A note on supersymmetric type II solutions of Lifshitz type

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ABSTRACT: We discuss a class of supersymmetric type II non-relativistic solutions with exact or asymptotic scale invariance. As already emerged from previous investigations, we find a clear correspondence between anisotropic d-dimensional vacua and relativistic solutions in $d+1$ dimensions. We will show that supersymmetric four-dimensional Poincaré invariant backgrounds in type IIB can descend to analogous solutions with anisotropic scaling in t and (x, y). This result can be applied to scale invariant theories, domain walls interpolating between four-dimensional Lifshitz vacua and more general solutions with only asymptotic, approximate scaling behaviour and hyperscaling violation.

Contents

1 Introduction

The gauge/gravity correspondence relates relativistic strongly coupled gauge theories to supergravity on Anti de Sitter or warped Minkowski space-times. Recently, the application of the ideas of holography to condensed matter systems have motivated the study of non relativistic geometries in string/supergravity theories. For instance, systems of strongly correlated electrons can exhibit critical points with an anisotropic rescaling between time and space

$$t \to \lambda^z t \qquad x^i \to \lambda x^i \quad i = 1 \dots D \, . \tag{1.1}$$

According to the holographic dictionary such behaviour should be described by a Lifshitz geometry

$$\mathrm{d}s^2 = -r^{2z}\mathrm{d}t^2 + r^2 \sum_{i=1}^{D}(\mathrm{d}x^i)^2 + \frac{\mathrm{d}r^2}{r^2} \, , \tag{1.2}$$

where r is the holographic energy direction.

Contrary to other non-relativistic solutions, like Schrödinger geometries, embedding Lifshitz solutions in string theory turned out to be a relatively non trivial issue. Lifshitz geometries appear in four dimensional models of gravity coupled to a topological term

or a massive vector [1, 2]. While initially some no-go theorems seemed to exclude the possibility of embedding Lifshitz solutions in the full supergravity theory,[1] explicit examples of 10-dimensional solutions with a Lifshitz factor were constructed in [5] and subsequently generalised in [6–8], and, in the context of $\mathcal{N} = 2$ gauged supergravity, in [9, 10].

The four-dimensional Lifshitz solutions found in [6] are supersymmetric and have scaling exponent $z = 2$. They can be obtained as circle reductions of Schrödinger solutions with $z = 0$, which represent a plane wave propagating on the world-volume of D3-branes transverse to a Ricci-flat space and with non-zero RR and NS magnetic 3-form flux. The examples found in [6] can be thought of as a reduction of an AdS_5 type IIB vacua with the addition of some fluxes. A similar result has been proven in [10] at the level of gauge supergravity: any $(d + 1)$-dimensional gauge supergravity admitting AdS_{d+1} vacua gives, upon reduction on a circle, a d-dimensional gauge supergravity admitting Lif_d vacua with $z = 2$. As we show below a similar result can also be obtained at the level of vacua in the 10-dimensional type II supergravities, avoiding all complications related to truncations and dimensional reductions.

In this note in fact we analyse in detail the supersymmetry constraints on a large class of non-relativistic solutions which include and generalise the examples found in [6]. We consider type II solutions corresponding to three-dimensional theories with anisotropic scaling in time t and space (x, y). This class of solutions will include scale invariant theories, domain walls interpolating between four-dimensional Lifshitz vacua and more general solutions with only asymptotic, approximate scaling behaviour or hyperscaling violation. We will show that many supersymmetric four-dimensional Poincaré invariant backgrounds in type IIB descend to analogous solutions with anisotropic scaling in t and (x, y). This result strengthens the correspondence between AdS_5 and Lif_4 vacua found in [6, 10]. In particular, starting from supersymmetric domain walls in type IIB interpolating between AdS_5 vacua, we often find solutions interpolating between the corresponding Lif_4 vacua.

More generally we provide a framework for constructing supersymmetric non-relativistic backgrounds which can be useful also to study solutions with only asymptotic scaling and different IR behaviour, corresponding for example to confining solutions. We hope to report on the subject in the next future.

For convenience, in the rest of the Introduction, we summarise the relevant facts we need about supersymmetry. In order to study non relativistic supersymmetric vacua we will use the formalism of Generalised Complex Geometry [11, 12]. In the case where the space is the (warped product)

$$ds^2 = e^{2A}ds_4^2 + ds_6^2 \tag{1.3}$$

of four-dimensional Minkowski or Anti de Sitter times an internal six-dimensional manifold M_6, the supersymmetry variations can be re-expressed as a set of differential equations on the internal manifold. More precisely, if η_+^1 and η_+^2 are two chiral spinors on M_6, we can define the bispinors

$$\Phi_\pm = \eta_+^1 \eta_\pm^{2\,\dagger}, \tag{1.4}$$

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[1]See for example [3, 4].

which, by Fierz identity, can be seen as sums of even, Φ_+, and odd, Φ_-, forms on M_6.[2] The supersymmetry variations can be shown to be equivalent to the following set of differential equations[3]

$$
\begin{aligned}
\mathrm{d}_H(e^{2A-\phi}\Phi_1) &= 0\,, \\
\mathrm{d}_H(e^{A-\phi}\,\mathrm{Im}\Phi_2) &= 0\,, \\
\mathrm{d}_H(e^{3A-\phi}\,\mathrm{Re}\Phi_2) &= \frac{1}{8}\,e^{4A}*\lambda(F)\,,
\end{aligned}
\tag{1.5}
$$

where F is the sum of the RR fluxes on the internal manifold, H is the NS flux, $\Phi_1 = \Phi_+$ and $\Phi_2 = \Phi_-$ in IIA while for IIB $\Phi_1 = \Phi_-$ and $\Phi_2 = \Phi_+$ [13, 14].

A similar approach has been very recently used to study generic ten-dimensional vacua [15]. In this case, the conditions for supersymmetry can be reformulated in terms of intrinsic objects constructed with the type II supersymmetry parameters: a ten-dimensional "spinor" Φ

$$
\Phi = \epsilon_1\bar{\epsilon}_2\,,
\tag{1.6}
$$

and two one-forms K_1 and K_2

$$
K_{1M} = \frac{1}{32}\bar{\epsilon}_1\Gamma_M\epsilon_1\,, \qquad K_{2M} = \frac{1}{32}\bar{\epsilon}_2\Gamma_M\epsilon_2\,,
\tag{1.7}
$$

which annihilate the supersymmetry parameters[4]

$$
K_1\cdot\epsilon_1 = K_2\cdot\epsilon_2 = 0\,.
\tag{1.8}
$$

Each K_i is null and gives rise to a basis of vielbeine $(e_{-i}\equiv K_i, e_{+i}, e_I)$ which we normalize as

$$
e_{-i}\cdot e_{+i} = \frac{1}{2}\,.
\tag{1.9}
$$

In this formalism the supersymmetry conditions are [15]

$$
\mathrm{d}_H(e^{-\phi}\Phi) = -(\tilde{K}\wedge +\iota_K)F\,,
\tag{1.10}
$$

$$
\mathrm{d}\tilde{K} = \iota_K H\,,
\tag{1.11}
$$

$$
(e_{+1}\cdot\Phi\cdot e_{+2}, \Gamma^{MN}[\pm\mathrm{d}_H(e^{-\phi}\Phi\cdot e_{+2}) + e^\phi\mathrm{d}^\dagger(e^{-2\phi}e_{+2})\Phi - F])\,,
\tag{1.12}
$$

$$
(e_{+1}\cdot\Phi\cdot e_{+2}, [\mathrm{d}_H(e^{-\phi}e_{+1}\cdot\Phi) - e^\phi\mathrm{d}^\dagger(e^{-2\phi}e_{+2})\Phi - F]\Gamma^{MN})\,,
\tag{1.13}
$$

plus the condition $L_K g = 0$. The vectors K and $\tilde{K}$ are

$$
K = \frac{K_1 + K_2}{2} \qquad \tilde{K} = \frac{K_1 - K_2}{2}\,.
\tag{1.14}
$$

In (1.12) and (1.13) we use the ten dimensional Mukai pairing $(A, B) = (A\wedge\lambda(B))_{10}$ and, in (1.12), the upper sign is for IIA and the lower is for IIB.

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[2]On $T(M_6)\oplus T^*(M_6)$, the polyforms Φ_+ and Φ_- correspond to pure spinors of positive and negative chirality, respectively.

[3]Here $\mathrm{d}_H = (\mathrm{d} - H\wedge)$ and $\lambda(C_p) = (-1)^{[p/2]}C_p$ if C_p is a form of degree p.

[4]$K_i\cdot\epsilon_i = K_{iM}\Gamma^M\epsilon_i$ is the Clifford multiplication.

In this paper we will apply this formalism to the study of non-relativistic solutions of Lifshitz type in type II supergravity. In section 2 we discuss a class of non relativistic solutions of type IIA, including and generalizing [6], and we reduce the supersymmetry constraints to those of an auxiliary four-dimensional Poincaré invariant vacuum of type IIB plus a set of algebraic and differential constraints on the fluxes. For reader convenience, we summarise our findings in section 2.3. In section 3 we discuss the parallel case of non relativistic type IIB solutions. In section 4 we discuss the particular case of SU(3) structures and we provide examples of Lif_4 solutions, domain walls and backgrounds with hyperscaling violation.

2 Non-relativistic solutions in type IIA

We look for static solutions in IIA supergravity corresponding to three-dimensional theories with anisotropic scaling in time t and space (x, y). We do not require to have exact scaling symmetry in order to accommodate also domain wall solutions interpolating between four-dimensional Lifshitz vacua and more general solutions with only asymptotic or approximate scaling behaviour. As usual, one of the transverse directions will play the role of the radial coordinate.

On the basis of the existing examples of Lifshitz solutions [6] and of the general structure of supersymmetric solutions in IIA [15], we choose a metric of the form

$$\mathrm{d}s_{10}^2 = -e^{2A_1}\mathrm{d}t^2 + e^{2A_2}(\mathrm{d}x^2 + \mathrm{d}y^2) + (e^1)^2 + \mathrm{d}s_6^2\,, \tag{2.1}$$

with

$$qe^1 = \mathrm{d}\varphi + \mu\,, \tag{2.2}$$

where φ denotes a generic angular direction, μ is a connection on M_6 with curvature $\alpha \equiv \mathrm{d}\mu$, and A_1, A_2 and q are functions on M_6.

According to this splitting we assume the following structure for the fluxes[5]

$$H^{IIA} = h + \mathrm{d}(e^{01})\,, \tag{2.3}$$

$$F^{IIA} = -q(e^1 f + e^{0xy} * \lambda(f)) + (1 + e^{01})(w + e^{xy} * \lambda(w))\,, \tag{2.4}$$

where we defined the three-dimensional vielbeine

$$e^0 = e^{A_1}\mathrm{d}t\,, \qquad e^x = e^{A_2}\mathrm{d}x\,, \qquad e^y = e^{A_2}\mathrm{d}y\,, \tag{2.5}$$

and f and w denote formal sums of odd and even forms on the internal space M_6,

$$f = f_1 + f_3 + f_5\,, \tag{2.6}$$

$$w = w_0 + w_2 + w_4 + w_6\,. \tag{2.7}$$

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[5]The RR fluxes are written in the democratic formulation [16] as a formal sum $F = \sum_{k=0}^{5} F_{(2k)}$, with k even in IIA and half-integer in IIB, subject to the condition $F = *_{10}\lambda(F)$. We have inherited from the democratic approach a somehow unnatural convention for the star product: $*C \wedge C = |C|^2\mathrm{Vol}$.

All star products are taken on the internal space M_6. For simplicity, we omit the wedge products and define $e^{abc\cdots} = e^a \wedge e^b \wedge e^c \wedge \cdots$. Also, for notational convenience, we write the type IIA dilaton ϕ_A in the form

$$e^{-\phi_A} = q\, e^{-\phi}\,. \tag{2.8}$$

The splitting of the metric is suggested by the conditions for supersymmetry. From (1.10)–(1.13) it follows that K must be dual to a Killing vector. For the static solutions we are considering we have a natural Killing vector corresponding to time translations. It is then natural to identify $K = e^0$. The choice of metric (2.1) corresponds to the general class of solutions where the second vector identifies a special direction in the transverse space, $\tilde{K} = e^1$.

The ansatz for the metric and fluxes makes it possible to reduce the search for vacua to a purely six-dimensional problem. The equations of motion and the Bianchi identities for the fluxes reduce to a set of differential constraints for the forms (α, w, h, f) on M_6 and the supersymmetry conditions can be written in terms of the six-dimensional pure spinors (1.4). One of the main results of our analysis is that the six-dimensional data $(\phi, h, f, \Phi_\pm)$ must satisfy the conditions for supersymmetry of a four-dimensional Poincaré invariant vacuum of type IIB, (1.5). This will allow to construct non-relativistic solutions with three space-time dimensions starting from known four-dimensional vacua. In the following sections we analyse the equations of motion and the supersymmetry constraints. We have summarised the results in section 2.3.

2.1 Bianchi identities and equations of motion

The fluxes have to satisfy the ten dimensional equations of motion and Bianchi identities

$$\mathrm{d}F^{\mathrm{IIA}} - H^{\mathrm{IIA}} \wedge F^{\mathrm{IIA}} = 0\,, \tag{2.9}$$

$$\mathrm{d}H^{IIA} = 0\,, \tag{2.10}$$

$$\mathrm{d}(e^{-2\phi_{IIA}} * H^{IIA}) = -\frac{1}{2} F^{IIA} \wedge *F^{IIA} \big|_8\,. \tag{2.11}$$

The previous conditions reduce to the following constraints on the internal forms h and f

$$\mathrm{d}_h f = 0\,,$$
$$\mathrm{d}_h(q\, e^{A_1 + 2A_2} * \lambda(f)) = 0\,,$$
$$\mathrm{d}h = 0\,,$$
$$\mathrm{d}(q e^{A_1 + 2A_2 - 2\phi} * h) = q\, e^{A_1 + 2A_2} f * f \big|_4\,, \tag{2.12}$$

and for the forms w and α

$$\mathrm{d}_h w = \alpha f\,,$$
$$\mathrm{d}_h(e^{2A_2} * \lambda(w)) = 0\,,$$
$$\mathrm{d}\,\alpha = 0\,,$$
$$\mathrm{d}(e^{2A_2 - 2\phi} * \alpha) = -e^{2A_2} f * w \big|_5\,, \tag{2.13}$$

plus the the algebraic relation

$$e^{-2\phi}\alpha * h = w * f\mid_5,\tag{2.14}$$

and the second order differential equation for q

$$* \, \mathrm{d}(qe^{2A_2+A_1-2\phi} * \mathrm{d}(qe^{-A_1})) = e^{2A_2}(e^{-2\phi}|\alpha|^2 + |w|^2)\,.\tag{2.15}$$

2.2 Supersymmetry conditions

The supersymmetry conditions (1.10)–(1.13) can also be reduced to a set of equations for the forms on M_6. We will first split the ten-dimensional spinors into the product of a two-dimensional and an eight-dimensional spinor, and then further reduce the eight-dimensional spinors to six-dimensional ones. The form (2.1) of the metric suggests to choose e^0 and e^1 as preferred directions for our basis of (real) ten-dimensional gamma matrices

$$\begin{aligned}
\Gamma^0 &= i\sigma_2 \otimes \mathbb{I}_{(8)}\,,\\
\Gamma^1 &= \sigma_1 \otimes \mathbb{I}_{(8)}\,,\\
\Gamma^I &= \sigma_3 \otimes \hat{\gamma}^I \qquad I = 1,\dots,8\,.
\end{aligned}\tag{2.16}$$

In type IIA the ten dimensional spinors ϵ_1 and ϵ_2 are Majorana-Weyl and have opposite chirality. Using the fact that $K = e^0$ and $\tilde{K} = e^1$ and the choice (2.16) of gamma matrices, the conditions (1.8) become

$$(\Gamma^0 + \Gamma^1)\epsilon_1 = 0\,,\tag{2.17}$$

$$(\Gamma^0 - \Gamma^1)\epsilon_2 = 0\,,\tag{2.18}$$

and imply

$$\epsilon_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\hat{\eta}_1 \qquad\qquad \epsilon_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\hat{\eta}_2\,,\tag{2.19}$$

where the eight-dimensional spinors $\hat{\eta}_1$ and $\hat{\eta}_2$ are real and have both positive chirality.[6] We further split the eight transverse directions into two plus six. For the gamma matrices this gives

$$\begin{aligned}
\hat{\gamma}^x &= \sigma_1 \otimes \mathbb{I}\,,\\
\hat{\gamma}^y &= \sigma_3 \otimes \mathbb{I}\,,\\
\hat{\gamma}^m &= \sigma_2 \otimes \gamma^m \qquad m = 1,\dots,6\,.
\end{aligned}\tag{2.21}$$

The spinors split accordingly

$$\hat{\eta}_i = \sqrt{\frac{C}{2}}\left[\begin{pmatrix} 1 \\ i \end{pmatrix}\eta_+^i + \begin{pmatrix} 1 \\ -i \end{pmatrix}\eta_-^i \right] \qquad i = 1,2\,,\tag{2.22}$$

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[6]The ten-dimensional and the six-dimensional chiral gammas are

$$\Gamma_{10} = \Gamma^0 \dots \Gamma^9 = \sigma_3 \otimes \hat{\gamma}_9 \qquad\qquad \gamma_7 = -i\gamma_1 \cdots \gamma_6\,.\tag{2.20}$$

where η_+^i are six-dimensional spinors of positive chirality and $\eta_- = (\eta_+)^*$. It is convenient to redefine $\hat{\eta}_1 \rightarrow \hat{\gamma}^{xy}\hat{\eta}_1$ or, equivalently, $\eta_1 \rightarrow -i\eta_1$.

By construction the vectors K_1 and K_2 are given by

$$K_{(1)M} = \frac{1}{32}\bar{\epsilon}_1\Gamma_M\epsilon_1 = \frac{C}{16}||\eta_+^1||^2(1,1,0,0,\ldots,0) = \frac{C}{16}||\eta_+^1||^2(e^0 + e^1)_M\,,$$

$$K_{(2)M} = \frac{1}{32}\bar{\epsilon}_2\Gamma_M\epsilon_2 = \frac{C}{16}||\eta_+^2||^2(1,-1,0,0,\ldots,0) = \frac{C}{16}||\eta_+^2||^2(e^0 - e^1)_M\,. \quad (2.23)$$

For simplicity, we restrict to the case where the norms of the six-dimensional spinors are equal: $||\eta_+^1|| = ||\eta_+^2|| = ||\eta_+||$. We expect that, analogously to relativistic vacua, this is the case of interest for backgrounds generated by D-branes. With equal norm for the spinors, the form of K and $\tilde{K}$ is particularly simple,

$$K = \frac{C}{16}||\eta_+||^2\, e^0 \qquad \tilde{K} = \frac{C}{16}||\eta_+||^2\, e^1\,. \qquad (2.24)$$

We still need to require that K is Killing. This fixes the functional dependence of C

$$C = 16\frac{e^{A_1}}{||\eta_+||^2}\,, \qquad (2.25)$$

and we finally have

$$K = e^{A_1}\, e^0 \qquad \tilde{K} = e^{A_1}\, e^1\,. \qquad (2.26)$$

The normalization of C is fixed in such a way that K is precisely dual to $\frac{\partial}{\partial t}$.

The ten dimensional "pure-spinor" also factorises into the product of four and six-dimensional forms

$$\Phi = \epsilon_1\bar{\epsilon}_2 = -\frac{1}{2}(1 + e^{01})\Phi_{(8)}\,, \qquad (2.27)$$

with

$$\Phi_{(8)} = \frac{16}{||\eta_+||^2}\, e^{A_1}\left\{\text{Im}[(1 + ie^{xy})\Phi_+] - \text{Re}[(e^x - ie^y)\Phi_-]\right\}\,. \qquad (2.28)$$

$\Phi_\pm$ are the six-dimensional pure spinors defined in (1.4)

$$\Phi_+ = \eta_+^1\eta_+^{2\,\dagger}\,, \qquad \Phi_- = \eta_+^1\eta_-^{2\,\dagger}\,. \qquad (2.29)$$

Let us now consider the supersymmetry conditions (1.10)–(1.13). Equation (1.11) is trivially satisfied. From equation (1.10) we obtain the six-dimensional equations

$$d_h\left(q\, e^{A_1-\phi}\frac{1}{||\eta_+||^2}\, \text{Im}\Phi_+\right) = 0\,, \qquad (2.30)$$

$$d_h\left(q\, e^{A_1+2A_2-\phi}\frac{1}{||\eta_+||^2}\, \text{Re}\Phi_+\right) = \frac{q}{8}\, e^{A_1+2A_2} * \lambda(f)\,, \qquad (2.31)$$

$$d_h\left(q\, e^{A_1+A_2-\phi}\frac{1}{||\eta_+||^2}\, \Phi_-\right) = 0\,. \qquad (2.32)$$

Notice that on the right hand side of equation (1.10) w disappears

$$(\tilde{K} \wedge +\iota_K)F = q\, e^{A_1}(1 + e^{01}) \wedge e^{xy} * \lambda(f)\,. \qquad (2.33)$$

The conditions (1.12) and (1.13) can also be simplified. The computation is more involved and we give it in appendix B. What is important for our analysis is that they reduce to two independent sets of conditions, one for f and one for α and w. All conditions for f are automatically satisfied once we fix

$$e^{A_1} = \frac{e^{2A_2}}{q}\,, \tag{2.34}$$

while the conditions for α and w are

$$\mathrm{Im}\left[\left(\bar{\Phi}_+, \frac{e^{-\phi}}{||\eta_+||^2}\alpha \cdot \Phi_+\right)_6\right] = \mathrm{Im}[(\bar{\Phi}_+, w)_6] = 0\,, \tag{2.35}$$

$$\mathrm{Re}\left[\left(\bar{\Phi}_+, \left(\frac{e^{-\phi}}{||\eta_+||^2}\alpha \cdot \Phi_+ - iw\right)\right)_6\right] = \mathrm{Re}\left[\left(\bar{\Phi}_+, \left(\frac{e^{-\phi}}{||\eta_+||^2}\Phi_+ \cdot \alpha + iw\right)\right)_6\right] = 0\,, \tag{2.36}$$

$$\mathrm{Im}\left[\left(\bar{\Phi}_+, \gamma^{mn}\left(\frac{e^{-\phi}}{||\eta_+||^2}\alpha \cdot \Phi_+ - iw\right)\right)_6\right] = \mathrm{Im}\left[\left(\bar{\Phi}_+, \left(\frac{e^{-\phi}}{||\eta_+||^2}\Phi_+ \cdot \alpha + iw\right)\gamma^{mn}\right)_6\right] = 0\,, \tag{2.37}$$

$$\left(\Phi_-, \gamma^n\left(\frac{4\,e^{-\phi}}{||\eta_+||^2}\alpha \wedge \Phi_+ - iw\right)\right)_6 = \left(\Phi_-, \left(\frac{4\,e^{-\phi}}{||\eta_+||^2}\alpha \wedge \bar{\Phi}_+ - iw\right)\gamma^n\right)_6 = 0\,, \tag{2.38}$$

where $\cdot$ denote the Clifford product and $(A, B)_6 = (A \wedge \lambda(B))_6$ is the six-dimensional Mukai pairing.

With the redefinition (2.34) the conditions (2.12) and (2.30)–(2.32) involving (f, h, ϕ) become identical to the equations for the fluxes and the supersymmetry conditions for a type IIB vacuum with four-dimensional Poincaré invariance. This is one of the advantages of our formalism: the fields (f, h, ϕ) in our non-relativistic solutions are the same as for four-dimensional relativistic solutions, where many supersymmetric backgrounds are known. In the next section we summarise our findings.

2.3 Summary: from four to three dimensions

Consider a supersymmetric type IIB background

$$\mathrm{d}s_{10}^2 = e^{2A}(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu)^2 + \mathrm{d}s_6^2\,, \qquad \mu = 0, \cdots, 3\,, \tag{2.39}$$

with dilaton ϕ and fluxes defined on M_6

$$H_{IIB} = h\,, \qquad F_{IIB} = f + e^{0xyz} * \lambda(f)\,, \tag{2.40}$$

and satisfying the Bianchi identities and the equations of motion

$$\begin{aligned} \mathrm{d}_h f &= 0\,, \\ \mathrm{d}_h(e^{4A} * \lambda f) &= 0\,, \\ \mathrm{d}h &= 0\,, \\ \mathrm{d}(e^{4A-2\phi} * h) &= e^{4A} f * f \mid_4\,. \end{aligned} \tag{2.41}$$

The type IIB supersymmetry parameters decompose as

$$\epsilon_i = \zeta_+ \eta_+^i + \zeta_- \eta_-^i \qquad i = 1, 2 \tag{2.42}$$

where ζ_+ are four-dimensional Weyl spinors and η_+^i are six-dimensional spinors on M_6 normalized as

$$||\eta_+^1|| = ||\eta_+^2|| = \frac{c_+}{2}e^A. \tag{2.43}$$

The conditions for supersymmetry can be written in terms of the six-dimensional pure spinors, $\Phi_\pm$ of (1.4), as

$$\mathrm{d}_h(e^{A-\phi}\,\mathrm{Im}\Phi_+) = 0, \tag{2.44}$$

$$\mathrm{d}_h(e^{3A-\phi}\,\mathrm{Re}\Phi_+) = \frac{c_+}{16}\,e^{4A}*\lambda f, \tag{2.45}$$

$$\mathrm{d}_h(e^{2A-\phi}\,\Phi_-) = 0. \tag{2.46}$$

Given such a four-dimensional supersymmetric type IIB vacuum, we can construct a non-relativistic supersymmetric solution in type IIA with metric (2.1) and

$$e^{A_1} = \frac{e^{2A}}{q}, \qquad e^{A_2} = e^A, \qquad e^{\phi_A} = \frac{e^\phi}{q}, \tag{2.47}$$

provided we find on M_6 a two-form α and a polyform $w = \sum_{k=0}^{3} w_{2k}$ satisfying the equations (coming from the 10-dimensional Bianchi identities and equations of motion)

$$\mathrm{d}_h w = \alpha f, \tag{2.48}$$

$$\mathrm{d}_h(e^{2A}*\lambda(w)) = 0, \tag{2.49}$$

$$\mathrm{d}\,\alpha = 0, \tag{2.50}$$

$$\mathrm{d}(e^{2A-2\phi}*\alpha) = -e^{2A}f*w\,|_5, \tag{2.51}$$

$$e^{-2\phi}\alpha*h = w*f\,|_5, \tag{2.52}$$

and the supersymmetry constraints

$$\mathrm{Im}[(\bar{\Phi}_+, e^{-\phi-A}\alpha\cdot\Phi_+)_6] = \mathrm{Im}[(\bar{\Phi}_+, w)_6] = 0, \tag{2.53}$$

$$\mathrm{Re}\left[\left(\bar{\Phi}_+, \left(e^{-\phi-A}\frac{2}{c_+}\alpha\cdot\Phi_+ - iw\right)\right)_6\right] = \mathrm{Re}\left[\left(\bar{\Phi}_+, \left(e^{-\phi-A}\frac{2}{c_+}\Phi_+\cdot\alpha + iw\right)\right)_6\right] = 0, \tag{2.54}$$

$$\mathrm{Im}\left[\left(\bar{\Phi}_+, \gamma^{mn}\left(e^{-\phi-A}\frac{2}{c_+}\alpha\cdot\Phi_+ - iw\right)\right)_6\right] = \mathrm{Im}\left[\left(\bar{\Phi}_+, \left(e^{-\phi-A}\frac{2}{c_+}\Phi_+\cdot\alpha + iw\right)\gamma^{mn}\right)_6\right] = 0, \tag{2.55}$$

$$\left(\Phi_-, \gamma^n\left(e^{-\phi-A}\frac{8}{c_+}\alpha\wedge\Phi_+ - iw\right)\right)_6 = \left(\Phi_-, \left(e^{-\phi-A}\frac{8}{c_+}\alpha\wedge\bar{\Phi}_+ - iw\right)\gamma^n\right)_6 = 0, \tag{2.56}$$

where $\cdot$ denote the Clifford product and $(A, B)_6 = (A\wedge\lambda B)_6$ is the six-dimensional Mukai pairing. The function q is determined by the second order differential equation

$$*\mathrm{d}(e^{4A-2\phi}*\mathrm{d}(q^2 e^{-2A})) = e^{2A}(e^{-2\phi}|\alpha|^2 + |w|^2). \tag{2.57}$$

3 Non-relativistic solutions in type IIB

The situation in type IIB is similar and we will be brief. Many solutions can be obtained by a plain T-duality. However there are a few interesting features of the solutions that deserve comments. We are interested in metrics of the form

$$\mathrm{d}s_{10}^2 = -2\,q\,e^{A_1}\mathrm{d}t\,\mathrm{d}\varphi + e^{2A_2}(\mathrm{d}x^2 + \mathrm{d}y^2) + q^2\,\mathrm{d}\varphi^2 + \mathrm{d}s_6^2, \tag{3.1}$$

where, as before, A_1, A_2 and q are functions on M_6. We introduce the null vielbeine

$$e^- = e^{A_1}(e^0 + e^1), \qquad e^+ = -\frac{1}{4}e^{-A_1}(e^0 - e^1), \tag{3.2}$$

in terms of

$$e^0 = e^{A_1}\mathrm{d}t, \qquad e^1 = q\,\mathrm{d}\varphi - e^{A_1}\mathrm{d}t. \tag{3.3}$$

Notice that the Killing vector $\frac{\partial}{\partial t}$ is now null and dual to $e^- = qe^{A_1}\mathrm{d}\varphi$.

The fluxes are

$$H^{IIB} = h - \frac{e^{-A_1}}{q}e^-\alpha, \tag{3.4}$$

$$F^{IIB} = f + 2e^{+-xy} * \lambda(f) + \frac{e^{-A_1}}{q}e^-(w + e^{xy} * \lambda(w)), \tag{3.5}$$

where we introduced formal sums f and w of odd and even forms defined on the internal space M_6 as in equations (2.6) and (2.7). Again all star products are taken in M_6.

Since the ten-dimensional supersymmetry parameters in type IIB have the same chirality, we now have

$$\epsilon_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\hat{\eta}_1 \qquad \epsilon_2 = -\begin{pmatrix} 1 \\ 0 \end{pmatrix}\hat{\eta}_2, \tag{3.6}$$

where the eight-dimensional spinors $\hat{\eta}_1$ and $\hat{\eta}_2$ are real and both with positive chirality. The sign in ϵ_2 is chosen for convenience. We can still split the eight-dimensional spinors in two plus six as in equation (2.22). However now $K_1 = K_2$, so that $\tilde{K} = 0$ and K is null

$$K = e^-. \tag{3.7}$$

Moreover, the ten-dimensional "pure-spinor" is now proportional to e^-

$$\Phi = \epsilon_1\bar{\epsilon}_2 = -\frac{1}{2}(e^0 + e^1)\Phi_{(8)}. \tag{3.8}$$

Exactly as before, given a four-dimensional supersymmetric type IIB vacuum corresponding to the pure spinors $\Phi_\pm$, with metric (2.39), dilaton ϕ and fluxes (h, f), we can construct a non-relativistic supersymmetric solution with metric (3.1) with

$$e^{A_1} = \frac{e^{2A}}{q}, \qquad e^{A_2} = e^A, \qquad e^{\phi_B} = e^\phi, \tag{3.9}$$

provided we find M_6 forms α, w satisfying the constraints (2.48)–(2.57).

There are a couple of interesting observations to be made. Since K is null in type IIB, the equations of motion are not necessarily a consequence of the supersymmetry conditions. As discussed in [17], all Einstein and dilaton equations of motion follow from supersymmetry except perhaps the $(0, M \neq 0)$ components of the Einstein equations. Indeed, in our case the $(0, M \neq 0)$ components of the Einstein equations imply the conditions (2.14) and (2.15) which are not consequences of supersymmetry nor the Bianchi identities and equations of motion for the fluxes.

It is interesting to notice also that both sides of equation (1.10) are proportional to e^- and therefore to $\mathrm{d}\varphi$. This allows to consider more general solutions where some of the functions and fluxes depend on φ. Examples of this type have been considered in [5, 6, 18, 19]. The generalization of our formalism to this case is straightforward. In this paper we consider the simplest example of such construction, obtained by sending

$$\eta_+^i \to e^{ik\varphi/2}\eta_+^i\,. \tag{3.10}$$

Since Φ is proportional to e^-, equation (1.10) is still satisfied. As discussed in appendix B, new terms appear instead in equations (1.12) and (1.13). As a result, in the supersymmetry conditions (2.35)–(2.37) we must replace

$$\frac{\alpha}{4}\cdot\Phi_+ \to \frac{\alpha}{4}\cdot\Phi_+ + i\frac{k}{2}\Phi_+\,, \qquad \Phi_+\cdot\frac{\alpha}{4} \to \Phi_+\cdot\frac{\alpha}{4} - i\frac{k}{2}\Phi_+\,. \tag{3.11}$$

We will discuss examples of this type in section 4.

4 SU(3) structure solutions

In the previous section we showed how, given a supersymmetric type IIB solution with four-dimensional Poincaré invariance (2.39) and (2.40), one can construct solutions, both in type IIA and type IIB supergravity, with a non-relativistic three-dimensional factor. In this section we will discuss some explicit examples of such construction. We will focus on a class of solutions where the internal manifold M_6 has SU(3) structure, since in this case the supersymmetry conditions for the forms α and w take a particularly simple form. We leave the study of SU(2) structure solutions for a future work.

In rewriting the supersymmetry variations in terms of pure spinors on the internal manifold M_6, we make the assumption that the six-dimensional supersymmetry parameters η_+^1 and η_+^2 are globally defined. Generically, two globally defined spinors reduce the structure group of M_6 to SU(2). However if they are parallel

$$\eta^1 = \eta_+\,, \qquad \eta_+^2 = e^{i\theta}\eta_+\,, \tag{4.1}$$

the structure group is SU(3). In six dimensions any spinor η_+ is annihilated by three gamma matrices

$$(\gamma^m + i\gamma^{m+3})\eta_+ = 0\,, \qquad m = 1, 2, 3\,, \tag{4.2}$$

and defines a complex structure. The SU(3) structure can then be equivalently expressed in terms of a $(1, 1)$ (with respect to the complex structure) two-form J and a $(3, 0)$ three-form Ω, and the pure spinors (1.4) have the form

$$\Phi_+ = \frac{1}{8}e^{-i\theta}||\eta_+||^2 e^{-iJ}\,, \qquad \Phi_- = -\frac{i}{8}e^{i\theta}||\eta_+||^2\Omega\,. \tag{4.3}$$

In ordinary four-dimensional vacua, the case $\theta = 0$ corresponds to a manifold M_6 which is (conformally) Calabi-Yau.

For SU(3) structure manifolds, the supersymmetry conditions for the forms α and w can be simplified using the formulae discussed in appendix A, in particular equation (A.6). With simple manipulations we can write the set of constraints (2.35)–(2.38) as

$$\eta_+^\dagger w\, \eta_+ = \eta_+^\dagger \alpha\, \eta_+ = 0\,, \tag{4.4}$$

$$\mathrm{Re}[e^{i\theta}\eta_+^\dagger \gamma^{mn}(e^{-\phi-i\theta}\alpha - iw)\eta_+] = \mathrm{Re}[e^{i\theta}\eta_+^\dagger(e^{-\phi-i\theta}\alpha + iw)\gamma^{mn}\eta_+] = 0\,, \tag{4.5}$$

$$\eta_-^\dagger \gamma^n(e^{-\phi-i\theta}\alpha - iw)\eta_+ = \eta_-^\dagger(e^{-\phi+i\theta}\alpha - iw)\gamma^n\eta_+ = 0\,, \tag{4.6}$$

or, equivalently,

$$(\bar{\Phi}_+, \alpha)_6 = (\bar{\Phi}_+, w)_6 = 0\,, \tag{4.7}$$

$$\mathrm{Im}[(\bar{\Phi}_+, \gamma^{mn}(e^{-\phi-i\theta}\alpha - iw))_6] = \mathrm{Im}[(\bar{\Phi}_+, (e^{-\phi-i\theta}\alpha + iw)\gamma^{mn})_6] = 0\,, \tag{4.8}$$

$$(\Phi_-, \gamma^n(e^{-\phi-i\theta}\alpha - iw))_6 = (\Phi_-, (e^{-\phi+i\theta}\alpha - iw)\gamma^n)_6 = 0\,. \tag{4.9}$$

Before discussing explicit solutions, let us analyze the content of the supersymmetry conditions (4.7)–(4.9). Consider first solutions where the polyform w only has a two-form component w_2. Then the general solution of the supersymmetry conditions is obtained by requiring that α and w_2 are $(1,1)$ and primitive.[7] The requirement that α and w_2 are primitive is just equation (4.4), or, equivalently, (4.7). If α and w_2 are also $(1,1)$ all other equations are automatically satisfied since any real two-form, C_2, $(1,1)$ and primitive satisfies

$$C_2\eta_+ = \eta_\pm^\dagger C_2 = 0\,. \tag{4.10}$$

In the special case $\theta = 0$, a $(2,0)$ component in $(e^{-\phi}\alpha - iw_2)$ will also be allowed. In fact, in this case

$$(e^{-\phi}\alpha - iw_2)\eta_+ = \eta_-^\dagger(e^{-\phi}\alpha - iw_2) = \eta_+^\dagger(e^{-\phi}\alpha + iw_2) = 0\,. \tag{4.11}$$

In the general case, α is still required to be primitive. The conditions on the other forms are more involved. Let us notice that all conditions can be satisfied at once if

$$\alpha\eta_+ = w\eta_+ = \lambda(w)\eta_+ = 0\,, \tag{4.12}$$

since then $\eta_\pm\alpha = \eta_\pm w = 0$.[8]

In type IIB, the modifications (3.11) one has to make to the supersymmetry conditions when the spinor depends on φ greatly simplify for $\theta = 0$, and become equivalent to a shift in the zero form component of w: $w_0 \to w_0 - 2ke^{-\phi}$.

4.1 Examples

Supersymmetric Lif_4 solutions based on AdS_5 vacua have been discussed in [5, 6, 10, 18]. As we will discuss now, solutions of this kind can be easily cast in our formalism. In the same spirit, we will show how flows among AdS_5 vacua can give rise to flows among the corresponding Lif_4 vacua.

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[7]A two-form C is primitive if its contraction with J is zero, $C_{mn}J^{mn} = \eta_+^\dagger C\eta_+ = 0$; this is equivalent to $(\bar{\Phi}_+, C) = C \wedge J^2 = 0$.

[8]For a generic even polyform A, $A\eta_+ = 0$ implies $\eta_+^\dagger\lambda(\bar{A}) = \eta_-^\dagger\lambda(A) = 0$.

4.1.1 Lif_4 from AdS_5 solutions

A first example of solutions fitting the ansatz of sections 2 and 3 is the family found in [6], which we review here. They represent type IIA and IIB Lif_4 backgrounds where the internal manifold is a U(1) fibration over a five-dimensional Sasaki-Einstein manifold Y. Both IIA and IIB solutions are based on the standard type IIB vacuum of the form $AdS_5 \times Y$

$$\mathrm{d}s_{10}^2 = r^2 (\eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu)^2 + \frac{1}{r^2}(\mathrm{d}r^2 + r^2 \mathrm{d}s_Y^2)\,, \tag{4.13}$$

where the internal manifold M_6 is a conformal Calabi-Yau cone over the Sasaki-Einstein Y. The dilaton ϕ is constant and set to zero, and, in the notations of sections 2 and 3, there is only a five-form flux

$$h = 0\,, \qquad *f = 4\frac{\mathrm{d}r}{r}\,. \tag{4.14}$$

The pure spinors are given by equation (4.3) with $\theta = 0$ and

$$J = \frac{1}{r^2} J_{\mathrm{CY}} \qquad\qquad \Omega = \frac{1}{r^2}\Omega_{\mathrm{CY}}\,. \tag{4.15}$$

The supersymmetry conditions (2.44)–(2.46) are trivially satisfied since $\mathrm{d}J_{CY} = \mathrm{d}\Omega_{CY} = 0$. We normalized the spinors with $c_+ = 2$ (this choice implies $||\eta_+||^2 = e^A$, cfr equation (2.43)).

Using (2.47) we can write the associated non-relativistic type IIA solution [6] as

$$\mathrm{d}s_{10}^2 = -\frac{r^4}{q^2}\mathrm{d}t^2 + r^2(\mathrm{d}x^2 + \mathrm{d}y^2) + \frac{\mathrm{d}r^2}{r^2} + \frac{1}{q^2}(\mathrm{d}\varphi + \mu)^2 + \mathrm{d}s_Y^2\,, \tag{4.16}$$

where $\mathrm{d}\mu = \alpha$. The dilaton is

$$e^{-2\phi} = q^2\,. \tag{4.17}$$

The forms α and w must satisfy equations (2.48)–(2.52). The simplest way to solve these constraints is to take only two-forms α and w_2 which are closed and co-closed on Y, or, equivalently harmonic on Y. These exist on all Sasaki-Einstein manifolds with $b_2 \neq 0$. They can be primitive and of type $(1,1)$ so that, as discussed above, they preserve supersymmetry. The fluxes then have the form [6]

$$B = \frac{r^2}{q^2}\mathrm{d}t \wedge (\mathrm{d}\varphi + \mu)\,, \tag{4.18}$$

$$F_2 = w_2\,, \tag{4.19}$$

$$F_4 = -4r^3 \mathrm{d}t \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}r + \frac{r^2}{q^2}\mathrm{d}t \wedge (\mathrm{d}\varphi + \mu) \wedge w_2\,. \tag{4.20}$$

The function q on Y must satisfy (2.57)

$$4q^2 - \Box_Y q^2 = |\alpha|^2 + |w_2|^2\,. \tag{4.21}$$

Explicit solutions of this equation have been found in [6] in the case of $T^{1,1}$ and some $Y^{p,q}$. For $T^{1,1}$ the function q is constant. Obviously we cannot find solutions of this type on S^5 since there are no harmonic forms on S^5.

It is interesting to look at the form of the supersymmetry parameters. They satisfy conditions (2.17) and (2.18). For the solutions with $\eta_+^1 = \eta_+^2$ we see from equation (2.19) that

$$\Gamma^{0xy}\epsilon_1 = \epsilon_2\,, \tag{4.22}$$

where we took into account the redefinition $\hat{\eta}_1 \to \hat{\gamma}^{xy}\hat{\eta}_1$. This condition is reminiscent of the original D3 brane condition.

The case of type IIB is analogous. The metric is now of the form (3.1)

$$\mathrm{d}s_{10}^2 = -2\,r^2\,\mathrm{d}t\,\mathrm{d}\varphi + r^2(\mathrm{d}x^2 + \mathrm{d}y^2) + \frac{\mathrm{d}r^2}{r^2} + q^2\,\mathrm{d}\varphi^2 + \mathrm{d}s_Y^2\,, \tag{4.23}$$

and the forms α and w enter in the NS-NS and RR-RR fluxes as in (3.4) and (3.5). Notice that the fluxes are proportional to $\mathrm{d}\varphi$ and the time-translation Killing vector is null. We still have solutions based on harmonic two-forms on Y [6].

In type IIB we can also have solutions with an explicit dependence on φ. These have been discussed in details in [5, 6, 10, 18, 19]. We just consider the simplest case. Set all the forms α and w equal to zero, except for w_0. The resulting solution then can exist also on S^5. A non-zero w_0 corresponds to a one-form flux

$$F_1 = w_0\mathrm{d}\varphi\,, \tag{4.24}$$

or, equivalently, to a linear profile for the axion $C_0 = w_0\varphi$. As discussed at the beginning of this section, with a φ dependent spinor we can compensate the non-zero value of w_0 (since $\theta = 0$) and have a supersymmetric solution. The equation (2.57) for q reads

$$4q^2 - \Box_Y q^2 = |w_0|^2\,, \tag{4.25}$$

and can be solved with a constant function q. When φ is compact, some quantization condition should be imposed on the parameters in order to have a consistent solution [18]. The supersymmetric solution with non-compact φ has been interpreted in [19] as describing a deformation of $\mathcal{N} = 4$ SYM (or, more generally, of the four-dimensional superconformal theory associated with Y) in the presence of a linear theta angle.

4.1.2 Asymptotically Lif_4 solutions

The ansatz described in sections 2 and 3 allows to show in an elegant way that many supersymmetric asymptotically AdS_5 backgrounds in type IIB descend to analogous solutions with asymptotic Lif_4 vacua. In particular, starting from supersymmetric domain walls in type IIB we expect to find solutions interpolating between Lif_4 vacua. In the gauge/gravity correspondence, a domain-wall connecting two AdS_5 vacua is interpreted as a RG flow between the corresponding CFT_4s [20, 21]. The interpolation between Lif_4 vacua has obviously an analogous interpretation in terms of RG flows between $NRCFT_3$.

As an example, we consider the case of 4d RG flows with SU(3) structure based on internal manifolds that are (non-conical) CY_6

$$\mathrm{d}s_{10}^2 = e^{2A}(\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu) + e^{-2A}\mathrm{d}s_{CY_6}^2\,, \qquad \mu = 0, .., 3\,. \tag{4.26}$$

The dilaton is constant and set to zero, and all fluxes are zero except for the five-form

$$* f = 4\mathrm{d}A \,, \tag{4.27}$$

where e^{-4A} is required to be a harmonic function on CY_6. The pure spinors are given again by equation (4.3) with $\theta = 0$, and

$$J = e^{-2A}J_{\mathrm{CY}} \qquad\qquad \Omega = e^{-2A}\Omega_{\mathrm{CY}} \,. \tag{4.28}$$

The first regular domain-wall which is conformally Calabi-Yau was found in [22] and interpolates between $AdS_5 \times T^{1,1}$ and $AdS_5 \times S^5$. It is obtained by displacing a large number of D3 branes on the resolved conifold. Near the position of the D3 branes a throat will emerge recreating $AdS_5 \times S^5$. The solution has a natural field theory interpretation in terms of baryonic VEVs. By moving in the baryonic moduli space of the conifold theory we can flow in the IR to pure $N = 4$ SYM.

Similar solutions exist for all resolved CY_6 [23]. In fact, it is well known that the web of four-dimensional quiver gauge theories associated to D3 branes sitting at conical Calabi-Yau singularities can be connected by RG flows induced by baryonic operators. On the geometrical side, the corresponding CY_6 are obtained by resolutions. When the resolution of the cone $C(Y_1)$ is only partial and the CY_6 has still local conical singularities of the form $C(Y_2)$ we can engineer a domain wall interpolating between $AdS_5 \times Y_1$ and $AdS_5 \times Y_2$ by putting D3 branes at the local singularity on the resolved Calabi-Yau.

From each of these solutions, we have type IIB domain-walls interpolating between the Lifshitz solutions corresponding to Y_1 and Y_2

$$\mathrm{d}s_{10}^2 = -2\,e^{2A}\mathrm{d}t\,\mathrm{d}\varphi + e^{2A}(\mathrm{d}x^2 + \mathrm{d}y^2) + q^2\,\mathrm{d}\varphi^2 + e^{-2A}\mathrm{d}s_{CY_6}^2 \,, \tag{4.29}$$

with a linear axion

$$F_1 = w_0 \mathrm{d}\varphi \,. \tag{4.30}$$

The function q is determined by

$$-\Box_{CY_6}(q^2 e^{-2A}) = e^{-4A}|w_0|^2 \,, \tag{4.31}$$

with q becoming constant in the asymptotic regions.

More generally, we can have type IIA and type IIB solutions with two-forms α and w_2. The type IIB metric would be still of the form (4.29) and in the type IIA case we have

$$\mathrm{d}s_{10}^2 = -e^{4A}\mathrm{d}t^2 + e^{2A}(\mathrm{d}x^2 + \mathrm{d}y^2) + (e^1)^2 + e^{-2A}\mathrm{d}s_{CY_6}^2 \,. \tag{4.32}$$

From equations (2.48)–(2.52) we see that the two-forms α and w_2 must be closed and co-closed on the CY_6

$$\mathrm{d}\alpha = \mathrm{d}w_2 = 0 \qquad \mathrm{d}(*_{CY_6}\alpha) = \mathrm{d}(*_{CY_6}w_2) = 0 \,. \tag{4.33}$$

Such harmonic forms exist on resolved CY_6 with $b_2 \neq 0$. Explicit solutions for the resolved conifold and its quotients can be found in [24, 25] and a general discussion in [23]. In this case q is determined by

$$-\Box_{CY_6}(q^2 e^{-2A}) = e^{-4A}(|\alpha|^2 + |w_2|^2) \,. \tag{4.34}$$

In the original example [22] the solution of this equation becomes singular in the IR, since S^5 has no harmonic two-forms. For more general solutions interpolating between different Sasaki-Einstein manifolds we expect the existence of regular solutions.

The field theory realization of the $NRCFT_3$ dual to the existing Lif_4 vacua is still unclear. We may expect to understand them in terms of Chern-Simons gauge theories (see [19] for an attempt). The existence of a map between (many) CFT_4s to $NRCFT_3$s and the corresponding flows suggests that it should be possible to understand and classify these $NRCFT_3$ in terms of the better known parent four-dimensional quiver gauge theories. For this reason, it would be interesting to perform a full scan of the Sasaki-Einstein manifolds that give rise to Lif_4 vacua and of the allowed (regular) flows between them. Moreover, as recently pointed out [26], three-dimensional theories with $z = 2$ have a trace anomaly which gives rise to a single central charge in models with a holographic Lif_4 dual. This central charge is expected to decrease along a holographic flow on general grounds [20, 21] and it would be very interesting to evaluate it for the models at hand.

4.1.3   Solutions with hyperscaling violation

The Lif_4 examples we have considered in section 4.1.1 are scale-invariant; the dilaton and all other scalar functions are required to be independent of the radial coordinate r. If we allow the dilaton, or other quantities, to have a non-trivial profile in r we can realize more general solutions with a dynamical critical exponent z and a hyperscaling violation exponent θ corresponding to the class of metrics [27, 28]

$$\mathrm{d}s^2 = u^{-2(1-\frac{\theta}{D})}\bigg(-u^{-2(z-1)}\mathrm{d}t^2 + \sum_{i=1}^{D}(\mathrm{d}x^i)^2 + \mathrm{d}u^2 \bigg), \tag{4.35}$$

with $u = 1/r$. The metric is conformal to the Lifschitz space-time but transforms as $\mathrm{d}s^2 \to \lambda^{2\theta/D}\mathrm{d}s^2$ under the rescaling

$$t \to \lambda^z t \qquad\qquad x^i \to \lambda x^i \qquad\qquad u \to \lambda u \,. \tag{4.36}$$

Obviously these solutions have singularities for small or large r, and can be only considered as an effective description of the physics in some range of the radial coordinate. For large and small r we may expect the solution to have a different form, corresponding to an AdS or Lifshitz vacuum, or to a more general regular solution. As pointed out in [28] a very simple physical realization of such system with $z = 1$ and $\theta = -\frac{1}{3}$ is given by D2 branes for a given range of values of r.

Starting with a general IIB solution of the form (4.26) with constant dilaton we can obtain a non-relativistic solution with running dilaton if q has an r-dependence (see (2.47) and (3.9)). The function q satisfies equation (2.57) which now reads

$$-\Box_{CY_6}(q^2 e^{-2A}) = e^{-4A}(|\alpha|^2 + |w|^2)\,. \tag{4.37}$$

The general solution of this equation is obtained from a particular one by adding the solution of the homogeneous equation $\Box_{CY_6}(q^2 e^{-2A}) = 0$.

Let us consider, for instance, the case of conic Calabi-Yau manifolds of section 4.1.1, with $e^A = r$. With zero internal forms $\alpha = w = 0$ and $q = e^{-A} = 1/r$, we obtain the non-relativistic type IIA solution

$$\mathrm{d}s_{10}^2 = -r^6 \mathrm{d}t^2 + r^2 (\mathrm{d}x^2 + \mathrm{d}y^2) + r^2 d\varphi^2 + \frac{\mathrm{d}r^2}{r^2} + \mathrm{d}s_Y^2 \,, \tag{4.38}$$

with $e^{\phi_A} = r$ and

$$H^{IIA} = \mathrm{d}(r^4 \mathrm{d}t \wedge \mathrm{d}\varphi) \,, \qquad F_4 = -4r^3 \mathrm{d}t \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}r \,. \tag{4.39}$$

There is also an analogous type IIB solution. Solutions of this kind, with $z = 3$ and $\theta = 2$,[9] have been discussed in [29, 30]. The solution only has a limited range of validity. For example, for large r the dilaton grows and invalidates the solution. We can have a different UV completion if we use the more general solution

$$q^2 = c_1 r^2 + c_2/r^2 \,. \tag{4.40}$$

In this case we obtain a metric that for large r is a sort of T-dual of the $AdS_5 \times Y$ type IIB solution and reduces in the IR to the metric with hyperscaling violation.

If we further add internal two- forms α and w_2 on Y we can have more general solutions with

$$q^2 = c_0 + c_1 r^2 + c_2/r^2 \,. \tag{4.41}$$

where c_0 is a radial independent solution of (4.21). For $c_2 = 0$ these solutions would interpolate between a T-dual of $AdS_5 \times Y$ in the UV and the Lif_4 solution discussed in section 4.1.1 in the IR.

5 Conclusions

We have discussed a general framework to determine supersymmetric type II non-relativistic solutions with exact or asymptotic scale invariance. As already emerged from previous investigations [6, 10], there is a clear correspondence between anisotropic d-dimensional vacua and relativistic solutions in $d + 1$ dimensions. The known supersymmetric Lif_4 solutions have $z = 2$ and descend from AdS_5 vacua. This correspondence between AdS_{d+1} and Lif_d vacua is certainly intriguing and deserve further study. In particular it may be useful in explicitly constructing the three-dimensional theories dual to Lifshitz vacua.

Beside clarifying the correspondence between $d + 1$ and d-dimensional vacua, our formalism can be applied to the search of new solutions. In this paper we have only considered the simplest generalizations of the solutions found in [6], based on CY_6 four-dimensional vacua. There are other obvious directions of investigation. For simplicity, we have only considered SU(3) structures. The case of SU(2) structure is considerably more involved.

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[9]The hyperscaling violation refers to the scaling of the effective $D+2 = 5$ dimensional metric in Einstein frame.

However, many interesting AdS_5 solutions with three-form fluxes, including the Pilch-Warner solution (PW) [31] and the beta-deformation of Sasaki-Einstein backgrounds [32], have SU(2) structure [33, 34] and we expect the existence of corresponding Lif_4 solutions.

The formalism could be applied also to the study of confining solutions. It would be quite interesting to see if known relativistic four-dimensional confining solutions descend to non-relativistic solutions with asymptotically Lifshitz scaling and a confining behaviour in the IR. In the case of the obvious candidates (the Klebanov-Strassler [35], the Maldacena-Nunez [36] and the interpolating baryonic branch [37] solutions) it is not immediately obvious how to find a set of polyforms (α, w) satisfying all constraints (2.48)–(2.52) and (2.53)–(2.56) and maintaining regularity. However, it is not excluded that a generalization of this construction will give interesting solutions. We leave the detailed analysis of these and similar cases to future work.

Acknowledgments

A   Notations and useful formulae

We use the notation of [15] to which we refer for more details. To a differential form we can associate a bispinor via the Clifford map

$$C_k \equiv \frac{1}{k!} C_{M_1 \ldots M_k} dx^{M_1} \wedge \ldots \wedge dx^{M_k} \; \longrightarrow \; \slashed{C}_k \equiv \frac{1}{k!} C_{M_1 \ldots M_k} \gamma^{M_1 \ldots M_k} \, . \tag{A.1}$$

The Clifford product is

$$\gamma^M C_k = (dx^M \wedge + \iota^M) C_k \, , \qquad C_k \gamma^M = (-)^k (dx^M \wedge - \iota^M) C_k \, , \tag{A.2}$$

where $\iota^M \equiv g^{MN} \iota_N \equiv g^{MN} \iota_{\partial/\partial x^N}$. In this formulae and in the main text the "slash" symbol is usually understood. We keep only when needed to clarify the origin of few signs. We take hermitian (imaginary and antisymmetric) six-dimensional gamma matrices. The six-dimensional chirality is defined through

$$\gamma_7 = -i\gamma^1 \cdots \gamma^6 \, , \tag{A.3}$$

which on bispinors gives

$$\gamma_7 \slashed{C} = -i * \lambda(\slashed{C}) \, , \tag{A.4}$$

where $\lambda(C_p) = (-1)^{[p/2]} C_p$, if C_p is a form of degree p. This equation is consistent with our definition of the star product

$$* C \wedge C = |C|^2 \mathrm{vol} \, . \tag{A.5}$$

The six-dimensional Mukai pairing reads

$$(A, B)_6 = (A \wedge \lambda B)_6 = -\frac{i}{8}(-1)^{\deg A}\mathrm{Tr}(\gamma_7 \, (\bar{A})^\dagger \mathcal{B})\,. \tag{A.6}$$

In using this equation, we must remember that the gamma matrices are imaginary so that one must be careful with signs when converting the conjugate odd forms into bispinor. The only relevant case for us comes with the complex conjugate of Φ_-

$$(\bar{\Phi_-}) = -\overline{(\Phi_-)}\,. \tag{A.7}$$

The pure spinors

$$\Phi_+ = \eta_+^1 \eta_+^{2\dagger}\,, \qquad\qquad \Phi_- = \eta_+^1 \eta_-^{2\dagger}\,, \tag{A.8}$$

satisfy the identities

$$(\bar{\Phi}_+, \Phi_+)_6 = (\bar{\Phi}_-, \Phi_-)_6\,, \qquad (\Phi_+, Z\Phi_-)_6 = (\Phi_+, Z\bar{\Phi}_-)_6 = 0\,, \qquad Z \in T \oplus T^*\,, \tag{A.9}$$

Using (A.6) we can also derive the following identities

$$(\Phi_+, X\Phi_- Y)_6 = -(-1)^{\left[\frac{\deg Y}{2}\right]}(\Phi_-, X\Phi_+ Y)_6 = \frac{i}{8}(\eta_-^{1\dagger} X\eta_+^1)(\eta_-^{2\dagger} Y\eta_-^2)\,,$$

$$(\bar{\Phi}_+, X\Phi_+ Y)_6 = (-1)^{\left[\frac{\deg Y}{2}\right]}(\bar{\Phi}_-, X\Phi_- Y)_6 = -\frac{i}{8}(\eta_+^{1\dagger} X\eta_+^1)(\eta_+^{2\dagger} Y\eta_+^2)\,, \tag{A.10}$$

$$(\Phi_+, X\bar{\Phi}_- Y)_6 = (-1)^{\left[\frac{\deg X}{2}\right]}(\bar{\Phi}_-, X\Phi_+ Y)_6 = -\frac{i}{8}(\eta_-^{1\dagger} X\eta_-^1)(\eta_+^{2\dagger} Y\eta_-^2)\,,$$

where X and Y are generic products of gamma matrices of the form $\gamma^{\{\mu_1 \cdots \mu_k\}}$ acting in the Mukai pairing via Clifford multiplication (A.2). It can be useful to remember that $Y^T = (-1)^{\left[\frac{\deg Y}{2}\right]}Y$ and, as obvious from (4.3), $\eta_+^{i\dagger} X\eta_+^i$ is non zero only for even X and $\eta_-^{i\dagger} X\eta_+^i$ only for odd X. Other useful identities which follow from (A.4) are

$$(\bar{\Phi}_\pm, \gamma^m * \lambda(f))_6 = (\bar{\Phi}_\pm, \gamma^m(-if))_6\,,$$
$$(\bar{\Phi}_\pm, *\lambda(f)\gamma^m)_6 = (\bar{\Phi}_\pm, if\gamma^m)_6\,. \tag{A.11}$$

B The conditions for supersymmetry

In this appendix we show how to simplify conditions (1.12) and (1.13). We will do it for the case of type IIA, type IIB being completely analogous. We start from K_1 and K_2 given in (2.23)

$$K_1 = \frac{C}{16}||\eta_+||^2(e^0 + e^1)\,, \qquad\qquad K_2 = \frac{C}{16}||\eta_+||^2(e^0 - e^1)\,. \tag{B.1}$$

Setting $K_i = e_{-i}$, we can define two basis of vielbeine as (e_{-i}, e_{+i}, e_I), with

$$e_{+1} = \frac{1}{4}e^{-A_1}(e^1 - e^0) \qquad\qquad e_{+2} = -\frac{1}{4}e^{-A_1}(e^0 + e^1)\,. \tag{B.2}$$

where we use the normalization of (1.9).

Using an identity analogous to (A.6) (see also formula (B.33) in [15])

$$(e_{+1} \cdot \Phi \cdot e_{+2}, C) = \frac{1}{32}(-1)^{\deg \Phi} \bar{\epsilon}_1 e_{+1} C e_{+2} \epsilon_2 \,, \tag{B.3}$$

where C is a generic bispinor, it is easy to verify that

$$(e_{+1} \cdot \Phi \cdot e_{+2}, C \cdot e_{+2}) = (e_{+1} \cdot \Phi \cdot e_{+2}, e_{+1} \cdot C) = 0 \tag{B.4}$$

and

$$(e_{+1} \cdot \Phi \cdot e_{+2}, \Gamma_{10} C) = (e_{+1} \cdot \Phi \cdot e_{+2}, C) \,. \tag{B.5}$$

The first identity follows from the vanishing of the square of the null vector e_{+i} and the second from the fact that ϵ_1 has positive chirality.

Let us now consider (1.12). The left-hand side of the Mukai paring gives

$$e_{+1} \cdot \Phi \cdot e_{+2} = \frac{1}{8} C e^{-2A_1}(1 - e^{01})\Phi_8 \,. \tag{B.6}$$

The d_H terms can be written as

$$\begin{aligned}
\mathrm{d}_H(e^{-\phi_A}\Phi \cdot e_{+2}) &= \mathrm{d}_H\left(e^{-\phi_A - A_1}\frac{C}{4}\Phi_8(e^0 + e^1)\right) \\
&= \mathrm{d}_h\left(e^{-\phi_A - A_1}\frac{C}{4}\Phi_8\right) \cdot (e^0 + e^1) + e^{-\phi_A - A_1}\frac{C}{4}\Phi_8 \mathrm{d}(e^0 + e^1) \\
&\quad -\mathrm{d}(e^{01}) \wedge e^{-\phi_A - A_1}\frac{C}{4}\Phi_8(e^0 + e^1) \\
&= \mathrm{d}_h\left(e^{-\phi_A - A_1}\frac{C}{4}\Phi_8\right) \cdot (e^0 + e^1) \\
&\quad + e^{-\phi_A - A_1}\frac{C}{4}\Phi_8\left(\mathrm{d}A_1 e^0 - \frac{\mathrm{d}q}{q}e^1 + \frac{\alpha}{q}(1 + e^{01})\right) \,, \tag{B.7}
\end{aligned}$$

while, from (B.2) it follows immediately that

$$\mathrm{d}^\dagger(e^{-2\phi_A}e_{+2}) = 0 \,. \tag{B.8}$$

Finally, the fluxes can be written as

$$F^{IIA} = (1 + \Gamma_{10})[-q\, e^1 f + (1 + e^{01})w] \,, \tag{B.9}$$

and, when inserted in (1.12), we can use (B.5) to replace Γ_{10} in (B.9) with the identity matrix. Notice also that the first term on the right hand side of equation (B.7) does not contribute to (1.12) because of the identities (B.4). Similarly, we can also manipulate the remaining terms dropping all pieces of the form $(\ ) \cdot (e^0 + e^1)$. We finally obtain

$$\left((1-e^{01})\Phi_8, \Gamma^{MN}\left[\left(e^{-\phi_A - A_1}\frac{C}{8}\left(\mathrm{d}A_1 + \frac{\mathrm{d}q}{q}\right)\Phi_8 + qf\right)(e^0 - e^1) + \left(e^{-\phi_A - A_1}\frac{C}{4q}\alpha\,\Phi_8 - 2w\right)(1 + e^{01})\right]\right). \tag{B.10}$$

The ten-dimensional Mukai pairing (B.10) can be reduced to a six-dimensional one using the explicit form of Φ_8 given in equation (2.28). We need to distinguish various cases for M and N.

When $M = 0$ or $M = 5$ the only non-zero contributions come from the term proportional to $e^0 - e^1$ on the right hand side. Not unexpectedly, these contributions are similar to those found in the analysis of four-dimensional vacua in [15]. When $N = x, y$ most of the terms vanish due to (A.9) and we obtain the constraint

$$(\Phi_-, f)_6 = 0 \,, \tag{B.11}$$

while $N = n$ gives

$$\text{Im}\left[\left(\bar{\Phi}_+, \gamma^n\left[e^{-A_1-\phi_A}\frac{C}{4}\left(dA_1 + \frac{dq}{q}\right)\Phi_+ - q * \lambda f\right]\right)\right] = 0 \,, \tag{B.12}$$

where we used (A.10) and (A.11). These two conditions are actually implies by the supersymmetry equations. Indeed, using (A.11) and (2.31), (B.11) can be written as

$$(\Phi_-, *f)_6 \sim (\Phi_-, d_h \Phi_+)_6 = (d_h \Phi_-, \Phi_+)_6 = 0 \tag{B.13}$$

due to (2.32) and (A.9). Since (2.30) and (2.31) can be combined in

$$d_h\left(\frac{C}{2}e^{A_2-\phi_A}\bar{\Phi}_+\right) = -\frac{C}{2}e^{A_2-\phi_A}dA_2\Phi_+ + qe^{A_1+A_2} * \lambda f \,, \tag{B.14}$$

we see that also (B.12) can be written as

$$\text{Im}\left[\left(\bar{\Phi}_+, \gamma^n d_h\left(\frac{C}{2}e^{A_2-\phi_A}\bar{\Phi}_+\right)\right)_6\right] = 0 \,, \tag{B.15}$$

provided we set $e^{A_1} = e^{2A_2}/q$. This equation is automatically satisfied since Φ_+ is a pure spinor. Indeed,

$$(\bar{\Phi}_+, X\bar{\Phi}_+) \tag{B.16}$$

can be different from zero only for an insertion X of six gamma matrices. On the contrary d_h can bring a maximum of three gamma since it is odd and d can only change the complex type (p, q) of a form by a maximum of two units in p and q.

For all other values of M and N, (B.10) gets contributions only from the term proportional to $(1 + e^{01})$. $M = 0$ and $N = 5$ and $M = x$, $N = y$ give

$$\left(\bar{\Phi}_+, \frac{C}{16\,q}e^{-A_1-\phi_A}\alpha \cdot \Phi_+ - iw\right)_6 = 0 \,, \tag{B.17}$$

where $\cdot$ denote the Clifford product, while for $M = m$ and $N = n$ we obtain

$$\text{Im}\left[\left(\bar{\Phi}_+, \gamma^{mn}\left(\frac{C}{16\,q}e^{-A_1-\phi_A}\alpha \cdot \Phi_+ - iw\right)\right)_6\right] = 0 \,. \tag{B.18}$$

Finally $M = x, y$, and $N = n$ give

$$\left(\Phi_-, \gamma^n\left(\frac{C}{4\,q}e^{-A_1-\phi_A}\alpha \wedge \Phi_+ - iw\right)\right)_6 = 0 \,. \tag{B.19}$$

In deriving the above conditions, we used repeatedly the identities (A.10). Using (2.25) we recover half of the conditions (2.35)–(2.38). Equation (1.13) can be treated in a similar way and gives the remaining half of the conditions.

The computation in type IIB is similar and it will not be reported here. Let us simply note that, with spinors depending on φ as in (3.10), there is an extra contribution from $\mathrm{d}_H(e^{-\phi_A}\Phi \cdot e_{+2})$ coming from the component of $\mathrm{d}\Phi_8$ along e^-; this term is responsible for the replacement (3.11).

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