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Letter

A non-perturbative contribution to jet quenching

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Abstract It has been argued by Caron-Huot that infrared contributions to the jet quenching parameter in hot QCD, denoted by $\hat{q}$, can be extracted from an analysis of a certain static-potential related observable within the dimensionally reduced effective field theory. Following this philosophy, the order of magnitude of a non-perturbative contribution to $\hat{q}$ from the colour-magnetic scale, g^2T/π, is estimated. The result is small; it is probably below the parametrically perturbative but in practice slowly convergent contributions from the colour-electric scale, whose all-orders resummation therefore remains an important challenge.

1 Introduction

One of the classic qualitative indications for the generation of a thermal medium in heavy ion collision experiments is the fact that high-p_T jets get quenched [1, 2]. In fact, not only do jets get quenched but they appear to do so more effectively than naive estimates suggest [3, 4]. This motivates not only a complete leading-order weak-coupling analysis [5], but also developing methods to go beyond the leading order [6], taking into account large corrections from the infrared (IR) scales that are characteristic of thermal field theory [7, 8]. (The weak-coupling regime has recently also been discussed in Ref. [9].)

In a weakly coupled non-Abelian plasma, there are two momentum scales in the IR: the colour-magnetic scale, g^2T/π, and the colour-electric scale, gT ($g^2 = 4\pi\alpha_s$ is the QCD gauge coupling). We refer to these through the gauge coupling, $g_{\rm E}^2/\pi$, and the mass parameter, $m_{\rm E}$, of the dimensionally reduced effective theory [10, 11]. The contributions of the two scales have been disentangled for several thermodynamic observables, and a particularly rich source of information is the "spectrum" measured through screening masses [12]. For instance, the smallest screening mass (at temperatures below ~10 GeV), which is parametrically of the form $2m_{\rm E} + \mathcal{O}(g_{\rm E}^2/\pi)$, gets a numerically insignificant contribution from the colour-magnetic scale [13], whereas the Debye screening mass, which is parametrically of the form $m_{\rm E} + \mathcal{O}(g_{\rm E}^2/\pi)$, is in practice all but dominated by the colour-magnetic scale [14]:

$$M_{\rm D} = m_{\rm E} + \frac{g_{\rm E}^2 N_{\rm c}}{4\pi}\left(\ln\frac{m_{\rm E}}{g_{\rm E}^2} + 6.9\right) + \mathcal{O}(g^3T) \simeq m_{\rm E} + 1.65 g_{\rm E}^2 + \mathcal{O}(g^3T). \quad (1)$$

The purpose of the current study is to estimate whether one of these extreme scenarios could be relevant for $\hat{q}$.

2 Colour-electric contribution

The rate of jet broadening (transverse momentum diffusion) is often parametrized by a phenomenological coefficient, called the jet quenching parameter and denoted by $\hat{q}$ [15]. Although, strictly speaking, the definition of $\hat{q}$ is ambiguous even at leading order [16], it can be argued that the ambiguities can be hidden by considering a quantity called the transverse collision kernel, $C(q_\perp)$; then

$$\hat{q} = \int_0^{q^*} \frac{{\rm d}^2\mathbf{q}_\perp}{(2\pi)^2} q_\perp^2 C(q_\perp). \quad (2)$$

It is now the choice of the upper bound q^* which reflects the ambiguities. The form of $C(q_\perp)$ for $q_\perp \sim \pi T$ was determined in Ref. [5] and for $q_\perp \sim gT$ in Ref. [6]; although the NLO contribution from $q_\perp \sim gT$ to $\hat{q}$ is parametrically suppressed by $\mathcal{O}(g)$, it is numerically large.

More precisely, denoting by $m_{\rm E} = gT\sqrt{N_{\rm c}/3 + N_{\rm f}/6} + \mathcal{O}(g^3T)$ the Debye mass parameter and by $g_{\rm E}^2 = g^2T + \mathcal{O}(g^4T)$ the coupling constant of the dimensionally reduced effective field theory, the next-to-leading order (NLO) expression for the IR part ($q_\perp \sim m_{\rm E}$) of the collision kernel can be written as [6]

$$C(q_\perp) = g_{\rm E}^2 C_F \left\{ \frac{1}{q_\perp^2} - \frac{1}{q_\perp^2 + m_{\rm E}^2} \right\}$$

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$$+ g_E^4 C_F N_c \Bigg\{ \frac{7}{32 q_\perp^3} - \frac{m_E + 2(q_\perp - \frac{m_E^2}{q_\perp}) \arctan(\frac{q_\perp}{m_E})}{4\pi (q_\perp^2 + m_E^2)^2} + \frac{m_E}{4\pi (q_\perp^2 + m_E^2)} \left[\frac{3}{q_\perp^2 + 4 m_E^2} - \frac{2}{q_\perp^2 + m_E^2} - \frac{1}{q_\perp^2} \right] - \frac{\arctan(\frac{q_\perp}{m_E})}{2\pi q_\perp (q_\perp^2 + m_E^2)} + \frac{\arctan(\frac{q_\perp}{2m_E})}{2\pi q_\perp^3} + \frac{m_E - (\frac{q_\perp}{2} + \frac{2m_E^2}{q_\perp}) \arctan(\frac{q_\perp}{2m_E})}{8\pi q_\perp^4} \Bigg\} + \mathcal{O}(g_E^6). \quad (3)$$

Here $C_F = (N_c^2 - 1)/(2N_c)$ is the Casimir of the fundamental representation. Choosing $m_E \ll q^* \ll \pi T$, the integral in Eq. (2) yields

$$\hat{q} = \frac{g_E^2 m_E^2 C_F}{2\pi} \ln \frac{q^*}{m_E} + \frac{g_E^4 m_E C_F N_c}{2\pi} \left\{ -\frac{q^*}{16 m_E} + \frac{3\pi^2 + 10 - 4\ln 2}{16\pi} + \mathcal{O}\left(\frac{m_E}{q^*}\right) \right\} + \mathcal{O}(g_E^6). \quad (4)$$

The terms involving q^* cancel against contributions from hard momenta, $q_\perp \gtrsim \pi T$ [5], and the q^*-independent middle term inside the curly brackets then represents the physical NLO contribution to $\hat{q}$ [6].

3 Colour-magnetic contribution

For small momenta, $q_\perp \ll m_E$, the NLO part of Eq. (3) behaves as

$$C^{\rm NLO}(q_\perp) = g_E^4 C_F N_c \left\{ \frac{7}{32 q_\perp^3} - \frac{1}{48\pi m_E q_\perp^2} - \cdots \right\}. \quad (5)$$

We note from Eqs. (3), (5) that, as also suggested by the operator definition [6], the small-$q_\perp$ limit ($q_\perp \ll m_E$) of $C(q_\perp)$ goes over into minus the momentum-space static potential of three-dimensional pure Yang–Mills theory:

$$\lim_{q_\perp \ll m_E} C(q_\perp) = -\tilde{V}(q_\perp), \quad (6)$$

where [17, 18]

$$\tilde{V}(q_\perp) = -\frac{g_E^2 C_F}{q_\perp^2} - \frac{7 g_E^4 C_F N_c}{32 q_\perp^3} + \mathcal{O}\left(\frac{g_E^6}{q_\perp^4}\right). \quad (7)$$

(In principle higher-dimensional “hybrid” potentials could also appear, but their contributions to $\hat{q}$ are suppressed by $(g_E^2/\pi m_E)^n$, with some positive n, with respect to those from $\tilde{V}(q_\perp)$.)

However, at the next order the perturbative expansion breaks down: a direct momentum-space computation of $\tilde{V}(q_\perp)$ at 2-loop order produces in $d = 3 - 2\epsilon$ dimensions the (gauge-independent) expression [17, 18]

$$\tilde{V}^{\rm NNLO}(q_\perp) = \frac{g_E^6 C_F N_c^2}{(4\pi)^{1-2\epsilon} q_\perp^{4+4\epsilon}} \left(\frac{1}{4\epsilon^2} + \frac{3}{4\epsilon} \right). \quad (8)$$

A comparison with Eq. (2) suggests the presence of a non-perturbative contribution to $\hat{q}$ at $\mathcal{O}(g_E^6)$, both because the integral is logarithmically divergent at the lower end, and because the coefficient in Eq. (8) is IR divergent.

For future reference we note that the Fourier transform of Eq. (7), $V(r) = \int_{\mathbf{q}_\perp} (e^{i\mathbf{q}_\perp \cdot \mathbf{r}} - 1) \tilde{V}(q_\perp)$, yields the coordinate space potential

$$V(r) = \frac{g_E^2 C_F}{2\pi} \ln \frac{r}{r_*} + \frac{7 g_E^4 C_F N_c}{64\pi} r + \mathcal{O}(g_E^6 r^2), \quad (9)$$

where r_* is a regularization-dependent constant.

4 Transformation to configuration space

In view the delicate nature of the IR contribution to $\hat{q}$, it is useful to re-express the small-$q_\perp$ part of Eq. (2) in another way. The idea is to make use of Eq. (6), in combination with a non-perturbative understanding of $V(r)$, in order to get a handle on IR physics.

To be concrete, let $\tilde{\theta}_\Lambda(q_\perp)$ be some cutoff function, where Λ is chosen to be formally in the range $g_E^2/\pi \ll \Lambda \ll m_E$. Then we can rephrase the IR part of Eq. (2), to be denoted by $\hat{q}_\Lambda$, as

$$\hat{q}_\Lambda = \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} q_\perp^2 C(q_\perp) \tilde{\theta}_\Lambda(q_\perp) = \int d^2\mathbf{r} \, \nabla^2 V(r) \theta_\Lambda(r), \quad (10)$$

where $\theta_\Lambda(r) \equiv \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{r}} \tilde{\theta}_\Lambda(q_\perp)$ is the configuration space version of the cutoff function. For instance, if $\tilde{\theta}_\Lambda(q_\perp) = \theta(\Lambda - q_\perp)$, then $\theta_\Lambda(r) = \Lambda J_1(\Lambda r)/2\pi r$. This very choice is not particularly convenient, however, since the Bessel function J_1 is oscillatory. We find it more transparent to choose a Gaussian,

$$\tilde{\theta}_\Lambda(q_\perp) \equiv \exp\left(-\frac{q_\perp^2}{\Lambda^2}\right), \qquad \theta_\Lambda(r) = \frac{\Lambda^2}{4\pi} \exp\left(-\frac{r^2 \Lambda^2}{4}\right), \quad (11)$$

because both functions are elementary and positive.

Now, because of rotational symmetry and the form of Eq. (9), the behaviour of $\nabla^2 V(r)$ appearing in Eq. (10) is

$$\nabla^2 V(r) = g_E^2 C_F \delta^{(2)}(\mathbf{r}) + \left[V''(r) + \frac{V'(r)}{r} \right]_{r>0} = g_E^2 C_F \delta^{(2)}(\mathbf{r}) + \left[\frac{2c(r)}{r^3} + \frac{F(r)}{r} \right]_{r>0}, \quad (12)$$

where we have adopted the notation $F(r) \equiv V'(r)$ and $c(r) \equiv r^3 V''(r)/2$ from Ref. [19].

We subsequently need to subtract the known perturbative terms, Eq. (9), from the IR contribution to $\hat{q}_\Lambda$, given that they were already included in Eqs. (3), (4). The corresponding potential, i.e. the term of $\mathcal{O}(g_{\rm E}^6 r^2)$ in Eq. (9), is denoted by δV. Thereby we obtain an expression for the remaining IR contribution, let us call it $\delta\hat{q}_\Lambda$:

$$\begin{aligned}\delta\hat{q}_\Lambda &= 2\pi \int_{0^+}^{\infty} \mathrm{d}r \left[F(r) + \frac{2c(r)}{r^2} - \frac{7 g_{\rm E}^4 C_F N_{\rm c}}{64\pi} \right] \theta_\Lambda(r) \\ &= \frac{1}{2}\int_{0^+}^{\infty} \mathrm{d}\hat{r}\, \Lambda \phi\left(\frac{\hat{r}}{\Lambda}\right) \exp\left(-\frac{\hat{r}^2}{4}\right), \end{aligned} \quad (13)$$

where we rescaled the integration variable as $\hat{r} = r\Lambda$ and defined

$$\phi(r) \equiv \delta V'(r) + r\delta V''(r) \quad (14)$$

$$= F(r) + \frac{2c(r)}{r^2} - \frac{7 g_{\rm E}^4 C_F N_{\rm c}}{64\pi}. \quad (15)$$

As Eq. (13) shows, only the short-distance part of $\phi(r)$, terms linear in r (modulo logarithms), contributes to $\delta\hat{q}_\Lambda$; this corresponds to terms quadratic in r in $\delta V(r)$.

The nature of the short-distance behaviour of $\delta V(r)$ can be discussed within a framework similar to the Operator Product Expansion [20, 21]. The lowest-dimensional condensate in three-dimensional pure SU(3) evaluates to

$$\begin{aligned}&\frac{1}{2g_{\rm E}^2}\langle \mathrm{Tr}[F_{kl}F_{kl}]\rangle_{\overline{\rm MS}} \\ &= \frac{6 g_{\rm E}^6 C_F N_{\rm c}^4}{(4\pi)^4} \left[\left(\frac{43}{12} - \frac{157}{768}\pi^2\right) \right. \\ &\quad \left. \times \left(\ln \frac{\bar{\mu}}{2 N_{\rm c} g_{\rm E}^2} - \frac{1}{3}\right) - 0.2 \pm 0.4^{(\rm MC)} \pm 0.4^{(\rm NSPT)} \right], \end{aligned} \quad (16)$$

where the errors come from Monte Carlo simulations (MC) [22] and a scheme conversion (NSPT) [23], and the numerical values apply for $N_{\rm c} = 3$. Just cancelling the scale dependence from Eq. (16) one might expect a short-distance behaviour of the form

$$\delta V(r) \sim g_{\rm E}^6 r^2 \ln\left(\frac{1}{g_{\rm E}^2 r}\right) + \mathcal{O}(g_{\rm E}^8 r^3). \quad (17)$$

However, according to Ref. [24] the shape at $g_{\rm E}^2 r \ll 1$ is really more complicated, $\sim g_{\rm E}^6 r^2 \ln[\ln(1/g_{\rm E}^2 r)]/\ln(1/g_{\rm E}^2 r)$, where $g_{\rm E}^2 N_{\rm c} \ln(1/g_{\rm E}^2 r)$ represents the difference of octet and singlet potentials. Note that such a tail does not contribute in Eq. (13) for $\Lambda \gg g_{\rm E}^2/\pi$. Unfortunately the analysis is not valid for the range $g_{\rm E}^2 r \sim 1$ that is relevant for us here, so we resort to modelling in the following.

5 Modelling of lattice data

In Fig. 1, numerical data for $r_0^2\phi(r)$ from Ref. [19] is shown. The parameter r_0 is defined from

$$r_0^2 F(r_0) = 1.65 \quad (18)$$

for any given lattice spacing [25]. In the continuum limit [19],

$$g_{\rm E}^2 r_0 \approx 2.2, \quad (19)$$

and we have inserted this estimate in order to combine the last term of Eq. (14) with lattice data. (Obviously, it would be nice to add data at shorter distances, but this task is non-trivial because strong cutoff effects appear and a careful extrapolation to the continuum limit is needed.)

Now, motivated by the naive Eq. (17) and the definition in Eq. (14), we describe the data through the function

$$r_0^2\phi(r) = \frac{ar}{r_0}\ln\frac{r_0}{r} + \frac{br}{r_0} + \frac{cr^2}{r_0^2}, \qquad r \lesssim 2r_0. \quad (20)$$

The data (at $\beta = 20$) are well modelled ($\chi^2/\mathrm{d.o.f.} = 0.18$) with $a = 0.72(2)$, $b = 0.55(1)$, $c = 0.18(1)$. We remark that around $r/r_0 \approx 2.0$ this function is close to the asymptotic value given by the non-perturbative string tension minus the perturbative subtraction, $\{[0.553(1)]^2 - 7/16\pi\} g_{\rm E}^4 r_0^2 \approx 0.81$ [26]. We have checked that adding a cubic term to the ansatz of Eq. (20) does not change the results in any significant way, for instance $a = 0.74(14)$, $b = 0.52(17)$ then. In the following it is therefore assumed that Eq. (20) reflects the magnitude of the true function for $g_{\rm E}^2 r \sim 1$.

Carrying out the integral in Eq. (13), we get

$$\delta\hat{q}_\Lambda = \frac{1}{r_0^3}\left[a\left(\ln\frac{\Lambda r_0}{2} + \frac{\gamma_{\rm E}}{2}\right) + b + \mathcal{O}\left(\frac{1}{\Lambda r_0}\right)\right]. \quad (21)$$

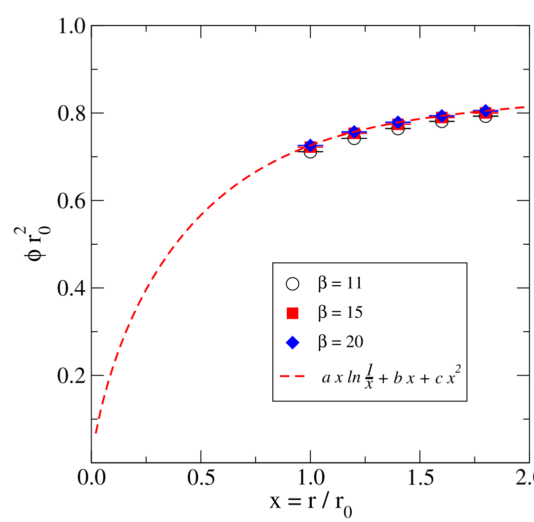

Fig. 1 The function ϕ defined by Eq. (14), in units of the parameter r_0, defined by Eq. (18). The lattice data is from tables 5 and 7 of Ref. [19]

The coefficient a/r_0^3, which comes together with a logarithmic dependence on the cutoff Λ, should be an analytically computable function, since the cutoff should cancel against an NNLO contribution from the colour-electric scale; in this sense it is not a "genuine" colour-magnetic contribution. In contrast, the coefficient b/r_0^3 represents a genuine colour-magnetic contribution *within the model* (although, because of the freedom in choosing the argument of the logarithm, this notion is somewhat ambiguous).

6 Phenomenological interpretation

Omitting from Eq. (21) terms vanishing for $\Lambda r_0 \gg 1$; replacing the cutoff inside the logarithm with the scale $\sim m_{\mathrm{E}}$ at which other physics sets in; inserting r_0 from Eq. (19); and estimating $m_{\mathrm{E}}/g_{\mathrm{E}}^2 \sim 1$ (cf. Fig. 1 of Ref. [13]); we get the order-of-magnitude estimate

$$\delta\hat{q}_\Lambda \simeq \frac{g_{\mathrm{E}}^6}{2.2^3}\left[0.72\left(\ln\frac{2.2}{2}+\frac{\gamma_{\mathrm{E}}}{2}\right)+0.55\right] \approx 0.08 g_{\mathrm{E}}^6. \quad (22)$$

This can be compared with the middle term from the curly brackets in Eq. (4),

$$\delta\hat{q} \approx \frac{g_{\mathrm{E}}^4 m_{\mathrm{E}} C_F N_{\mathrm{c}}}{32\pi^2}\left(3\pi^2+10-4\ln 2\right) \approx 0.47 g_{\mathrm{E}}^4 m_{\mathrm{E}}. \quad (23)$$

The colour-magnetic contribution, Eq. (22), is clearly below the NLO perturbative contribution from the colour-electric scale, Eq. (23).

In conclusion, the contribution to $\hat{q}$ from the colour-magnetic scale, $q_\perp \sim g_{\mathrm{E}}^2/\pi$, may well be of modest magnitude. Thus the phenomenological motivation for its theoretically consistent determination may be feeble; rather, it should probably be measured as a part of the total IR contribution from both the colour-electric and colour-magnetic scales, e.g. along the lines suggested in Ref. [6].

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Note added in proof Recently a paper appeared [27] in which, as acknowledged there, some basic ideas of the current study, communicated to one of the authors several years ago, were revealed. Unfortunately the practical implementation, citing e.g. a peculiar temperature dependence of $\delta\hat{q}_\Lambda$, appears to suffer from misunderstandings. (After the appearance of the current paper and further email correspondence, many of these problems have been rectified in v2.)

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