \le T_2^*$. Since $TC_2'(T)$ is an increasing function, TC_2'

$(t_1) \ge TC_2'(T_2^*)$ and $TC_2'(t_1 - t_2) \le TC_2'(T_2^*)$. So, $TC_2'(t_1) \ge 0$ and $TC_2'(t_1 - t_2) \le 0$. Since $TC_2'(T_2^*) = 0$, $TC_2(T)$ is decreasing on $[t_1 - t_2, T_2^*]$ and increasing on $[T_2^*, t_1]$. $2A \ge \Delta_2$ can be implied from $T_3^* \ge t_1 - t_2$. Since $TC_3'(T)$ is an increasing function, $TC_3'(T_3^*) \ge TC_3'(t_1 - t_2)$ which implies that $TC_3'(t_1 - t_2) \le 0$. Since $TC_3'(T)$ is increasing on $[0, t_1 - t_2]$, $TC_3'(T) \le 0$ for $T \in [0, t_1 - t_2]$. Therefore, $TC_3(T)$ is decreasing on $T \in [0, t_1 - t_2]$. Hence, we have

(1) $TC_1(T)$ is decreasing on $[t_1, \infty)$.
(2) $TC_2(T)$ is decreasing on $[t_1 - t_2, T_2^*]$ and increasing on $[T_2^*, t_1]$.
(3) $TC_3(T)$ is decreasing on $[0, t_1 - t_2]$.

Therefore, TC(T) attains minimum at $T^* = T_2^*$ and $TC^*(T) = TC_2(T_2^*)$.

(3) $2A \le \Delta_2$ implies that $2A \le \Delta_1$. $2A \le \Delta_1$ implies that $TC_1(T)$ is increasing on $[t_1, \infty)$. $2A \le \Delta_2$ can be implied from $t_1 - t_2 \ge T_2^*$. Since $TC_2'(T)$ is increasing, $TC_2'(t_1 - t_2) \ge TC_2'(T_2^*)$. Therefore, $TC_2'(t_1 - t_2) \ge 0$. $2A \le \Delta_1$ is implied from $t_1 \ge T_2^*$. Thus, $TC_2'(t_1) \ge TC_2'(T_2^*)$. Therefore, $TC_2'(t_1) \ge 0$. Therefore, $TC_2(T)$ is increasing on $[t_1 - t_2, t_1]$. $2A \le \Delta_2$ implies that $TC_3(T)$ is a convex function on $[0, t_1 - t_2]$. Therefore, $TC_3(T)$ is decreasing on $[0, T_3^*]$ and increasing on $[T_3^*, t_1 - t_2]$. From the above discussion, we have

(1) $TC_1(T)$ is increasing on $[t_1, \infty)$.
(2) $TC_2(T)$ is increasing on $[t_1, t_2, t_1]$.
(3) $TC_3(T)$ is decreasing on $[0, T_3^*]$ and increasing on $[T_3^*, t_1 - t_2]$.

Hence, TC(T) attains minimum at $T^* = T_3^*$ and $TC^*(T) = TC_3(T_3^*)$.

When retail price discount rate is endogenous

Here, the retailer determines the optimal replenishment cycle time T^* and the optimal price discount r^* to minimize TC(T). With the consideration that the retailer 1's demand due to advance sales discount program and the fraction of other retailers' customers who switch to retailer 1 under advance sales discount program are linearly increasing with retail price discount r, let $Y_1(r) = \alpha.r$ and $Y_1(r) = \beta.r$ as in Tsao (2009). The problem here is to minimize

$$TC(T,r) = \begin{cases} TC_1(T,r) & \text{if} & t_1 \leq T \\ TC_2(T,r) & \text{if} & T \leq t_1 \leq T + t_2 \,. \\ TC_3(T,r) & \text{if} & T + t_2 \leq t_1 \end{cases} \quad (7)$$

To solve this problem, the closed form solution for each $r_i(T)$ is found by solving $\frac{\partial TC_i(T,r)}{\partial r} = 0$, i =1,2,3. Substituting these $r_i(T)$ to the corresponding $TC_i(T,r)$, $TC_i(T,r)$ reduces to single variable function as $TC_i(T)$. The optimal value of T_i^* is determined by solving the $\frac{dTC_i(T,r_i(T))}{\partial T} = 0$, $i = 1,2,3$. The optimal values of T^* and r^* are such that $TC(T^*,r^*) = \min\{TC_1(T_1^*,r_1^*), TC_2(T_2^*,r_2^*), TC_3(T_3^*,r_3^*)\}$.

The second derivative of $TC_i(T,r)$ with respect to r is $\frac{\partial^2 TC_i(T,r)}{\partial r^2} = 2(1-\delta)(\alpha D_1 + \beta D_2)pI_e(t_1-t_2) > 0$. Thus, $TC_i(T,r)$, $i = 1,2,3$, is a convex function of r for a fixed value of T. Solving the partial differential equation $\frac{\partial TC_i(T,r)}{\partial r} = 0$, $i = 1,2,3$, we get

$$r_1(T) = \frac{1}{4(1-\delta)(\alpha D_1 + \beta D_2)pI_e(t_1-t_2)}$$
$$\times \left[\begin{array}{l} 2(1-\delta)(\alpha D_1 + \beta D_2)pI_e(t_1-t_2) + \frac{\alpha D_1}{T}\left(\begin{array}{l} cI_p t_1{}^2 - pI_e(t_1-t_2)^2 \\ -pI_p t_2(2t_1-t_2) \end{array} \right) \\ +\alpha D_1 T(H + cI_p) + 2\alpha D_1(pI_p t_2 - cI_p t_1) \end{array} \right]$$

$$(8)$$

$$r_2(T) = \frac{1}{4(1-\delta)(\alpha D_1 + \beta D_2)pI_e(t_1-t_2)}$$
$$\times \left[\begin{array}{l} 2(1-\delta)(\alpha D_1 + \beta D_2)pI_e(t_1-t_2) + \frac{\alpha D_1}{T}\left((I_e + I_p)p(t_1-t_2)^2 \right) \\ +\alpha D_1 T(H + cI_p) - 2\alpha D_1 pI_p(t_1-t_2) \end{array} \right]$$

$$(9)$$

$$r_3(T) = \frac{1}{4(1-\delta)(\alpha D_1 + \beta D_2)pI_e(t_1-t_2)}$$
$$\times [\alpha D_1 TH - 2\alpha D_1 pI_e(t_1-t_2-T/2)$$
$$+ 2(1-\delta)(\alpha D_1 + \beta D_2)pI_e(t_1-t_2)]$$

$$(10)$$

Theorem 2

(a) If

$$-\frac{\alpha D_1}{2T}\left(cI_p t_1{}^2 - PI_e(t_1-t_2)^2 - pI_p t_2(2t_1-t_2) \right) - \frac{\alpha D_1}{2}T(H + cI_p)$$
$$- \alpha D_1\left(PI_p t_2 - cI_p t_1 \right) + (1-\delta)(\alpha D_1 + \beta D_2)PI_e(t_1-t_2) > 0,$$

then the unique optimal solution $r_1^*(T)$ lies in the interval (0,1).

(b) If

$$-\frac{\alpha D_1}{2T}\left((I_e + I_p)p(t_1-t_2)^2 \right) - \frac{\alpha D_1}{2}T(H + pI_p) + \alpha D_1 pI_p(t_1-t_2)$$
$$+ (1-\delta)(\alpha D_1 + \beta D_2)pI_e(t_1-t_2) > 0,$$

then the unique optimal solution $r_2^*(T)$ lies in the interval (0,1).

(c) If

$$-\frac{\alpha D_1}{2T}\left((I_e + I_p)p(t_1-t_2)^2 \right) - \frac{\alpha D_1}{2}T(H + pI_p) + \alpha D_1 pI_p(t_1-t_2)$$
$$+ (1-\delta)(\alpha D_1 + \beta D_2)pI_e(t_1-t_2) > 0,$$

then the unique optimal solution $r_3^*(T)$ lies in the interval (0,1).

Proof

(a) Let

$$G_1(r) = \frac{\partial TC_1(T,r)}{\partial r} = -\frac{\alpha D_1}{2T}$$
$$\times \left(cI_p t_1{}^2 - pI_e(t_1-t_2)^2 - pI_p t_2(2t_1-t_2) \right)$$
$$-\frac{\alpha D_1}{2}T(H + cI_p) - \alpha D_1\left(pI_p t_2 - cI_p t_1 \right)$$
$$- (1-\delta)(\alpha D_1 + \beta D_2)p(1-2r)I_e(t_1-t_2).$$

Since $TC_1(T,r)$ is a convex function of r, $G_1(r)$ is an increasing function of r.

$$G_1(0) = -\frac{\alpha D_1}{2T}\left(cI_p t_1{}^2 - pI_e(t_1-t_2)^2 - pI_p t_2(2t_1-t_2) \right)$$
$$-\frac{\alpha D_1}{2}T(H + cI_p) - \alpha D_1\left(pI_p t_2 - cI_p t_1 \right)$$
$$- (1-\delta)(\alpha D_1 + \beta D_2)pI_e(t_1-t_2) < 0$$

$$G_1(1) = -\frac{\alpha D_1}{2T}\left(cI_p t_1{}^2 - PI_e(t_1-t_2)^2 - pI_p t_2(2t_1-t_2) \right)$$
$$-\frac{\alpha D_1}{2}T(H + cI_p) - \alpha D_1\left(PI_p t_2 - cI_p t_1 \right)$$
$$+ (1-\delta)(\alpha D_1 + \beta D_2)PI_e(t_1-t_2)$$

If $G_1(1) > 0$, then $r_1^*(T)$ lies in the interval (0,1).

(b) Let

$$G_2(r) = \frac{\partial TC_2(T,r)}{\partial r} = -\frac{\alpha D_1}{2T}\left((I_e + I_p)p(t_1-t_2)^2 \right)$$
$$-\frac{\alpha D_1}{2}T(H + pI_p) + \alpha D_1 pI_p(t_1-t_2)$$
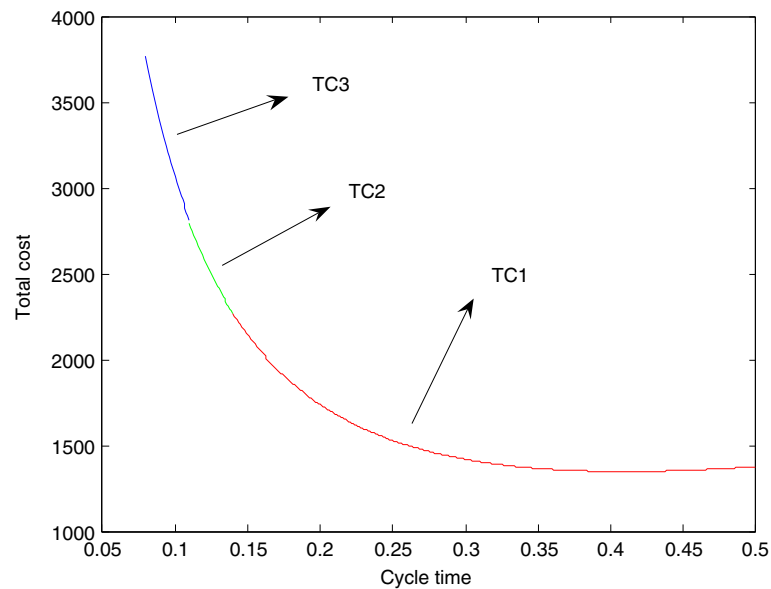$$-(1-\delta)(\alpha D_1 + \beta D_2)p(1-2r)I_e(t_1-t_2).$$

Figure 4 Graphic representation of TC$_i$ for Example 1.

Since TC$_2$(T,r) is a convex function of r, G$_2$(r) is an increasing function of r.

$$G_2(0) = -\frac{\alpha D_1}{2T}\left((I_e + I_p)p(t_1 - t_2)^2\right) - \frac{\alpha D_1}{2}TH$$
$$- \frac{\alpha D_1 pI_p}{2}(T - 2(t_1 - t_2)) + \alpha D_1 pI_p(t_1 - t_2)$$
$$-(1-\delta)(\alpha D_1 + \beta D_2)pI_e(t_1 - t_2) < 0$$

$$G_2(1) = -\frac{\alpha D_1}{2T}\left((I_e + I_p)p(t_1 - t_2)^2\right) - \frac{\alpha D_1}{2}T\left(H + pI_p\right)$$
$$+ \alpha D_1 pI_p(t_1 - t_2) + (1-\delta)(\alpha D_1 + \beta D_2)pI_e(t_1 - t_2).$$

If G$_2$(1) > 0, then $r_2^*(T)$ lies in the interval (0,1).

(c) Similar to (b)

First, we find $r_1(T)$, $r_2(T)$, and $r_3(T)$ using Equations (8), (9), and (10), respectively. Substituting $r_1(T)$, $r_2(T)$, and r_3(T) into TC$_1$(r,T), TC$_2$(r,T), and TC$_3$(r,T), respectively, each

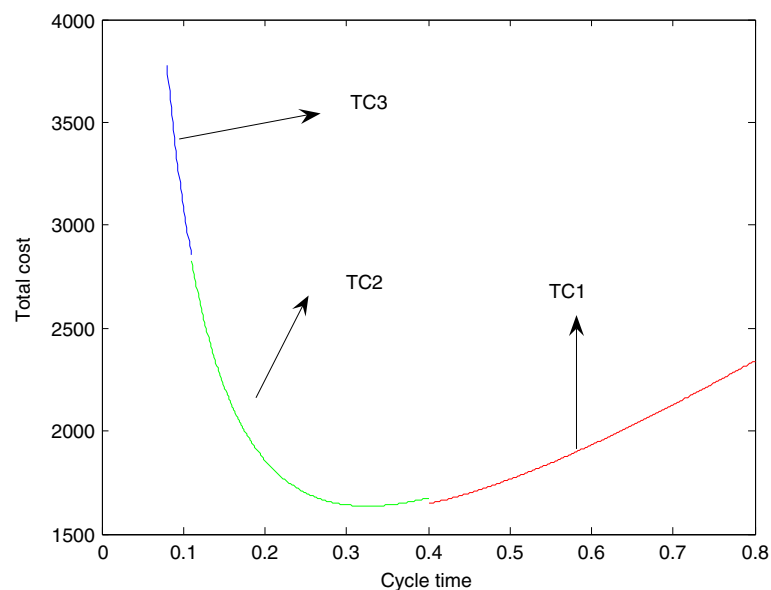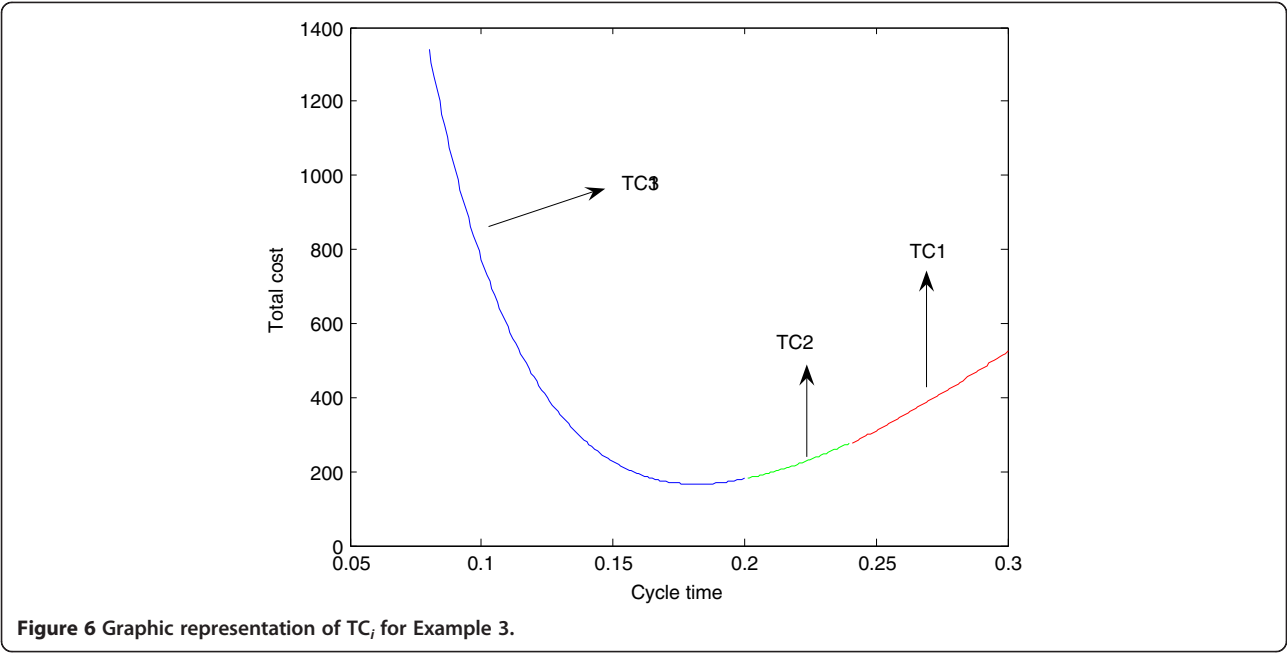**Figure 5 Graphic representation of TC$_i$ for Example 2.**

Figure 6 Graphic representation of TC$_i$ for Example 3.

TC$_i$(r,T) becomes a function of T alone rather than a function of r and T, since r is a function of T. Solving the equations $\frac{d\text{TC}_i}{dT} = 0, (i = 1, 2, 3)$, we get the optimal solutions T_1^*, T_2^*, and T_3^*.

Find the minimum of $\{\text{TC}_1(r_1(T_1^*),T_1^*),\ \text{TC}_2(r_2(T_2^*),T_2^*),\ \text{TC}_3(r_3(T_3^*),T_3^*)\}$ and the resultant is the optimal cost and corresponding T_i^* and $r_i(T_i^*)$ are the optimal solutions. While we are solving the differential equation $\frac{d\text{TC}_i}{dT} = 0$, if we get multiple solutions, then we have to check the $\frac{d^2\text{TC}_i}{dT^2} > 0$ for optimality.

Numerical analysis

Here, we find the optimal solutions for various cases to illustrate the solution procedures and obtain sensitivity analysis on order cancellation rate (δ), ordering cost (A), and holding cost (H), trade credit periods t_1 and t_2.
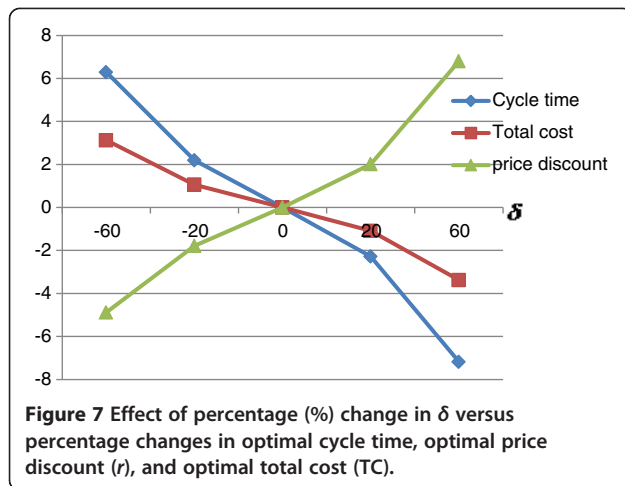
Example 1

Let $A = 300$, $D_1 = 2,000$, $D_2 = 2,000$, $\delta = 0.1$, $H = 1$, $I_p = 0.15$, $I_e = 0.2$, $t_1 = 0.14$, $t_2 = 0.10$, $p = 11$, $c = 10$. First, let r be fixed, say $r = 0.56$. We get $\Delta_1 = 453.80$ and $\Delta_2 = 7.25$. Clearly, $2A > \Delta_1$; by Theorem 1, we obtain that $T_1^* = 0.4096$, and the total cost TC$_1 = 1,347.60$. If r is a decision variable, then we utilize the solution procedure in 'When retail price discount rate is endogenous' section. The optimal solutions are $T_1^* = 0.5560$, $r_1^* = 0.8963$, and $\text{TC}_1^* = 1,214.80$. A graphic representation of TC$_i$ is shown in Figure 4.

Example 2

Let $A = 300$, $D_1 = 3,000$, $D_2 = 3,000$, $\delta = 0.1$, $H = 1$, $I_p = 0.15$, $I_e = 0.2$, $t_1 = 0.14$, $t_2 = 0.10$, $p = 11$, $c = 10$. First, let r be fixed, say $r = 0.56$. We get $\Delta_1 = 680.7$ and $\Delta_2 = 10.9$. Clearly, $\Delta_1 > 2A$ and $2A < \Delta_2$; by Theorem 1, we obtain

**Table 1 Sensitivity analysis with respect to the model parameters**

Parameter	Value	T^*	r^*	TC^*
δ	0.1	0.4899	0.8613	1,486.70
	0.2	0.4710	0.8893	1,457.00
	0.25	0.4609	0.9055	1,441.70
	0.3	0.4504	0.9237	1,426.00
	0.4	0.4278	0.9671	1,393.60
H	1	0.4899	0.8613	1,486.70
	1.2	0.4667	0.8712	1,539.30
	1.25	0.4613	0.8736	1,552.10
	1.3	0.4561	0.8760	1,564.80
	1.4	0.4461	0.8808	1,589.70
A	300	0.4899	0.8613	1,486.70
	310	0.4953	0.8656	1,508.40
	315	0.4979	0.8677	1,519.10
	320	0.5005	0.8698	1,529.70
	330	0.5057	0.8740	1,550.70
t_1	0.11	0.3359	0.8465	1,735.60
	0.12	0.3358	0.8465	1,684.90
	0.13	0.3357	0.8465	1,633.90
	0.14	0.3354	0.8465	1,582.50
	0.15	0.3351	0.8465	1,530.80
t_2	0.05	0.3333	0.6468	1,320.30
	0.06	0.3339	0.6468	1,373.50
	0.07	0.3343	0.6468	1,426.30
	0.08	0.3348	0.6468	1,478.70
	0.09	0.3351	0.6468	1,530.80

Figure 7 Effect of percentage (%) change in δ versus percentage changes in optimal cycle time, optimal price discount (r), and optimal total cost (TC).

Figure 9 Effect of % change in A versus % changes in optimal cycle time (T) optimal price discount (r), and optimal total cost (TC).

$T_2^* = 0.3354$ and the total cost $TC_2 = 1,582.50$. If r is a decision variable, then we utilize the solution procedure in 'When retail price discount rate is endogenous' section. The optimal solutions are $r_2^* = 0.8613$, $T_2^* = 0.4899$, and $TC_1^* = 1,486.70$. A graphic representation of TC_i is shown in Figure 5.

Example 3

Let $A = 300$, $D_1 = 8,000$, $D_2 = 8,000$, $\delta = 0.1$, $H = 1$, $I_p = 0.15$, $I_e = 0.2$, $t_1 = 0.28$, $t_2 = 0.09$, $p = 11$, $c = 10$. First, let r be fixed, say $r = 0.56$. We get $\Delta_1 = 5,292$ and $\Delta_2 = 655$. Clearly, $\Delta_2 > 2A$; by Theorem 1, we obtain $T_3^* = 0.1818$, and the total cost $TC_2 = 165.53$. If r is a decision variable, then we utilize the solution procedure in 'When retail price discount rate is endogenous' section. The optimal solutions are $r_2^* = 0.05611$, $T_2^* = 0.5573$, and $TC_1^* = 245.80$. A graphic representation of TC_i is shown in Figure 6.
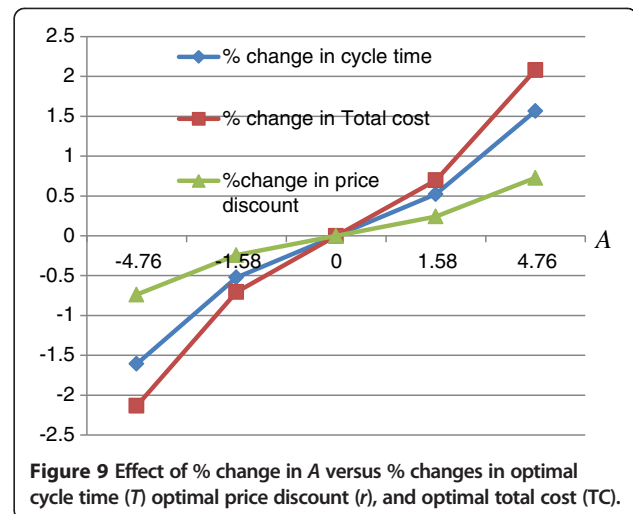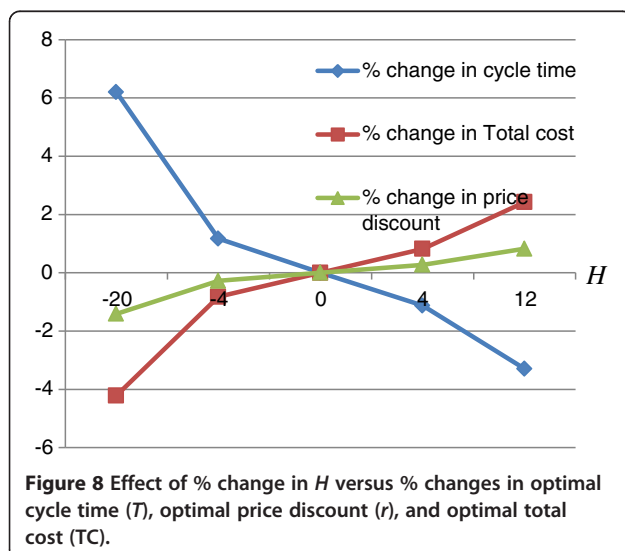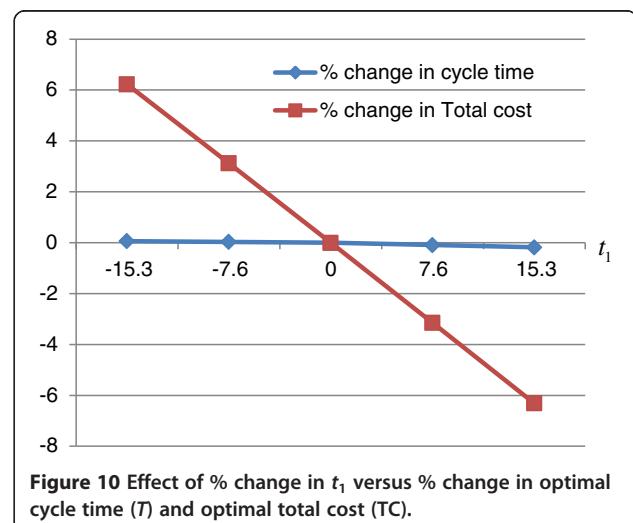
Sensitivity analysis

Here, we consider the data as in numerical Example 2. Sensitivity analysis on various parameters is presented in Table 1. It is also illustrated in Figures 7, 8, 9, 10, and 11.

Based on the results in Table 1 and Figures 7, 8, 9, 10 and 11, the following are observed:

- When partial order cancellation rate δ increases, the cycle time and total cost are decreased where as the optimal price discount rate increases marginally in order to increase the sales marginally.
- When holding cost H increases, the retailer will increase the price discount and shorten the cycle time. Total cost marginally increases due to the marginal increase in price discount rate offered by the retailer.
- When the ordering cost A increases, the retailer will increase his price discount rate and the inventory cycle
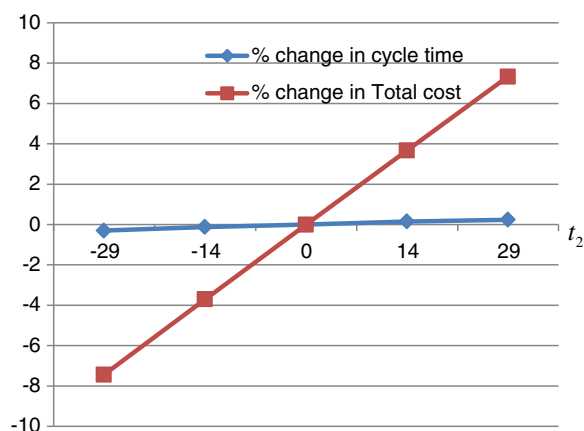

Figure 8 Effect of % change in H versus % changes in optimal cycle time (T), optimal price discount (r), and optimal total cost (TC).

Figure 10 Effect of % change in t_1 versus % change in optimal cycle time (T) and optimal total cost (TC).

Figure 11 Effect of % change in t_2 versus % change in optimal cycle time (T) and optimal total cost (TC).

time is also increased. To reduce the frequency of replenishment, the retailer lengthens the cycle time.

- When supplier provides a longer credit period t_1, the retailer replenishes the goods more often. In other words, the retailer will minimize the inventory cycle time to take advantage of longer credit period.
- When the retailer provides longer credit period t_2, the retailer's cycle time will be increased. Thus, the retailer will replenish the goods not often to decrease the loss.

Conclusions and future research

Unlike the existing research in two-level trade credit EOQ models, this paper considers partial order cancellation during advance sales period in the retailer's inventory system. The objective behind this consideration is that the market customers' decisions upon their orders play a vital role in trade. Since the payment time of the customers has an impact in the interest earned and interest payable by the retailer, the total costs are estimated under the investigation of exact payment time. The solution procedures are obtained for two cases: (a) when price discount rate is fixed and (b) when price discount rate is a decision variable. Using the derived mathematical theorems, the optimal solutions for price discount and replenishment cycle time are found out for both cases when price discount rate is fixed and endogenous. The sensitivity analyses are made for various inventory parameters.

In future research, one can implement the effect of inflation, perishability to this paper. Considering the trade credit periods t_1 and t_2 as decision variables could be a good extension of this paper. Further, one can improve the paper by considering integrated supplier-retailer inventory system.

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