Saimaa University of Applied Sciences
Technology, Lappeenranta
Double Degree Program in Civil and Construction Engineering

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Strength analysis of wooden curved beams

Thesis 2017

Abstract

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The purpose of the study was to calculate two versions of wooden curved beam which is a part of a frame structure. The chosen material is stated as glued-laminated timber. The study demands examination through comparison of such results as values of forces, bending moments and deflections in order to determine a more effective cross-section from the economical point of view of using timber as a structural material. The work was accomplished as a final project of bachelor degree education.

The base for the calculations was Eurocode norms with consideration of Finnish National Annex. This study was carried out in several designing programs such as AutoCAD and SCAD Office. AutoCAD was used in order to create the initial design model, and then it was exported to SCAD where the needed loads were applied and the calculations were completed. Theoretical information was gathered from literature, handbooks and the Internet; the calculations are based on official Eurocode 5 norms.

The aim achieved was to compare two wooden beams from the efficient cross-section value point of view. The comparison of the schemes also proved the presumable differences in forces values and values of bending moments. Further study is required to evaluate the real possibility of applying these results from economical point of view in an entire construction project for the whole life cycle.

Keywords: construction, wooden, timber, long-span, structure, struts, beam, curved, cross-section, glued-laminated timber, glulam.

Table of contents

Terminology

Symbols:

C_e	exposure coefficient
C_t	thermal coefficient
C_{dir}	the directional factor
C_{season}	the season factor
$C_e(z)$	exposure factor
$C_r(z)$	the roughness factor
$C_o(z)$	the orography factor
F_w	the wind loadings per unit length
S_k	characteristic value of snow on the ground at the relevant site
S	snow load on the roof
V_b	the basic wind velocity
$V_{b,o}$	the fundamental value of the basic wind velocity
$V_m(z)$	the mean wind velocity
W_{fin}	final deflection
Z_{min}	the minimum height based on terrain category
Z_{max}	the maximum height based on terrain category
Z_o	the roughness length
Z	height above ground
b	cross-section width

$c_s c_d$	structural factor
c_{pe}	external pressure coefficient
c_{pi}	internal pressure coefficient
h	cross-section height
k_{mod}	a modification factor taking into account the effect of the load duration and moisture content
k_{r}	terrain factor
l	length of the beam
$I_v(z)$	the turbulence intensity
r_{in}	inner radius of the beam
q_b	the basic velocity pressure
$q_p(z)$	the peak velocity pressure
t	thickness of the laminate
z	height
w_e	external wind pressure:
w_i	internal wind pressure:
γ_m	partial factor for a material property
ρ	air density
μ	snow load shape coefficient

1 Introduction

1.1 Background

Timber as a building material has been used since ancient times. This is explained by a big number of forests, simple manufacture and transportation to the building site. Moreover, timber has good constructive qualities - significant strength, elasticity at relatively little weight. These structures are used widely in framing and covering [1]:

- industrial buildings – frames and trusses up to 18 m;
- sport centers, exhibition and other public buildings (to increase architectural attraction and gain social effect) – arches and frames up to 60 m;
- industrial and storage buildings with chemical aggressive environment – arches up to 45 m.
- low-rise housing – beams and trusses.

Sometimes timber is not used rationally, as shown on Figure 1, which represents Russian timber market, when about 50% of timber goes into producing assembling forks and less than 10% into manufacturing bearing structures.

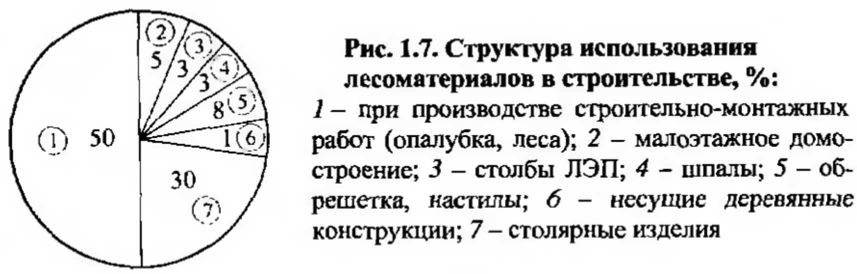


Figure 1. Diagram of usage timber in construction,%: 1 – producing assembling works (formwork, scaffold); 2 – low-rise housing; 3 – power lines; 4 – railroad ties; 5 – lathing; 6 – bearing wooden structures; 7 – joiner's goods [1].

Nowadays a significant experience in design, manufacture, assembling and maintenance of wooden structures is accumulated. This includes also knowledges of disadvantages of timber structures such as liability to rot, changing sizes (because of its porosity, amount of water inside is changing), structures buckle and crack. One of the most important parts of manufacturing is to protect with fire-proof mixtures. Little solidity across the grain leads to crushing at places where the load is applied. Rigidity and solidity are less than of steel, therefore they can yield more under the load weight. The latter often demands measures to improve.

1.2 The objective description

A previous research for the similar theme was carried out by the author for the other bachelor thesis for Saint Petersburg State University of Architecture and Civil Engineering (SPSUACE). The previous study was accomplished on the basis of Russian norms for construction of wooden elements, which is called SP 64.13330.2011. The calculations and graphical analysis were done using SCAD Office software. The aim of the current thesis is to follow the changes that the strutted system brings into the structure including the shortening of the values of forces and bending moment and the maximum deflection. The task is to find out how introducing the metal parts into a wooden structure will influence on its cross-section at the beginning of the design stage.

1.3 Methodology

In this study two variants of wooden systems are designed and calculated also by means of AutoCAD and SCAD Office as in the previous work. The first variant is designed without using struts, the second structure will be with this system. There will also be a comparison of the cases in order to choose the most efficient way to design a structure provided by the differences given due to the presence of a strutted system. The calculations were accomplished according to Eurocode 5. This system considers two types of limit states:

1. ultimate limit state – loss of equilibrium, failure due to excessive deformations, loss of stability;
2. serviceability limit state – deformations which affect the appearance of the structure or the effective use, damage (including cracking) that affects durability of the building [2].

2 Theoretical fundamentals

2.1 Strut systems

Strut systems are bar systems which consist of wooden structures capable to perform independently and besides contain additional elements designed to shorten bending moments of the main elements loaded with extranodal load. Strut systems are statically indeterminate.

To provide adequate density of connections of strut system elements tensioning of the bottom chord is accomplished, which is achieved by means of screws in supporting ties or by turnbuckle. Also it can be done with lowering the under-spring chain along the support using special ring. This kind of arrangement does not take a lot of effort to tension the chain and is handy for pulling it during the maintenance [1].

The simplest strut systems span from 9 to 15 meters, the most difficult can span up to 40 meters. One of the common strut systems is shown on Figure 2.

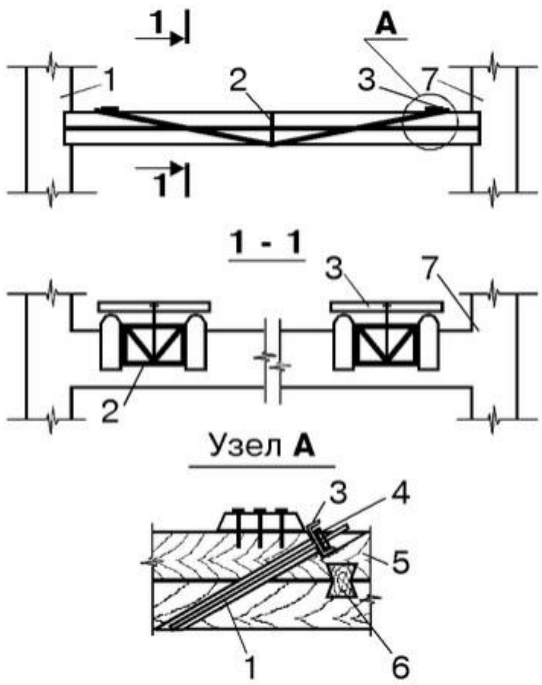


Figure 2. Strengthening beams with metal tightening. 1 – metal tightening; 2 – metal frame; 3 – supporting pad; 4 – nut; 5 – strengthened wooden beam; 6 – dowel; 7 – existing walls. [3]

During the maintenance of the wooden structures the necessity to strengthen them sometimes occurs. One of the possible ways to strengthen wooden beams and triangle cross-tie systems is to turn them into strut systems.

2.1.1 Necessity to strengthen

Necessity to strengthen wooden structures are as follows:

1. Bad condition of beam structure. It is a consequence of wooden beam being damaged: increased humidity, temperature swings, presence of bark beetles, cracking.

2. Deflection is too high. Under dead weight, dead and live loads floor beams can deflect. According to the norms, if the deflection is, for example, within the limits of 1:300, then the condition of the beam is acceptable. If the value of the deflection is bigger, then there is a need to strengthen the structure.

3. The need to increase the bearing capacity which can be connected with renovating the attic into garret or living room. For instance, such kind of renovation will lead to increasing dead and live loads on the second floor, which obviously demands the changes in cross-sections of existing wooden beams [3].

The main demands for strengthening structures are as follows:

- To provide needed loadbearing capacity, reliability and durability;
- Including in working process strengthening elements, providing their collaboration with the main structure;
- Strengthening elements should not change the gravity center position of the main section and interrupt the centering of elements in structures' joints [1].

2.1.2 Methods of strengthening wooden structures

The need to repair or to strengthen structures occurs with changes of maintenance conditions, sizes of a building, increase of technological loads due to planned renovation, or other reasons.

Strengthening is carried out with absence of live loads: snow load – on decking, technological loads – on floors.

The main strengthening methods are summarized in the table below [1].

Table 1. Methods of realizing strengthening

Name of the method	Method of realizing strengthening
1. Indirect strengthening or changes in maintenance conditions	1.1. Usage of loadbearing capacity reserve at the expense of correction: • actual acting dead and live loads; • actual strength properties of wood; • actual design model; • nature of combined work of bearing and envelope structures; • bearing capacity according to the modern norms. 1.2. Limitation of technological and live loads: • replacement of existing envelope structures by the new ones with less weight; • replacement of old technological equipment by new equipment with less weight; • timely cleaning overnormative deposits of production waste on galleries and horizontal and inclined surfaces of structures; • cleaning floors from building litter and removed equipment; • regular cleaning floors from snow in wintertime; • replacement of old insulation by modern lighter insulation; • placing additional bearing structures between existing structures for unloading the latter.
2. Changing static working scheme of the structure	2.1. Placing additional supports, hangs. If the existing logs are unbroken, then in order to increase bearing capacity one can increase their amount. Installation of the

	additional wooden beams allows increasing the load on the structure. When installing new logs, it is necessary to protect their ends with ruberoid to prevent damaging. 2.2. Turning continuous systems into split and vice versa. 2.3. Placing additional elements: tightening, struts, rods. 2.4. Placing additional braces, distributing systems.
3. Increasing the cross-section area of the element	Adding to existing element additional elements which increase their area.
4. Local strengthening	Placing cover plates, spanning local defects, steel artificial devices at supporting ties of the structure.
5. Connections strengthening	Placing additional bolts and dowels, changing working scheme of joint connection, increasing cover plates.

Nowadays it is also possible to use some modern techniques of strengthening, for instance, reinforcing with carbon fiber (carbon-filled plastic). Carbon fiber (strips, plates, needles, fabric) is glued in several layers until the required gauges of beam rigidity will be achieved. Performance convenience and material lightness lead to obtaining popularity of carbon fiber as an effective means for beams and other building structures renovation.

However, the most effective methods are changing static (constructive) scheme (adding tightening, installing additional supports, raking braces). These methods are recommended mostly when unsatisfactory technical condition of a structure or presence of free space under strengthening structure takes place.

2.2 Glued-laminated timber

In this study long-span timber structure is used, therefore it is necessary to choose the material the future structure will be made of. As the idea is to create a curved structure, glulam is considered as the most suitable structural material in such a case.

The history of this material goes long ago. The first glulam patents were approved in Germany around 1900 and this technology was spread to Scandinavia at the beginning of the 20th century [4]. Nowadays it is used mostly all over the world, because it can be applied to different kind of buildings, for example, public buildings such as large swimming pools, art galleries, libraries, and sport centers, where the attractive appearance of the material can be easily observed and provides a pleasant impression of the place. Figures 3 - 5 show some examples of applying glued laminated timber in public buildings.

Figure 3. Hall in the swimming pool in St. Petersburg with 18 m span. [5]

Figure 4. Shopping centre Aura in St. Petersburg. [5]

Figure 5. Storage for materials preventing ice-covering with diameter of 24 m in St. Petersburg. [5]

Glulam has various undoubtable advantages, for example:

1. The most important is that glulam has better strength and stiffness properties in comparison with ordinary structural timber, also it is said to be stronger than even steel. This is explained in a way that the ordinary timber has a critical value for strength of the weakest cross-section (nod, finger joint), while glulam has laminates with various strengths and they are mixed, so the possibility of failure of some laminates with similar flaws in the same beam to occur is minimized.

2. The second most important is that any architectural form can be achieved using this material; it can span long distances with minimum supports, so architects and engineers have no limits in design.

3. Moreover, it has high fire-resistant properties, perfect insulation characteristics and the life cycle is quite long in chemically aggressive environments.

The material itself consists of a number of individual laminates of structural timber which are joined by means of effective and ecological glue under controlled conditions to provide required length and height. The laminates, when cut, are placed on top of each other. To reduce the internal stresses the laminates are turned so that the core sides face the same way throughout the cross-section [4]. In the period after applying glue and before it hardens, the package is put under pressure and bent if needed. After the glue hardens the product is transported to the place where the planning works will take place. After that the holes for future connections are made, the components are checked visually and marked, then they are wrapped and passed to the transportation. The technology of manufacturing glulam structures allows producing different shapes of cross-section. The most common is usage of rectangular cross-sections, but hollow, T, I or L sections are also available which are shown on Figure 6.

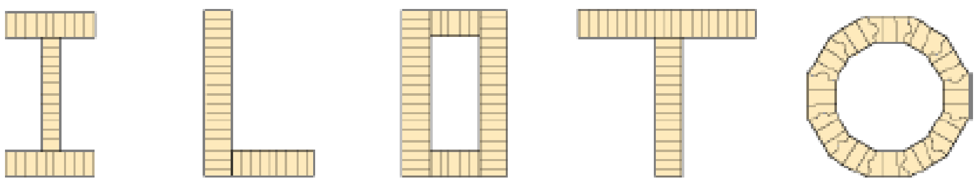


Figure 6. Examples of composite glulam sections [4].

The initial width of the rectangular cross-section is normally 45 mm, and after sides planning it can be reduced by a few millimeters, it depends on sawmill standard range. The width is also influenced by whether the sides were only planed and sanded or just planed, then occasional patches are present, however, accepted [4].

The glue must be of high strength and durability in long-term loading. Because of different colors that the glues give, the joints can appear either dark or as thin lines on the surfaces. There are some kinds of glue in use:

1. PRF – phenol-resorcinol-formaldehyde – is type I and is approved to be used in any climate (indoors and outdoors) and gives dark red-brown colour [4].

2. MUF – melamine-urea-folmaldehyde – is used more frequently, also is type I. The joints from this glue are light after applying and they darker in time [4].

L-marked glulam – is a term in Nordic countries for glulam with at least four laminates, fabricated under proper conditions. The mark means that the product is approved and was prepared under control.

The elements are supplied at a moisture content of 12% and the drying process is largely fueled with sawdust and other bi-products, which reduces the electricity use. It can be called ecological in sense of that on the building site it does not demand any additional works of assembling, and the wrapping can be reclaimed. Also the material can be re-used if its properties are saved during the life-cycle [4].

Glulam standard strength classes are GL24, GL28, GL32 and GL36; the numbers refer to the characteristic bending strength of each class, for example, for GL28 it is 28 N/mm^2. Also the material can be of homogeneous lay-up (all the laminations are of the same strength class), for example, GL28h, or combined (one-sixth of the depth on both sides of the neutral axis of a beam – outer laminations – are of higher strength class), for example, GL28c [6]. The latter are more efficient, since they allow higher strength laminations to be placed where bending stresses are higher. However, it is also suitable to use “h” and “c” classes partially in the same structure to ensure the profitability of the material.

3 Calculations

3.1 SCAD Office

As the aim of the thesis is to calculate a wooden structure using SCAD software, then it has to be explained, because such a software was made by Ukrainian authors ScadSoft and is used mostly by Russian, Ukrainian, Belorussian as it already has programmed construction norms from Russia, Ukraine, Belarus which can be chosen if necessary to have a more reliable background for calculations. This information includes also values for easier loads computation, for example, it has in its base all Russian cities with their wind and snow regions and belonging values, which can be then put into calculation by inserting the measurements of the building and by giving more precise details about the constructive scheme of the building. These calculations can be completed in WeST – additional part contained in SCAD Office package, and then exported into MS Word in forms of graphs and tables to simplify following works. As in this study only Eurocode norms are examined which do not correspond with Russian SP or SNiP, thus it is possible only to do loads collection manually and later apply them directly on the design scheme.

According to the official web-site, SCAD Office is a modern system designed by engineers and developed by a team of professional programmers. The system includes a high-performance software SCAD and some additional programs that allow everybody to solve problems of analysis and design structure. The system is still updating by improving interfaces and capabilities, adding new design components.

Figures 7 - 8 show some interface views of SCAD and Figure 9 shows the interface of additional package program WeST.

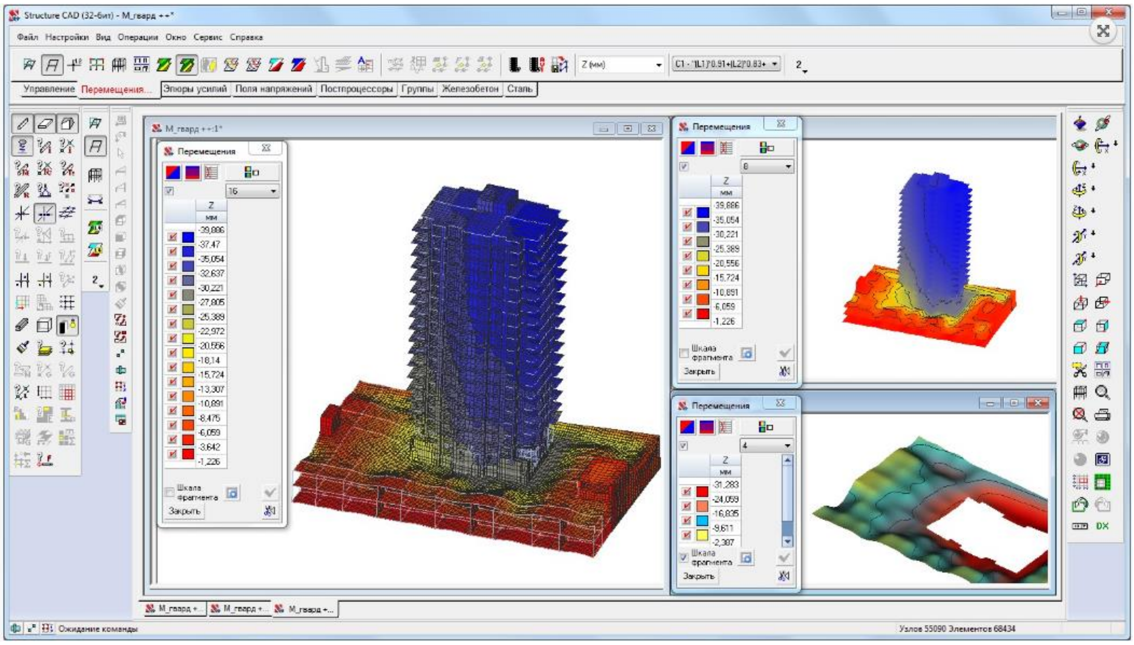


Figure 7. Representation of deflections for a whole structure and details [7]

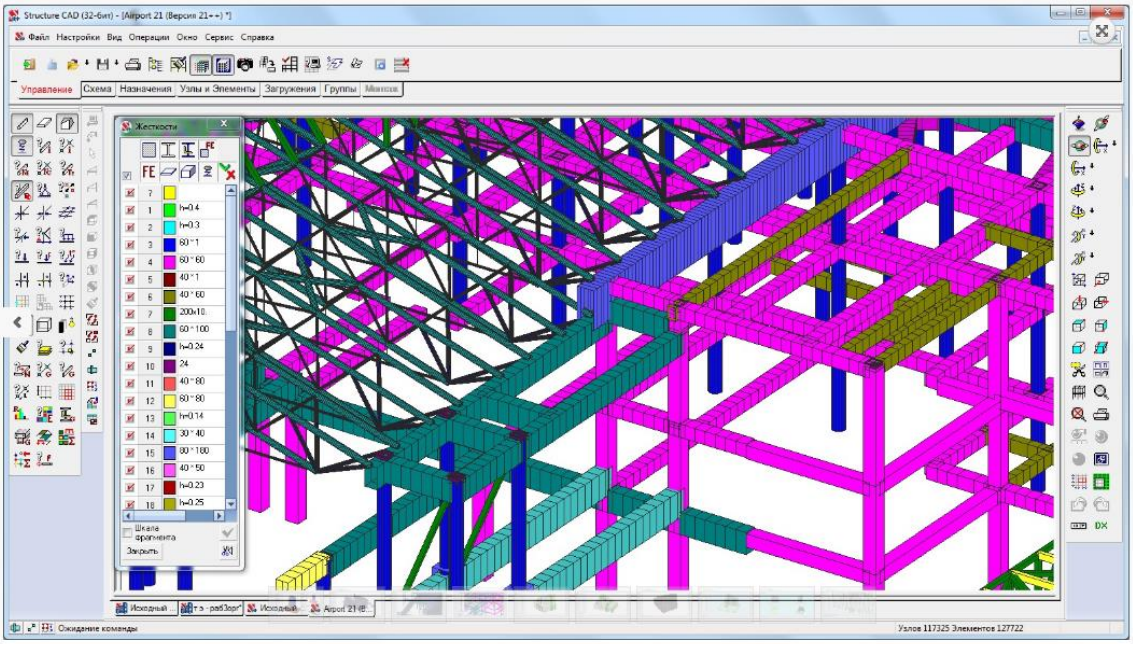


Figure 8. Representation of cross-sections [7]

Figure 9. Representation ComeIN system [7]

SCAD Office includes an elaborated library of finite elements for modeling bar, plate, solid and combined structures, modules of stability analysis, building design stress combinations, verifying stressed state of structural elements according to various failure theories, calculating forces and displacement caused by loading combinations. The system also includes possibilities for choosing reinforcement and for verifying cross-sections of steelwork structural elements.

The SCAD Office package includes the following programs, some of them are also available in English:

1. SCAD – an integrated system for finite element structural analysis;
2. Kristall – analysis of steel structural elements;
3. ARBAT – selection of reinforcement and expert investigation of reinforced structural elements;
4. ComeIn – analysis of masonry and reinforced masonry structural elements;
5. Decor – analysis of wooden structural members;

6. Zapros – analysis of members of foundations and beds;
7. WeST – calculation of loadings in compliance with SP "Loadings and influences";
8. Monolit – design of monolithic rib-reinforced floors;
9. Comet – design and analysis of steel structural joints;
10. Cross – calculation of coefficients of subgrade reaction for structures on elastic foundations;
11. Section Designer – creation and analysis of geometric properties of sections made of rolled profiles and plates;
12. Consul – construction of any types of cross-sections and calculation of their geometric properties using a theory of solid bars;
13. Tonus – construction of any types of cross-sections and calculation of their geometric properties using a theory of thin-walled bars;
14. Sezam – a tool to find equivalent cross-sections;
15. KoKon – hand-book of stress concentration coefficients and of stress intensity coefficients;
16. Kust – theoretical-practical hand-book for a designer [7].

To be more exact, this program offers, which is the most important, good graphics tools for building and editing the geometry, describing physical and mechanical properties of materials, specifying special conditions, and specifying loads upon a structure. Also it contains a large set of structural prototypes including frames, trusses, beams, surfaces specified by equations.

It is much easier to work with SCAD Office, when you have a design model completed in, for example, AutoCAD, this method is accomplished in this thesis work, then converted to dxf format which SCAD is able to export and recognize, and then one can change the model if needed or just apply required to the plan loads, proceed with calculations in order to achieve results in forms of graphic analysis or tables. The graphic form shows results of deflections or movement analysis as a deformed structure, there is also available color or digital indication of displacement values in ties, even animated representation of how the structure performs under loads can be taken from it. Analysis results in table form can be exported into the texts of Microsoft Word or Microsoft Excel sheets.

3.2 Interoperability between SCAD and other graphic software

3.2.1 Import

Figure 10 shows the SCAD interface on the import tab. It represents the possibility to import into SCAD several kinds of data format. They are, for example, DXF, DWG, IFC, R2S, ANSYS CDB, STAAD, and FEMAP.

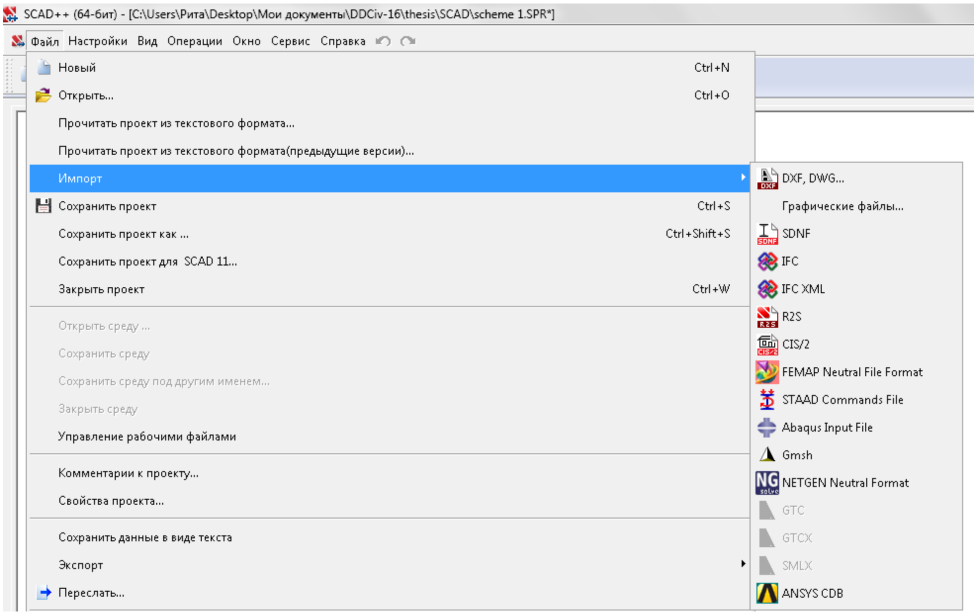


Figure 10. SCAD interface on the import page

<u>Graphic files import:</u>

1. DXF, DWG – geometry description can be imported from AutoCAD or other systems supporting dxf and dwg formats. The following types of graphic primitives can be arranged: 3DFACE, SOLID, TRACE, LINE, POLYLINE, LWPOLYLINE, ELLIPSE, CIRCLE, ARC.

2. Autodesk 3D Studio Max (.3ds) – allows to take geometry of calculation schemes of shell structures and their fragments.

3. SGI Inventor (.iv) – provides transmitting into the complex of information about geometry of three-dimensional objects, consisting of bars and shell elements [7].

<u>Plugins:</u>

In most cases SCAD provides the ability of direct reading, for example, DXF, IFC, SDNF. However, to interact with several programs (Revit, ArchiCAD, Tekla, AllPlan) special plugins are invented, which work in the corresponding program environment and as a result create an intermediate file with R2S format, which can be imported into SCAD [7].

1. Plugin Tekla provides data export into R2S format, which can be imported into SCAD program. While exporting such kinds of model data are given:

- ties coordinates and overlay braces;
- for bar elements geometry information (coordinates of the beginning and end of each element), information about local axis orientation, pins, physical material properties;
- for curved bars approximation of curved bars is accomplished by a pack of usual bars.
- export of the following cross-section types: parametric, rolled steel and user-defined.

For user-defined cross-sections, while exporting, a representation in Konsul pack of SCAD is created. While exporting rolled steel sections, a list of compliance between SCAD and Tekla gages is used. For lamellar elements information about geometry (including holes), thickness and material data is exported.

All loads are exported into SCAD with saving of their names. In each load a list of the following types of loads is realized:

- Nodal loads – forces and moments;
- Concentrated and distributed (uniform and trapeziform) loads on bars in global and local coordinate systems;
- Concentrated loads on plates in local coordinate system;
- Distributed by the load line on a plate rib;
- Evenly distributed and trapeziformed loads on plates in local coordinate system [8].

2. Plugin ArchiCAD also provides export into R2S which can be imported into SCAD.

- for bar elements – geometry information, information about local axis orientation. For bars with circular or rectangular cross-section during export corresponding stiffness parameters will be created. For cross-sections with a different shape from rectangular, bar element will be created, but the shape for its cross-section will not be identified.
- For lamellar elements – information about geometry (including holes), thickness, this is a sum of thicknesses of bearing layers. Finishing layers of multi-layer materials are not considered neither during thickness calculation, nor at plate vertexes coordinates calculation. Approximation of curved plates is done with a set of usual plates. For plates with no constant thickness, export medium thickness is considered.

3. Plugin Revit, while exporting, transmits analytical model data – nodes coordinates and bracings.

- For bar elements geometry information, information about local axis orientation, pins, physical material properties; for curved bars approximation of curved bars is accomplished by a pack of usual bars.
- For lamellar elements information about geometry (including holes), thickness and material data is exported.

All loads are exported with saving of their names. During export only ”Hosted” loads are considered – these are directly connected to the element or nod [7].

Figures 11-12 show some examples of conversion between the programs.

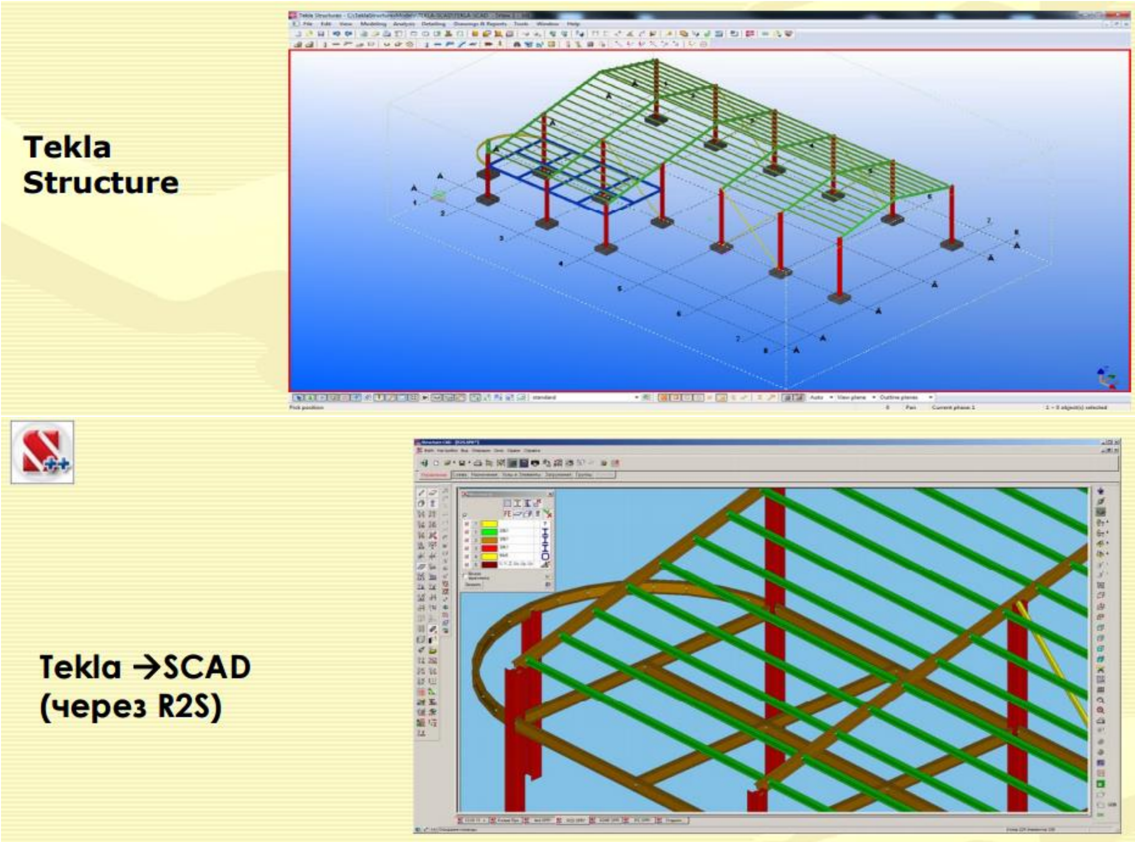


Figure 11. Import from Tekla Structure into SCAD Office [9]

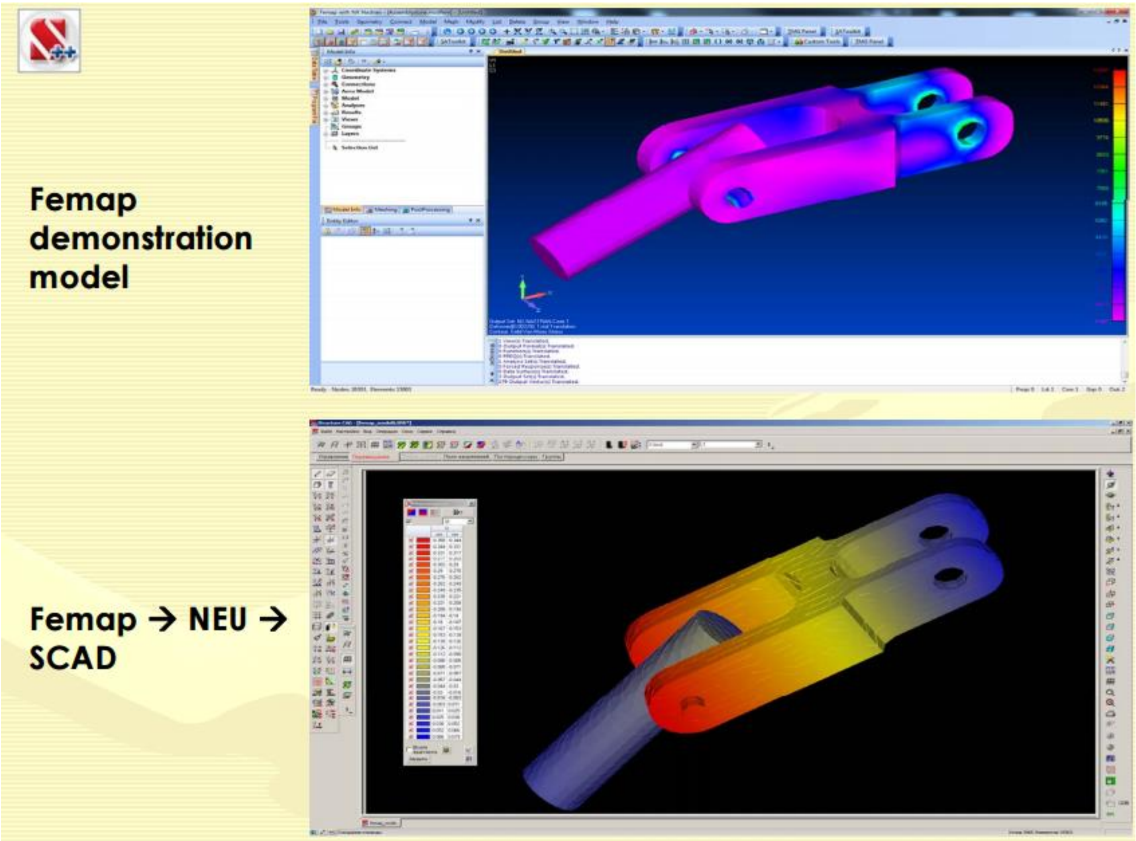


Figure 12. Import from Femap into SCAD Office [9]

3.2.2 Export

SCAD offers a possibility to export created drawings and calculations into several formats to cooperate with other software and users. They are stated as follows:

1. IFC

While exporting rod elements, depending on the position in space is given by the form of the rod - the beam (horizontal element) or column (the element that occupies any position in a space apart from the vertical). In terms of the IFC, this means IfcBeam or IfcColumn. Information about the coordinates of the beginning and end of the rod, the orientation of the local axes and cross-sectional shape also transmits. For parametrically defined cross-sections its form and dimensions are exported. Profiles of rolled metal are transmitted in the form of polygons. For the type of stiffening rod, which is incompatible with IFC format (for example, the numerical description) created fictitious transverse section in the form of a square of 10 cm from the side [7].

2. SDNF implies the use of cross-sections of the rods only in the form of rolled metal profiles; for export, a special table of compliance between the bases of rolled metal SCAD and Tekla software; for SCAD bar elements with cross-section shape different from the rolled metal only the information about the coordinates of the beginning and end of the final element is exported, pins and rigid inserts [7].

3. While exporting to GMSH, the user can choose a type of creating msf-file: text (ASCII) or Binary. Information about nods coordinates and geometry of rod, plate and volumetric elements is exported [7].

4. FNFF (Femap neutral file format) exports data on:

- Cross-section shape – area, perimeter, moments of inertia, moment of inertia for torsion, conditional cut areas.
- Physical properties - type: isotropic or orthotropic; Young's modulus, Poisson's ratio, the shear modulus, the coefficient of thermal expansion, density.

- Loads - nodal forces and moments, transfer of the trapezoid and uniformly distributed loads on bars (local or global coordinate systems), trapezoidal transmission and uniformly distributed load on plates (local or global coordinate systems); for loads, defined in the local coordinate system, for export they are transformed into a global coordinate system [7].

Figure 13 shows an example of conversion between the programs.

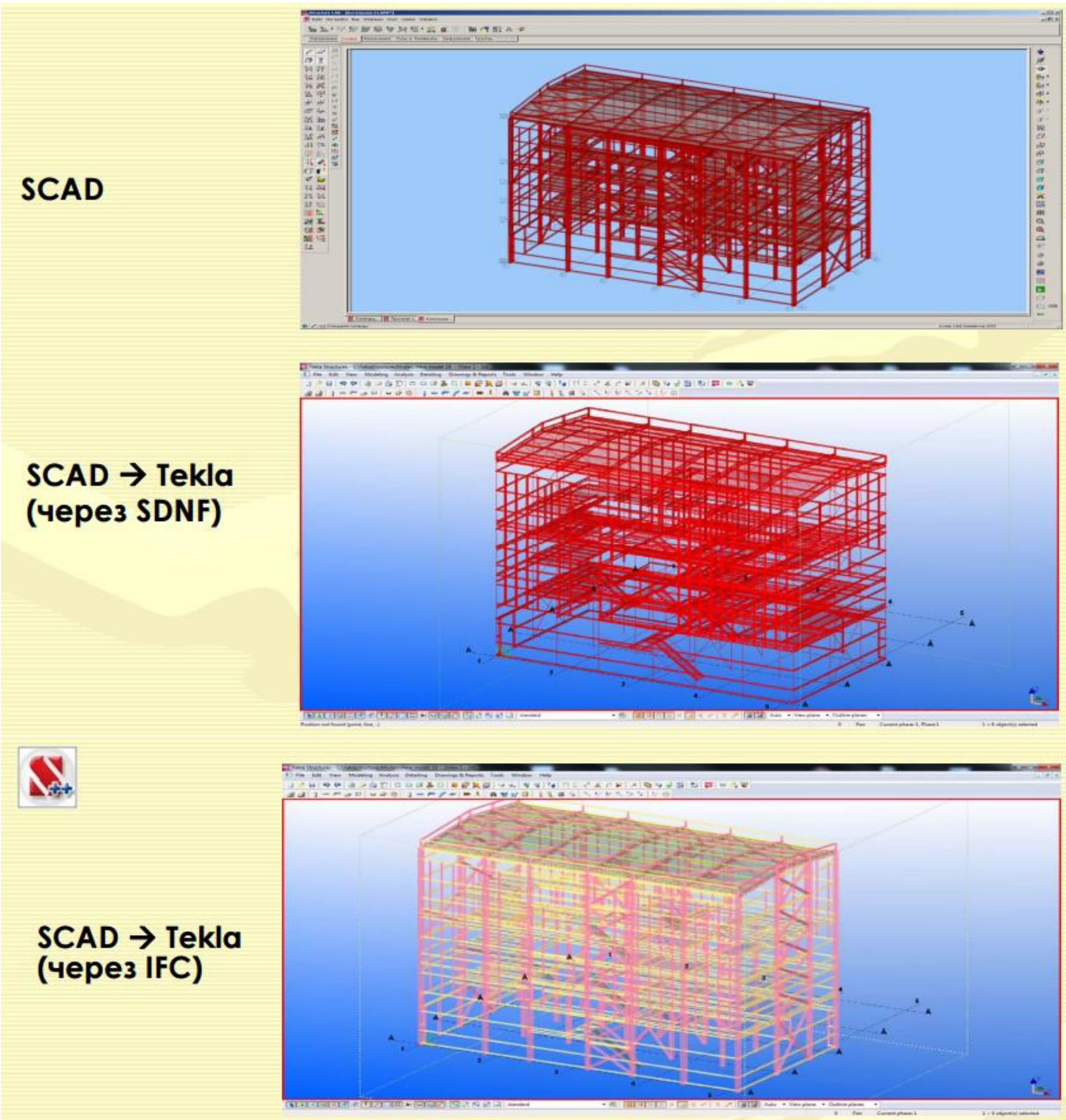


Figure 13. Export from SCAD into Tekla [9]

3.2.3 Export limitations

1. IFC does not provide the presence of three-dimensional elements. It does not have a way to add the most of loads available in SCAD, so the loads cannot be exported. IFC does not have a standard method of adding physical properties to the material, thus, the materials cannot be exported [7].
2. SDNF does not allow to transmit information about special finite elements, load export is not realized (because SDNF considers only one kind of trapezoid load), does not provide material setting, therefore, materials are also not exported [7].
3. GMSH does not support such data as loads, pins, physical material properties; therefore, they are not exported [7].
4. FNFF does not provide information about loads (moments), does not consider transmitting the load on a part of a rod, therefore, it is assumed that the load acts on the whole rod length; evenly distributed loads are ignored [7].

3.3 Description of the structure

The current study was accomplished on the base of the previous study which is shown on Figures 14-17. It was completed in 3D AutoCAD.

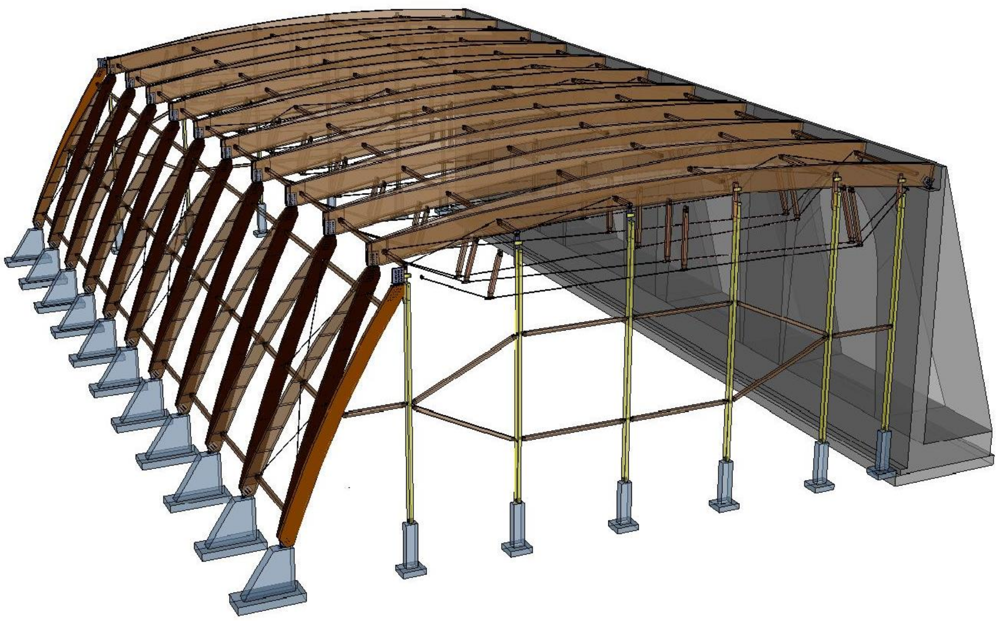


Figure 14. Three-dimensional representation of timber structure

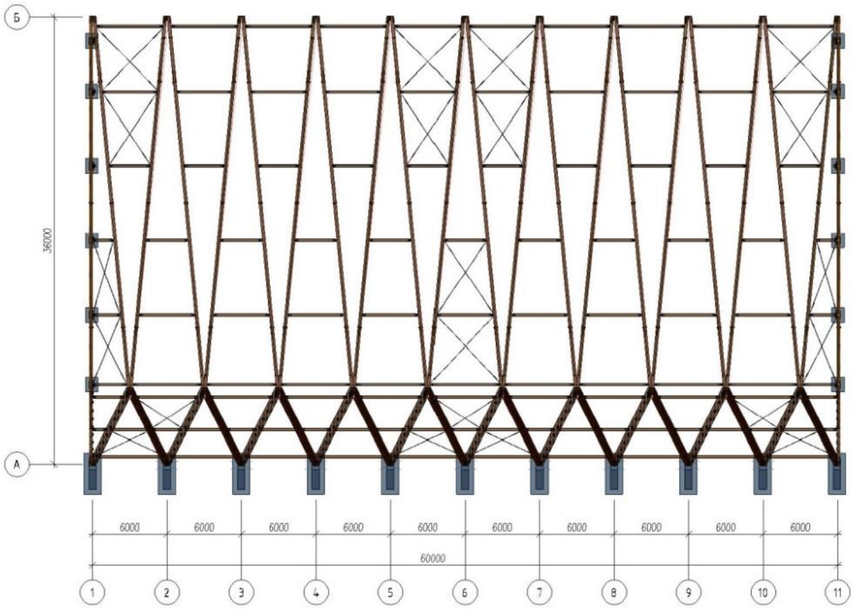


Figure 15. Plan of timber structure

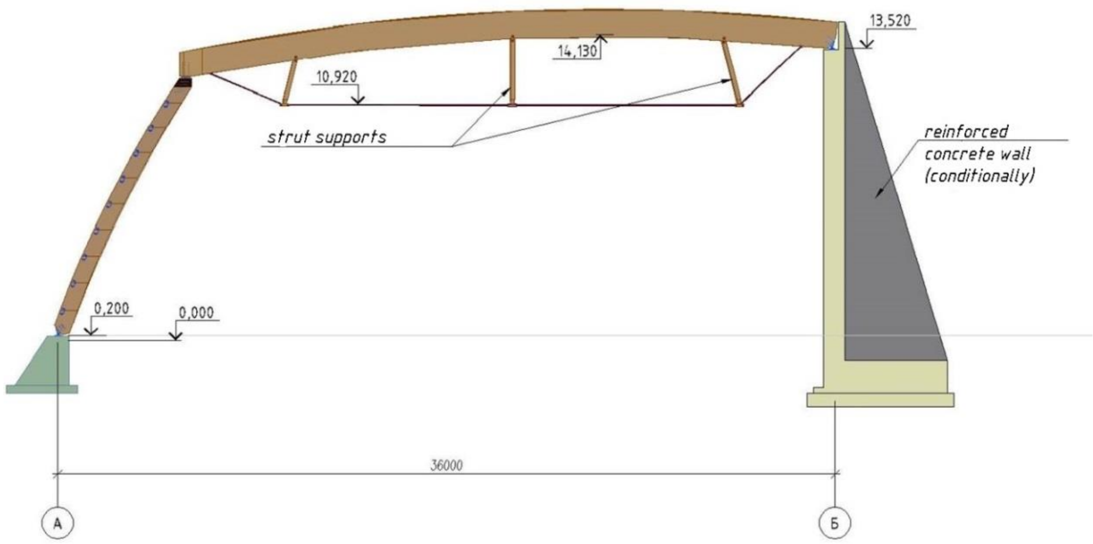


Figure 16. Section of timber structure

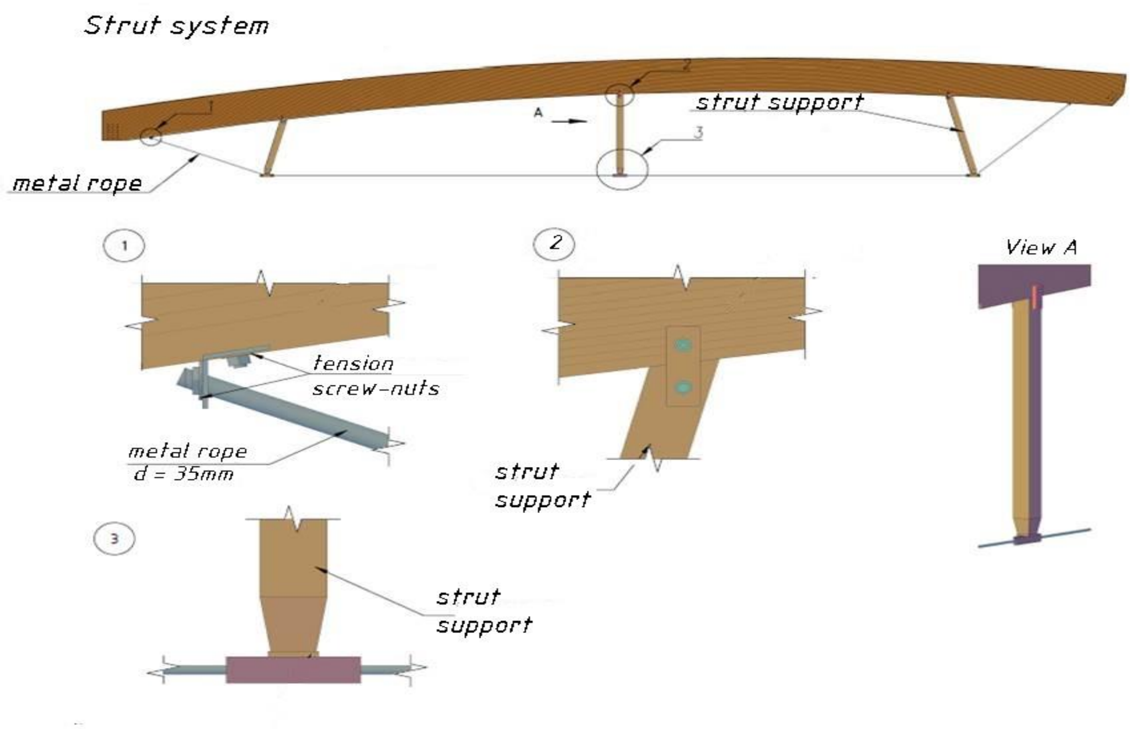


Figure 17. Detailed representation of the beam

The initial structure was inspired by a Water Sports Palace in Kazan, Russia which is shown on Figure 18.

Figure 18. Water sports palace in Kazan, Russia [9]

For the study the library with dimensions 60 x 36 m was chosen. The distance of 36 m is spanned by a frame according to the architectural expressiveness. In some way this frame can be considered as a wooden glulam beam on wooden support, which will be later turned into metal and wood composite truss. Such a method as shown on Figure 19 is applied in order to provide a more open space for the building content, for instance, bookshelves or in case of limited sizes of length of the building. The curved beam spans a distance of 30 m. The frames among themselves are connected by braces on the whole surface to provide stability of the whole building structure and for future installation of the covering.

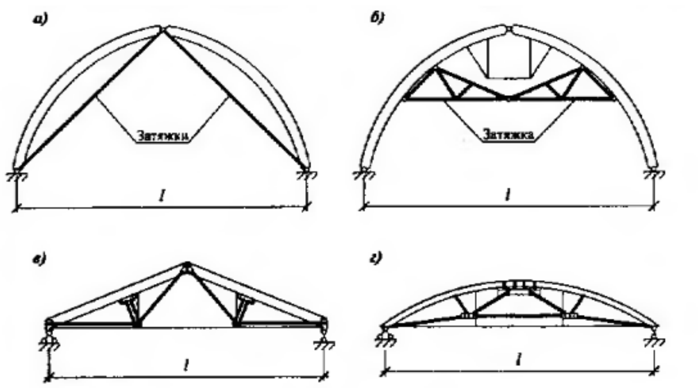


Рис. 14.19. Усиление арок:
а, *б* – введение затяжек; *в* – превращение металлодеревянной арки в ферму;
г – превращение арки в металлодеревянную ферму

Figure 19. Strengthening arches: a, b (top row) – introducing tightening; c, d (bottom row) – turning an arch into metal and wood composite truss [1].

This study implies calculations only of one beam from both constructive variants: the first variant is designed without adding struts into the timber structure; the second variant will contain it. The latter is designed on the base of an example from a school-book by Shmidt A. B. which is presented on Figure 20.

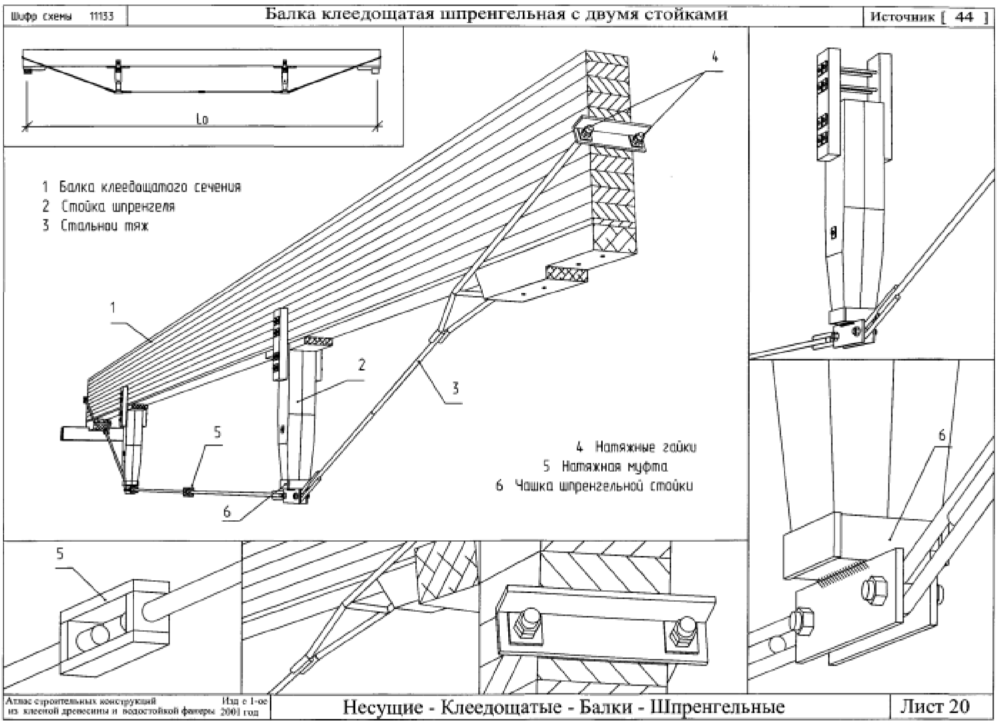


Figure 20. Beam with struts with two supports. [11]

Figure 21 shows the appearance of the geometrical models which were used in calculations. For a wall with buttress in the model there is no detailed elaboration, the anchorage is set at the ends of the structure. The additional support in the middle is used to avoid the sag of the metal rope because of its big length.

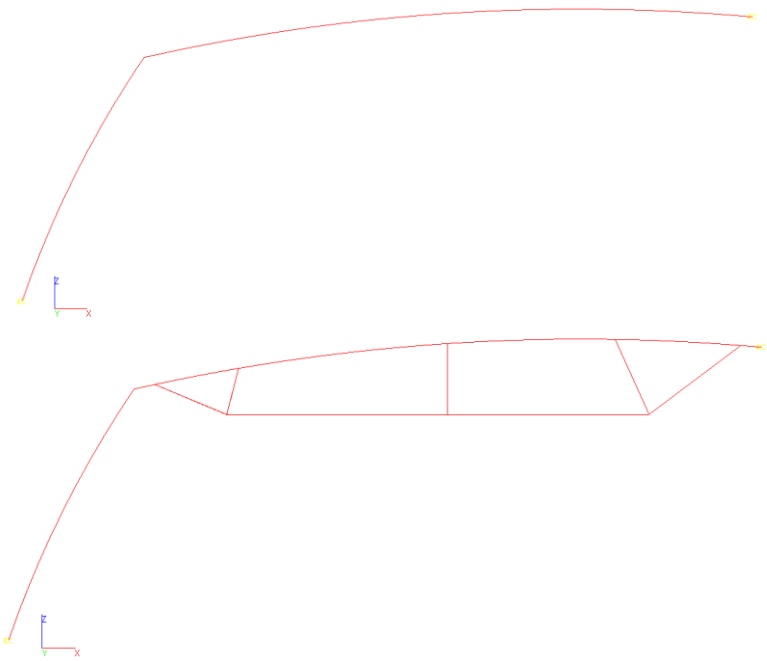


Figure 21. Geometrical models of the structures (from the top – scheme 1, scheme 2).

3.4 General information according to the Finland National Annex

Collection of loads was provided according to the chosen territory of Lappeenranta, Finland. For the structure service class 1 is used as a structure in heated rooms or in corresponding moisture conditions. Material – Glulam GL28h (according to EN 14080), γ_M = 1,2 [12]. Load duration class 'medium-term' (snow) k_{mod} = 0,8. If the wind load is not the leading variable action, it may be disregarded in the loading combinations of serviceability limit states. [12]

3.5 Load collection

3.5.1 Snow loads

The minimum snow load according to Finland National Annex to standard SFS-EN 1991-1-3: 2003, snow load for the city of Lappeenranta equals 2,5 kN/m^2. With normal topography for load arrangements C_e = 1,0.

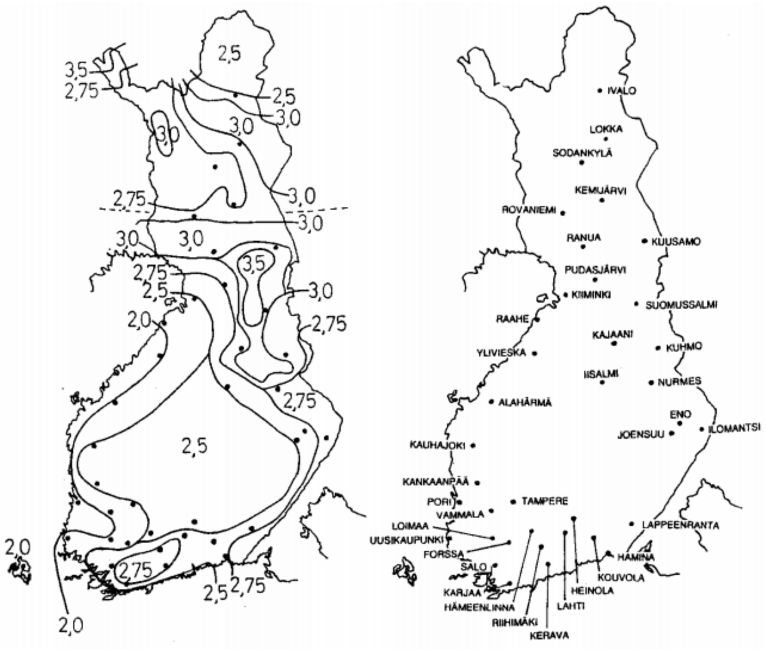


Figure 4.1 (FI): Snow loads on the ground in Finland. When the construction site is located in an area where the value is not constant, the intermediate values are obtained by linear interpolation in proportion to distances from the closest curves.

Figure 22. Snow loads in Finland [13]

Snow load on roofs determines as

$$S = \mu \cdot C_e \cdot C_t \cdot S_k = 0{,}87 \cdot 1{,}0 \cdot 1{,}0 \cdot 2{,}5 = 2{,}175 \mathrm{kN/m^2} = 0{,}22\ \mathrm{T/m^2} \qquad (1)$$

μ = 0,87 – the snow load shape coefficient (determined according to the Figure 23);

S_k = 2,5 kN/m^2 – the characteristic value of snow load on the ground;

C_e = 1,0 – the exposure coefficient;

C_t = 1,0 – the thermal coefficient.

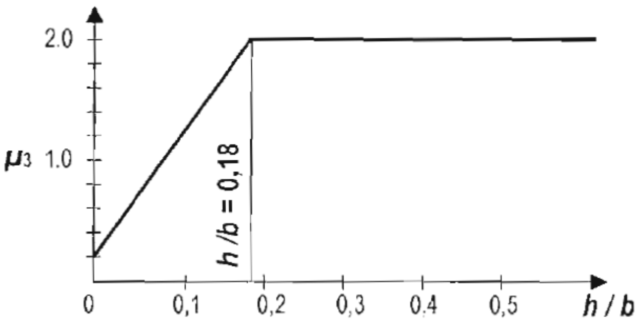


Figure 23. Recommended snow load shape coefficient for cylindrical roofs of differing rise to span ratios (for β < 60°) [13]

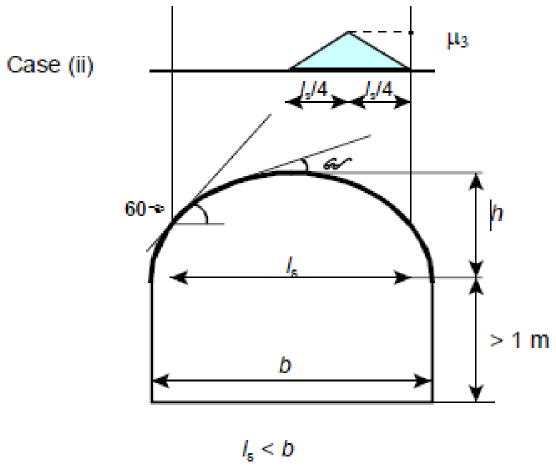


Figure 24. Snow load shape coefficient for cylindrical roof [13]

Snow with spacing of 6 m:

$$S = 2{,}175 \cdot 6 = 13{,}05\,\mathrm{kN/m} = 1{,}33\,\mathrm{T/m} \quad (2)$$

After applying the determined snow load to the geometrical scheme, the distribution is shown on Figure 25. The view is the same on both schemes.

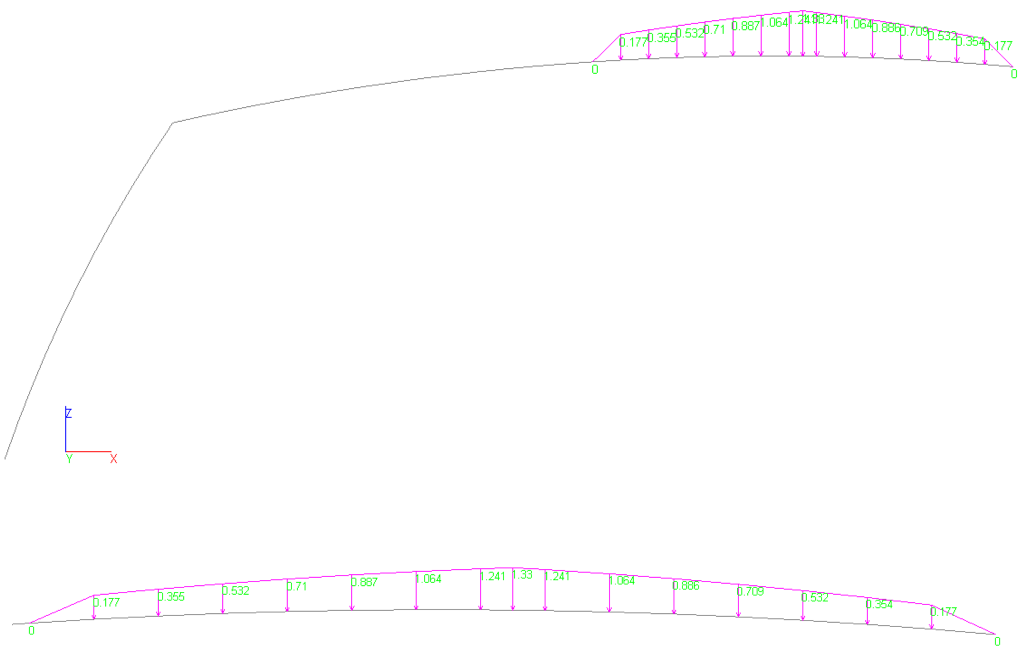


Figure 25. Distribution of the snow load (from the top – the whole distribution according to the National Annex; scaled image).

3.5.2 Wind loads

According to Finland National Annex to standard SFS-EN 1991-1-4, for fundamental value of the basic wind velocity for sea areas the value $v_{b,o}$ = 22 m/s [14].

- The basic wind velocity:

$$V_b = C_{dir} \cdot C_{season} \cdot V_{b,o} = 1{,}0 \cdot 1{,}0 \cdot 22 = 22 \text{ m/s} \quad (3)$$

where V_b – is the basic wind velocity;

$V_{b,o}$ – the fundamental value of the basic wind velocity;

C_{dir} – is the directional factor;

C_{season} – is the season factor.

- Mean wind velocity

$$V_m(z) = C_r(z) \cdot C_o(z) \cdot V_b = 1{,}24 \cdot 1 \cdot 22 = 27{,}28 \text{ m/s} \quad (4)$$

$V_m(z)$ – the mean wind velocity;

$C_r(z)$ – the roughness factor;

$C_o(z)$ – the orography factor = 1,0.

- Terrain roughness

Z_{min} = 1 m – the minimum height based on terrain category;

Z_{max} = 200 m – the maximum height based on terrain category;

Z_o = 0,01 m – the roughness length;

Z = 14 350 mm – height above ground.

Terrain factor depending on the roughness length Z_o:

$$k_r = 0{,}19 \cdot \left(\frac{z_o}{z_{o,II}}\right)^{0{,}07} = 0{,}19 \cdot \left(\frac{0{,}01}{0{,}05}\right)^{0{,}07} = 0{,}17 \quad (5)$$

The roughness factor:

$$C_r(z) = k_r \cdot \ln\left(\frac{z}{z_o}\right) = 0{,}17 \cdot \ln\left(\frac{14{,}35}{0{,}01}\right) = 1{,}24 \quad (6)$$

- Wind turbulence

The turbulence intensity $I_v(z)$ at height z is defined as the standard deviation of the turbulence divided by the mean wind velocity. The standard deviation of the turbulence σ_v may be determined as

$$\sigma_v = k_r \cdot V_b \cdot k_I = 0{,}17 \cdot 22 \cdot 1{,}0 = 3{,}74 \text{ m/s} \quad (7)$$

$$l_v(z) = \frac{\sigma_v}{V_m(z)} = \frac{3{,}74}{27{,}28} = 0{,}14 \quad (8)$$

- Peak velocity pressure

The peak velocity pressure $q_p(z)$ at height z, which includes mean and short-term velocity fluctuations, should be determined.

$$q_p(z) = [1 + 7l_v(z)]\frac{1}{2}pV_m^2(z) = [1 + 7 \cdot 0{,}14] \cdot \frac{1}{2} \cdot 1{,}25 \cdot 27{,}28^2 = 0{,}921 \text{ kN/m}^3 \quad (9)$$

where ρ – air density, which depends on the altitude, temperature and barometric pressure to be expected in the region;

$C_e(z)$ – exposure factor, determined in (10)

$$C_e(z) = \frac{q_p(z)}{q_b} = \frac{921}{302{,}5} = 3 \tag{10}$$

q_b – the basic velocity pressure, determined in (11)

$$q_b = \frac{1}{2} \cdot \rho \cdot V_b^2 = \frac{1}{2} \cdot 1{,}25 \cdot 22^2 = 302{,}5\ \mathrm{N/m^2} \tag{11}$$

- Wind actions.

The calculations for the determination of wind actions is given in Table 2.

Table 2. The summary of calculations for the determination of wind actions

Parameter	Value
Peak velocity pressure q_p	
Basic wind velocity v_b	22 m/s
Reference height z_e	14,35 m
Terrain category	I
Characteristic peak velocity pressure q_p	0,921 kN/m^2
Turbulence intensity I_v	0,14
Mean wind velocity v_m	27,28
Orography coefficient $c_o(z)$	1,0
Roughness coefficient $c_r(z)$	1,24
Wind pressure	
External pressure coefficient $c_{pe\ (roof)}$. A: B: C:	(Figure 26) -0,156 -0,778 -0,511
External wind pressure: $w_e = q_p c_{pe\ (roof)}$ A: B: C:	(Figure 26) -0,144 kN/m^2 -0,717 kN/m^2 -0,471 kN/m^2
Internal pressure coefficient c_{pi}	-0,3
Internal wind pressure: $w_i = q_p c_{pi}$	-0,28 kN/m^2
Wind forces	
Structural factor: $c_s c_d$	1,0

- Wind pressure on external surfaces

$$w_e = q_p(z_{pe}) \cdot C_{pe} \tag{12}$$

where C_{pe} – the pressure coefficient for the external pressure.

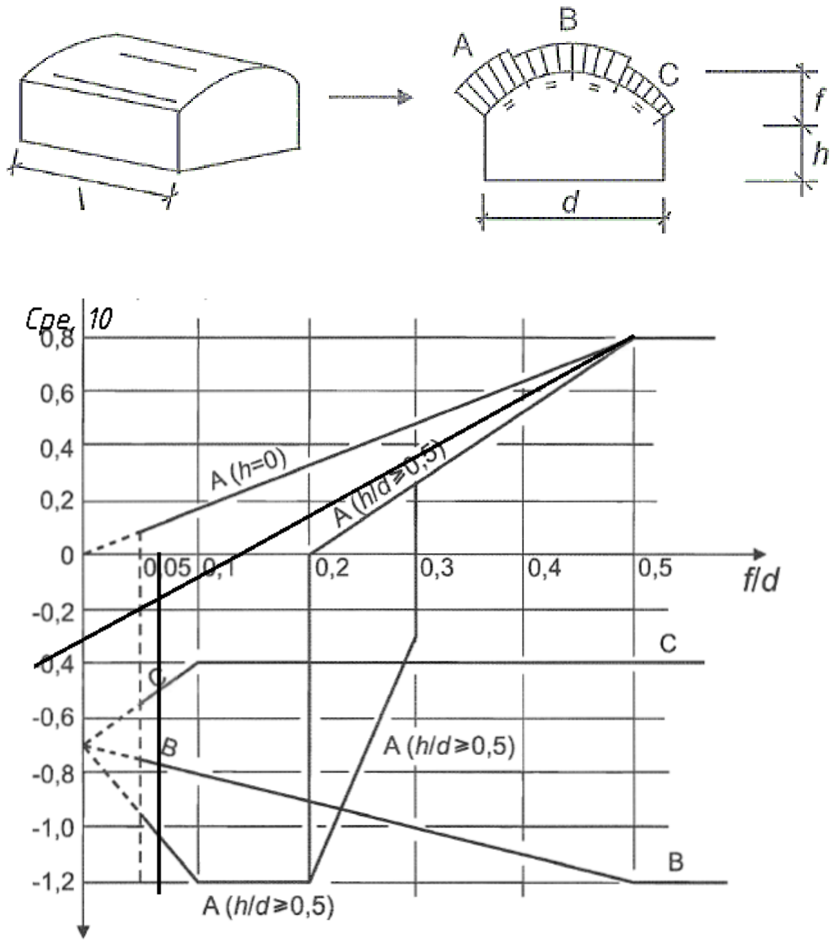


Figure 26. Determination of coefficient c_{pe} for vaulted roofs. [14]

- Wind pressure on internal surfaces

$$w_i = q_p(z_{pi}) \cdot C_{pi} = 0{,}921 \cdot (-0{,}3) = -0{,}28 \text{ kN/m2} \tag{13}$$

- Pressure on vertical walls:

e = 2h = 24 m < 36 m.

$C_{pe(D)}$ = +0,7 (Table 3)

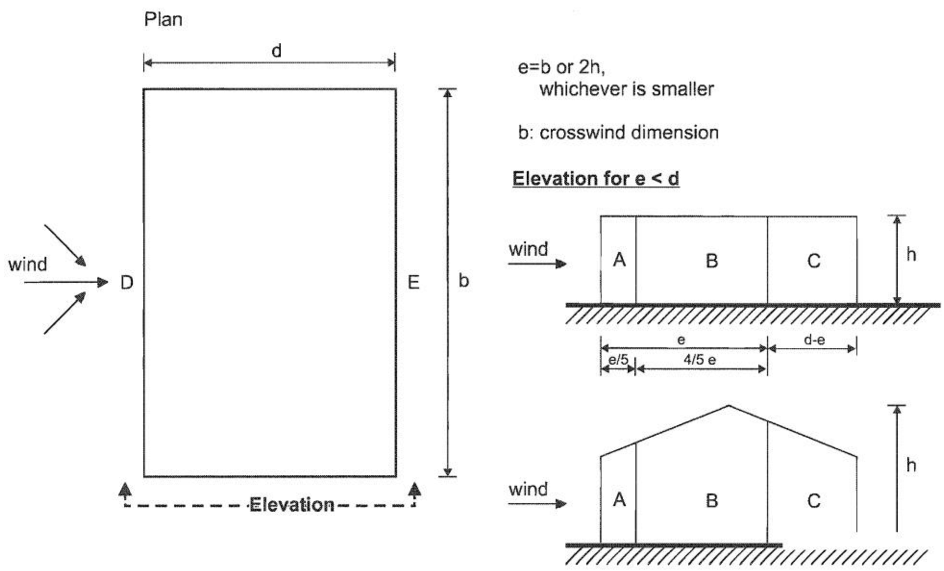


Figure 27. Pressure on vertical walls. [14]

Table 3. Recommended values of external pressure coefficients for vertical walls of rectangular plan buildings

Zone	A		B		C		D		E	
h/d	$c_{pe,10}$	$c_{pe,1}$	$c_{pe,10}$	$c_{pe,1}$	$c_{pe,10}$	$c_{pe,1}$	$c_{pe,10}$	$c_{pe,1}$	$c_{pe,10}$	$c_{pe,1}$
5	-1,2	-1,4	-0,8	-1,1	-0,5		+0,8	+1,0	-0,7	
1	-1,2	-1,4	-0,8	-1,1	-0,5		+0,8	+1,0	-0,5	
≤ 0,25	-1,2	-1,4	-0,8	-1,1	-0,5		+0,7	+1,0	-0,3	

- Wind forces

The wind loadings per unit length F_w (in kN/m):

$$F_w = (c_{pe} + c_{pi}) \cdot q_p \cdot \mathrm{S} \tag{14}$$

where s – spacing (influence width) = 6 m.

For wall the wind load equals:

$$F_w = (c_{pe} + c_{pi}) \cdot q_p \cdot \mathrm{S} = (0{,}7 - 0{,}3) \cdot 0{,}921 \cdot 6 = 2{,}21\,\mathrm{kN/m} = 0{,}225\,\mathrm{T/m} \tag{15}$$

For beam the wind load equals according to Figure 27:

- Zone A

$$F_w = (c_{pe} + c_{pi}) \cdot q_p \cdot S = (-0{,}156 - 0{,}3) \cdot 0{,}921 \cdot 6 = -2{,}52\,\mathrm{kN/m} = -0{,}257\ \mathrm{T/m} \quad (16)$$

- Zone B

$$F_w = (c_{pe} + c_{pi}) \cdot q_p \cdot S = (-0{,}778 - 0{,}3) \cdot 0{,}921 \cdot 6 = -5{,}957\,\mathrm{kN/m} = -0{,}607\ \mathrm{T/m} \quad (17)$$

- Zone C

$$F_w = (c_{pe} + c_{pi}) \cdot q_p \cdot S = (-0{,}511 - 0{,}3) \cdot 0{,}921 \cdot 6 = -4{,}482\,\mathrm{kN/m} = -0{,}457\ \mathrm{T/m} \quad (18)$$

The distribution of the wind load is shown on Figure 28. The view is the same on both schemes.

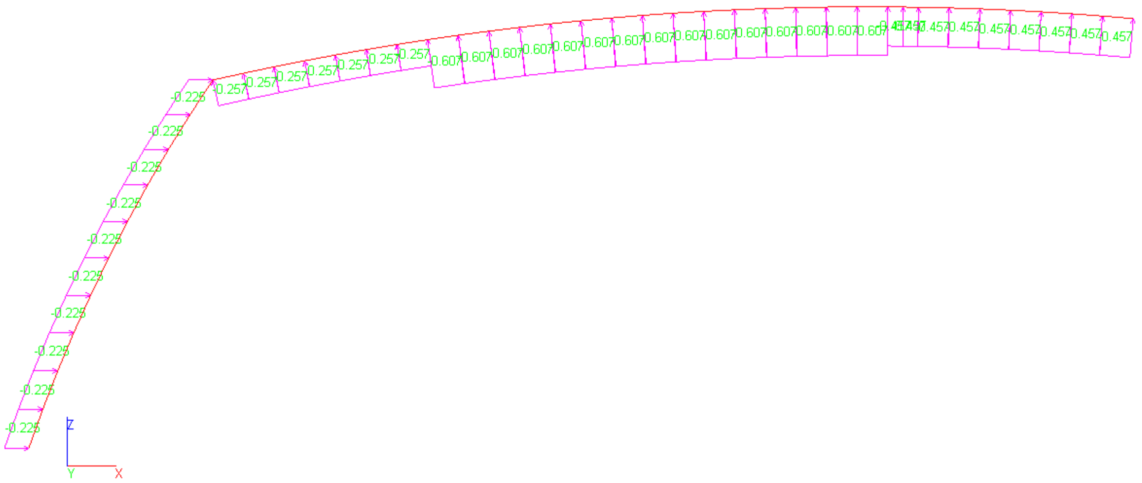


Figure 28. Distribution of the wind load

3.6 Constructional scheme 1

3.6.1 Properties:

b = 240 mm, *h* = h_{ap} = 1920 mm, *l* = 30 000 mm, *r* $_{in}$ = 101 400 mm, *t* = 40 mm.

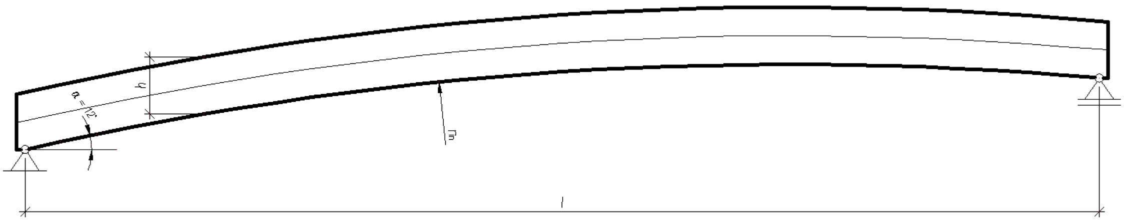


Figure 29. Dimensions of the beam

Table 4 shows characteristic strength and stiffness values in N/mm^2 and densities in kg/m^3 for selected strength classes of softwood glulam. [6]

Table 4. Characteristic strength and stiffness values, and densities for selected strength classes of softwood glulam

Property		Strength classes			
		GL28h	GL32h	GL28c	GL32c
Bending strength	$f_{m,k}$	28	32	28	32
Tension strength	$f_{t,0,k}$	19.5	22.5	16.5	19.5
Tension strength perp. to grain	$f_{t,90,k}$	0.45	0.50	0.40	0.45
Compression strength	$f_{c,0,k}$	26.5	29	24	26.5
Compression strength perp. to grain	$f_{c,90,k}$	3.0	3.3	2.7	3.0
Shear strength	$f_{v,k}$	3.2	3.8	2.7	3.2
Mean modulus of elasticity	$E_{0,mean}$	12600	13700	12600	13700
Lower 5-percentile modulus of elasticity	$E_{0,05}$	10200	11100	10200	11100
Mean modulus of elasticity perp. to grain	$E_{90,mean}$	420	460	390	420
Shear modulus	G_{mean}	780	850	720	780
Characteristic density [a]	ρ_k	410	430	380	410
Mean density [b]	ρ_{mean}	460	500	440	480
[a] Used for verifying mechanically fastened connections [b] Used for calculating self-weight of members					

3.6.2 SCAD calculations

Figure 30 shows a 3-D model of calculated structure created by SCAD automatically using the physical parameters given in the beginning.

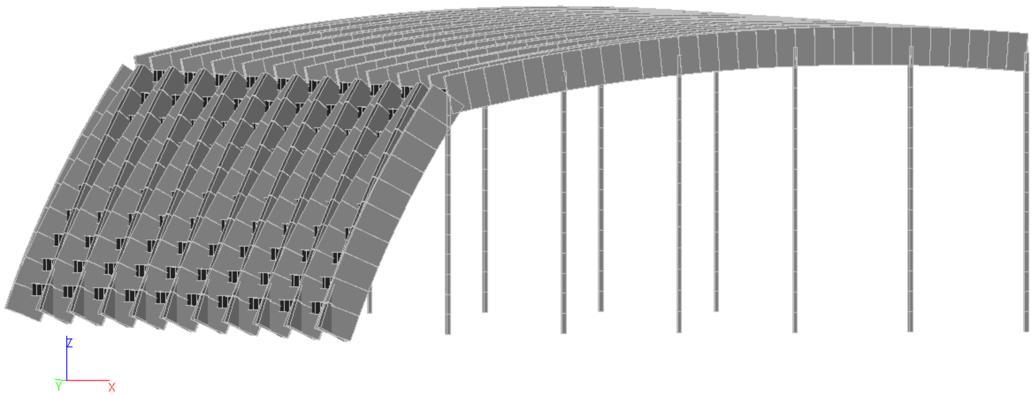


Figure 30. Three-dimensional model of the whole structure completed in SCAD

Dead weight is also calculated automatically by SCAD and represented in Figure 31. The beam is divided into one meter parts, so the dead weight is a distribution of load per one meter. The equation is as follows:

$$g = \frac{b \cdot h \cdot 1\text{m} \cdot \rho}{1\text{m}} = \frac{0.24 \cdot 1.92 \cdot 1\text{m} \cdot 0.46}{1\text{m}} = 0.212\ \text{T/m} \tag{19}$$

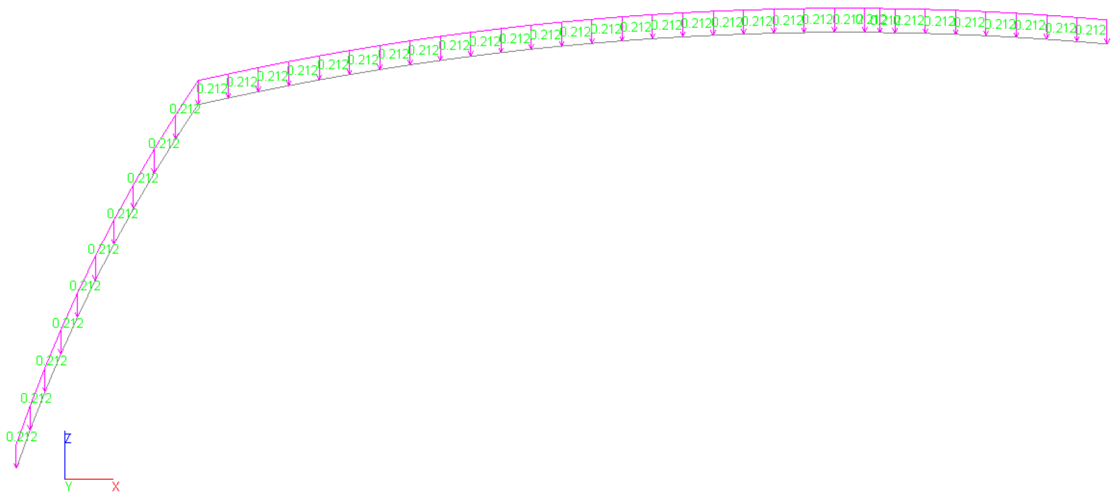


Figure 31. Dead weight

Figure 32 shows the chosen combination for future calculations due to being more crucial. The combination is based on an example of a curved beam calculation according to [2], where the coefficient for the dead load is 1,35 and for the snow load it is stated as 1,5.

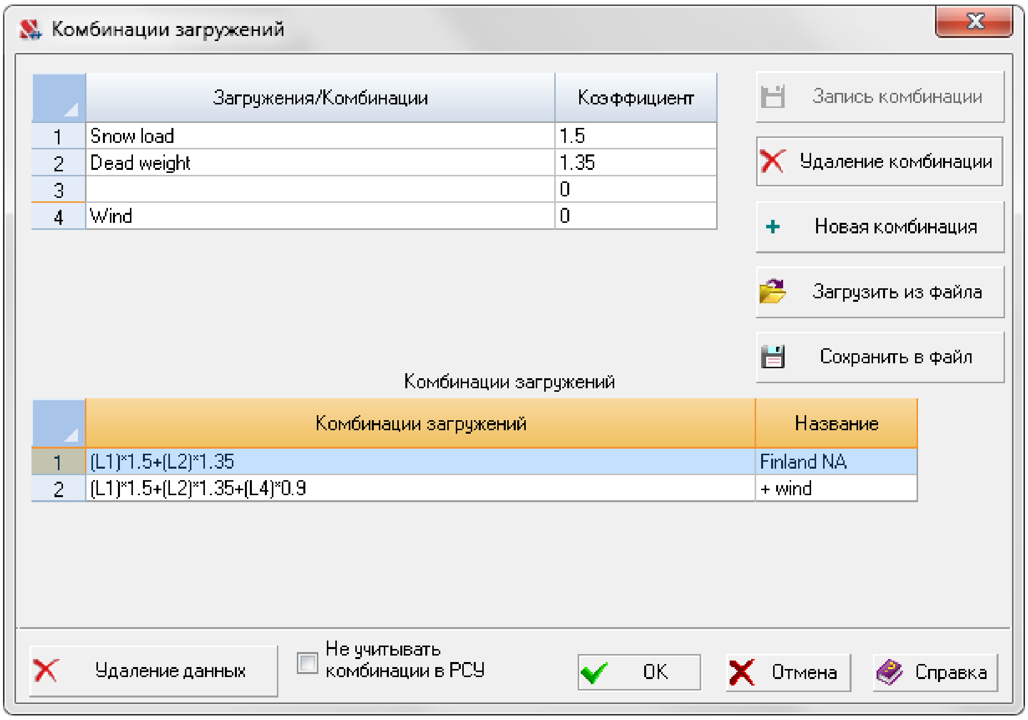
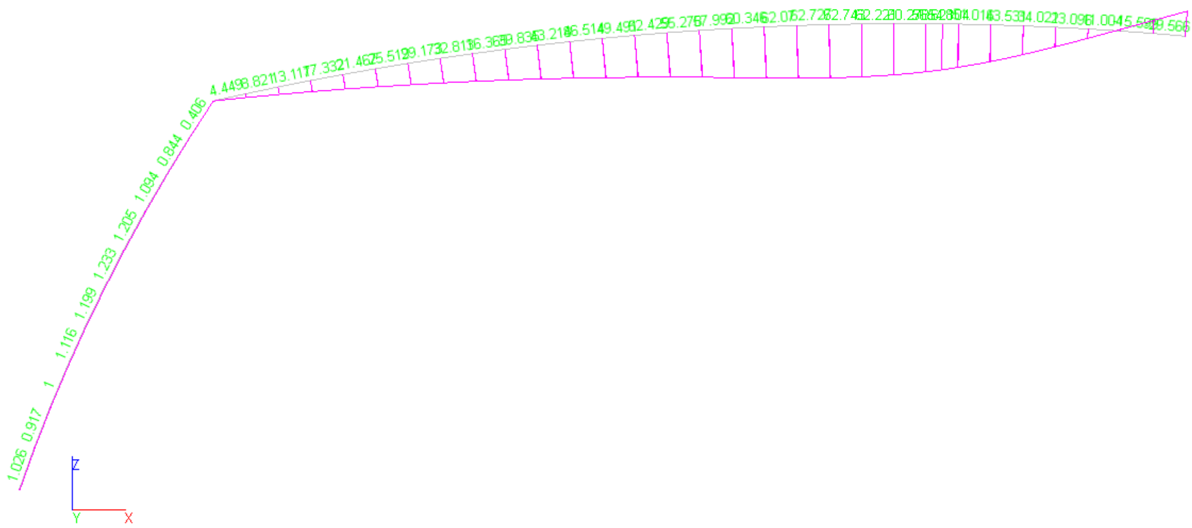


Figure 32. Load combination used for calculations

The same load combination was used in the next scheme. The received moment and forces distribution are presented in Figures 33 - 35.



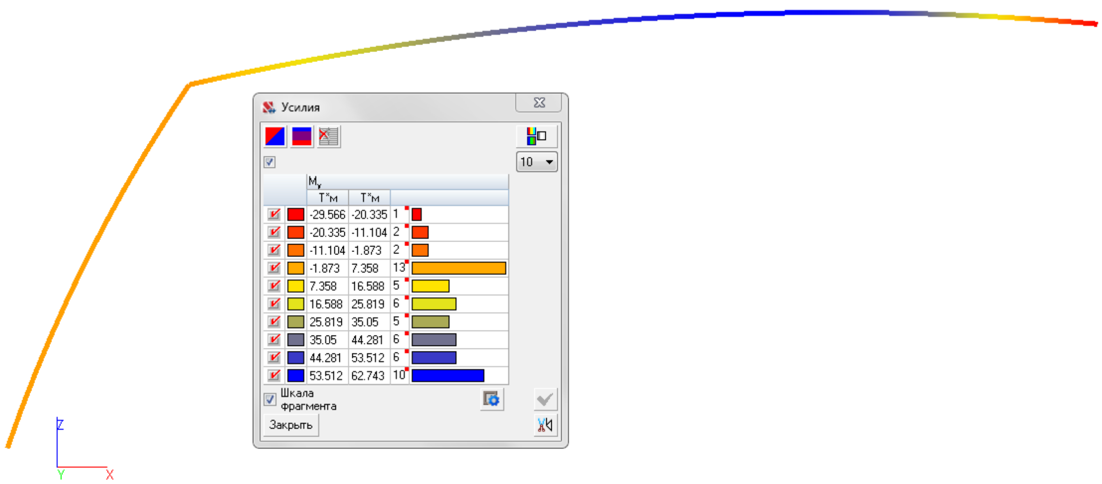


Figure 33. Distribution of the bending moment M in T·m (from the top – scheme with values; coloured indication)

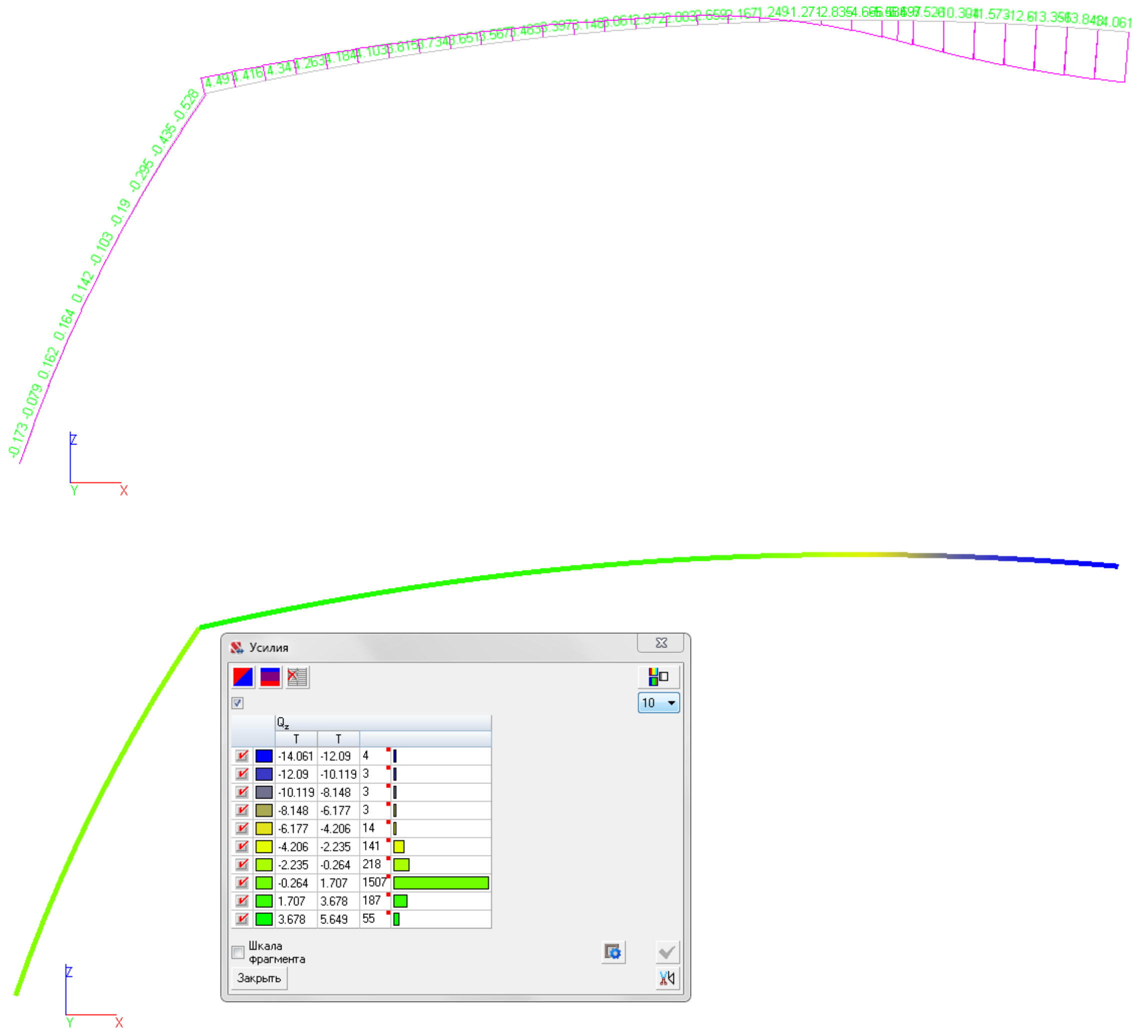


Figure 34. Distribution of the shear force Q in T (from the top – scheme with values; coloured indication)

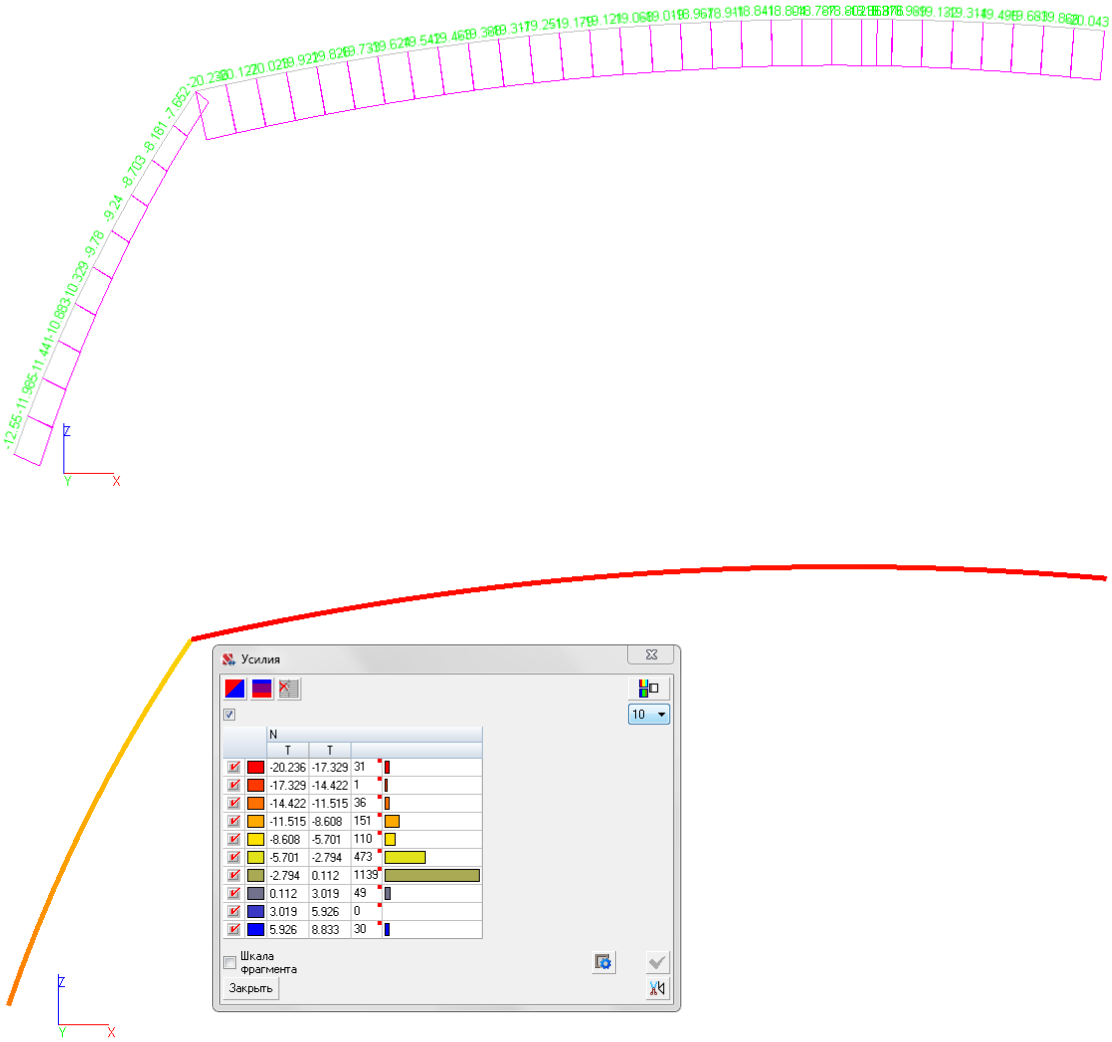


Figure 35. Distribution of the normal force N in T (from the top – scheme with values; colored indication)

3.6.3 Theoretical verification

<u>Design values:</u>

$$f_{m,d} = k_{mod} \cdot \frac{f_{m,k}}{\gamma_m} = 0{,}8 \cdot \frac{28{,}0}{1{,}2} = 18{,}67 \ \text{N/mm}^2 \tag{20}$$

$$f_{t,90,d} = k_{mod} \cdot \frac{f_{t,90,k}}{\gamma_m} = 0{,}8 \cdot \frac{0{,}45}{1{,}2} = 0{,}3 \ \text{N/mm}^2 \tag{21}$$

$$M_{max,d} = M_{ap,d} = \ 62{,}73 \ \cdot 9{,}81 = 615{,}4 \ \text{kN·m} \tag{22}$$

<u>Bending stress:</u>

$$\sigma_{m,max,d} = \sigma_{m,ap,d} = k_l \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} \tag{23}$$

$$k_l = k_1 + k_2 \cdot \left(\frac{h_{ap}}{r}\right) + k_3 \cdot \left(\frac{h_{ap}}{r}\right)^2 + k_4 \cdot \left(\frac{h_{ap}}{r}\right)^3 \tag{24}$$

$$k_1 = 1 + 1{,}4 \cdot \tan \alpha_{ap} + 5{,}4 \cdot \tan^2 \alpha_{ap} = 1 + 1{,}4 \cdot \tan 0 + 5{,}4 \cdot \tan^2 0 = 1 \tag{25}$$

$$k_2 = 0{,}35 - 8 \cdot \tan \alpha_{ap} = 0{,}35 \tag{26}$$

$$k_3 = 0{,}6 + 8{,}3 \cdot \tan \alpha_{ap} - 7{,}8 \cdot \tan^2 \alpha_{ap} = 0{,}60 \tag{27}$$

$$k_4 = 6 \cdot \tan^2 \alpha_{ap} = 0 \tag{28}$$

$$r = r_{in} + 0{,}5 \cdot h_{ap} = 101\,400 + 960 = 102\,360 \text{ mm} \tag{29}$$

$$\begin{aligned} k_l &= k_1 + k_2 \cdot \left(\frac{h_{ap}}{r}\right) + k_3 \cdot \left(\frac{h_{ap}}{r}\right)^2 + k_4 \cdot \left(\frac{h_{ap}}{r}\right)^3 \\ &= 1{,}0 + 0{,}35 \cdot \left(\frac{1920}{102\,360}\right) + 0{,}6 \cdot \left(\frac{1920}{102\,360}\right)^2 = 1{,}007 \end{aligned} \tag{30}$$

$$\begin{aligned} \sigma_{m,max,d} = \sigma_{m,ap,d} &= k_l \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} = 1{,}007 \cdot \frac{6 \cdot 615{,}4}{240 \cdot 1920^2} \\ &= 4{,}2 \text{ N/mm}^2 \end{aligned} \tag{31}$$

For curved beams:

$$\frac{r_{in}}{t} = \frac{101\,200}{40} = 2530 > 240, \;\; k_r = 1{,}0 \tag{32}$$

$$\frac{\sigma_{m,ap,d}}{k_r \cdot f_{m,d}} = \frac{4{,}2}{1{,}0 \cdot 18{,}67} = 0{,}23 < 1{,}0 \tag{33}$$

3.6.4 Tension stresses perpendicular to grain

$$\sigma_{t,90,ap,d} = k_p \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} \tag{34}$$

$$k_p = k_5 + k_6 \cdot \left(\frac{h_{ap}}{r}\right) + k_7 \cdot \left(\frac{h_{ap}}{r}\right)^2 \tag{35}$$

$$k_5 = 0{,}2 \cdot \tan\alpha_{ap} = 0 \tag{36}$$

$$k_6 = 0{,}25 - 1{,}5 \cdot \tan\alpha_{ap} + 2{,}6 \cdot \tan^2\alpha_{ap} = 0{,}25 \tag{37}$$

$$k_7 = 2{,}1 \cdot \tan\alpha_{ap} - 4 \cdot \tan^2\alpha_{ap} = 0 \tag{38}$$

$$k_p = k_5 + k_6 \cdot \left(\frac{h_{ap}}{r}\right) + k_7 \cdot \left(\frac{h_{ap}}{r}\right)^2 = 0{,}25 \cdot \left(\frac{1920}{102\ 360}\right) = 0{,}005 \tag{39}$$

$$\sigma_{t,90,ap,d} = k_p \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} = 0{,}005 \cdot \frac{6 \cdot 615{,}4}{240 \cdot 1920^2} = 0{,}02\ \mathrm{N/mm^2} \tag{40}$$

With the reference volume V_o = 0,01 m^3 and

$$V = \frac{\beta \cdot \pi}{180} \cdot b \cdot \left(h_{ap}^2 + 2 \cdot r_{in} \cdot h_{ap}\right) = \frac{12 \cdot \pi}{180} \cdot 0{,}24 \cdot (1{,}920^2 + 2 \cdot 101{,}4 \cdot 1{,}920)$$
$$= 19{,}75\ \mathrm{m^3} \tag{41}$$

the volume factor is:

$$k_{vol} = \left(\frac{V_o}{V}\right)^{0.2} = \left(\frac{0{,}01}{19{,}75}\right)^{0.2} = 0{,}22 \ \text{ and} \tag{42}$$

k_{dis} = 1,4 for curved beams

$$\frac{\sigma_{t,90,ap,d}}{k_{dis} \cdot k_{vol} \cdot f_{t,90,d}} = \frac{0{,}02}{1{,}4 \cdot 0{,}22 \cdot 0{,}3} = 0{,}21 < 1 \tag{43}$$

3.6.5 Shear force

Maximum shear force value from the calculation equals Q = -14,061 T.

Static cross-section moment:

$$S = h^2 \cdot b/6 = 192^2 \cdot 24/6 = 147456\ \mathrm{sm^3} \tag{44}$$

Inertia moment of the cross-section:

$$I = h^3 \cdot b/12 = 192^3 \cdot 24/12 = 14155776\ \mathrm{sm^4} \tag{45}$$

Shear, design value:

$$f_{v,d} = k_{mod} \cdot \frac{f_{v,k}}{\gamma_m} = 0{,}8 \cdot \frac{3{,}2}{1{,}2} = 2{,}13 \ \mathrm{N/mm^2} \tag{46}$$

Stress equals:

$$\tau = \frac{\mathrm{Q} \cdot S}{I \cdot b} = \frac{-14{,}061 \cdot 147456}{14155776 \cdot 24} = -6{,}1\frac{\mathrm{kg}}{\mathrm{sm^2}} = -0{,}6\frac{\mathrm{N}}{\mathrm{mm^2}} \leq f_{v,d} = 2{,}1\frac{\mathrm{N}}{\mathrm{mm^2}} \tag{47}$$

3.6.6 Combined tension perpendicular to grain and shear force

1. Maximum shear force value and bending moment value at this point:

$$\frac{\tau_d}{f_{v,d}} + \frac{\sigma_{t,90,d}}{k_{dis} \cdot k_{vol} \cdot f_{t,90,d}} = \frac{0{,}6}{2{,}13} + \frac{0{,}01}{1{,}4 \cdot 0{,}22 \cdot 0{,}3} = 0{,}39 \leq 1 \tag{48}$$

$$\sigma_{t,90,ap,d} = k_p \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} = 0{,}005 \cdot \frac{6 \cdot 29{,}57}{0{,}24 \cdot 1{,}92^2} = 0{,}01 \ \mathrm{N/mm^2} \tag{49}$$

2. Maximum bending moment value and shear force value at this point:

$$\frac{\tau_d}{f_{v,d}} + \frac{\sigma_{t,90,d}}{k_{dis} \cdot k_{vol} \cdot f_{t,90,d}} = \frac{0{,}05}{2{,}13} + \frac{0{,}02}{1{,}4 \cdot 0{,}22 \cdot 0{,}3} = 0{,}24 \leq 1 \tag{50}$$

$$\tau = \frac{\mathrm{Q} \cdot S}{I \cdot b} = \frac{1{,}27 \cdot 147456}{14155776 \cdot 24} = 0{,}55\frac{\mathrm{kg}}{\mathrm{sm^2}} = -0{,}05\frac{\mathrm{N}}{\mathrm{mm^2}} \leq f_{v,d} = 2{,}1\frac{\mathrm{N}}{\mathrm{mm^2}} \tag{51}$$

3.6.7 Compression perpendicular to grain

$$\sigma_{c,90,d} \leq k_{\mathrm{c,90}} \cdot f_{c,90,d} = 3{,}5 \ \mathrm{N/mm^2} \tag{52}$$

where:

$\sigma_{c,90,d}$ – the design compressive stress in the contact area perpendicular to the grain;

$f_{c,90,d}$ – the design compressive strength perpendicular to the grain;

$k_{c,90}$ – a factor taking into account the load configuration, possibility of splitting and degree of compressive deformation = 1,75.

$$f_{c,90,d} = 0{,}8 \cdot \frac{3}{1{,}2} = 2 \ \mathrm{N/mm^2} \tag{53}$$

$$\sigma_{c,90,d} = \frac{F_{c,90,d}}{A_{ef}} = \frac{14{,}61}{0{,}24(0{,}2 + 0{,}06)} = 234{,}1\frac{\mathrm{T}}{\mathrm{m^2}} = 2{,}3 \ \mathrm{N/mm^2} \leq 3{,}5\mathrm{N/mm^2} \tag{54}$$

$F_{c,90,d}$ – the support load;

A_{ef} – effective area at support point.

3.6.8 Lateral buckling

From bending:

Relative slenderness:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{crit}}} = \sqrt{\frac{28}{33,4}} = 0,92 \tag{55}$$

The critical bending stress for softwood with rectangular cross-section:

$$\sigma_{m,crit} = \frac{0,78 \cdot b^2}{h \cdot l_{ef}} \cdot E_{0,05} = \frac{0,78 \cdot 240^2}{1920 \cdot 0,9 \cdot 30000} \cdot 10200 = 8,84\ \mathrm{N/mm^2} \tag{56}$$

where

l_{ef} – effective beam length = 0,9·l;

$E_{0,05}$ – 5% value of elasticity modulus.

Combination of bending and compression:

$$\left(\frac{\sigma_{m,d}}{k_{crit} \cdot f_{m,d}}\right)^2 + \frac{\sigma_{c,d}}{k_{c,z} \cdot f_{c,d}} = \left(\frac{4.2}{1 \cdot 18,67}\right)^2 + \frac{3,15}{0,89 \cdot 17,67} = \ 0,25\ \leq 1 \tag{57}$$

$\sigma_{m,d}$ – the design bending stress;

$\sigma_{c,d}$ – the design compressive stress;

$f_{c,d}$ – the design compressive strength parallel to grain;

$k_{c,z}$ – given by expression:

$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z^2 - \lambda_{rel,z}^2}} = \frac{1}{0,84 + \sqrt{0,84^2 - 0,79^2}} = 0,89 \tag{58}$$

$$k_z = 0,5\left(1 + \beta_c\left(\lambda_{rel,z} - 0,3\right) + \lambda_{rel,z}^2\right) = 0,5 \cdot (1 + 0,1(0,79 - 0,3) + 0,79^2) = 0,84 \tag{59}$$

$\beta_c = 0,1$ – factor for members within the straightness limits for glued laminated timber;

$\lambda_{rel,z}$ – the relative slenderness:

$$\lambda_{rel,z} = \frac{\lambda_z}{\pi} \cdot \sqrt{\frac{f_{c,0,k}}{E_{0,05}}} = \frac{48,7}{\pi} \cdot \sqrt{\frac{26,5}{10200}} = 0,79 \tag{60}$$

$$\lambda_z = \frac{l_{ef}}{r} = \frac{27}{0,289 \cdot 1,92} = 48,7 \tag{61}$$

3.7 Constructional scheme 2

3.7.1 Properties

This variant has a metal part which is pretensioned with, for instance, 10 tones. The section of the metal rope is 35 mm. Thus, we can reduce the cross-section height of the beam to 1120 mm.

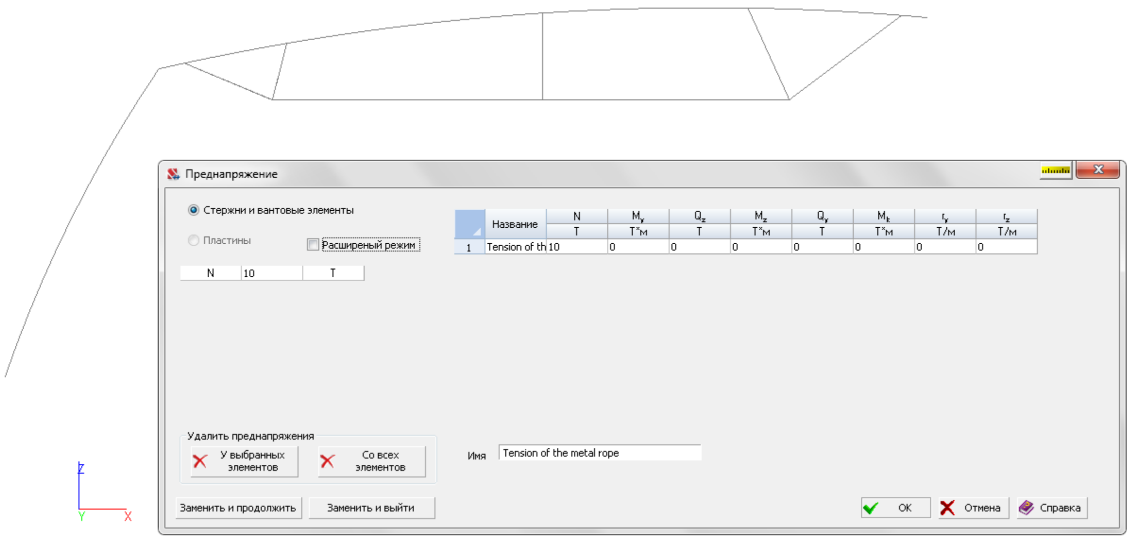


Figure 36. Tension of the metal rope

Second case parameters:

b = 240 mm, h = h_{ap} = 1120 mm, l = 30 000 mm, r_{in} = 101 400 mm, t = 40 mm.

3.7.2 SCAD calculations

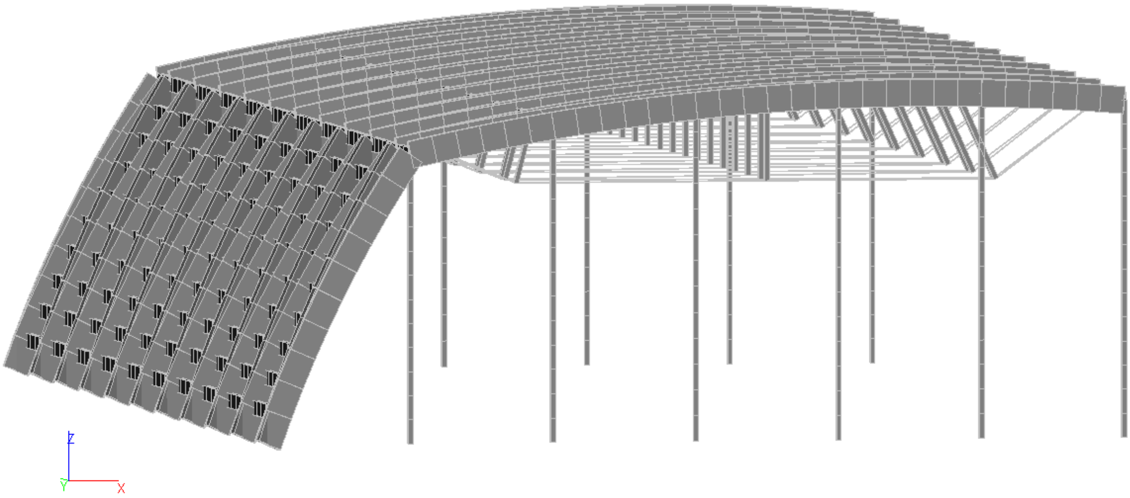


Figure 37. Three dimensional model of the whole structure

Dead weight is also calculated automatically by SCAD and represented in Figure 38. The beam is divided into one meter parts, so the dead weight is a distribution of load per one meter. The equation is as follows:

$$g = \frac{b \cdot h \cdot 1\text{m} \cdot \rho}{1\text{m}} = \frac{0.24 \cdot 1.12 \cdot 1\text{m} \cdot 0.46}{1\text{m}} = 0.124\ \text{T/m} \tag{62}$$

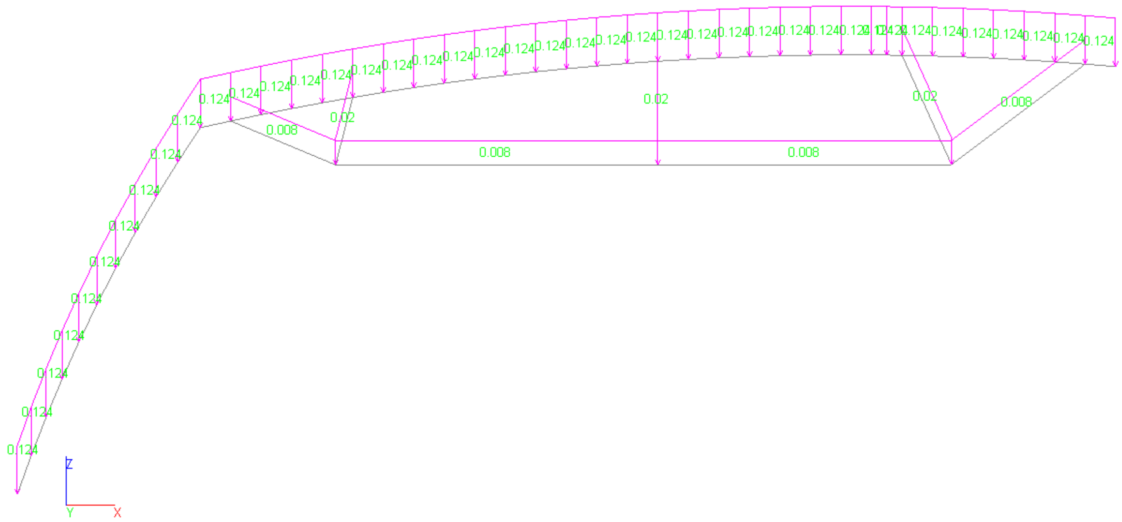
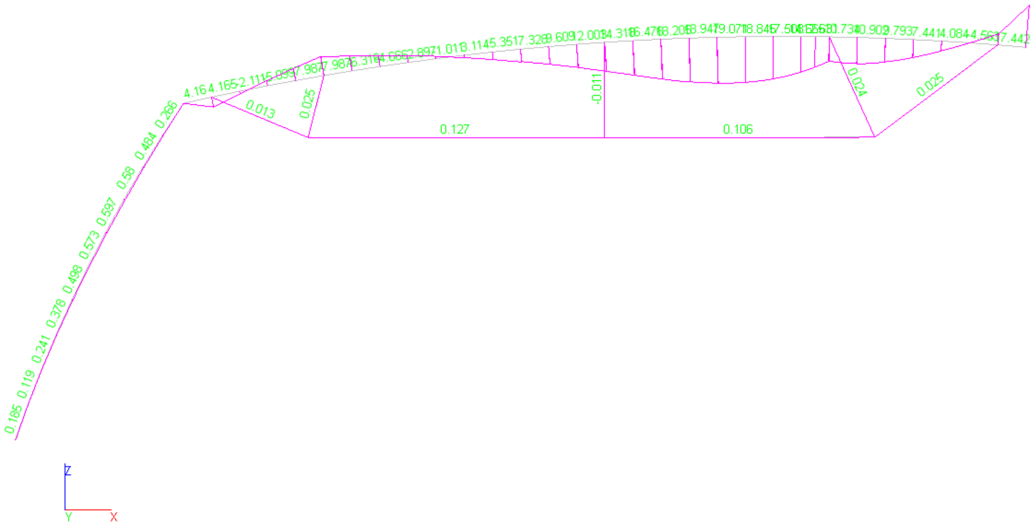


Figure 38. Dead weight

The received moment and forces distribution are presented in Figures 39 - 41.



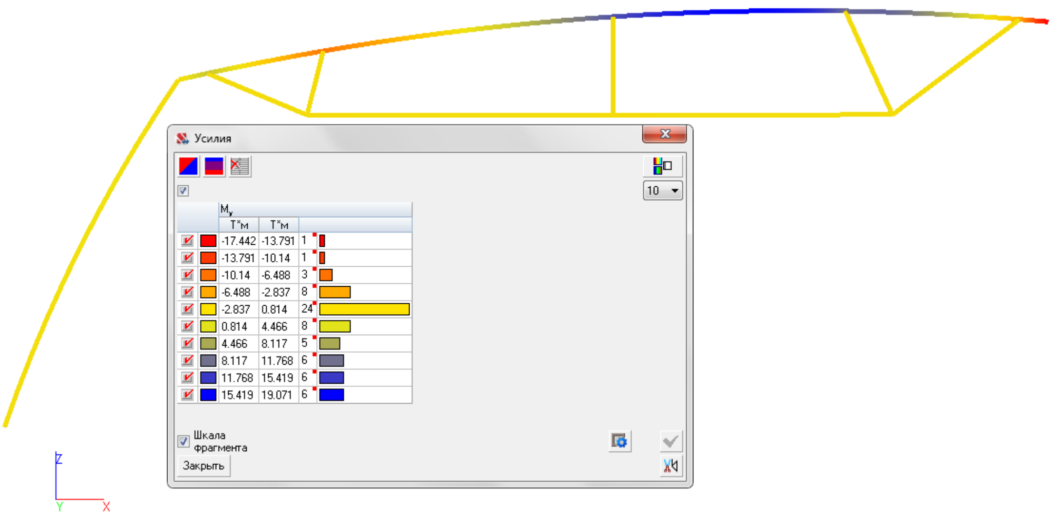
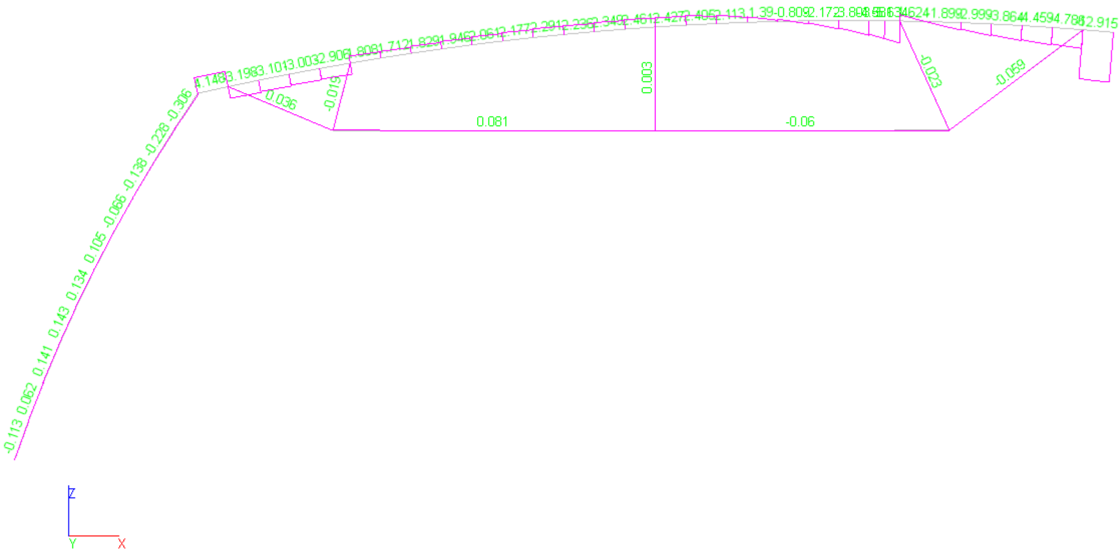


Figure 39. Distribution of the bending moment M in T·m (from the top – scheme with values; coloured indication)



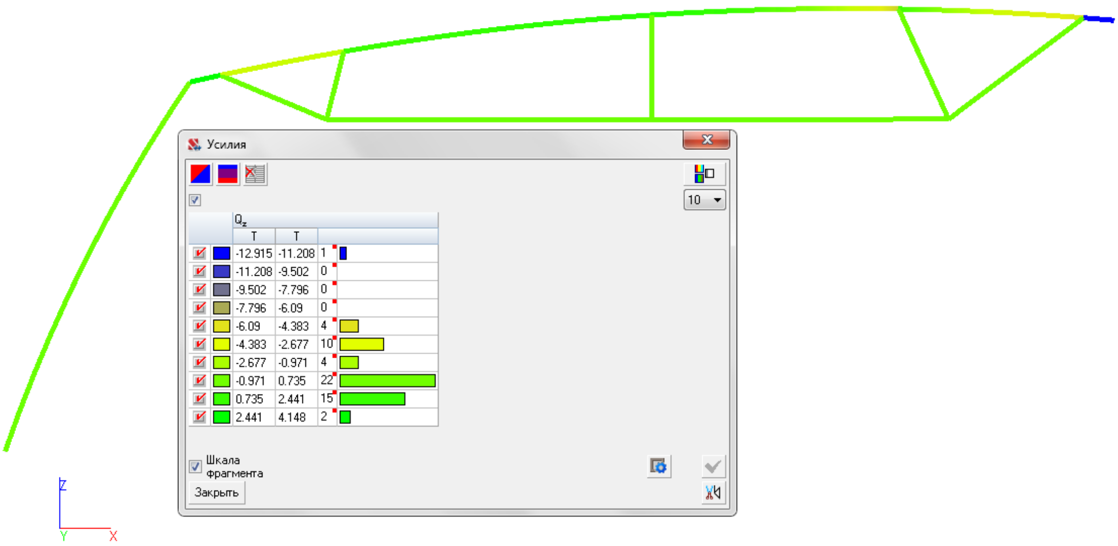
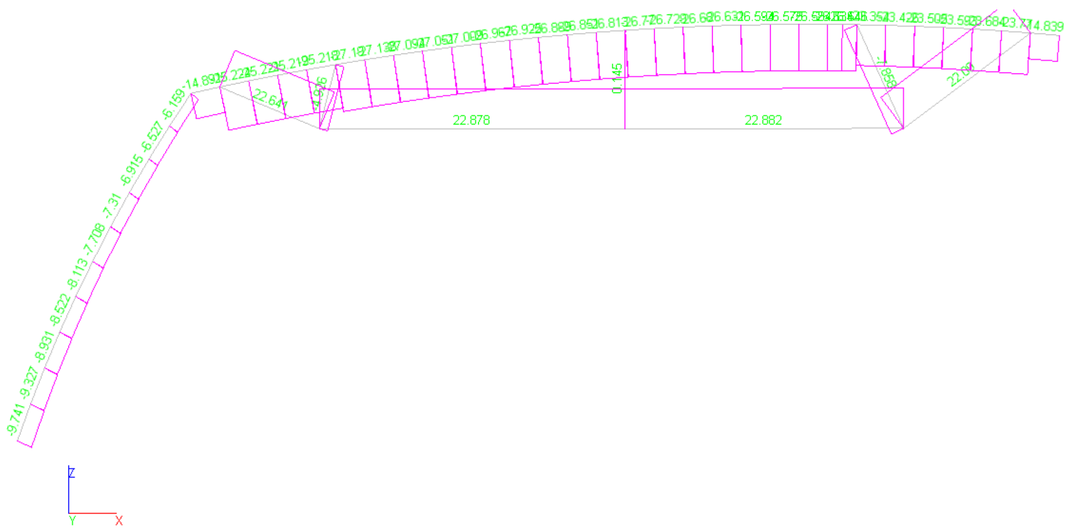


Figure 40. Distribution of the shear force Q in T (from the top – scheme with values; coloured indication)



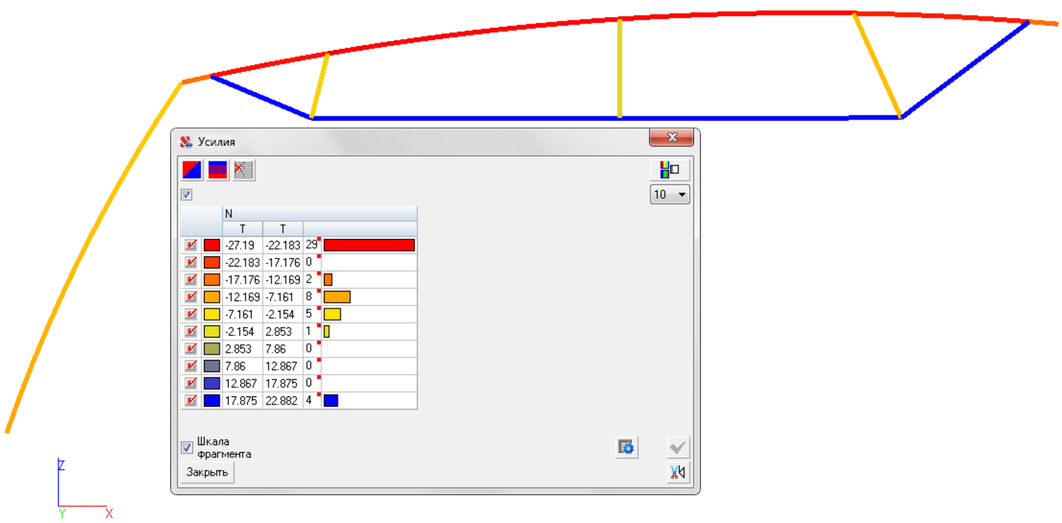


Figure 41. Distribution of the normal force N in T (from the top – scheme with values; coloured indication)

3.7.3 Theoretical verification

Design values:

$$f_{m,d} = k_{mod} \cdot \frac{f_{m,k}}{\gamma_m} = 0{,}8 \cdot \frac{28{,}0}{1{,}2} = 18{,}67 \;\; \mathrm{N/mm^2} \tag{63}$$

$$f_{t,90,d} = k_{mod} \cdot \frac{f_{t,90,k}}{\gamma_m} = 0{,}8 \cdot \frac{0{,}45}{1{,}2} = 0{,}3 \;\; \mathrm{N/mm^2} \tag{64}$$

$$M_{max,d} = M_{ap,d} = 187{,}09 \; \mathrm{kNm} \tag{65}$$

Bending stress:

$$\sigma_{m,max,d} = \sigma_{m,ap,d} = k_l \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} \tag{66}$$

$$k_l = k_1 + k_2 \cdot \left(\frac{h_{ap}}{r}\right) + k_3 \cdot \left(\frac{h_{ap}}{r}\right)^2 + k_4 \cdot \left(\frac{h_{ap}}{r}\right)^3 \tag{67}$$

$$k_1 = 1 + 1{,}4 \cdot \tan \alpha_{ap} + 5{,}4 \cdot \tan^2 \alpha_{ap} = 1 + 1{,}4 \cdot \tan 0 + 5{,}4 \cdot \tan^2 0 = 1 \tag{68}$$

$$k_2 = 0{,}35 - 8 \cdot \tan \alpha_{ap} = 0{,}35 \tag{69}$$

$$k_3 = 0{,}6 + 8{,}3 \cdot \tan \alpha_{ap} - 7{,}8 \cdot \tan^2 \alpha_{ap} = 0{,}60 \tag{70}$$

$$k_4 = 6 \cdot \tan^2 \alpha_{ap} = 0 \tag{71}$$

$$r = r_{in} + 0{,}5 \cdot h_{ap} = 101\,400 + 560 = 101\,960 \; \mathrm{mm} \tag{72}$$

$$\begin{aligned} k_l &= k_1 + k_2 \cdot \left(\frac{h_{ap}}{r}\right) + k_3 \cdot \left(\frac{h_{ap}}{r}\right)^2 + k_4 \cdot \left(\frac{h_{ap}}{r}\right)^3 \\ &= 1{,}0 + 0{,}35 \cdot \left(\frac{1120}{101\,960}\right) + 0{,}6 \cdot \left(\frac{1120}{101\,960}\right)^2 = 1{,}004 \end{aligned} \tag{73}$$

$$\begin{aligned} \sigma_{m,max,d} = \sigma_{m,ap,d} &= k_l \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} = 1{,}004 \cdot \frac{6 \cdot 187{,}09}{240 \cdot 1120^2} \\ &= 3{,}74 \; \mathrm{N/mm^2} \end{aligned} \tag{74}$$

For curved beams:

$$\frac{r_{in}}{t} = \frac{101\ 200}{40} = \ 2530 > 240,\ \ k_r = 1{,}0 \tag{75}$$

$$\frac{\sigma_{m,ap,d}}{k_r \cdot f_{m,d}} = \frac{3{,}74}{1{,}0 \cdot 18{,}67} = \ 0{,}2 < 1{,}0 \tag{76}$$

3.7.4 Tension stresses perpendicular to grain

$$\sigma_{t,90,ap,d} = k_p \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} \tag{77}$$

$$k_p = k_5 + k_6 \cdot \left(\frac{h_{ap}}{r}\right) + k_7 \cdot \left(\frac{h_{ap}}{r}\right)^2 \tag{78}$$

$$k_5 = 0{,}2 \cdot \tan \alpha_{ap} = \ 0 \tag{79}$$

$$k_6 = 0{,}25 - 1{,}5 \cdot \tan \alpha_{ap} + 2{,}6 \cdot \tan^2 \alpha_{ap} = 0{,}25 \tag{80}$$

$$k_7 = 2{,}1 \cdot \tan \alpha_{ap} - 4 \cdot \tan^2 \alpha_{ap} = 0 \tag{81}$$

$$k_p = k_5 + k_6 \cdot \left(\frac{h_{ap}}{r}\right) + k_7 \cdot \left(\frac{h_{ap}}{r}\right)^2 = 0{,}25 \cdot \left(\frac{1120}{101\ 960}\right) = 0{,}003 \tag{82}$$

$$\sigma_{t,90,ap,d} = k_p \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} = 0{,}003 \cdot \frac{6 \cdot 187{,}09}{240 \cdot 1120^2} = 0{,}01\ \mathrm{N/mm^2} \tag{83}$$

With the reference volume V_o = 0,01 m^3 and

$$V = \frac{\beta \cdot \pi}{180} \cdot b \cdot \left(h_{ap}^2 + 2 \cdot r_{in} \cdot h_{ap}\right) = \frac{12 \cdot \pi}{180} \cdot 0{,}24 \cdot (1{,}120^2 + 2 \cdot 101{,}4 \cdot 1{,}120)$$
$$= 11{,}47\ \mathrm{m}^3 \tag{84}$$

the volume factor is:

$$k_{vol} = \left(\frac{V_o}{V}\right)^{0.2} = \left(\frac{0{,}01}{11{,}47}\right)^{0.2} = \ 0{,}24\ \text{ and} \tag{85}$$

$k_{dis} = 1{,}4$ for curved beams

$$\frac{\sigma_{t,90,ap,d}}{k_{dis} \cdot k_{vol} \cdot f_{t,90,d}} = \frac{0{,}01}{1{,}4 \cdot 0{,}22 \cdot 0{,}3} = \ 0{,}1 < 1 \tag{86}$$

3.7.5 Shear force

Maximum shear force from the calculation equals Q = -12,915 T.

Static cross-section moment:

$$S = h^2 \cdot b/6 = 112^2 \cdot 24/6 = 50176\ \mathrm{sm}^3 \tag{87}$$

Inertia moment of the cross-section:

$$I = h^3 \cdot b/12 = 112^3 \cdot 24/12 = 2809856\ \mathrm{sm}^4 \tag{88}$$

Lateral stress equals:

$$\tau = \frac{\mathrm{Q} \cdot S}{I \cdot b} = \frac{-12{,}915 \cdot 50176}{2809856 \cdot 24} = 0{,}94\,\frac{\mathrm{N}}{\mathrm{mm}^2} \leq f_{v,d} = 2{,}13\,\frac{\mathrm{N}}{\mathrm{mm}^2} \tag{89}$$

3.7.6 Combined tension perpendicular to grain and shear force

1. Maximum shear force value and bending moment value at this point:

$$\frac{\tau_d}{f_{v,d}} + \frac{\sigma_{t,90,d}}{k_{dis} \cdot k_{vol} \cdot f_{t,90,d}} = \frac{0{,}94}{2{,}13} + \frac{0{,}01}{1{,}4 \cdot 0{,}24 \cdot 0{,}3} = 0{,}54 \leq 1 \tag{90}$$

$$\sigma_{t,90,ap,d} = k_p \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} = 0{,}005 \cdot \frac{6 \cdot 17{,}442}{0{,}24 \cdot 1{,}92^2} = 0{,}01\ \mathrm{N/mm}^2 \tag{91}$$

2. Maximum bending moment value and shear force value at this point:

$$\frac{\tau_d}{f_{v,d}} + \frac{\sigma_{t,90,d}}{k_{dis} \cdot k_{vol} \cdot f_{t,90,d}} = \frac{0{,}06}{2{,}13} + \frac{0{,}01}{1{,}4 \cdot 0{,}24 \cdot 0{,}3} = 0{,}13 \leq 1 \tag{92}$$

$$\tau = \frac{\mathrm{Q} \cdot S}{I \cdot b} = \frac{0{,}8 \cdot 50176}{2809856 \cdot 24} = 0{,}59\,\frac{\mathrm{kg}}{\mathrm{sm}^2} = 0{,}06\,\frac{\mathrm{N}}{\mathrm{mm}^2} \leq f_{v,d} = 2{,}1\,\frac{\mathrm{N}}{\mathrm{mm}^2} \tag{93}$$

3.7.7 Compression perpendicular to grain

$$\sigma_{c,90,d} \leq k_{\mathrm{c,90}} \cdot f_{c,90,d} = 3{,}5\ \mathrm{N/mm}^2 \tag{94}$$

where:

$\sigma_{c,90,d}$ – the design compressive stress in the contact area perpendicular to the grain;

$f_{c,90,d}$ – the design compressive strength perpendicular to the grain;

$k_{c,90}$ – a factor taking into account the load configuration, possibility of splitting and degree of compressive deformation = 1,75.

$$f_{c,90,d} = 0{,}8 \cdot \frac{3}{1{,}2} = 2\,\frac{\mathrm{N}}{\mathrm{mm}^2} \tag{95}$$

$$\sigma_{c,90,d} = \frac{F_{c,90,d}}{A_{ef}} = \frac{12{,}915}{0{,}24(0{,}2+0{,}06)} = 206{,}9\,\mathrm{T/m^2} = 2{,}03\,\mathrm{N/mm^2} \leq 3{,}5\mathrm{N/mm^2} \tag{96}$$

$F_{c,90,d}$ – the support load;

A_{ef} – effective area at support point.

3.7.8 Lateral buckling

<u>From bending</u>

Relative slenderness:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{crit}}} = \sqrt{\frac{28}{17{,}32}} = 1{,}27 \tag{97}$$

The critical bending stress for softwood with rectangular cross-section:

$$\sigma_{m,crit} = \frac{0{,}78 \cdot b^2}{h \cdot l_{ef}} \cdot E_{0,05} = \frac{0{,}78 \cdot 240^2}{1120 \cdot 0{,}9 \cdot 30000} \cdot 10200 = 15{,}15\,\mathrm{N/mm^2} \tag{98}$$

where

l_{ef} – effective beam length = $0{,}9 \cdot l$;

$E_{0,05}$ – 5% value of elasticity modulus.

<u>Combination of bending and compression</u>

$$\left(\frac{\sigma_{m,d}}{k_{crit} \cdot f_{m,d}}\right)^2 + \frac{\sigma_{c,o,d}}{k_{c,z} \cdot f_{c,o,d}} = \left(\frac{0{,}19}{1 \cdot 18{,}67}\right)^2 + \frac{4{,}73}{0{,}5 \cdot 17{,}67} = 0{,}54 \leq 1 \tag{99}$$

$\sigma_{m,d}$ – the design bending stress;

$\sigma_{c,0,d}$ – the design compressive stress;

$f_{c,0,d}$ – the design compressive strength parallel to grain;

$k_{c,z}$ – given by expression:

$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z^2 - \lambda_{rel,z}^2}} = \frac{1}{1{,}46 + \sqrt{1{,}46^2 - 1{,}35^2}} = 0{,}5 \tag{100}$$

$$k_z = 0{,}5\left(1 + \beta_c\left(\lambda_{rel,z} - 0{,}3\right) + \lambda_{rel,z}^2\right) = 0{,}5 \cdot (1 + 0{,}1(1{,}35 - 0{,}3) + 1{,}35^2)$$
$$= 1{,}46 \tag{101}$$

$\beta_c = 0{,}1$ – factor for members within the straightness limits for glued laminated timber;

$\lambda_{rel,z}$ – the relative slenderness:

$$\lambda_{rel,z} = \frac{\lambda_z}{\pi} \cdot \sqrt{\frac{f_{c,0,k}}{E_{0,05}}} = \frac{83}{\pi} \cdot \sqrt{\frac{26{,}5}{10200}} = 1{,}35 \tag{102}$$

$$\lambda_z = \frac{l_{ef}}{r} = \frac{27}{0{,}289 \cdot 1{,}12} = 83 \tag{103}$$

3.8 Deflections

W_{fin} – final maximum deflection for curved structures on two supports equals $l/300$ according to Table 5 [2].

Table 5. Examples of limiting values for deflections of beams

	w_{inst}	$w_{net,fin}$	w_{fin}
Beam on two supports	$\ell/300$ to $\ell/500$	$\ell/250$ to $\ell/350$	$\ell/150$ to $\ell/300$
Cantilevering beams	$\ell/150$ to $\ell/250$	$\ell/125$ to $\ell/175$	$\ell/75$ to $\ell/150$

Therefore, for the study it is necessary to fulfill the condition:

$$W_{fin} = \frac{30000}{300} = 100 \text{ mm} \tag{104}$$

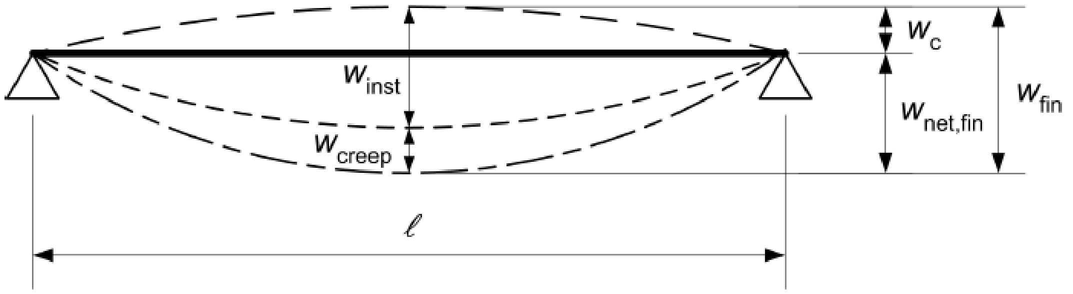


Figure 42. Components of total deflection [12]

Figure 43 shows the maximum deflection curve and the value of 35,968 mm.

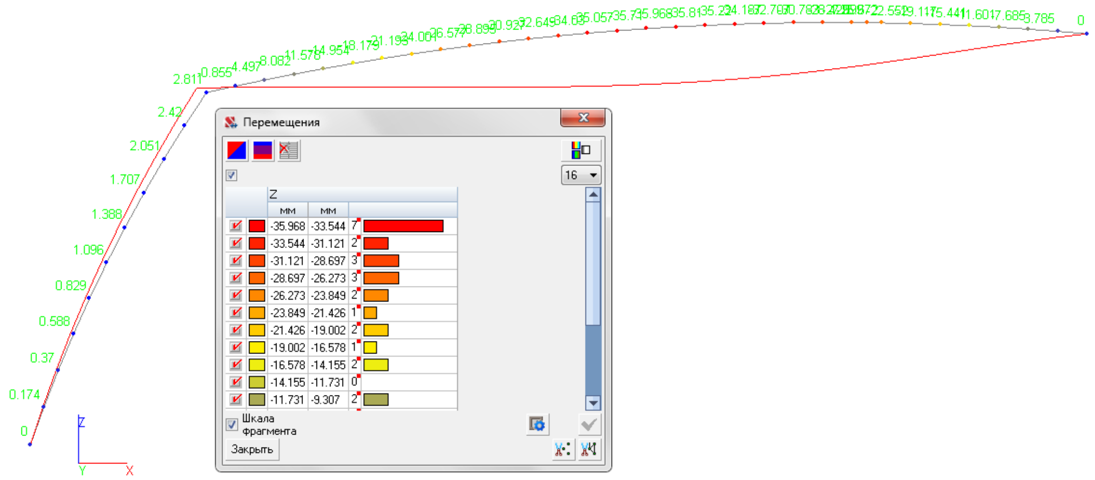


Figure 43. Deflection of the beam – scheme 1

Figure 44 shows the maximum deflection curve and the value of 36,013 mm.

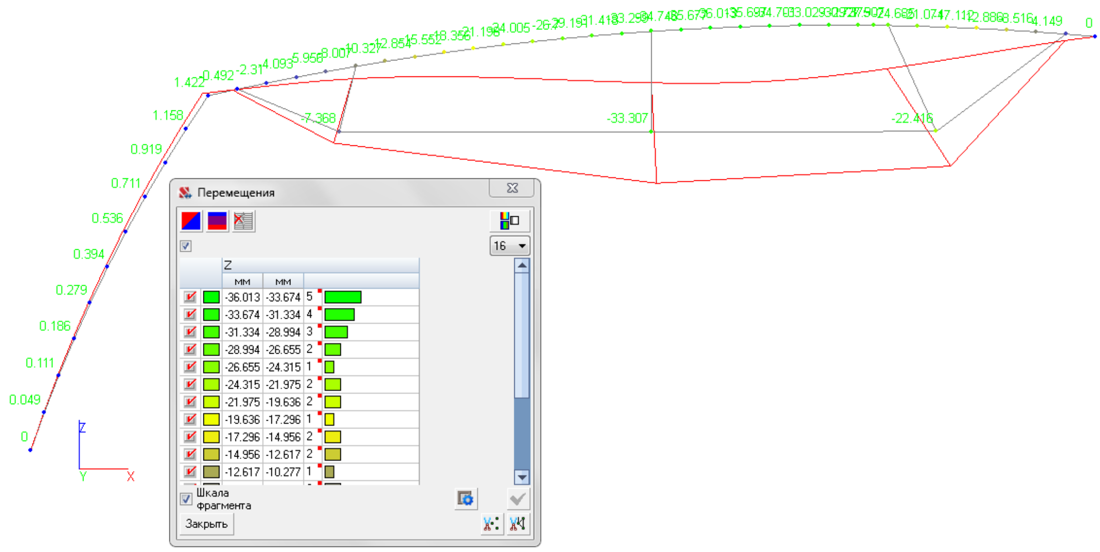


Figure 44. Deflection of the beam – scheme 2

According to Figures 43, 44 the maximum deflection at chosen cross-sections does not exceed the limited value of 100 mm.

3.9 Comparison

To compare the two schemes the parameter of the maximum deflection was chosen. It equals approximately 36 mm. The value has its base on possible additional loads in the future calculations, for example, covering and equipment such as lighting, if this structure is applicable to public buildings. The calculations were influenced by this parameter. The criteria are listed in Table 6.

Table 6. Comparison of the schemes

№	Parameter	Scheme 1	Scheme 2	Maximum allowed values
1	Cross-section, b x h, mm	240 x 1920	240 x 1120	
2	Bending moment from SCAD, kN·m	615,4	187,09	
3	Normal force from SCAD, T	-20,236	-27,19	
		+8,833	+22,882	
4	Shear force from SCAD, T	-14,061	-12,915	
		+5,679	+4,818	
5	Deflection from SCAD, mm	35,968	36,013	100
6	Bending stress, N/mm^2	4,2	3,74	18,67
7	Tension stress perpendicular to grain, N/mm^2	0,02	0,01	0,3
8	Shear stress, N/mm^2	-0,6	0,94	2,13
9	Combination of tension and shear stresses (maximum)	0,39	0,54	1
10	Compression perpendicular to grain, N/mm^2	2,3	2,03	3,5
11	Lateral buckling from bending, N/mm^2	8,84	15,15	18,67
12	Combination of bending and compression	0,25	0,54	1

4 Conclusion

Timber structures are used in various buildings and structures from private houses to long-span stadiums, swimming pools and bridges. So, the applicability of timber in modern construction seems to be very wide-spread, therefore, new methods to improve standard versions of structures need to be implemented.

The technologies also allow nowadays to calculate such difficult systems using computer programs, therefore, such a tool as SCAD Office was introduced into these calculations and this study.

The results of this study show that glue a laminated structure can be applied in various situations, and if necessary it can contain a metal additional part which is called strut system. This system allows constructors and customers save the money on the costs of wooden parts. The calculation shows that without such systems the cross-section height increases by 800 mm, which is quite a lot for one beam, not to mention ten structures of this kind, for example. The comparison of the schemes also proved the presumable differences in forces values and values of bending moments.

As only wooden beam was calculated in this study without any roofing systems or any additional equipment, therefore, no roof loads were taken into account. Main goal was to compare two kinds of wooden systems. This study is theoretical; as a result, it requires further calculations and investigations, for example, in the structure costs in case when all the rest loads not provided in this study may occur.

Figures

Tables

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