THE ALEXANDER POLYNOMIAL OF A TORUS KNOT WITH TWISTS

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ABSTRACT

This note gives an explicit calculation of the doubly infinite sequence \( \Delta(p, q, 2m), m \in \mathbb{Z} \) of Alexander polynomials of the \((p, q)\) torus knot with \(m\) extra full twists on two adjacent strings, where \(p\) and \(q\) are both positive. The knots can be presented as the closure of the \(p\)-string braids \((\delta_p)^q \sigma_1^{2m}\), where \(\delta_p = \sigma_{p-1} \sigma_{p-2} \cdots \sigma_2 \sigma_1\), or equally of the \(q\)-string braids \((\delta_q)^p \sigma_1^{2m}\). As an application we give conditions on \((p, q)\) which ensure that all the polynomials \(\Delta(p, q, 2m)\) with \(|m| \geq 2\) have at least one coefficient \(a\) with \(|a| > 1\). A theorem of Ozsvath and Szabo then ensures that no lens space can arise by Dehn surgery on any of these knots. The calculations depend on finding a formula for the multivariable Alexander polynomial of the 3-component link consisting of the torus knot with twists and the two core curves of the complementary solid tori.

Keywords: torus knot, twist, Dehn surgery, multi-variable Alexander polynomial.

1 Introduction

The calculations for the sequence \(\Delta(p, q, 2m), m \in \mathbb{Z}\) of Alexander polynomials of the \((p, q)\) torus knot with \(m\) extra full twists on two adjacent strings were initially done for the \((7, 17)\) torus knot in response to a query of Yoav Moriah [4] about their Alexander polynomials. The results in this case allowed him to deduce, from work of Ozsvath and Szabo [7], that the only knots in this sequence which can give a lens space after Dehn surgery are those with \(m = 0, \pm 1\).

In his thesis [1] and a subsequent paper [2] John Dean studies a more general class of knots lying on the surface of a standard genus 2 surface, which he calls twisted torus knots. He gives a condition, which he terms primitive/Seifert fibred, on the knot in relation to the two complementary handlebodies. Knots satisfying this condition yield small Seifert fibre spaces (with base \(S^2\) and at most 3 exceptional fibres) under some Dehn surgery. The knots considered in this paper are simple
examples of Dean’s twisted torus knots, which are primitive/Seifert fibred only in the cases \( m = \pm 1 \) or \( q = 3 \) or \( q = \pm 2 \mod p \).

My original method for the \((7, 17)\) calculation was simply to use the skein relation for the Conway polynomial to produce a recursive relation for the Conway polynomials \( f_k(z) \) of any sequence of knots differing only in having \( k \) half twists at one spot in two directly oriented strands.

In the Conway skein a single half-twist \( \sigma \) satisfies the quadratic equation

\[
\sigma^2 = z\sigma + 1
\]

with roots \( s, -s^{-1} \), where \( s - s^{-1} = z \). This leads to the relation

\[
f_{k+2} = (s - s^{-1})f_{k+1} + f_k.
\]

Solving the recurrence relation gives a formula \( f_k = cs^k + d(-s)^{-k} \) in terms of \( s \), where \( c \) and \( d \) are rational functions to be determined; the Alexander polynomial is given by setting \( s^2 = t \).

Knowing the Alexander polynomials for say \( k = 0 \) and \( k = 2 \) determines \( c \) and \( d \), and hence the whole sequence of Alexander polynomials (by setting \( s^2 = t \)). For the case of \((7, 17)\) an explicit Maple calculation of \( f_0 \) and \( f_2 \) was enough to find the sequence and to answer Moriah’s original question.

2 Use of the reduced Burau matrix

Attempts to simplify and generalise the calculations led first to the corresponding recurrence formula for the suitably normalised multivariable Alexander polynomial \( a_k \) of a sequence of links with several components, differing by \( k \) half twists in two directly oriented strands. Where the two strands involved in the twisting belong to components both labelled with the same variable \( t = s^2 \) the polynomials again satisfy a recurrence relation with solution \( a_k = cs^k + d(-s)^{-k} \) for some rational functions \( c \) and \( d \) determined by \( a_0 \) and \( a_1 \). This relation holds for the properly normalised form of the Alexander polynomial, as given for example by Murakami [6]. Frequently, however, the Alexander polynomial has been multiplied by a power of the variables, and a variant of this relation may work systematically.

One such variant occurs naturally when the multivariable polynomial of a closed \( n \)-braid \( \bar{\beta} \) and its axis \( A \) is realised as the characteristic polynomial of the reduced Burau matrix of \( \beta \), as in [5]. We can assume that the sequence of links is presented as the closure of a sequence of braids \( \beta \sigma_1^k \), in which the twists take place in the first two strands, both labelled by the same meridian element \( t \). In this representation the reduced Burau matrix for \( \sigma_1 \) is the \((n - 1) \times (n - 1)\) block matrix

\[
S = \begin{pmatrix}
-t & 1 \\
0 & 1
\end{pmatrix} \oplus I_{n-3},
\]

which has eigenvalues \(-t\) once and \(1\) repeated \( n - 2 \) times. It satisfies the equation \( S^2 = (1 - t)S + tI \).

Let \( B \) be the reduced multivariable Burau matrix of \( \beta \). Then \( BS^k \) is the reduced Burau matrix of \( \beta \sigma_1^k \), and

\[
BS^{k+2} = (1 - t)BS^{k+1} + tBS^k.
\]
Since the exterior powers of \( S \) all have the two eigenvalues 1 and \(-t\), and characteristic polynomials are formed by taking traces of exterior powers it follows that the polynomials \( \Delta_k = \det(I - xBS^k) \) also satisfy the recurrence relation

\[
\Delta_{k+2} = (1 - t)\Delta_{k+1} + t\Delta_k.
\]

This gives the formula

\[
\Delta_{k+1} - \Delta_k = (-t)^k(\Delta_1 - \Delta_0),
\]

and hence

\[
\Delta_k = (1 - t + t^2 - \cdots + (-t)^{k-1})(\Delta_1 - \Delta_0).
\]

For the case of \( k = 2m \), with \( m \geq 0 \) full twists, this will also give a recurrence relation leading to the formula

\[
\Delta_{2m} = (1 + t^2 + \cdots + t^{2m-2})(\Delta_2 - \Delta_0)
\]

for the multivariable polynomials of the sequence of links.

### 3 The multivariable Alexander polynomial

Use of the multivariable Alexander polynomial can be taken a stage further, by the application of two basic principles, due essentially to Torres [8] and Fox [3].

Suppose that \( L \) is an oriented link with several components, \( L_1, \ldots, L_n \). Write \( H_1(S^3 - L) \cong (\mathbb{Z}_\infty)^n \) multiplicatively, with positive meridian generator \( t_i \) corresponding to the component \( L_i \). The Alexander polynomial \( \Delta_L \) is an element of the group ring \( \mathbb{Z}[H_1(S^3 - L)] \), in other words, a Laurent polynomial in \( t_1, \ldots, t_n \).

**Theorem 1 (Fox)** If \( f : S^3 - L \to S^3 - L' \) is a homeomorphism of link exteriors, and \( f_* \) is the induced map on \( H_1 \) then

\[
\Delta_{L'} = f_*(\Delta_L).
\]

We want to find the Alexander polynomials of the sequence of links \( L'(k) \) shown here, which consist of the \((p, q)\) torus knot with \( k \) inserted half-twists lying on or near a standard torus \( T \), along with the core curves \( L_1 \) and \( L_2 \) of each complementary solid torus.
We label the meridians of the components by $t$, $x$ and $y$ as shown.

Now apply theorem 1 to the sequence of links $L(k)$, shown below,

after choosing an orientation preserving homeomorphism $f$ of the complement of the core curves which carries $T$ to itself and takes $L_3(k)$ to $L'_3(k)$ for all $k$ as follows. Let $A$ be the oriented arc on $T$, which runs from one side of $L_3$ to the other and gives, along with the coherently oriented part of $L_3$, an oriented curve isotopic to the meridian of $L_2$.

Choose $f$ to carry the curve $L_3$ on $T$ to the $(p, q)$ torus knot and $A$ to the arc which joins two adjacent strings in the $(p, q)$ knot as shown.
This homeomorphism \( f \) of the complement of \( L_1 \) and \( L_2 \) then carries each \( L(k) \) to \( L'(k) \).

Now \( f \) is determined by its effect on the torus \( T \), which is given by a \( 2 \times 2 \) unimodular matrix \( \begin{pmatrix} p & r \\ q & s \end{pmatrix} \). We can find \( r \) and \( s \) explicitly in terms of \( p \) and \( q \), knowing that \( f \) carries the oriented graph \( L_3 \cup A \) to the \((p,q)\) torus knot \( L'_3 \) together with the arc between adjacent strings in its braid presentation. Following this oriented arc on \( T \) with the coherently oriented part of \( L'_3 \) gives a curve whose linking number with \( L_1 \) must lie between 1 and \( p-1 \), as it will form one component of a \( p \)-string closed braid with axis \( L_1 \) made from putting the half-twist in the adjacent strings. Since \( A \) together with the coherently oriented part of \( L_3 \) is isotopic to the meridian \( y \) of \( L_2 \), we know that \( f \) carries this to a curve whose linking number with \( L_1 \) is \( r \). Consequently \( 0 < r < p \) (and \( 0 < s < q \)). This determines \( r \) and \( s \), since \( s \equiv p-1 \mod q \) and \( r \equiv -q-1 \mod p \).

To find the Alexander polynomial \( \Delta_k' \) for the link \( L'(k) \) with \( k \) half-twists it is enough to find the polynomial \( \Delta_k \) for the link \( L(k) \) and then substitute \( f^*(x) \) and \( f^*(y) \) for \( x \) and \( y \).

In terms of the homology of \( S^3 - L' \) the original meridian \( x \) becomes \( f^*(x) = x^p y^t \) and \( y \) becomes \( f^*(y) = x^r y^s \), since the image of the meridian \( x \) lies in the solid torus with core \( L_1 \) and represents \( q \) times the core, so its linking number with \( L'_3 \) is \( q \) times the linking number of \( L_1 \) with \( L'_3 \) giving the term \( t^p q \), while the image of the meridian \( y \) represents \( r \) times the core of \( L_2 \), giving the term \( t^r q \).

The basic link \( L(0) \) has polynomial \( \Delta_L(0) = 1 - x \), using for example the characteristic polynomial of the reduced Burau matrix for the identity braid on 2 strings \((L_2 \text{ and } L_3)\) with axis \( L_1 \). Substituting \( f^*(x) \) for \( x \) gives \( \Delta_{L'(0)} = 1 - x^p y^t \).

We already have \( \Delta_0 = 1 - x \), so it is enough to find \( \Delta_1 \) or \( \Delta_2 \), or indeed \( \Delta_{-1} \). In fact \( L(-1) \) is the fairly simple link shown here.

\[ \beta \]

This yields \( \Delta_{-1} = (1 - y)(1 - x(yt)^{-1}) \), and gives

\[ \Delta_{-1} - \Delta_0 = -t(\Delta_0 - \Delta_{-1}) = (1 + t)x - ty - xy^{-1} \]

and \( \Delta_2 - \Delta_0 = (1 - t^2)x - t(1 - t)y - (1 - t)xy^{-1} \).
Then

\[
\Delta_{2m} = 1 - t^{2m}x - (1 - t)(1 + t^2 + \cdots + t^{2m-2})(ty + xy^{-1})
\]

for \(m > 0\), and so

\[
\Delta'_{2m} = 1 - t^{2m}x^py^qt^{pq} - (1 - t)(1 + t^2 + \cdots + t^{2m-2})(x^r y^st^{rq+1} + x^{p-r} y^{q-s} t^{(p-r)q}).
\]

The corresponding formula for \(m < 0\) is

\[
\Delta_{-2m} = 1 - t^{-2m}x + (1 - t)(t^{-2} + t^{-4} + \cdots + t^{-2m})(ty + xy^{-1})
\]

giving

\[
\Delta'_{-2m} = 1 - t^{-2m}x^py^qt^{pq} + (1 - t)(t^{-2} + t^{-4} + \cdots + t^{-2m})(x^r y^st^{rq+1} + x^{p-r} y^{q-s} t^{(p-r)q}).
\]

To find the Alexander polynomial of the \((p, q)\) torus knot with \(2m\) half-twists we apply \(f_s\) as above to get \(\Delta'_{2m}\), and then use the second general result which gives the Alexander polynomial of a sublink starting from the polynomial of the link.

**Theorem 2 (Torres)** The Alexander polynomial of the sublink of \(L\) given by deleting a component \(L_1\) with meridian \(x\), leaving a link of more than one component, is found by setting \(x = 1\) in \(\Delta_{L}\) and dividing by \(1 - X\), where the component \(L_1\) represents \(X\) in the homology of the residual link \(L - L_1\). If only one component remains, with meridian \(t\), the Alexander polynomial of this knot is the expression above (which will be a rational function of \(t\)) multiplied by \(1 - t\).

In our case, deleting both \(L_1\) and \(L_2\) from \(L'(2m)\) will involve dividing \(\Delta_{2m}\) by \((1 - t^p)(1 - t^q)\) and multiplying by \(1 - t\), after setting \(x = y = 1\).

Equivalently set \(x = t^p, y = t^q\) in \(\Delta_{2m}(1 - t)/(1 - t^p)(1 - t^q)\) to get an explicit formula for the Alexander polynomial \(\Delta(p, q, 2m)\) for the \((p, q)\) torus knot with \(m > 0\) full twists in adjacent strings.

\[
\Delta(p, q, 2m) = \frac{1 - t}{(1 - t^p)(1 - t^q)} \times (1 - (1 - t)(1 + t^2 + \cdots + t^{2m-2})(t^{rq+1} + t^{(p-r)q}) - t^{pq+2m}).
\]

This form works well for \(m \geq 0\), as it gives the Alexander polynomial as a genuine polynomial, with non-zero constant term. Indeed it is well-adapted for power series expansion. The two critical powers of \(t\) which contribute to the changes of the polynomial with \(m\) are \(t^{rq+1} = t^{ps}\) and \(t^{(p-r)q}\). If the roles of \(p\) and \(q\) are reversed then these terms change places, since \(p - r \equiv q^{-1} \mod p\) and \(s \equiv p^{-1} \mod q\). We shall assume that we have ordered \(p\) and \(q\) so that \(ps\) is the smaller of the two exponents. Equivalently we have arranged that \(s < \frac{1}{2}q\) (and hence \(r < \frac{1}{2}p\)).

The formula for \(\Delta_{2m}\) can be derived without using the recurrence relation from the multivariable polynomial of the 4-component link shown.
Using the presentation of this link as the closure of the braid
\[ \sigma_3\sigma_2\sigma_1^2\sigma_2^{-2}\sigma_1^2\sigma_2\sigma_3^{-1}\sigma_2\sigma_1^2\sigma_2, \]
its multivariable polynomial can be found using the multivariable Burau calculation procedure [5]. In terms of the meridians \(x, y, t, w\), it is
\[
(1 - t^2)(1 - xw) - (1 - t)(1 - w)(yt + xy^{-1}).
\]
The polynomial for the link \(L(2m)\) can then be derived, using theorems 1 and 2. First put \(m\) full twists on the two strings through the unknotted component with meridian \(w\), where the effect on the polynomial, by theorem 1, is to replace \(w\) by \(wt^{2m}\). Then delete this unknotted component leaving the link \(L(2m)\). By theorem 2 the polynomial is then given by setting \(w = 1\) and dividing by \(1 - t^2\), to get
\[
\Delta_{2m} = 1 - xt^{2m} - \frac{(1 - t)(1 - t^{2m})}{1 - t^2}(yt + xy^{-1})
\]
for all \(m \in \mathbb{Z}\).

4 Sequences of polynomials whose coefficients are not all 0, ±1

In this section we give conditions on \(p, q > 0\) which ensure that the only possible Alexander polynomials in the sequence \(\Delta(p, q, 2m)\) with all their coefficients 0, ±1 are those with \(|m| \leq 1\), and hence by [7] at most three knots in the sequence yield lens spaces after Dehn surgery.

We start with a result for the part of the sequence with \(m \geq 0\).

**Theorem 3** Suppose that \(s < \frac{1}{3}q\), where \(s \equiv p^{-1} \mod q\) and \(0 < s < q\). Then the coefficient of \(t^{ps+2}\) in \(\Delta(p, q, 2m)\) is \(\leq -2\) for all \(m \geq 2\).
For example, if \( \{p, q\} = \{7, 17\} \) we have \( 5 \equiv 7^{-1} \mod 17 \) and the coefficient of \( t^{37} \) is \(-2\) for \( m \geq 2 \).

**Proof.** Under the given conditions \( ps < (p - r)q \), and \( p, q > 3 \). For \( m \geq 2 \) the only terms that can contribute to \( t^{ps+2} \) are

\[
\frac{1 - t}{(1 - t^p)(1 - t^q)}(1 - (1 - t)(1 + t^2)t^{ps}).
\]

Expand \((1 - t^p)(1 - t^q))^{-1} as \((1 + t^p + t^{2p} + \cdots)(1 + t^q + t^{2q} + \cdots) = A(p, q)\), say.

We must examine the coefficient of \( t^{ps+2} \) in \((1 - t)A(p, q) - t^{ps}(1 - t)^2A(p, q)\). Now \((1 - t)^2(1 + t^2)A(p, q) = 1 - 2t + 2t^2 \) up to terms in \( t^2 \), and will contribute \(-2\) to the coefficient of \( t^{ps+2} \).

It is then enough to show that the coefficient of \( t^{ps+2} \) in \((1 - t)A(p, q) \) is \( \leq 0 \). This in turn will be guaranteed by showing that the coefficient of \( t^{ps+2} \) in \( A(p, q) \) is zero. Now this coefficient counts the number of solutions of the equation \( ap + bq = ps + 2 \) in non-negative integers \( a, b \).

Since \( ps \equiv 1 \mod q \) we have \( ap \equiv 3 \mod q \) and so \( 3ps - ap \equiv 0 \mod q \). Then \( 3s \equiv a \mod q \), but this is not possible since \( 0 \leq a \leq s < 3s < q \), by hypothesis. \( \square \)

The formula for the Alexander polynomial \( \Delta(p, q, -2m) \) of the \( (p, q) \) torus knot with \( m \) negative full twists in adjacent strings (where \( p, q > 0 \)) is given from \( \Delta_{-2m} \)

above as

\[
\Delta(p, q, -2m) = \frac{1 - t}{(1 - t^p)(1 - t^q)} \times (1 + (1 - t)(t^{-2} + t^{-4} + \cdots + t^{-2m})(t^{rq+1} + t^{(p-r)q} - t^{pq-2m}).
\]

This can be adapted for power series computation by considering

\[
t^{2m}\Delta(p, q, -2m) = \frac{1 - t}{(1 - t^p)(1 - t^q)} \times (t^{2m} + (1 - t)(1 + t^2 + \cdots + t^{2m-2})(t^{rq+1} + t^{(p-r)q} - t^{pq}).
\]

Again we shall assume that we have ordered \( p \) and \( q \) so that \( ps \) is the smaller of the two critical powers \( rq + 1 = ps \) and \( (p - r)q \) of \( t \) which contribute to the changes with \( m \).

The following general result for negative twists complements the previous result, under the same conditions.

**Theorem 4** Suppose that \( s \leq \frac{1}{3}q \), where \( s \equiv p^{-1} \mod q \) and \( 0 < s < q \). Then the coefficient of at least one of the terms \( t^{ps+1}, t^{ps+2}, t^{ps+3} \) in \( t^{2m}\Delta(p, q, -2m) \) is \( \pm 2 \) for all \( m \geq 2 \).

**Proof.** Under the given conditions \( ps < (p - r)q \), and \( p, q > 3 \). For \( m \geq 2 \) we have

\[
t^{2m}\Delta(p, q, -2m) = \frac{1 - t}{(1 - t^p)(1 - t^q)}(t^{2m} + (1 - t)(1 + t^2)t^{ps}).
\]
to terms in $t^{ps+3}$. Expand $((1 - tp)(1 - t^q))^{-1}$ as $A(p, q) = \sum a_i t^i$, where $a_i$ counts the number of ways to write $i = ap + bq$ with non-negative integers $a, b$. For $i \leq pq$ we know that $a_i = 0$ or $1$. Furthermore, $i = ps$ is the first time that two consecutive coefficients $a_{i-1}$ and $a_i$ are both $1$, as $s \equiv p^{-1} \mod q$.

Since we have assumed that $3s < q$ it also follows that we can’t have $a_i = a_{i+2} = 1$ with $i < ps$. Thus in any four consecutive coefficients of $\sum_{i=0}^{ps} a_it^i$ there are two consecutive coefficients which are equal (either to 0, or 1), and so among any 3 consecutive coefficients of $(1 - t)\sum_{i=0}^{ps} a_it^i$ at least one of them is zero.

Now consider the coefficients of the three consecutive terms $t^{ps+1}, t^{ps+2}, t^{ps+3}$ in $t^{2m}\Delta(p, q, -2m)$. The contribution from $(1 - t)^2(1 + t^2)A(p, q)t^{ps}$ is $(1 - 2t + 2t^2 - 2t^3)t^{ps}$, while the contribution from $t^{2m}(1 - t)A(p, q)$ involves three consecutive coefficients of $(1 - t)A(p, q)$ up to degree at most $ps$. At least one of these must be zero, leaving one of the coefficients as ±2.

(Of course, once $2m > ps + 1$ the coefficient of $t^{ps+1}$ will be −2, and the lowest degree term in the whole polynomial will be $t^{ps}$ so that in standard polynomial form the Alexander polynomial is $1 - 2t + \cdots$.)

\[ \square \]

5 Some contrasting examples.

The conditions on $p$ and $q$ in theorems 3 and 4 can be phrased simply in terms of the continued fraction expansion of $p/q = [a_0, a_1, \ldots, a_k] = a_0 + 1/(a_1 + 1/(\cdots + 1/a_k))$, where each $a_i \geq 1$ and $a_k \geq 2$.

**Definition.** A Laurent polynomial with integer coefficients is **thick** if it has some coefficient $a$ with $|a| > 1$.

A knot whose Alexander polynomial is thick admits no lens space surgery, [7].

**Theorem 5 (rephrasing theorems 3, 4)** If $p, q > 3$ and $p/q = [a_0, a_1, \ldots, a_k]$ with $a_k \geq 3$ then $\Delta(p, q, 2m)$ is thick for all $m$ with $|m| \geq 2$.

**Proof.** If $p > q$ and $p/q = [a_0, a_1, \ldots, a_k]$ then $q/p = [0, a_0, a_1, \ldots, a_k]$. We can then assume, by swapping $p$ and $q$ if necessary, that $k$ is odd. Then

$$
\begin{pmatrix}
1 & a_0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
a_1 & 1
\end{pmatrix}
\cdots
\begin{pmatrix}
1 & 0 \\
a_k & 1
\end{pmatrix}
= \begin{pmatrix}
p & r \\
q & s
\end{pmatrix}
$$

where $0 < r < p$ and $0 < s < q$. Hence $\begin{pmatrix}
p & r \\
q & s
\end{pmatrix} = \begin{pmatrix}
a & b \\
c & d
\end{pmatrix}\begin{pmatrix}
1 & 0 \\
a_k & 1
\end{pmatrix}$ for some non-negative $a, b, c, d$. It follows that $q = c + da_k > da_k = sa_k$ unless $s = 1$ and $q = a_k$. When $a_k \geq 3$ and $p, q > 3$ we have $3s < q$ as required for theorems 3 and 4. \[ \square \]

The methods in theorem 4 show also that, except in the case $q = 2$, when the term $t^{(p-q)q}$ also contributes to the coefficient of $t^{ps+1}$, the Alexander polynomial $\Delta(p, q, -2m)$ will start $1 - 2t + \cdots$ for sufficiently large $m$.

In contrast to this if $(p - 1)(q - 1) < 2ps < pq$ then all the knots with $2m > 0$ half-twists have coefficients $0, \pm 1$. This follows since the adjustments in the series
in passing from $2m$ to $2m + 2$ occur after the half-way stage $(p - 1)(q - 1)/2$ in the Alexander polynomial, and inductively all the terms must be $0, \pm 1$, by symmetry of the Alexander polynomial.

This happens, when $q = 3$, and in some cases when $a_k = 2$, for example when $p \equiv \pm 2 \text{ mod } q$. This includes the case $(5, 8)$ but not the next Fibonacci pair $(8, 13)$, which has some some coefficients $\pm 2$ for certain positive values of $m$.

Noting that the cases where $m = \pm 1, q = 3$ or $q = \pm 2 \text{ mod } p$ are those which satisfy Dean’s primitive/Seifert fibred condition it is interesting to speculate on how far this condition identifies knots with thin Alexander polynomial among Dean’s general twisted torus knots.

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References


