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Oscillation of solutions of some generalized nonlinear α-difference equations

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Abstract
In this paper, the authors discuss the oscillation of solutions of some generalized nonlinear α-difference equation

$$\Delta_{\alpha(\ell)}(p(k)\Delta_{\alpha(\ell)}u(k)) + q(k)f(u(k-\tau(k))) = 0, \tag{1}$$

$k \in [a,\infty)$, where the functions p, q, f and τ are defined in their domain of definition and $\alpha > 1$, ℓ is a positive real. Further, $uf(u) > 0$ for $u \neq 0$, $p(k) > 0$ and $\lim_{k\to\infty}(k-\tau(k)) = \infty$, where $R_k = \sum_{r=0}^{[\frac{k-\ell}{\ell}]} \frac{1}{\alpha^r p(r\ell)} \to \infty$ as $k \to \infty$ and $u(k)$ is defined for $k \geq \min_{i\geq 0}(i-\tau(i))$ for all $k \in [a,\infty)$ for some $a \in [0,\infty)$.

MSC: 39A12

Keywords: generalized α-difference equation; generalized α-difference operator; oscillation and nonoscillation

1 Introduction

The basic theory of difference equations is based on the operator Δ defined as $\Delta u(k) = u(k+1) - u(k)$, $k \in \mathbb{N} = \{0,1,2,3,\ldots\}$. Even though many authors [1–4] have suggested the definition of Δ as

$$\Delta u(k) = u(k+\ell) - u(k), \quad k \in \mathbb{R}, \ell \in \mathbb{R} - \{0\}, \tag{2}$$

there was no significant progress in this area. But recently, [5] considered the definition of Δ as given in (2) and developed the theory of difference equations in a different direction. For convenience, the operator Δ defined by (2) is labeled as Δ_ℓ, and by defining its inverse Δ_ℓ^{-1}, many interesting results and applications in number theory (see [5–7]) were obtained. By extending the study related to the sequences of complex numbers and ℓ being real, some new qualitative properties of the solutions like rotatory, expanding, shrinking, spiral and weblike were obtained for difference equation involving Δ_ℓ. The results obtained using Δ_ℓ can be found in [5–7]. Popenda and Szmanda [8, 9] defined Δ as

$$\Delta_\alpha u(k) = u(k+1) - \alpha u(k), \tag{3}$$

and based on this definition, they studied the qualitative properties of a particular difference equation, and no one else has handled this operator. Recently Manuel *et al.* [6, 10]

considered the definition of Δ_ℓ as given in (3), and by defining its inverse, some interesting results on number theory were obtained.

In [11], Szafranski and Szmanda obtained sufficient conditions for the oscillation of a similar difference equation involving Δ. In this paper the theory is extended from Δ to $\Delta_{\alpha(\ell)}$ for all real $k \in [a, \infty)$, and we discuss the oscillatory behavior of solutions of generalized nonlinear α-difference equation (1).

Throughout this paper, we make use of the following assumptions.

(a) $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$, $\mathbb{N}(a) = \{a, a+1, a+2, \ldots\}$;
(b) $\mathbb{N}_\ell(a) = \{a, a+\ell, a+2\ell, \ldots\}$;
(c) $\lceil x \rceil$ and $[x]$ denote upper integer and integer part of x, respectively;
(d) $j = k - k_i - [\frac{k-k_i}{\ell}]\ell$, $k_i \in [0, \infty)$.

2 Preliminaries

In this section, we present some preliminaries which will be useful for future discussion.

Definition 2.1 [12] The inverse of the generalized α-difference operator denoted by $\Delta^{-1}_{\alpha(\ell)}$ on $u(k)$ is defined as follows. If $\Delta_{\alpha(\ell)} v(k) = u(k)$, then

$$\Delta^{-1}_{\alpha(\ell)} u(k) = v(k) - \alpha^{\lceil \frac{k}{\ell} \rceil} v(j), \tag{4}$$

where $k \in \mathbb{N}_\ell(j)$, $j = k - [\frac{k}{\ell}]\ell$.

Lemma 2.2 [13] *If the real-valued function $u(k)$ is defined for all $k \in [a, \infty)$ and $\alpha > 1$, then*

$$\Delta^{-1}_{\alpha(\ell)} u(k) = \sum_{r=0}^{[\frac{k-a-j-\ell}{\ell}]} \frac{u(a+j+r\ell)}{\alpha^{\lceil \frac{a+j+\ell-k+r\ell}{\ell} \rceil}} + \alpha^{\lceil \frac{k-a}{\ell} \rceil} u(a+j) \tag{5}$$

for all $k \in \mathbb{N}_\ell(j)$, $j = k - a - [\frac{k-a}{\ell}]\ell$.

Definition 2.3 [7] The solution $u(k)$ of (1) is called oscillatory if for any $k_1 \in [a, \infty)$ there exists $k_2 \in \mathbb{N}_\ell(k_1)$ such that $u(k_2)u(k_2+\ell) \leq 0$. The difference equation itself is called oscillatory if all its solutions are oscillatory. If the solution $u(k)$ is not oscillatory, then it is said to be nonoscillatory (*i.e.*, $u(k)u(k+\ell) > 0$ for all $k \in [k_1, \infty)$).

3 Main results

In this section we present conditions for the oscillation of equation (1).

Theorem 3.1 *Assume that*

(i) $q(k) \geq 0$ *and* $\sum_{r=0}^{\infty} \alpha^r q(r\ell) = \infty$,
(ii) $\liminf_{|u(k)| \to \infty} |f(u(k))| > 0$.

Then every solution of Equation (1) *is oscillatory.*

Proof Assume that Equation (1) has a nonoscillatory solution $u(k)$, and we assume that $u(k)$ is eventually positive. Then there is a positive integer k_1 such that

$$u\big(k - \tau(k)\big) > 0 \quad \text{for } k \geq k_1. \tag{6}$$

From Equation (1) we have

$$\Delta_{\alpha(\ell)}\big(p(k)\Delta_{\alpha(\ell)}u(k)\big) = -q(k)f\big(u\big(k-\tau(k)\big)\big), \quad k \geq k_1,$$

and so $p(k)\Delta_{\alpha(\ell)}u(k)$ is eventually nonincreasing. We first show that

$$p(k)\Delta_{\alpha(\ell)}u(k) \geq 0 \quad \text{for } k \geq k_1.$$

In fact, if there is $k_2 \geq k_1$ such that $p(k_2)\Delta_{\alpha(\ell)}u(k_2) = c < 0$ and $p(k)\Delta_{\alpha(\ell)}u(k) \leq c$ for $k \geq k_2$, that is,

$$\Delta_{\alpha(\ell)}u(k) \leq \frac{c}{p(k)},$$

hence by Lemma 2.2,

$$u(k) = \alpha^{\lceil \frac{k-k_2}{\ell} \rceil} u(k_2 + j) + c \sum_{r=0}^{\frac{k-k_2-\ell-j}{\ell}} \frac{1}{\alpha^{\lceil \frac{k_2+j+\ell-k+r\ell}{\ell} \rceil} p(k_2 + j + \ell + r\ell)} \to -\infty \quad \text{as } k \to \infty,$$

which contradicts the fact that $u(k) > 0$ for $k \geq k_2$. Hence, $p(k)\Delta_{\alpha(\ell)}u(k) \geq 0$ for $k \geq k_1$. Therefore we obtain

$$u\big(k-\tau(k)\big) > 0, \qquad \Delta_{\alpha(\ell)}u(k) \geq 0, \qquad \Delta_{\alpha(\ell)}\big(p(k)\Delta_{\alpha(\ell)}u(k)\big) \leq 0 \quad \text{for } k \geq k_2.$$

Let $L = \lim_{k\to\infty} u(k)$.

Then $L > 0$ is finite or infinite.

Case 1. $L > 0$ is finite.

From the function $f(k)$ defined in its domain of definition, we have

$$\lim_{k\to\infty} f\big(u\big(k-\tau(k)\big)\big) = f(L) > 0.$$

Thus, we may choose a positive integer k_4 ($\geq k_1$) such that

$$f\big(u\big(k-\tau(k)\big)\big) > \frac{1}{2}f(L), \quad k \geq k_4. \tag{7}$$

By substituting (7) in Equation (1) we obtain

$$\Delta_{\alpha(\ell)}\big(p(k)\Delta_{\alpha(\ell)}u(k)\big) + \frac{1}{2}f(L)q(k) \leq 0, \quad k \geq k_4. \tag{8}$$

By Lemma 2.2, we obtain

$$p(k+\ell)\Delta_{\alpha(\ell)}u(k+\ell) - \alpha^{\lceil \frac{k-k_4}{\ell} \rceil} p(k_4+j)\Delta_{\alpha(\ell)}u(k_4+j) + \frac{1}{2}f(L) \sum_{r=0}^{\frac{k-k_4-\ell-j}{\ell}} \frac{p(k_4+j+r\ell)}{\alpha^{\lceil \frac{k_4+j+\ell-k+r\ell}{\ell} \rceil}} \leq 0,$$

and so

$$\frac{1}{2}f(L)\sum_{r=0}^{\frac{k-k_4-\ell-j}{\ell}}\frac{p(k_4+j+r\ell)}{\alpha^{\lceil\frac{k_4+j+\ell-k+r\ell}{\ell}\rceil}}\le\alpha^{\lceil\frac{k-k_4}{\ell}\rceil}p(k_4+j)\Delta_{\alpha(\ell)}u(k_4+j),\quad k\ge k_4,$$

which contradicts (i).

Case 2. $L=\infty$. For this case, from condition (ii) we have

$$\liminf_{k\to\infty}f\bigl(u\bigl(k-\tau(k)\bigr)\bigr)>0,$$

and so we may choose a positive constant c and a positive integer k_5 sufficiently large such that

$$f\bigl(u\bigl(k-\tau(k)\bigr)\bigr)\ge c\quad\text{for }k\ge k_5. \tag{9}$$

Substituting (9) into Equation (1) we have

$$\Delta_{\alpha(\ell)}\bigl(p(k)\Delta_{\alpha(\ell)}u(k)\bigr)+cq(k)\le 0,\quad k\le k_5.$$

Using a similar argument as in Case 1, we obtain a contradiction to condition (i). This completes the proof. □

Example 3.2 For the generalized α-difference equation

$$\Delta_{\alpha(\ell)}\left(\frac{1}{k}\Delta_{\alpha(\ell)}u(k)\right)=\left(\frac{4k^2+6k\ell+\ell^2}{(k+\ell)_\ell^{(2)}}\right)(-\alpha)^{\lceil\frac{k+2\ell}{\ell}\rceil},$$

all the conditions of Theorem 3.1 hold and hence all the solutions are oscillatory. In fact $u(k)=(-\alpha)^{\lceil\frac{k}{\ell}\rceil}k$ is one such solution.

Theorem 3.3 *Assume that*

(iii) $q(k)\ge 0$ *and* $\sum_{r=0}^{\infty}\alpha^r R(r\ell)q(r\ell)=\infty$.

Then every bounded solution of (1) *is oscillatory.*

Proof Proceeding as in the proof of Theorem 3.1, with the assumption that $u(k)$ is a bounded nonoscillatory solution of (1), we get inequality (8), and so we obtain

$$R(k)\Delta_{\alpha(\ell)}\bigl(p(k)\Delta_{\alpha(\ell)}u(k)\bigr)+\frac{1}{2}f(L)R(k)q(k)\le 0,\quad k\ge k_4. \tag{10}$$

It is easy to see that

$$\begin{aligned}R(k)&\Delta_{\alpha(\ell)}\bigl(p(k)\Delta_{\alpha(\ell)}u(k)\bigr)\\&\ge\Delta_{\alpha(\ell)}\bigl(R(k)p(k)\Delta_{\alpha(\ell)}u(k)\bigr)-\alpha p(k)\Delta_{\alpha(\ell)}u(k)\Delta_{\alpha(\ell)}R(k).\end{aligned}\tag{11}$$

Using (10) in (11), (11) reduces to

$$\begin{aligned}
&R(k)p(k)\Delta_{\alpha(\ell)}u(k)-\alpha^{\lceil\frac{k-k_2}{\ell}\rceil}R(k_2+j)p(k_2+j)\Delta_{\alpha(\ell)}u(k_2+j)\\
&\quad-\alpha\sum_{r=0}^{\frac{k-k_4-j-\ell}{\ell}}\frac{p(k_2+j+r\ell)}{\alpha^{\lceil\frac{k_2-k+j+\ell+r\ell}{\ell}\rceil}}\Delta_{\alpha(\ell)}u(k_2+j+r\ell)\Delta_{\alpha(\ell)}R(k_2+j+r\ell)\\
&\quad+\frac{1}{2}f(L)\sum_{r=0}^{\frac{k-k_4-j-\ell}{\ell}}\frac{R(k_2+j+r\ell)}{\alpha^{\lceil\frac{k_2-k+j+\ell+r\ell}{\ell}\rceil}}q(k_2+j+r\ell)\le 0,
\end{aligned}$$

which implies

$$\begin{aligned}
&\frac{1}{2}f(L)\sum_{r=0}^{\frac{k-k_4-j-\ell}{\ell}}\frac{R(k_2+j+r\ell)}{\alpha^{\lceil\frac{k_2-k+j+\ell+r\ell}{\ell}\rceil}}q(k_2+j+r\ell)\\
&\quad\le u(k+\ell)+\alpha^{\lceil\frac{k-k_4}{\ell}\rceil}R(k_4+j)p(k_4+j)\Delta_{\alpha(\ell)}u(k_4+j+r\ell)-\alpha^{\lceil\frac{k-k_4}{\ell}\rceil}u(k_4+j),\quad k\ge k_4.
\end{aligned}$$

Hence, there exists a constant c such that

$$\sum_{r=0}^{\frac{k-k_4-j-\ell}{\ell}}\frac{R(k_4+j+r\ell)}{\alpha^{\lceil\frac{k_2-k+j+\ell+r\ell}{\ell}\rceil}}q(k_4+j+r\ell)\le c\quad\text{for all } k\ge k_4,$$

which is a contradiction to condition (iii) which completes the proof. □

Example 3.4 For the generalized α-difference equation

$$\Delta_{\alpha(\ell)}\bigl(k\Delta_{\alpha(\ell)}u(k)\bigr)=\left(\frac{(\alpha 2^{\ell}+1)(\alpha k+k+\ell)}{2^{k+2\ell}}\right)(-1)^{\lceil\frac{k+2\ell}{\ell}\rceil},$$

all the conditions of Theorem 3.3 hold and hence all the solutions are oscillatory. In fact $u(k)=\frac{(-1)^{\lceil\frac{k}{\ell}\rceil}}{2^k}$ is one such solution.

Theorem 3.5 *Assume that*

(iv) $(k-\tau(k))$ *is nondecreasing, where* $\tau(k)\in[0,\infty)$,
(v) *there exists* $p(k_n)$ *such that* $p(k_n)\le 1$ *for* $k_n\in[0,\infty)$,
(vi) $\sum_{r=0}^{\infty}\alpha^r q(r\ell)=\infty$,
(vii) f *is nondecreasing and there is a nonnegative constant* M *such that*

$$\limsup_{s\to 0}\frac{s}{f(s)}=M. \tag{12}$$

Then the difference $\Delta_{\alpha(\ell)}u(k)$ *of every solution* $u(k)$ *of Equation* (1) *oscillates.*

Proof If not, then Equation (1) has a solution $u(k)$ such that its difference $\Delta_{\alpha(\ell)}u(k)$ is nonoscillatory. Assume first that the sequence $\Delta_{\alpha(\ell)}u(k)$ is eventually negative. Then there is a positive integer k_1 such that

$$\Delta_{\alpha(\ell)}u(k)<0,\quad k>k_1,$$

and so $u(k)$ is decreasing for $k \geq k_1$, which implies that $u(k)$ is also nonoscillatory. Set

$$w(k) = \frac{p(k)\Delta_{\alpha(\ell)}u(k)}{f(u(k-\tau(k)))} k \geq k_2 \geq k_1. \tag{13}$$

Then

$$\begin{aligned}\Delta_{\alpha(\ell)}w(k) &= \frac{p(k+\ell)\Delta_{\alpha(\ell)}u(k+\ell)}{f(u(k+\ell-\tau(k+\ell)))} - \alpha\frac{p(k)\Delta_{\alpha(\ell)}u(k)}{f(u(k-\tau(k)))} = \frac{\Delta_{\alpha(\ell)}(p(k)\Delta_{\alpha(\ell)}u(k))}{f(u(k-\tau(k)))} \\ &\quad + p(k+\ell)\Delta_{\alpha(\ell)}u(k+\ell)\frac{f(u(k-\tau(k))) - f(u(k+\ell-\tau(k+\ell)))}{f(u(k+\ell-\tau(k+\ell)))f(u(k-\tau(k)))} \\ &\leq \frac{\Delta_{\alpha(\ell)}(p(k)\Delta_{\alpha(\ell)}u(k))}{f(u(k-\tau(k)))} = -q(k), \quad k \geq k_2. \end{aligned} \tag{14}$$

By Lemma 2.2, we have

$$w(k+\ell) - \alpha^{\lceil \frac{k-k_2}{\ell} \rceil} w(k_2+j) \leq -\sum_{r=0}^{\frac{k-k_2-j}{\ell}} \frac{q(k_2+j+r\ell)}{\alpha^{\lceil \frac{k_2+j-k+r\ell}{\ell} \rceil}},$$

and by (vi) we get

$$\lim_{k\to\infty} w(k) = -\infty, \tag{15}$$

which implies that eventually

$$f\big(u\big(k-\tau(k)\big)\big) > 0 \text{ and therefore } k-\tau(k) > 0. \tag{16}$$

By (15), we can choose k_3 ($\geq k_2$) such that

$$w(k) \leq -(M+\ell), \quad k \geq k_3.$$

That is,

$$p(k)\Delta_{\alpha(\ell)}u(k) + (M+\ell)f\big(u\big(k-\tau(k)\big)\big) \leq 0, \quad k \geq k_3. \tag{17}$$

Set $\lim_{k\to\infty} u(k) = L$. Then $L \geq 0$. Now we prove that $L = 0$. If $L > 0$, then we have

$$\lim_{k\to\infty} f\big(u\big(k-\tau(k)\big)\big) = f(L) > 0$$

since $f(k)$ is defined in its domain of definition. Choosing k_4 sufficiently large such that

$$f\big(u\big(k-\tau(k)\big)\big) > \frac{1}{2}f(L), \quad k \geq k_4, \tag{18}$$

and substituting (19) into (17), we obtain

$$\Delta_{\alpha(\ell)}u(k) + \frac{1}{2p(k)}(M+\ell)f(L) \leq 0, \quad k \geq k_4. \tag{19}$$

From Lemma 2.2, we have

$$u(k+\ell)-\alpha^{\lceil\frac{k-k_4}{\ell}\rceil}u(k_4+j)+\frac{1}{2}(M+\ell)f(L)\sum_{r=0}^{\frac{k-k_4-\ell-j}{\ell}}\frac{1}{\alpha^{\lceil\frac{k_4-k+j+r\ell}{\ell}\rceil}p(k_4-k+j+r\ell)}\leq 0,$$

which implies that $\lim_{k\to\infty}u(k)=-\infty$.

This contradicts (16). Hence $\lim_{k\to\infty}u(k)=0$.

By the assumptions we have

$$\limsup_{k\to\infty}\frac{u(k-\tau(k))}{f(u(k-\tau(k)))}\leq M.$$

From this we can choose k_4 such that

$$\frac{u(k-\tau(k))}{f(u(k-\tau(k)))}<M+\ell,\quad k\geq k_5.$$

That is, $u(k-\tau(k))<(M+\ell)f(u(k-\tau(k)))$, $k\geq k_5$, and so from (17) we get

$$p(k)\Delta_{\alpha(\ell)}u(k)+u\bigl(k-\tau(k)\bigr)<0,\quad k\geq k_5.$$

In particular, for a function $p(k_n)$ satisfying condition (v), we have

$$u(k_n+l)-\alpha u(k_n)+x_{k_n}-\tau(k_n)\leq p(k_n)\bigl(u(k_n+l)-\alpha u(k_n)\bigr)+u\bigl(k_n-\tau(k_n)\bigr)<0$$

for k sufficiently large, which implies that

$$0<u(k_n+l)+\bigl(u\bigl(k_n-\tau(k_n)\bigr)-\alpha u(k_n)\bigr)<0$$

for all large k. This is a contradiction. The case that $\Delta_{\alpha(\ell)}u(k)$ is eventually positive can be treated in a similar fashion and this completes the proof of the theorem. □

Example 3.6 For the generalized α-difference equation

$$\Delta_{\alpha(\ell)}\left(\frac{1}{k}\Delta_{\alpha(\ell)}u(k)\right)=\left(\frac{(2^\ell+1)((2^\ell+1)k+\ell)}{(k+\ell)_\ell^{(2)}}\right)(-\alpha)^{\lceil\frac{k+2\ell}{\ell}\rceil}2^k,$$

all the conditions of Theorem 3.5 hold and hence all the solutions are oscillatory. In fact $u(k)=(-\alpha)^{\lceil\frac{k}{\ell}\rceil}2^k$ is one such solution.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally to the manuscript and read and approved the final draft.

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