Intersymbol Decorrelating Detector for Asynchronous CDMA Networks with Multipath

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Most reported multiuser detection techniques for CDMA systems need the channel estimation including the delay spread and the parameters of the multipath channel of the desired user. This paper proposes an intersymbol decorrelating detector that makes use of the cross-correlation matrix constructed by the consecutively received symbols. The proposed detector is attractive for its simplicity because no channel estimation is required except for the synchronization of the desired user. Compared with other reported multiuser detectors, simulation results show that the proposed detector provides a good performance when the active users have significant intersymbol interference.

Keywords and phrases: blind multiuser detection, cross-correlation matrix, multipath, decorrelating detector.

1. INTRODUCTION

Blind multiuser detection for code-division multiple-access (CDMA) systems has flourished rapidly in recent years. The general ground work of blind multiuser detection was first reported in [1], in which it was assumed that the detector had no priori knowledge except for the signature waveform and timing of the desired user. Without considering the multipath, a blind minimum output energy (MOE) detector was proposed in [1], and later the canonical subspace representation of decorrelating detector and minimum mean square error (MMSE) detector were reported in [2]. In [3], a reduced-rank MOE detector was proposed by using array processing techniques. However, these detectors would not work properly when the intersymbol interference (ISI) could not be ignored. In asynchronous CDMA systems, the ISI is generated by the delays and the multipath channels of the active users. In order to overcome the influence of the ISI, several approaches for channel estimation were developed in [4, 5, 6],

and a number of improved subspace detectors were proposed by exploiting the channel estimation in [7, 8, 9]. Since the channel estimation increases the computational complexity of the detectors substantively, a reduced computational constrained optimization solution was proposed with a sacrifice of the performance (inferior to MMSE detector) [10]. However, all these methods need to estimate the delay spread and the parameters of the multipath channel of the desired user. Hence, one possible drawback for these methods is that the channel estimation may suffer from variations of channel and noise environment, which definitely deteriorates the performance of the detection.

Attempt is reported in this paper to avoid channel estimation based on a new multiuser detection scheme to exploit the intersymbol information, which has not been considered in the previously reported detectors. It is noted that in asynchronous CDMA systems, the current received signal has certain correlation with its preceeding and succeeding ones due to the existence of ISI and MAI. This paper presents an intersymbol decorrelating detector for asynchronous CDMA systems with multipath. The basic idea of the proposed detector is to construct an intersymbol cross-correlation matrix of the consecutively received symbols, and then to derive the

decorrelating detector [11] for the desired user. It is shown that the proposed detector can be implemented conveniently without the estimation of the delay spread and the multipath channel of the desired user. Simulation results show that the proposed detector achieves a promising performance especially when the active users have significant intersymbol interference.

The paper is organized as follows. In the next section, signal model is presented to express asynchronous CDMA system in fading multipath channels. By constructing the crosscorrelation matrix, Section 3 develops an intersymbol decorrelating detector. In Section 4, simulation examples are provided to demonstrate the performance of the proposed detector. Conclusions are given in Section 5.

2. SIGNAL MODEL

We consider a DS-CDMA system with K users and a normalized spreading factor of N chips per symbol. The transmitted signal due to the kth user is given by

$$y_k(t) = A_k \sum_{i=-\infty}^{\infty} b_k[i]s_k(t - iT), \tag{1}$$

where T is the symbol duration, A_k and $b_k[i] \in \{+1, -1\}$ are, respectively, the amplitude and the symbol stream of the kth user. The signature waveform $s_k(t)$ is of the form

$$s_k(t) = \sum_{j=0}^{N-1} c_k[j]\psi(t - jT_c), \quad 0 \le t \le T, \tag{2}$$

where $c_k[j] = \pm 1$ $(0 \le j \le N - 1)$ is the spreading sequence allocated to the kth user and $\psi(t)$ is the normalized chip waveform with a duration $T_c = T/N$. The discrete-time expression of the transmitted signal of user k at the chip rate is obtained by a multirate convolution

$$y_k(n) = \sum_{i=-\infty}^{\infty} b_k(i)\bar{c}_k(n - iN), \tag{3}$$

where $\bar{c}_k(n) = A_k c_k(n)$ for $n = 0, \dots, N - 1$. By propagating through the asynchronous multipath channel that is assumed to have a maximum length of M $(M < N)$, the received signal due to user k can be expressed by [12]

$$r_k(n) = \sum_{m=-\infty}^{\infty} y_k(m)g_k(n - d_k - m), \tag{4}$$

where $g_k(m) \ne 0$, $0 \le m \le M$, is the mth complex multipath parameter for user k and $0 \le d_k < N$ is the delay of user k in terms of the chip duration. Based on (3) and (4), we obtain

$$r_k(n) = \sum_{m=-\infty}^{\infty} b_k(m)h_k(n - d_k - mN), \tag{5}$$

where

$$h_k(n) = \sum_{j=-\infty}^{\infty} \bar{c}_k(j)g_k(n - j). \tag{6}$$

Then, the received signal is the superposition of the signals from all K users plus additive complex white Gaussian noise, that is,

$$r(n) = \sum_{k=1}^{K} r_k(n) + v(n), \tag{7}$$

where $v(n)$ is a zero-mean complex Gaussian variable with a variance of σ^2. From (6) and denoting $\mathbf{h}_{k,\text{all}} = [h_k(0), h_k(1), \dots, h_k(N - 1 + M)]^H$ as the signature vector of user k, we obtain

$$\mathbf{h}_{k,\text{all}} = \bar{\mathbf{C}}_k \mathbf{g}_k, \tag{8}$$

where

$$\bar{\mathbf{C}}_k = \begin{bmatrix} \bar{c}_k(0) & & \mathbf{0} \\ \vdots & \ddots & \bar{c}_k(0) \\ \bar{c}_k(N-1) & \ddots & \vdots \\ \mathbf{0} & \ddots & \bar{c}_k(N-1) \end{bmatrix}, \qquad \mathbf{g}_k = \begin{bmatrix} g_k(0) \\ \vdots \\ g_k(M) \end{bmatrix}; \tag{9}$$

$\mathbf{g}_k$ is the kth user's multipath channel vector. According to (5), we denote

$$\mathbf{r}_k(n) = \begin{bmatrix} r_k(nN) \\ \vdots \\ r_k(nN + N - 1) \end{bmatrix}, \qquad \underline{\mathbf{h}}_k = \begin{bmatrix} \mathbf{0} \\ h_k(0) \\ \vdots \\ h_k(N - d_k - 1) \end{bmatrix}, \tag{10}$$

$$\underline{\bar{\mathbf{h}}}_k = \begin{bmatrix} h_k(N - d_k) \\ \vdots \\ h_k(N + M - 1) \\ \mathbf{0} \end{bmatrix}. \tag{11}$$

The nth received symbol vector for user k can be given by

$$\mathbf{r}_k(n) = \underline{\mathbf{h}}_k b_k(n) + \underline{\bar{\mathbf{h}}}_k b_k(n - 1), \tag{12}$$

and $\underline{\mathbf{r}}(n) = [r(nN), \dots, r(nN + N - 1)]^T$, which is the vector of the received symbols from all users, is given by

$$\underline{\mathbf{r}}(n) = \sum_{k=1}^{K} \underline{\mathbf{r}}_k(n) + \underline{\mathbf{v}}(n) = \underline{\mathbf{H}}\mathbf{b}(n) + \underline{\mathbf{v}}(n), \tag{13}$$

where $\underline{\mathbf{H}} = [\underline{\mathbf{h}}_1, \dots, \underline{\mathbf{h}}_K, \underline{\bar{\mathbf{h}}}_1, \dots, \underline{\bar{\mathbf{h}}}_K]$ is the signature matrix of all users with full column rank $2K$, $\mathbf{b}(n) = [b_1(n), \dots, b_K(n), b_1(n - 1), \dots, b_K(n - 1)]^T$ contains the received bits of K users, and $\underline{\mathbf{v}}(n) = [v(nN), \dots, v(nN + N - 1)]^T$ is the independent white Gaussian noise vector.

3. INTERSYMBOL DECORRELATING DETECTOR

We assume without loss of generality that user 1 is the desired user, and that the receiver is synchronized to user 1, that is, $d_1 = 0$. Then, the signal in (13) can be written as

$$\underline{\mathbf{r}}(n) = \underline{\mathbf{h}}_1 b_1(n) + \bar{\underline{\mathbf{h}}}_1 b_1(n-1) + \underline{\widetilde{\mathbf{H}}}\tilde{\mathbf{b}}(n) + \underline{\mathbf{v}}(n), \qquad (14)$$

where $\underline{\widetilde{\mathbf{H}}} = [\underline{\mathbf{h}}_2,\ldots,\underline{\mathbf{h}}_K, \bar{\underline{\mathbf{h}}}_2,\ldots,\bar{\underline{\mathbf{h}}}_K]$ and $\tilde{\mathbf{b}}(n) = [b_2(n),\ldots, b_K(n), b_2(n-1),\ldots,b_K(n-1)]^T$ are, respectively, the signature matrix and the received bits of the interfering users. It is noted from (14) that only the first M entries of $\bar{\underline{\mathbf{h}}}_1$ are nonzero. Thus, the ISI caused by $\bar{\underline{\mathbf{h}}}_1$ can be removed by truncating the first $G \geq M$ entries of $\underline{\mathbf{r}}(n)$. For convenience and without loss of generality, it is assumed that $G \geq M$ to obtain

$$\mathbf{r}(n) = \mathbf{h}_1 b_1(n) + \widetilde{\mathbf{H}}\tilde{\mathbf{b}}(n) + \mathbf{v}(n), \qquad (15)$$

where $\mathbf{r}(n)$ is an $(N - G) \times 1$ vector which consists of the last $N - G$ entries of $\underline{\mathbf{r}}(n)$. Similarly, $\mathbf{h}_1$, $\widetilde{\mathbf{H}}$, and $\mathbf{v}(n)$ are composed of the last $N - G$ rows of $\underline{\mathbf{h}}_1$, $\underline{\widetilde{\mathbf{H}}}$, and $\underline{\mathbf{v}}(n)$, respectively. It is assumed throughout the paper that $[\mathbf{h}_1\widetilde{\mathbf{H}}]$ is of full column rank. The signal in (15) is a truncation of the signal in (14) which indicates that the ISI of the desired user is eliminated. In the following analysis, G is selected as a fixed number which is assumed to be not smaller than M. It should be pointed out that when the selected G is not smaller than M, the ISI of the desired user in (15) cannot be totally removed. In this case, the performance of the proposed detector will suffer deterioration. The influence of the selection of G on the detector will be simulated and analyzed in Section 4.

In (15), the ISI of the interfering users still exist as their nonzero delays. This is an advantageous property for the following proposed intersymbol cross-correlation matrix.

Based on the signal model of (15), a linear detector for user 1 can be represented by a vector $\mathbf{w}_1$, which is applied to the received signal $\mathbf{r}(n)$. Then, the nth bit of user 1 can be estimated according to the following rule:

$$\hat{b}_1[n] = \text{sgn}\left\{\Re\left(\mathbf{w}_1^H \mathbf{r}(n)\right)\right\}. \qquad (16)$$

In this paper, a linear decorrelating detector is developed to utilize the intersymbol information of (15). We first construct the autocorrelation matrix of the received signal $\mathbf{r}(n)$, which is given by

$$\mathbf{R} = E\{\mathbf{r}(n)\mathbf{r}(n)^H\} = \mathbf{h}_1 \mathbf{h}_1^H + \widetilde{\mathbf{H}}\widetilde{\mathbf{H}}^H + \sigma^2 \mathbf{I}_{N-G}, \qquad (17)$$

where $\mathbf{I}_{N-G}$ is an $(N-G) \times (N-G)$ unitary matrix. A useful cross-correlation matrix of the received signal $\mathbf{r}(n)$ is constructed to obtain the intersymbol information, that is,

$$\bar{\mathbf{R}} = E\{\mathbf{r}(n)\mathbf{r}(n+1)^H\} + E\{\mathbf{r}(n)\mathbf{r}(n-1)^H\}$$
$$= \widetilde{\mathbf{H}}\begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix}\widetilde{\mathbf{H}}^H. \qquad (18)$$

In (18), $\bar{\mathbf{R}}$ is made up of the cross-correlation of the consecutively received signals. It is noted that $\bar{\mathbf{R}}$ only contains the signature matrix of all interfering users except for the desired user because the ISI of the desired user has been truncated in (15). By performing an eigendecomposition of the matrix $\mathbf{R}$ and $\bar{\mathbf{R}}$, we obtain

$$\mathbf{R} = [\mathbf{U}_s, \mathbf{U}_n]\begin{bmatrix} \Lambda_s & \\ & \Lambda_n \end{bmatrix}[\mathbf{U}_s, \mathbf{U}_n]^H$$
$$= \mathbf{U}_s\Lambda_s\mathbf{U}_s^H + \mathbf{U}_n\Lambda_n\mathbf{U}_n^H, \qquad (19)$$

$$\bar{\mathbf{R}} = [\bar{\mathbf{U}}_s, \bar{\mathbf{U}}_n]\begin{bmatrix} \bar{\Lambda}_s & \\ & \mathbf{0} \end{bmatrix}[\bar{\mathbf{U}}_s, \bar{\mathbf{U}}_n]^H$$
$$= \bar{\mathbf{U}}_s\bar{\Lambda}_s\bar{\mathbf{U}}_s^H, \qquad (20)$$

where $\Lambda_s = \text{diag}(\lambda_1,\ldots,\lambda_{2K-1})$ contains $(2K - 1)$ largest eigenvalues of the signal subspace in $\mathbf{R}$, and $\mathbf{U}_s$ contains the corresponding orthonormal eigenvectors. Both $\Lambda_n = \sigma^2 \mathbf{I}_{N-G-2K+1}$ and $\mathbf{U}_n$ are, respectively, eigenvalues and orthonormal eigenvectors of the noise subspace. Similarly, $\bar{\Lambda}_s = \text{diag}(\bar{\lambda}_1,\ldots,\bar{\lambda}_{2K-2})$ contains $(2K - 2)$ nonzero eigenvalues and $\bar{\mathbf{U}}_s$ contains the corresponding eigenvectors. Obviously, $\bar{\mathbf{R}}$ contains ISI and MAI of all interfering users.

The construction of $\bar{\mathbf{R}}$ plays a key role for the following derived decorrelating detector. Since $\bar{\mathbf{R}}$ is formed from the cross-correlation matrix of consecutively received signals, the proposed detector is then named as intersymbol decorrelating detector.

We consider the relationship between the subspace spanned by $\bar{\mathbf{R}}$ and the subspace spanned by $\widetilde{\mathbf{H}}$.

Proposition 1. *The null space of $\bar{\mathbf{U}}_s$ in (20) is equal to the null space of $\widetilde{\mathbf{H}}$ in (15), that is,*

$$\text{null}(\widetilde{\mathbf{H}}) = \text{null}(\bar{\mathbf{U}}_s) = \text{range}\left(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H\right). \qquad (21)$$

For the proof, see Appendix A.

Proposition 1 shows that $\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H$ projects any signal onto the null space of $\widetilde{\mathbf{H}}$. Since $\widetilde{\mathbf{H}}$ only contains the signature vectors of interfering users, it is natural to explore the optimal linear detector by removing the influence generated by $\widetilde{\mathbf{H}}$. In [2], the decorrelating detector for user 1 is defined as the unique signal $\mathbf{d} \in \text{range}(\mathbf{U}_s)$, such that $\mathbf{d}^H\mathbf{h}_1 > 0$ and $\mathbf{d}^H\widetilde{\mathbf{H}} = \mathbf{0}$, where $\mathbf{U}_s$ is given by (19). Clearly, the decorrelating detector for user 1 is orthonormal to $\widetilde{\mathbf{H}}$. Then, considering the property of Proposition 1, another form of the decorrelating detector is proposed for the desired user, which is given by the following proposition

Proposition 2. *In terms of the matrix $\bar{\mathbf{R}}$ in (18), the decorrelating detector for user 1 in (15) can be given by*

$$\mathbf{d}_1 = (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1. \qquad (22)$$

For the proof, see Appendix B.

Proposition 2 shows that $\mathbf{d}_1$ is equivalent to the decorrelating detector for the desired user.

We now consider the blind implementation of $\mathbf{d}_1$. Utilizing the property of Proposition 1, (15) is directly projected onto $\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H$ to obtain

$$
\begin{aligned}
\tilde{\mathbf{r}}(n) &= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{r}(n) \\
&= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)[\mathbf{h}_1 b_1(n) + \tilde{\mathbf{H}}\tilde{\mathbf{b}}(n) + \mathbf{v}(n)] \quad (23) \\
&= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)[\mathbf{h}_1 b_1(n) + \mathbf{v}(n)],
\end{aligned}
$$

where the third equality follows from Proposition 1, that is, $(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\tilde{\mathbf{H}} = \mathbf{0}$. It is clear from (23) that the MAI and ISI of all interfering users are removed from $\tilde{\mathbf{r}}(n)$. The autocorrelation matrix of $\tilde{\mathbf{r}}(n)$ can be given by

$$
\begin{aligned}
\tilde{\mathbf{R}} &= E\{\tilde{\mathbf{r}}(n)\tilde{\mathbf{r}}^H(n)\} \\
&= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)(\mathbf{h}_1\mathbf{h}_1^H + \sigma^2\mathbf{I}_{N-G})(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H) \\
&= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1\mathbf{h}_1^H(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H) \\
&\quad + \sigma^2(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H).
\end{aligned} \quad (24)
$$

We next premultiply $\mathbf{d}_1$ by $\tilde{\mathbf{R}}$ to obtain

$$
\begin{aligned}
\tilde{\mathbf{R}}\mathbf{d}_1 &= \tilde{\mathbf{R}}(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1 \\
&= [(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1\mathbf{h}_1^H(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H) \\
&\quad + \sigma^2(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)](\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1 \\
&= [\mathbf{h}_1^H(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1 + \sigma^2](\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1.
\end{aligned} \quad (25)
$$

Clearly, the decorrelating detector of the desired user $\mathbf{d}_1$, that is $(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1$, is an eigenvector of $\tilde{\mathbf{R}}$ with the corresponding eigenvalue $\mathbf{h}_1^H(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1 + \sigma^2$. In fact, $\mathbf{h}_1^H(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1 + \sigma^2$ is the largest eigenvalue of $\tilde{\mathbf{R}}$. In order to prove this, we consider the rest of the eigenvectors of $\tilde{\mathbf{R}}$. Denoting $\mathbf{u}$ as an eigenvector of $\tilde{\mathbf{R}}$ which is orthogonal to $\mathbf{d}_1$, that is $\mathbf{u}^H(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1 = 0$, we have

$$
\begin{aligned}
\tilde{\mathbf{R}}\mathbf{u} &= [(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1\mathbf{h}_1^H(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H) \\
&\quad + \sigma^2(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)]\mathbf{u} \\
&= \sigma^2(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{u},
\end{aligned} \quad (26)
$$

where the second equality follows from $\mathbf{u}^H(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1 = 0$. It is clear from (26) that $\mathbf{h}_1^H(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1 + \sigma^2$ is the largest eigenvalue of $\tilde{\mathbf{R}}$, and all of the other eigenvalues of $\tilde{\mathbf{R}}$ are smaller than σ^2. Therefore, the proposed detector $\mathbf{d}_1$ can be implemented by calculating the principle eigenvector of $\tilde{\mathbf{R}}$, that is,

$$
\begin{aligned}
\mathbf{d}_1 &= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{h}_1 \\
&= \arg\max \tilde{\mathbf{R}} \\
&= \arg\max (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{R}(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H).
\end{aligned} \quad (27)
$$

Algorithm 1 summarizes the implementation of the proposed intersymbol decorrelating detector. It is observed from Algorithm 1 that the realization of the detector is very simple.

Sampling frame: $\mathbf{r}(0), \mathbf{r}(1), \ldots, \mathbf{r}(P)$.

Step 1. Compute autocorrelation and cross-correlation:

$\mathbf{R} = \dfrac{1}{P+1}\sum_{n=0}^{P}\mathbf{r}(n)\mathbf{r}(n)^H$,

$\bar{\mathbf{R}} = \dfrac{1}{P}\left(\sum_{n=0}^{P}\mathbf{r}(n-1)\mathbf{r}(n)^H + \sum_{n=1}^{P}\mathbf{r}(n)\mathbf{r}(n-1)^H\right)$.

Step 2. Compute eigendecomposition of $\bar{\mathbf{R}}$:

$\bar{\mathbf{R}} = [\bar{\mathbf{U}}_s, \bar{\mathbf{U}}_n]\begin{bmatrix}\bar{\Lambda}_s & \mathbf{0} \\ & \end{bmatrix}[\bar{\mathbf{U}}_s, \bar{\mathbf{U}}_n]^H = \bar{\mathbf{U}}_s\bar{\Lambda}_s\bar{\mathbf{U}}_s^H$.

Step 3. Form decorrelating detector:

$\mathbf{d}_1 = \text{Max-eigenvector }(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)\mathbf{R}(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s\bar{\mathbf{U}}_s^H)$.

ALGORITHM 1: Blind intersymbol decorrelating detector.

No information is needed on background noise level, the delay spread, the parameters of the multipath channel, or even the spreading sequence of the desired user. The only requirement is the synchronization of the desired user. In contrast, some of these parameters must be known or estimated for the detectors of [7, 8, 12]. It should be pointed out that the basic idea of the proposed detector is to utilize the eigendecomposition of the cross-correlation matrix $\bar{\mathbf{R}}$ to null out the ISI and MAI of all interfering users. According to (15), $\bar{\mathbf{R}}$ has $2(K-1)$ eigenvectors spanned by $\mathbf{h}_k\bar{\mathbf{h}}_k^H + \bar{\mathbf{h}}_k\mathbf{h}_k^H$ for $k = 2, \ldots, K$. However, if the kth interfering user has similar delays to that of the desired user, that is, $d_k \approx 0$, the eigenvalues corresponding to the kth user in $\bar{\mathbf{R}}$ will be small since only the first d_k entries of $\bar{\mathbf{h}}_k$ are nonzero. Because it is usually difficult to estimate the small eigenvalue's subspace accurately, more errors will occur on the estimation of the kth user's eigenvectors in step 2 of Algorithm 1 for small d_k. In this case, the estimated $\bar{\mathbf{U}}_s$ may lose a part of the kth user's information and the proposed detector cannot totally null out the interference of the kth user. Then, the performance of the proposed detector will be deteriorated, which will be shown in the next section.

Remark 1. Since the proposed detector relies heavily on the perfect synchronization of the desired user, errors of the synchronization may deteriorate the performance of the proposed detector. However, the synchronization of the desired user or, equivalently, the available delay information of the desired user is a basic prerequirement for all linear multiuser detectors. All of the detectors mentioned in this paper require the available knowledge of the delay of the desired user, and the error of the synchronization will definitely deteriorate the performance of these detectors. In practical CDMA systems such as CDMA 2000, special synchronization channel is allocated in order to obtain the precise timing information of the desired user.

Remark 2. Compared with the conventional subspace decorrelating detector in [2], the proposed intersymbol decorrelating detector has a simpler implementation. By assuming the synchronization of the desired user, the decorrelating detector in [2] also needs to estimate the multipath channel response vector of the desired user $\mathbf{g}_1$ and the variance of
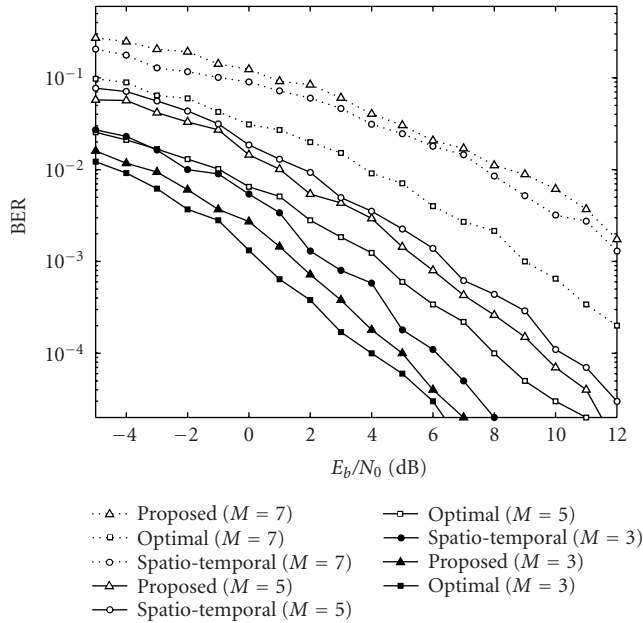
FIGURE 1: BER of various decorrelating detectors: $G = 5$, $K = 8$, $d_k \geq 8$ ($k = 2, \ldots, 8$).

FIGURE 2: BER of various decorrelating detectors: $G = 5$, $K = 8$, $d_2 = 3$, $d_3 = 2$, $d_k \geq 8$ ($k = 4, \ldots, 8$).

the background noise σ^2. However, the channel estimation will increase the complexity of the detector and the channel estimation error will deteriorate the performance of the detector. Such required channel estimations are removed in the proposed detector by introducing an intersymbol cross-correlation matrix $\bar{\mathbf{R}}$, which leads to a simpler implementation. In addition, the proposed method makes an unstructured assumption on the received desired waveform. It is generally true that an estimator under structured assumptions performs better than that under unstructured assumptions. Since the conventional detectors in [2] use a structured assumption on the received desired waveform in which only the multipath coefficients are not known, these detectors normally achieve better performance compared with the proposed detctor.

4. SIMULATION RESULTS

In this section, experimental results are provided to illustrate the performance of the proposed intersymbol decorrelating detector. The proposed method is tested in an asynchronous CDMA system with $K = 8$ users and spreading gain $N = 31$. The spreading sequences for all users are generated by Gold codes. For both cases, we simulate a severe near-far situation that each interfering user is at least with a power of 10 dB more than that of the desired user. Meanwhile, user 1 is assumed to be synchronized and all interfering users have the delay smaller than the duration of one symbol. The multipath gains in each user's channel are randomly chosen and kept fixed. The multipath gains have been normalized with equal
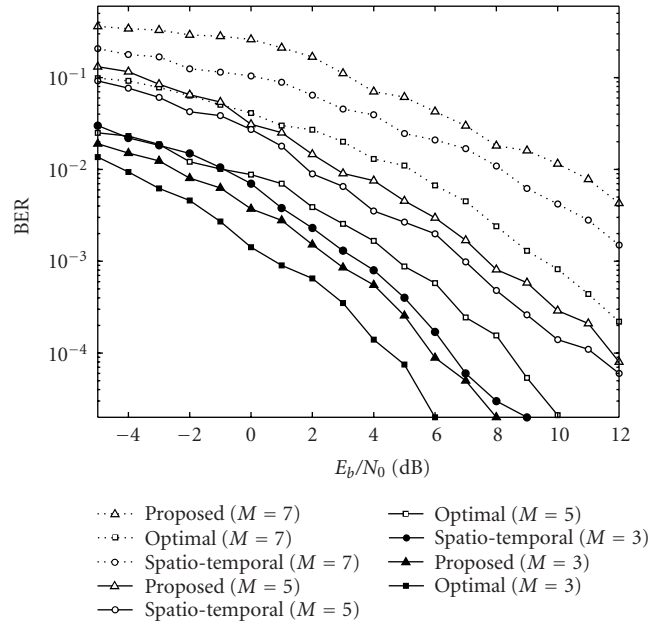
power. We compare the performance of three decorrelating detectors, that is, the optimal decorrelating detector, the proposed detector, and the spatio-temporal decorrelating detector in [7]. The proposed detector is implemented by Algorithm 1, where the length of signal frame is $P = 500$. The truncating window of the proposed detector is $G = 5$, that is, the first five entries of the received signal vector are truncated to remove the ISI of the desired user.

Figure 1 compares the performance of these detectors under different delay spreads of the multipath channel. In this example, the initial delay d_k of each interfering user is assumed to be randomly distributed between 8 and 31 in terms of the chip cycle T_c, that is, the interfering users have significant intersymbol interference. It is seen that the BERs of the proposed detector are close to the true optimal decorrelating detector and are always better than that of spatio-temporal decorrelating detector [7] when the delay spreads of the multipath channel are 3 and 5. This is because the signal channel is ideal for the proposed detector, that is, the ISI of the desired user is totally removed because $M \leq G$, and all interfering users have significant intersymbol interference as the delay $d_k \geq 8$. When the delay spread of the multipath channel increases to 7, the performance of the proposed detector suffers more deterioration and is close to that of the spatio-temporal decorrelating detector. The reason for such deterioration is that the signal in (15) cannot totally remove the ISI of the desired user in the case of $M > G$. Hence, the remanent ISI of the desired user in $\bar{\mathbf{R}}$ will influence the accuracy of the proposed detector. Figure 2 also compares the performance of the above detectors with different delay spreads of the multipath channel. In this example, two users are assumed to have similar delays to that of user 1, that is, $d_2 = 3$ and $d_3 = 2$,

while the other five interfering users' delays are larger than or equal to 8 T_c. It is seen that the performances of the optimal detector and the spatio-temporal detector in Figure 2 are similar to their performances in Figure 1. However, the interfering user's delay has obvious influence on the performance of the proposed detector. In Figure 2, the proposed detector has higher BERs compared with that in Figure 1. Meanwhile, the performance of the proposed detector is inferior to that of the spatio-temporal detector except when the multipath length $M = 3$. As explained in the previous section, this is because the eigenvalues corresponding to user 2 and user 3 in $\bar{\mathbf{R}}$ are close to zero due to small delays. Thus, more errors will occur on the estimation of the eigenvectors of user 2 and user 3 within limited signal frame, which leads to the deterioration of the proposed detector.

In general, the performance of the proposed detector is more sensitive to the delays of the interfering users compared with the optimal decorrelating detector and the spatio-temporal decorrelating detector. However, our method is very simple and only requires the delay of the desired user, while the spatio-temporal decorrelating detector in [7] requires the knowledge of the delay, the signature sequence of the desired user, and the desired user's multipath channel $\mathbf{g}_1$.

5.   CONCLUSIONS

A blind intersymbol decorrelating detector for asynchronous CDMA systems is proposed. The detector makes use of an important cross-correlation matrix between adjacent symbols to mitigate the influence of the interfering users. It is shown that the detector can be easily implemented without the estimations of the multipath channel and the multipath length, while a good performance is achieved when the system has significant intersymbol interference.

APPENDICES

A.   PROOF OF PROPOSITION 1

Denote an $(N - G) \times 1$ vector by $\mathbf{x}$. The proof of Proposition 1 is equivalent to the proof of the following items:

(1) if $\mathbf{x}^H \widetilde{\mathbf{H}} = \mathbf{0}$, this implies $\mathbf{x}^H \bar{\mathbf{U}}_s = \mathbf{0}$,
(2) if $\mathbf{x}^H \bar{\mathbf{U}}_s = \mathbf{0}$, this implies $\mathbf{x}^H \widetilde{\mathbf{H}} = \mathbf{0}$.

We consider $\bar{\mathbf{R}} \bar{\mathbf{R}}^H$, which is given by

$$
\begin{aligned}
\bar{\mathbf{R}} \bar{\mathbf{R}}^H &= \widetilde{\mathbf{H}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \widetilde{\mathbf{H}}^H \widetilde{\mathbf{H}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \widetilde{\mathbf{H}}^H \\
&= \widetilde{\mathbf{H}} \mathbf{W} \widetilde{\mathbf{H}}^H \\
&= \bar{\mathbf{U}}_s \bar{\Lambda}_s \bar{\mathbf{U}}_s^H \bar{\mathbf{U}}_s \bar{\Lambda}_s \bar{\mathbf{U}}_s^H \\
&= \bar{\mathbf{U}}_s \bar{\Lambda}_s^2 \bar{\mathbf{U}}_s^H,
\end{aligned}
\tag{A.1}
$$

where

$$
\mathbf{W} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \widetilde{\mathbf{H}}^H \widetilde{\mathbf{H}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix}
\tag{A.2}
$$

and the third equality follows from (20). Obviously, $\bar{\mathbf{R}} \bar{\mathbf{R}}^H$ is positive and has the same eigenvectors as that of $\bar{\mathbf{R}}$. To prove the first item, we have

$$
\begin{aligned}
\mathbf{x}^H \widetilde{\mathbf{H}} = \mathbf{0} &\Longrightarrow \mathbf{x}^H \widetilde{\mathbf{H}} \mathbf{W} \widetilde{\mathbf{H}}^H \mathbf{x} = 0 \\
&\Longrightarrow \mathbf{x}^H \bar{\mathbf{U}}_s \bar{\Lambda}_s^2 \bar{\mathbf{U}}_s^H \mathbf{x} = 0 \\
&\Longrightarrow \mathbf{x}^H \bar{\mathbf{U}}_s = \mathbf{0},
\end{aligned}
\tag{A.3}
$$

where the last step follows from the fact that $\bar{\Lambda}_s^2$ is a positive diagonal matrix. For the second item, the proof is as follows:

$$
\begin{aligned}
\mathbf{x}^H \bar{\mathbf{U}}_s = \mathbf{0} &\Longrightarrow \mathbf{x}^H \bar{\mathbf{U}}_s \bar{\Lambda}_s^2 \bar{\mathbf{U}}_s^H \mathbf{x} = 0 \\
&\Longrightarrow \mathbf{x}^H \widetilde{\mathbf{H}} \mathbf{W} \widetilde{\mathbf{H}}^H \mathbf{x} = 0 \\
&\Longrightarrow \mathbf{x}^H \widetilde{\mathbf{H}} = \mathbf{0}.
\end{aligned}
\tag{A.4}
$$

Similarly, the last step in (A.4) is derived because $\mathbf{W}$ is also positive and of full rank. By using items (1) and (2), we have $(\bar{\mathbf{U}}_s) = (\widetilde{\mathbf{H}})$. In addition, $(\bar{\mathbf{U}}_s)$ can be easily expressed by its projection matrix, that is, $(\bar{\mathbf{U}}_s) = \text{range}(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)$, where $\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H$ projects any signal onto the null space of $\widetilde{\mathbf{H}}$.

B.   PROOF OF PROPOSITION 2

Based on (15) and the eigendecomposition of (19), the decorrelating detector for user 1 must satisfy the follow conditions [11]:

$$
\begin{aligned}
\mathbf{d}_1 = (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 \in \text{range}(\mathbf{U}_s), \\
\mathbf{d}_1^H \mathbf{h}_1 > 0, \quad \mathbf{d}_1^H \widetilde{\mathbf{H}} = \mathbf{0}.
\end{aligned}
\tag{B.1}
$$

With (19), the proof of the first condition is equivalent to the proof of $\mathbf{U}_n^H \mathbf{d}_1 = \mathbf{0}$ since range($\mathbf{U}_s$) = ($\mathbf{U}_n$). It is easily achieved as follows:

$$
\begin{aligned}
\mathbf{U}_n^H \mathbf{d}_1 &= \mathbf{U}_n^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 \\
&= \mathbf{U}_n^H \mathbf{h}_1 - \mathbf{U}_n^H \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H \mathbf{h}_1 \\
&= \mathbf{0},
\end{aligned}
\tag{B.2}
$$

where the third equality follows from the fact that $\mathbf{h}_1 \in$ range($\mathbf{U}_s$) is orthogonal to $\mathbf{U}_n$, and range($\mathbf{U}_n$) = ($\widetilde{\mathbf{H}}, \mathbf{h}_1$) belongs to the subspace of ($\widetilde{\mathbf{H}}$) = ($\bar{\mathbf{U}}_s$), that is, $\mathbf{U}_n^H \bar{\mathbf{U}}_s = \mathbf{0}$.

Now we consider the second condition. With (20) and Proposition 1, it is easy to obtain

$$
\begin{aligned}
\mathbf{d}_1^H \mathbf{h}_1 = \mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 = \mathbf{h}_1^H \bar{\mathbf{U}}_n \mathbf{U}_n^H \mathbf{h}_1 > 0, \\
\mathbf{d}_1^H \widetilde{\mathbf{H}} = \mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \widetilde{\mathbf{H}} = \mathbf{0}.
\end{aligned}
\tag{B.3}
$$

Therefore, $\mathbf{d}_1$ is the decorrelating detector of user 1.

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