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# The *Ceteris Paribus* Structure of Logics of Game Forms

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## Abstract

The article introduces a ceteris paribus modal logic, called CP, interpreted on the equivalence classes induced by finite sets of propositional atoms. This logic is studied and then used to embed three logics of strategic interaction, namely atemporal STIT, the coalition logic of propositional control (CL−PC) and the starless fragment of the dynamic logic of propositional assignments (DL−PA). The embeddings highlight a common ceteris paribus structure underpinning the key operators of all these apparently very different logics and show, we argue, remarkable similarities behind some of the most influential formalisms for reasoning about strategic interaction.

## 1. Introduction

The logical analysis of agency and games—for an expository introduction to the field see van der Hoek and Pauly's overview paper (2007)—has boomed in the last two decades giving rise to a plethora of different logics in particular within the multi-agent systems field.[1] At the heart of these logics are always representations of the possible *choices* (or actions) of groups of players (or agents) and their *powers* to force specific outcomes of the game. Some logics take the former as primitives, like STIT (the logic of *seeing to it that*, Belnap, Perloff, & Xu, 2001; Horty, 2001), some take the latter like CL (*coalition logic*, Pauly, 2002; Goranko, Jamroga, & Turrini, 2013) and ATL (*alternating-time temporal logic*, Alur, Henzinger, & Kupferman, 2002).

In these formalisms the power of players is modeled in terms of the notion of effectivity. In a strategic game, the $\alpha$-effectivity of a group of players consists of those sets of outcomes of the game for which the players have some collective action which forces the outcome of the game to end up in that set, no matter what the other players do (Moulin & Peleg, 1982). So, if a set of outcomes $X$ belongs to the $\alpha$-effectivity of a set of players $J$, *there exists* an individual action for each agent in $J$ such that, *for all* actions of the other players, the outcome of the game will be contained in $X$. If we keep the actions of the other agents

1. The richness of this logical landscape was the object of the IJCAI'13 invited talk by A. Herzig *Logics for Multi-Agent Systems: a Critical Overview.*

fixed, then the selection of an individual action for each agent in $J$ corresponds to a choice of $J$ under the assumption that the other agents stick to their choices.

It was already observed by van Benthem, Girard, and Roy (2009) that this formalization of choice and power in games is of an 'all other things being equal', or *ceteris paribus*, nature. Considering which outcomes of a game are possible for a set of players $J$ once the other players have fixed their actions, amounts to considering what may be the case under the ceteris paribus condition *'all actions of the agents not in $J$ being equal (to their current ones)'*. In the aforementioned work van Benthem et al. also show how this intuition can be used, for instance, to give a modal formulation of Nash equilibria of one-shot games.[2] In the current paper we leverage this intuition further and show how it can provide a novel systematization of many of the most influential formalisms in the field of logic and games.

### 1.1 Scientific Context

Formal relationships linking the logics (or fragments thereof) we mentioned above have been object of several publications. Notable examples are: the embedding of CL into the next-time fragment of ATL (Goranko, 2001) and the embedding of CL into NCL (*normal coalition logic*, Broersen, Herzig, & Troquard, 2007; Balbiani, Gasquet, Herzig, Schwarzentruber, & Troquard, 2008a), the embedding of CL and ATL into STIT (Broersen, Herzig, & Troquard, 2005, 2006). Earlier contributions have also attempted more comprehensive systematizations of the field of logic and games. Two in particular are worth mentioning: Goranko and Jamroga's work (2004), which compares game logics based on the computation tree abstraction like ATL and its variants; and Herzig's work (2014), which provides a conceptual and syntax-based—while we favor here semantic methods—comparison of all the main formalisms in the literature.

### 1.2 Aim of the Paper

The aim of the paper is to provide a technical contribution towards a unification of the field of logic and games. We set out to develop a series of embeddings which highlight a common structure in the representation of choice and power, which underpins the semantics of all the logics mentioned above.

We focus on the components of the semantics of those logics that have directly to do with the representation of choice and power, and we abstract away from the representation of time and repeated interaction. So the logics we will be working with are: the atemporal fragment of STIT, logic CL−PC (*coalition logic of propositional control*, van der Hoek & Wooldridge, 2005) and the starless fragment of DL−PA (*dynamic logic of propositional assignments*, van Eijck, 2000; Balbiani, Herzig, & Troquard, 2013). These logics cover, arguably, a large spectrum of the most influential existing formalisms.[3] Logic STIT is often considered a standard in the literature, as it embeds both CL and ATL (Broersen et al., 2005,

2. We refer the reader to Osborne and Rubinstein's textbook (1994) for an introduction to the basic notions of game theory.
3. It is worth stressing that we focus here on logics of choice and power (that is, on the notion of effectivity) and not on formalisms incorporating also an explicit representation of how choice and power are implemented (that is, an explicit notion of strategy), such as for instance ATL with explicit strategies (Walther, van der Hoek, & Wooldridge, 2007).

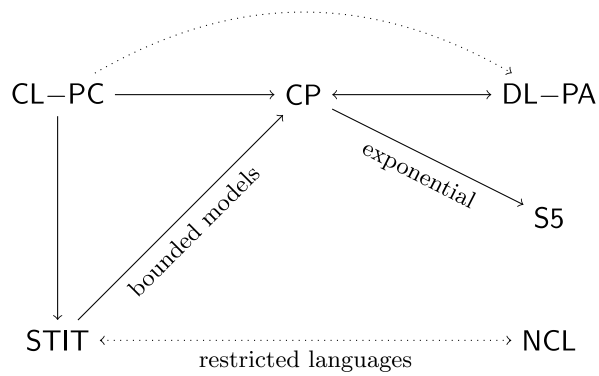



Figure 1: *Summary of the embeddings established in the paper and known from the literature* An arrow indicates that each formula of the source logic is satisfiable if and only if a suitable translation of that formula is satisfiable in the target logic. DL−PA denotes the starless version of dynamic logic of propositional assignments, NCL and STIT denote the atemporal version of, respectively, normal coalition logic and the seeing-to-it logic. S5 denotes the normal modal logic of equivalence relations. Dotted lines indicate embeddings known in the literature: from CL−PC to DL−PA (Balbiani et al., 2013) and from STIT to NCL (and vice versa) with respect to fragments of the respective languages (Lorini & Schwarzentruber, 2011). The embedding from STIT to CP assumes a bound on the STIT-models. All embeddings are polynomial except for the one from CP to S5.

2006), so we use it as a natural starting point. Logic CL−PC is an influential extension of CL and has strong formal ties (Dunne, van der Hoek, Kraus, & Wooldridge, 2008) with the, equally influential, Boolean games model (Harrenstein, van der Hoek, Meyer, & Witteveen, 2001) in multi-agent systems. Finally, logic DL−PA is an extension of PDL (*propositional dynamic logic*, Harel, Kozen, & Tiuryn, 2000), which has recently been proposed as a new standard for the representation of choice and power (Herzig, Lorini, Moisan, & Troquard, 2011; Balbiani et al., 2013).

To articulate our analysis, whose main technical tool consists of satisfiability-preserving embeddings, the paper introduces and studies—in its axiomatization and complexity—a simple ceteris paribus logic based on propositional equivalence, which we call CP. Such logic is the yardstick allowing us to compare and unify STIT, CL−PC and DL−PA.

### 1.3 Outline and Summary of Results

Section 2 introduces logic CP. The logic will be compared to S5 and axiomatized.
Section 3 provides a study of the relationship between the atemporal version of STIT and CP. We show that CP embeds atemporal group STIT—the fragment of atemporal STIT in which actions of both individuals and groups are represented—under the assumption that the agents' choices are bounded. We call the latter atemporal 'bounded' group STIT. Moreover, we show that CP embeds atemporal individual STIT—the variant of atemporal

STIT in which only the actions of individuals are represented. The former embedding is used to transfer complexity results to CP. We also present an embedding in CP of a variant of atemporal group STIT in which groups are nested (i.e., given two sets of agents $J$ and $J'$ either $J \subseteq J'$ or viceversa).
Section 4 provides an embedding of the coalition logic of propositional control into atemporal 'bounded' group STIT—and therefore, indirectly, into CP—as well as a direct embedding of CL−PC into CP.
Section 5 provides an embedding of the *starless* fragment of DL−PA into CP as well as an embedding of CP into DL−PA and therefore, indirectly, of STIT (on bounded models) and CL−PC into DL−PA.
Finally, in Section 6 we discuss the obtained results, put them in perspective with related work and draw some general implications for the field. We conclude in Section 7. Longer proofs are collected in a technical appendix at the end of the paper.

Figure 1 gives a graphical presentation of the embeddings established in the paper—as well as relevant ones already known in the literature. Two embeddings are known for the above logics: the embedding of CL−PC into DL−PA (Balbiani et al., 2013), and the embedding of STIT into NCL, and vice versa, when the language of STIT (and of NCL) are restricted to a fragment which does not allow nesting modalities.[4]

## 2. A Ceteris Paribus Logic Based on Propositional Equivalence

In this section we introduce and study the logic that will be used as target logic in all the embeddings we will present. The section starts with the definition of equivalence modulo a set of atoms. Then we present our ceteric paribus logic CP whose semantics is based on these equivalence relations. The section finishes with an exponentially embedding of ceteris paribus logic CP into S5 proving that the CP-satisfiability problem is decidable.

### 2.1 Equivalence Modulo a Set of Atoms

Consider a structure $(W, V)$ where $W$ is a set of states, and $V : \mathbf{P} \longrightarrow 2^W$ a valuation function from a countable set of atomic propositions $\mathbf{P}$ to subsets of $W$.[5]

**Definition 1.** *(Equivalence modulo $X$) Given a pair $(W, V)$, $X \subseteq \mathbf{P}$ and $|X| < \omega$, the relation $\sim^V_X \subseteq W^2$ is defined as:*

$$w \sim^V_X w' \iff \forall p \in X : \big(w \in V(p) \Longleftrightarrow w' \in V(p)\big)$$

*When $X$ is a singleton (e.g. $p$), we will often write $\sim^V_p$ instead of $\sim^V_{\{p\}}$. Also, in order to avoid clutter, we will often drop the reference to $V$ in $\sim^V_X$ when clear from the context.*

Intuitively, two states $w$ and $w'$ are equivalent up to set $X$, or $X$-equivalent, if and only if they satisfy the same atoms in $X$ (according to a given valuation $V$). The finiteness of

4. The reader is referred to Lorini and Schwarzentruber's paper (2011) for the BNF of this language.
5. In the literature game logics are sometimes defined over a countable set of atoms (e.g., Balbiani et al., 2013) and sometimes over a finite set of atoms (e.g., van der Hoek & Wooldridge, 2005). Here we opt for generality and define the language of CP over a countable set. Under the assumption of a finite supply of atoms some of the results we present later would trivialize (for instance the CP satisfiability problem would be in PTIME) and would therefore hide some of the interesting technical features of CP.

$X$ is clearly not essential in the definition. It is assumed because, as we will see, each set $X$ will be taken to model a set of actions of some agent in a game form and sets of actions are always assumed to be finite.

We state the following simple fact without proof. It highlights some interesting features of the notion of propositional equivalence modulo subsets of $\mathbf{P}$, some of which will be of use later on in the paper.

**Fact 1.** *(Properties of $\sim_P$) The following holds for any set of states $W$, valuation $V : \mathbf{P} \longrightarrow 2^W$ and finite sets $X, Y \subseteq \mathbf{P}$:*

*(i)* $\sim_X$ *is reflexive, transitive and symmetric;*

*(ii) if* $X \subseteq Y$ *then* $\sim_Y \subseteq \sim_X$*;*

*(iii) if* $X$ *is a singleton,* $\sim_X$ *induces a partition of* $W$ *with at most 2 cells;*

*(iv)* $\sim_X \cap \sim_Y = \sim_{X \cup Y}$*;*

*(v)* $\sim_\emptyset = W^2$.

Intuitively: *(i)* states that each $\sim_X$ is an equivalence relation; *(ii)* states that the larger the set of atoms, the more refined is the equivalence relation indexed by it; *(iii)* states that if the set of atom is a singleton, then its equivalence relation would induce a partition of one (if the proposition in the singleton is globally true or globally false in the model) or two cells (otherwise); *(iv)* states that the relation indexed by the union of two sets of atoms is the relation one obtains by intersecting the relations of the two sets; finally *(v)* states that the relation of the empty set of atoms is the global relation.

### 2.2 A Modal Logic of the $\sim_X$ Relation

In this section we consider a simple modal language interpreted on relations $\sim_X$ and axiomatize its logic on the class of structures $(W, V)$. The key modal operator of the language will be $\langle X \rangle$, whose intuitive meaning is '$\varphi$ is the case in some state which is $X$-equivalent to the current one' or, to stress a ceteris paribus reading, '$\varphi$ is possible *all things expressed in $X$ being equal*'. We call the resulting logic *propositional ceteris paribus logic*, CP in short.

#### 2.2.1 SYNTAX OF CP.

Let $\mathbf{P}$ be a countable set of atomic propositions. The language $\mathcal{L}_{\mathsf{CP}}(\mathbf{P})$ is defined by the following BNF:

$$\mathcal{L}_{\mathsf{CP}}(\mathbf{P}) : \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle X \rangle \varphi$$

where $p$ ranges over $\mathbf{P}$ and $X$ is a finite subset of atomic propositions ($X \subseteq \mathbf{P}$ and $X$ finite). Note that as the set of finite subsets of atomic propositions is countable, the language $\mathcal{L}_{\mathsf{CP}}(\mathbf{P})$ is also countable. The Boolean connectives $\top, \vee, \rightarrow, \leftrightarrow$ and the dual operators $[X]$ are defined as usual. Although we have taken diamond operators as primitive, we will for convenience also make use of box operators to state some results in later sections.

The set $SF(\varphi)$ of subformulas of a formula $\varphi$ is defined inductively as follows:

- $SF(p) = \{p\}$;

- $SF(\neg\varphi) = \{\neg\varphi\} \cup SF(\varphi)$;
- $SF(\varphi \wedge \psi) = \{\varphi \wedge \psi\} \cup SF(\varphi) \cup SF(\psi)$;
- $SF(\langle X \rangle\varphi) = \{\langle X \rangle\varphi\} \cup SF(\varphi)$.

We say that a signature $X$ appears in $\varphi$ if there exists a formula $\psi$ such that $\langle X \rangle\psi \in SF(\varphi)$.

#### 2.2.2 Semantics of CP

This is the class of models we will be working with:

**Definition 2.** *(CP-models) Given a countable set $\mathbf{P}$, a CP-model for $\mathcal{L}_{\mathsf{CP}}(\mathbf{P})$ is a tuple $\mathcal{M} = (W, V)$ where:*

- *$W$ is a non-empty set of states;*
- *$V : \mathbf{P} \longrightarrow 2^W$ is a valuation function.*

*A CP-model is called* universal *if $W = 2^{\mathbf{P}}$ and $V$ is s.t. $V(p) = \{w \mid p \in w\}$. It is called* non-redundant *if $\sim_{\mathbf{P}}$ is the identity relation in $W^2$.*

Intuitively, a CP-model consists just of a state-space and a valuation function for a given set of atoms. The satisfaction relation is defined as follows:

**Definition 3.** *(Satisfaction for CP-models) Let $\mathcal{M} = (W, V)$ be an CP-model for $\mathcal{L}_{\mathsf{CP}}(\mathbf{P})$, $w \in W$ and $\varphi, \psi \in \mathcal{L}_{\mathsf{CP}}(\mathbf{P})$:*

$$\begin{aligned}
\mathcal{M}, w \models_{\mathsf{CP}} p &\iff w \in V(p); \\
\mathcal{M}, w \models_{\mathsf{CP}} \neg\varphi &\iff \mathcal{M}, w \not\models_{\mathsf{CP}} \varphi; \\
\mathcal{M}, w \models_{\mathsf{CP}} \varphi \wedge \psi &\iff \mathcal{M}, w \models_{\mathsf{CP}} \varphi \text{ AND } \mathcal{M}, w \models \psi; \\
\mathcal{M}, w \models_{\mathsf{CP}} \langle X \rangle\varphi &\iff \exists w' \in W : w \sim_X^V w' \text{ AND } \mathcal{M}, w' \models_{\mathsf{CP}} \varphi
\end{aligned}$$

*Formula $\varphi$ is CP-satisfiable, if and only if there exists a model $\mathcal{M}$ and a state $w$ such that $\mathcal{M}, w \models_{\mathsf{CP}} \varphi$. Formula $\varphi$ is valid in $\mathcal{M}$, noted $\mathcal{M} \models_{\mathsf{CP}} \varphi$, if and only if for all $w \in W$, $\mathcal{M}, w \models_{\mathsf{CP}} \varphi$. Finally, $\varphi$ is CP-valid, noted $\models_{\mathsf{CP}} \varphi$, if and only if it is valid in all CP-models. The logical consequence of formula $\varphi$ from a set of formulae, noted $\Phi \models_{\mathsf{CP}} \varphi$, is defined as usual.*

So, modal operators are interpreted on the equivalence relations $\sim_X$ induced by the valuation of the model. It is worth observing that the logic of this class of models is not closed under uniform substitution,[6] that is, logic CP is not uniform.[7] To witness that, notice that formula $[\{p\}]p \vee [\{p\}]\neg p$ is valid, whereas $[\{p\}]\varphi \vee [\{p\}]\neg\varphi$ is not.

Let us give a simple illustration of the above semantics.

**Example 1.** *Let us consider the following model $\mathcal{M}$ made up of 5 states $w, x, u, y, z$:*

6. For the definition of uniform substitution the reader is referred to the textbook by Blackburn, de Rijke, and Venema (2001, Def. 1.18).
7. The terminology comes from Goldblatt's work (1992).

| | | | | |
|---|---|---|---|---|
| $w : p, q$ | $x : p$ | $u : p, q, r$ | $z : q$ | $y$ |

*For instance, we have* $\mathcal{M}, w \models_{\mathsf{CP}} \langle\{p, q\}\rangle r$ *and* $\mathcal{M}, z \models_{\mathsf{CP}} [\{p\}]\neg r$.

The following lemmas state simple facts concerning the relation of logic CP with logic S5 and isolate an interesting class of CP-models.

**Lemma 1.** *Let* $\mathcal{L}^{\emptyset}_{\mathsf{CP}}(\mathbf{P})$ *the set of formulae* $\varphi \in \mathcal{L}_{\mathsf{CP}}(\mathbf{P})$ *containing only* $\langle\emptyset\rangle$ *operators. The set of formulae of* $\mathcal{L}^{\emptyset}_{\mathsf{CP}}(\mathbf{P})$ *which are* CP*-valid is the modal logic of Kripke frames* $(W, W^2)$*, i.e., logic* S5.

*Proof.* It follows directly from Fact 1 item *(v)*. □

In other words, the $\langle\emptyset\rangle$ operator of $\mathcal{L}_{\mathsf{CP}}$ is nothing but the global modality (Blackburn et al., 2001, pp. 367–370). The next lemma states that CP is actually the logic of the class of *relevant* CP-models.

**Lemma 2.** *Every satisfiable* CP*-formula is satisfiable on a non-redundant model.*

*Proof.* Assume $\mathcal{M}, w \models \varphi$. We show that $\mathcal{M}_{\mathbf{P}}, |w|_{\mathbf{P}} \models \varphi$ of $\mathcal{M}$ where $\mathcal{M}_{\mathbf{P}}$ is the quotient of $\mathcal{M}$ by the equivalence relation $\sim_{\mathbf{P}}$ (defined in the natural way) and $|w|_{\mathbf{P}}$ is the set of states which are $\sim_{\mathbf{P}}$-equivalent to $w$. We proceed by induction on the structure of $\varphi$. The propositional and Boolean cases are obvious. Let $\varphi = \langle X\rangle\psi$ with $X \subseteq \mathbf{P}$. From the assumption and the semantics of CP operators we have that there exists $v$ such that $w \sim_X v$ and $\mathcal{M}, v \models \psi$. By construction we directly have that $|w|_{\mathbf{P}} \sim_X |v|_{\mathbf{P}}$. By IH $\mathcal{M}_{\mathbf{P}}, |v|_{\mathbf{P}} \models \psi$, and therefore $\mathcal{M}_{\mathbf{P}}, |w|_{\mathbf{P}} \models \langle X\rangle\psi$. □

### 2.2.3 Axiomatics of CP

We can obtain an axiom system for CP by a standard reduction technique exploiting Lemma 1. The axiom system is given in Figure 2. The first thing to notice is that the system consists of the usual S5 axioms plus the `Reduce` axiom. Logic S5 is known to be sound and strongly complete for the class of models where the accessibility relation is the total relation $W^2$ (Blackburn et al., 2001), and modality $\langle\emptyset\rangle$ can therefore be axiomatized as the (dual of) the global modality.

Having said this, soundness and strong completeness of the above system are easy to establish.

**Theorem 1.** *The axiom system given in Figure 2 is sound and strongly complete for the class of* CP*-models.*

*Proof.* Soundness It suffices to show that `Reduce` is CP-valid, which follows straightforwardly from Definition 1. Completeness To obtain completeness we proceed as customary in DEL (van Ditmarsch, Kooi, & van der Hoek, 2007; Wang & Cao, 2013). We first fix a

| | |
|---|---|
| (P) | all tautologies of propositional calculus |
| (K) | $[\emptyset](\varphi \rightarrow \psi) \rightarrow ([\emptyset]\varphi \rightarrow [\emptyset]\psi)$ |
| (T) | $\varphi \rightarrow \langle\emptyset\rangle\varphi$ |
| (4) | $\langle\emptyset\rangle\langle\emptyset\rangle\varphi \rightarrow \langle\emptyset\rangle\varphi$ |
| (5) | $\langle\emptyset\rangle\varphi \rightarrow [\emptyset]\langle\emptyset\rangle\varphi$ |
| (Reduce) | $[X]\varphi \leftrightarrow \bigwedge_{Y \subseteq X} \left( \left( \bigwedge_{p \in Y} p \wedge \bigwedge_{p \in X \setminus Y} \neg p \right) \rightarrow [\emptyset] \left( \left( \bigwedge_{p \in Y} p \wedge \bigwedge_{p \in X \setminus Y} \neg p \right) \rightarrow \varphi \right) \right)$ |
| (MP) | IF $\vdash_{\mathsf{CP}} \varphi$ AND $\vdash_{\mathsf{CP}} \varphi \rightarrow \psi$ THEN $\vdash_{\mathsf{CP}} \psi$ |
| (N) | IF $\vdash_{\mathsf{CP}} \varphi$ THEN $\vdash_{\mathsf{CP}} [\emptyset]\varphi$ |

Figure 2: Axiom schemas and rules of CP. $X, Y$ range over finite elements of $2^{\mathbf{P}}$, $\varphi, \psi$ over $\mathcal{L}_{\mathsf{CP}}(\mathbf{P})$, and $p$ over $\mathbf{P}$. As usual, $\vdash_{\mathsf{CP}}$ means that there exists a sequence of formulae each of which are either an axiom or are obtained from previous formulae through the application of an inference rule.

translation $tr_0 : \mathcal{L}_{\mathsf{CP}}(\mathbf{P}) \longrightarrow \mathcal{L}^{\emptyset}_{\mathsf{CP}}(\mathbf{P})$ as follows:

$$\begin{aligned} tr_0(p) &= p \\ tr_0(\neg\varphi) &= \neg tr_0(\varphi) \\ tr_0(\varphi \wedge \psi) &= tr_0(\varphi) \wedge tr_0(\psi) \\ tr_0([X]\varphi) &= \bigwedge_{Y \subseteq X} \left( \left( \bigwedge_{p \in Y} p \wedge \bigwedge_{p \in X \setminus Y} \neg p \right) \rightarrow [\emptyset] \left( \left( \bigwedge_{p \in Y} p \wedge \bigwedge_{p \in X \setminus Y} \neg p \right) \rightarrow tr_0(\varphi) \right) \right) \end{aligned}$$

We also write $tr_0(\Phi)$ for $\{tr_0(\varphi) \mid \varphi \in \Phi\}$. Notice that the translation removes occurrences of $\langle X\rangle$ and $[X]$ operators from formulae where $X \neq \emptyset$ and it has the same structure of axiom Reduce. Consider then the following rule of substitution of provable equivalents (REP):

(REP) IF $\vdash_{\mathsf{CP}} \varphi \leftrightarrow \varphi'$ THEN $\vdash_{\mathsf{CP}} \psi \leftrightarrow \psi[\varphi/\varphi']$

where $\psi[\varphi/\varphi']$ is the formula that results from $\psi$ by replacing zero or more occurrences of $\varphi$, in $\psi$, by $\varphi'$. We have that rule REP is derivable in the axiom system of Figure 2 (†). The proof of this claim is provided in Appendix A. By using axiom Reduce and rule REP we obtain by (†) that, for any $\varphi \in \mathcal{L}_{\mathsf{CP}}(\mathbf{P})$, $\vdash_{\mathsf{CP}} \varphi \leftrightarrow tr_0(\varphi)$ (‡). We can then proceed as follows: if $\Phi \models_{\mathsf{CP}} \varphi$ then $tr_0(\Phi) \models_{\mathsf{CP}} tr_0(\varphi)$ by (‡); by Lemma 1 and the strong completeness of S5 we thus obtain $tr_0(\Phi) \vdash_{\mathsf{S5}} tr_0(\varphi)$ and therefore $tr_0(\Phi) \vdash_{\mathsf{CP}} tr_0(\varphi)$; finally by (‡) again it follows that $\Phi \vdash_{\mathsf{CP}} \varphi$, which proves strong completeness. □

The crux of the above reduction argument lies in the use of axiom `Reduce`. What the axiom does is to enable the reduction of $[X]$-formulae by taking care of all the possible truth-value combinations of the atoms in $X$. If a given combination, e.g., $\left(\bigwedge_{p\in Y} p \wedge \bigwedge_{p\in X\setminus Y} \neg p\right)$, is true at a given state (for some $Y$), then in all accessible states, if that combination is true, then what occurs in the scope of $[X]$ is also true.

We opted for the above axiomatization in virtue of its simplicity, but alternative systems are of course possible. One in particular is worth mentioning. It first reduces $\langle p\rangle$ operators by axiom:

$$\langle p\rangle\varphi \leftrightarrow ((p \wedge \langle\emptyset\rangle(p \wedge \varphi)) \vee (\neg p \wedge \langle\emptyset\rangle(\neg p \wedge \varphi))) \tag{1}$$

This states that $\langle p\rangle\varphi$ is equivalent to either the case in which the current state satisfies $p$ and there exists a (possibly different) $p$-state where $\varphi$ is true, or the case where $\neg p$ is true and there exists a (possibly different) $\neg p$-state where $\varphi$ is true (recall property *(iii)* in Fact 1). Given the above reduction, one can then use axioms to enforce the appropriate behavior of $\sim_X$ relations where $X$ consists of more than one atom. To this aim, axioms can be used that are known to be canonical for properties *(ii)* and *(iv)* of Fact 1, namely:

$$\langle X \cup Y\rangle\varphi \rightarrow \langle X\rangle\varphi \tag{2}$$

$$\langle X\rangle i \wedge \langle Y\rangle i \rightarrow \langle X \cup Y\rangle i \tag{3}$$

where $i$ ranges over a set of nominals. A complete system could then be obtained by axiomatizing the behavior of nominals—through axioms and rules used in hybrid logic (Areces & Ten Cate, 2006). From that system, a named canonical model could be built (i.e., a canonical model where all maximal consistent sets contain exactly one nominal) where the axioms in Formulae 1-3 would enforce the desirable properties on the canonical relations.

### 2.3 Exponentially Embedding $\mathsf{CP}$ into $\mathsf{S5}$

The property expressed by axiom `Reduce` enables a truth-preserving translation of $\mathsf{CP}$ into $\mathsf{S5}$ via the translation $tr_0$ provided in the proof of Theorem 1. This translation is, however, such that the length of the translated formula grows exponentially by a tower of exponents of height equal to the modal depth of the original formula.

In this section we propose a translation that is single exponential and preserves satisfiability. Take the standard modal language $\mathcal{L}_\Box(\mathbf{P})$ with one modal operator $\Box$ defined on the set of atoms $\mathbf{P}$. $\mathsf{S5}$-models are structures $\mathcal{M} = (W, V)$ where $W$ is a set of states, and $V : \mathbf{P} \longrightarrow 2^W$ a valuation function. Given an $\mathsf{S5}$-model $\mathcal{M} = (W, V)$ and a state $w \in W$, the truth conditions are defined as follows:

$$\mathcal{M}, w \models_{\mathsf{S5}} \Box\varphi \iff \forall u \in W : \mathcal{M}, u \models_{\mathsf{S5}} \varphi$$

$\mathsf{S5}$-satisfiability is defined as usual. It is possible to define an exponential truth-preserving reduction $tr : \mathcal{L}_{\mathsf{CP}}(\mathbf{P}) \longrightarrow \mathcal{L}_\Box(\mathbf{P})$ as follows:

$$tr(\varphi_0) \quad = \quad p_{\varphi_0} \wedge \bigwedge_{\varphi \in SF(\varphi_0)} \Box(p_\varphi \leftrightarrow tr_1(\varphi))$$

where $p_\varphi$ are fresh atomic proposition (note that $p_{\varphi_0}$ is $p_\varphi$ when $\varphi$ is the formula $\varphi_0$ itself, which is also a subformula of $\varphi_0$)[8], $\varphi$ ranging over $SF(\varphi_0)$ and $tr_1$ is defined as follows:

$$\begin{array}{rcl} tr_1(p) & = & p \quad \text{FOR } p \in \mathbf{P} \\ tr_1(\neg\varphi) & = & \neg tr_1(\varphi) \\ tr_1(\varphi \wedge \psi) & = & tr_1(\varphi) \wedge tr_1(\psi) \\ tr_1([\emptyset]\varphi) & = & \Box p_\varphi \\ tr_1([X]\varphi) & = & \bigwedge_{Y \subseteq X} \left( \left( \bigwedge_{p \in Y} p \wedge \bigwedge_{p \in X \setminus Y} \neg p \right) \to \Box \left( \left( \bigwedge_{p \in Y} p \wedge \bigwedge_{p \in X \setminus Y} \neg p \right) \to p_\varphi \right) \right) \end{array}$$

Intuitively, the translation is designed to operate like axiom `Reduce` but avoiding exponential blow-up to pile up with the modal depth of the formula. The atomic propositions $p_\varphi$ in $tr_1([X]\varphi)$ avoid the non-elementary size of $tr(\varphi_0)$. The definition of $tr_1([\emptyset]\varphi)$ corresponds to the degenerated case of $tr_1([X]\varphi)$ where $X = \emptyset$.The following theorem states the satisfiability preservation. The proof is given in Appendix B.

**Theorem 2.** *(tr preserves satisfiability) Let $\varphi_0$ be a* CP*-formula. The two following statements are equivalent: $\varphi_0$ is* CP*-satisfiable; $tr(\varphi_0)$ is* S5*-satisfiable.*

As a consequence, we also obtain the following result.

**Corollary 1.** *(Decidability) The satisfiability problem for* CP *is decidable and in NEXPTIME.*

*Proof.* The satisfiability problem for S5 is decidable and in NP (Blackburn et al., 2001, Ch. 6). The result follows from Theorem 2 and a decision procedure may work as follows: in order to check that $\varphi$ is satisfiable we compute the formula $tr(\varphi)$ and we apply a NP-decision procedure to check whether $tr(\varphi)$ is S5-satisfiable or not. □

Notice that if the cardinality of each $X$ that appears in operators $[X]$ of $\varphi$ is bounded by a fixed integer, then the translation $tr$ becomes polynomial in the size of $\varphi$. Thus, as S5-satisfiability problem is NP-complete, the CP-satisfiability problem with a bounded cardinality restrictions over set of atomic propositions in modal operators is in NP. As it is trivially NP-hard, it is NP-complete.

In Section 3, we will embed the atemporal version of STIT (the logic of *seeing to it that*) into CP thereby obtaining lower bounds results.

## 3. The *Ceteris Paribus* Structure of STIT Logic

In this section, we investigate the possibility of embedding the logic of agency STIT into CP. STIT logic (the logic of *seeing to it that*, Belnap et al., 2001; Horty, 2001) is one of the most prominent logical accounts of agency. It is the logic of constructions of the form "agent $i$ (or group $J$) sees to it that $\varphi$". STIT has a non-standard modal semantics based on the concepts of *moment* and *history.* However, as shown by Balbiani, Herzig, and Troquard (2008b) and Herzig and Schwarzentruber (2008), the basic STIT language without temporal operators can be 'simulated' in a standard Kripke semantics.

8. Such a use of fresh atomic propositions to obtain more efficient satisfiability preserving translations is based on the propositional logic technique known as *Tseitin transformation* (Tseitin, 1968).

### 3.1 Atemporal Group STIT

First let us recall the syntax and the semantics of atemporal group STIT. The language of this logic is built from a countable set of atomic propositions $\mathbf{P}$ and a finite set of agents $AGT = \{1, \ldots, n\}$ and is defined by the following BNF:

$$\mathcal{L}_{G-\mathsf{STIT}}(\mathbf{P}, AGT) : \varphi \ ::= \ p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid [J : stit]\varphi$$

where $p$ ranges over $\mathbf{P}$ and $J$ ranges over $2^{AGT}$. The construction $[J : stit]\varphi$ is read "group $J$ sees to it that $\varphi$ is true regardless of what the other agents choose". We define the dual operator $\langle J : stit\rangle\varphi \stackrel{\mathtt{def}}{=} \neg[J : stit]\neg\varphi$. When $J = \emptyset$, the construction $[\emptyset : stit]\varphi$ is read "$\varphi$ is true regardless of what every agent chooses" or simply "$\varphi$ is necessarily true".

**Definition 4** (STIT-Kripke model, Herzig & Schwarzentruber, 2008)**.** *A* STIT*-Kripke model* $\mathcal{M} = (W, \{R_J\}_{J \subseteq AGT}, V)$ *is a 3-tuple where:*

- $W$ *is a non-empty set of worlds;*
- *for all* $J \subseteq AGT$, $R_J$ *is an equivalence relation such that:*
  - *i)* $R_J \subseteq R_\emptyset$*;*
  - *ii)* $R_J = \bigcap_{j \in J} R_{\{j\}}$*;*
  - *iii) for all* $w \in W$ *and* $(w_1, \ldots, w_n) \in W^n$*, if* $u_1 \in R_{\{1\}}(w), \ldots, u_n \in R_{\{n\}}(w)$ *then* $\bigcap_{1 \le j \le n} R_{\{j\}}(u_j) \neq \emptyset$*;*
- $V : \mathbf{P} \to 2^W$ *is a valuation function for atomic propositions;*

*with* $R_J(w) = \{u \in W : (w, u) \in R_J\}$ *for any* $J \in 2^{AGT}$*.*

The partition induced by the equivalence relation $R_J$ is the set of possible choices of the group $J$.[9] Indeed, in STIT a choice of a group $J$ at a given world $w$ is identified with the set of possible worlds $R_J(w)$. We call $R_J(w)$ the set of possible outcomes of group $J$'s choice at world $w$, in the sense that group $J$'s current choice at $w$ forces the possible worlds to be in $R_J(w)$. The set $R_\emptyset(w)$ is simply the set of possible outcomes at $w$, or said differently, the set of outcomes of the current game at $w$. According to Condition *(i)*, the set of possible outcomes of a group $J$'s choice is a subset of the set of possible outcomes. Condition *(ii)*, called *additivity*, means that the choices of the agents in a group $J$ is made up of the choices of each individual agent and no more. Condition *(iii)* corresponds to the property of *independence of agents*: whatever each agent decides to do, the set of outcomes corresponding to the joint action of all agents is non-empty. More intuitively, this means that agents can never be deprived of choices due to the choices made by other agents. In Lorini and Schwarzentruber's work (2011) determinism for the group $AGT$ was assumed. That is to say that the set of outcomes corresponding to a joint action of all agents is a singleton. Horty's group STIT logic (Horty, 2001) does not suppose this. Here we deal with Horty's version of STIT. So a STIT model is a game form in which a joint action of all agents might determine more than one outcome.

9. One can also see the partition induced by the equivalence relation $R_j$ as the set of actions that agent $j$ can *try*, where the notion of *trying* corresponds to the notion of *volition* studied in philosophy of action (e.g., O'Shaughnessy, 1974; McCann, 1974).

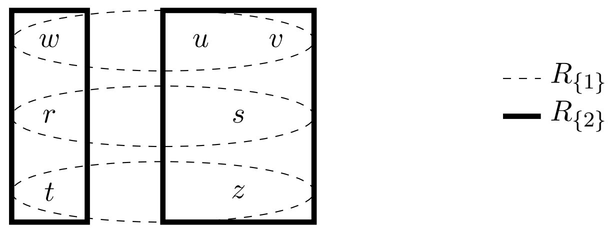



Figure 3: The $\mathsf{STIT}$-model $\mathcal{M}$

**Example 2.** *The tuple* $\mathcal{M} = (W, R_\emptyset, R_{\{1\}}, R_{\{2\}}, R_{\{1,2\}}, V)$ *defined by:*

- $W = \{w, u, v, r, s, t, z\}$;
- $R_\emptyset = W \times W$;
- $R_{\{1\}} = \{w, u, v\}^2 \cup \{r, s\}^2 \cup \{t, z\}^2$;
- $R_{\{2\}} = \{w, r, t\}^2 \cup \{u, v, s, z\}^2$;
- $R_{\{1,2\}} = \{(w, w), (r, r), (s, s), (t, t), (z, z), (u, u), (v, v), (u, v), (v, u)\}$;
- *for all* $p \in \mathbf{P}$, $V(p) = \emptyset$.

*is a* $\mathsf{STIT}$*-Kripke model. Figure 3 shows the model* $\mathcal{M}$*. The equivalence classes induced by the equivalence relation* $R_{\{1\}}$ *are represented by ellipses and correspond to the choices of agent* 1*. The equivalence classes induced by the equivalence relation* $R_{\{2\}}$ *are represented by rectangles and correspond to the choices of agent* 2*. The choice of group* $\{1, 2\}$ *at a given world is determined by the intersection of the choice of agent* 1 *and the choice of agent* 2 *at this world. For example, the choice of agent* 1 *at world* $u$ *is* $\{w, u, v\}$ *whereas the choice of agent* 2 *at world* $u$ *is* $\{u, v, s, z\}$*. The choice of group* $\{1, 2\}$ *at* $u$ *is* $\{u, v\}$*. Note that Condition* (iii) *of Definition 4 ensures that for any choice of agent* 1 *and for any choice of agent* 2 *the intersection between these two choices is non-empty. That is, for any equivalence class induced by the relation* $R_{\{1\}}$ *and for any equivalence class induced by the relation* $R_{\{2\}}$*, the intersection between these two equivalence classes is non-empty.*

Given a $\mathsf{STIT}$-Kripke model $\mathcal{M} = (W, \{R_J\}_{J \subseteq AGT}, V)$ and a world $w$ in $\mathcal{M}$, the truth conditions of $\mathsf{STIT}$ formulae are the following:

$$\begin{aligned}
\mathcal{M}, w \models_{\mathsf{STIT}} p &\iff w \in V(p);\\
\mathcal{M}, w \models_{\mathsf{STIT}} \neg\varphi &\iff \mathcal{M}, w \not\models_{\mathsf{STIT}} \varphi;\\
\mathcal{M}, w \models_{\mathsf{STIT}} \varphi \wedge \psi &\iff \mathcal{M}, w \models_{\mathsf{STIT}} \varphi \text{ AND } \mathcal{M}, w \models_{\mathsf{STIT}} \psi;\\
\mathcal{M}, w \models_{\mathsf{STIT}} [J : stit]\varphi &\iff \forall v \in R_J(w) : \mathcal{M}, v \models_{\mathsf{STIT}} \varphi
\end{aligned}$$

where $R_J(w) = \{u \in W \mid (w, u) \in R_J\}$.

### 3.2 Embedding Atemporal STIT into CP

We are not able to embed group STIT into CP because of many reasons. The first one is that the group STIT satisfiability problem is undecidable if there are more than 3 agents (Herzig & Schwarzentruber, 2008).[10] The second one is that group STIT does not have the finite model property. Indeed Herzig and Schwarzentruber (2008) provide a translation from the product logic $\mathsf{S5}^n$ to group STIT logic, and as $\mathsf{S5}^n$ does not have the finite model property (Gabbay, Kurucz, Wolter, & Zakharyaschev, 2003), so atemporal group STIT will not have it. On the contrary CP inherits the finite model property from S5. Indeed, if a formula $\varphi$ is CP-satisfiable, Theorem 2 says that $tr(\varphi)$ is S5-satisfiable. But as S5 has the polynomial model property, there exists a polynomial-sized S5-model for $tr(\varphi)$ in the size of $tr(\varphi)$. In other words, there exists an exponential S5-model for $tr(\varphi)$ in the size of $\varphi$. Theorem 2 ensures that there exists an exponential CP-model for $\varphi$ in the size of $\varphi$.

We will nevertheless embed a variant of group STIT under the assumption that every agent has a finite and bounded number of actions in his repertoire. For every agent $j$, a $R_j$-equivalence class $R_j(u)$ corresponds to an action of agent $j$. We say that agent $j$ has $k_j$ actions in a STIT model if and only if there are exactly $k_j$ $R_j$-equivalence classes in $\mathcal{M}$.

The game structure in STIT-models should be enforced in CP-models. That is why we introduce special atomic propositions to encode the game structure. Without loss of generality, we assume that the set $\mathbf{P}$ contains special atomic propositions $rep^j_1, rep^j_2, \ldots$ for all agents $j$ which are used to represent the actions of the agents. Let $k$ be the maximal number of actions: $k = max_{j \in AGT} k_j$. For every agent, we represent its actions by numbers $\ell$ in $\{0, \ldots, k-1\}$ and some atomic propositions encode the binary representation of $\ell$. Let $m$ be an integer that represents the number of digits we need to represent an action. For instance let $m = \lceil log_2 k \rceil$ (the ceiling of the logarithm of $k$). For a given agent $j$, $\mathcal{R}^j_m = \{rep^j_1, \ldots, rep^j_m\}$ is the set atomic propositions that represent the binary digits of an action of agent $j$. We suppose that if $j \neq i$ then $\mathcal{R}^j_m \cap \mathcal{R}^i_m = \emptyset$.

**Example 3.** *For example, in the model of Example 2, agent* 1 *has* $k_1 = 3$ *actions and agent* 2 *has* $k_2 = 2$ *actions. So* $k = 3$ *and* $m = \lceil log_2 3 \rceil = 2$. *We have* $\mathcal{R}^1_m = \{rep^1_1, rep^1_2\}$ *and* $\mathcal{R}^2_m = \{rep^2_1, rep^2_2\}$. *Then for instance, we may represent the action of agent 1 corresponding to* $R_{\{1\}}(w) = \{w, u, v\}$ *by the valuation* $\neg rep^1_1 \wedge \neg rep^1_2$, *the action of agent 1 corresponding to* $\{r, s\}$ *by* $rep^1_1 \wedge \neg rep^1_2$, *the action of agent 1 corresponding to* $\{t, z\}$ *by* $\neg rep^1_1 \wedge rep^1_2$, *the action of agent 2 corresponding to* $\{w, r, t\}$ *by* $\neg rep^2_1 \wedge \neg rep^2_2$ *and the action of agent 2 corresponding to* $\{u, v, s, z\}$ *by* $rep^2_1 \wedge \neg rep^2_2$.

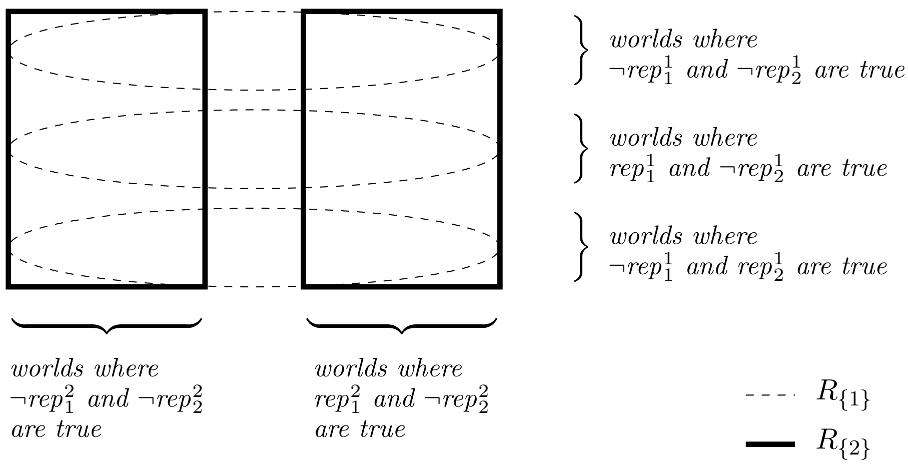
Let $\mathcal{R}_m = \bigcup_{j \in AGT} \mathcal{R}^j_m$ be the set of all atomic propositions used to denote actions. Let us define the following CP formula:

$$GRID_m \stackrel{\texttt{def}}{=} \bigwedge_{x \in \mathcal{R}_m} [\emptyset]((x \to \langle \mathcal{R}_m \setminus \{x\} \rangle \neg x) \wedge (\neg x \to \langle \mathcal{R}_m \setminus \{x\} \rangle x)) \tag{4}$$

This formula enforces a CP model to be *universal* (over $\mathcal{R}_m$), that is, to contain all possible valuations over $\mathcal{R}_m$ (recall Definition 2). A model that satisfies $GRID_m$ is then interpreted as a game form where each valuation of $\mathcal{R}^j_m$ represents an action of player $j$.

10. See Lorini and Schwarzentruber's paper (2011) for a study of some decidable fragments of group STIT.

**Example 4.** *For instance, if some world of a* CP *model* $\mathcal{M}'$ *satisfies* $GRID_2$*, then the 'skeleton' of* $\mathcal{M}'$ *should have the following form (we intentionally draw the 'skeleton' of the model* $\mathcal{M}'$ *so that it looks like the model* $\mathcal{M}$*):*




We now define a translation from $\mathcal{L}_{G-\mathsf{STIT}}$ to $\mathcal{L}_{\mathsf{CP}}(\mathbf{P})$ as follows:

$$\begin{aligned} tr_2(p) &= p \quad \text{FOR } p \in \mathbf{P} \\ tr_2(\neg\varphi) &= \neg tr_2(\varphi) \\ tr_2(\varphi \wedge \psi) &= tr_2(\varphi) \wedge tr_2(\psi) \\ tr_2([J : stit]\varphi) &= [\bigcup_{j \in J} \mathcal{R}_m^j] tr_2(\varphi) \end{aligned}$$

The translation $tr_2$ should be parameterized by $m$. For notational convenience, in what follows we write $tr_2$ instead of $tr_2^m$ leaving implicit the parameter $m$.

The set $\bigcup_{j \in J} \mathcal{R}_m^j$ represents all the atomic propositions used to represented actions of the coalition $J$.

**Example 5.** *For instance, with* $m = 2$,

$$tr_2([\{1\} : stit][\{1,2\} : stit]p) = [\{rep_1^1, rep_2^1\}][\{rep_1^1, rep_2^1, rep_1^2, rep_2^2\}]p.$$

We then obtained the desired satisfiability-preservation result. The proof is given in Appendix C.

**Theorem 3.** *Let us consider a group* STIT *formula* $\varphi$*. Let* $m$ *be an integer. Then the following items are equivalent:*

1. $\varphi$ *is* STIT*-satisfiable in a* STIT*-model where each agent has at most* $2^m$ *actions;*

2. $\varphi$ *is* STIT*-satisfiable in a* STIT*-model where each agent has exactly* $2^m$ *actions;*

3. $GRID_m \wedge tr_2(\varphi)$ *is* CP*-satisfiable.*

### 3.3 Atemporal Individual STIT

In this subsection, we consider the following fragment of STIT called atemporal individual STIT [11]:

$$\mathcal{L}_{I-\mathsf{STIT}}(\mathbf{P}, AGT) : \varphi \ ::= \ p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid [\{j\} : stit]\varphi$$

where $p$ ranges over $\mathbf{P}$ and $j$ ranges over $AGT$.

This fragment of STIT, axiomatized by Xu (1998), has the exponential finite model property (see Lemma 7 in Balbiani et al., 2008b). Moreover, as the following theorem highlights, it can be embedded in the logic CP.

**Theorem 4.** *Let us consider a* STIT *formula* $\varphi$ *of the individual* STIT *fragment. Let* $m$ *be the length of* $\varphi$*. Then the following three items are equivalent:*

1. $\varphi$ *is* STIT*-satisfiable*
2. $\varphi$ *is* STIT*-satisfiable in a model where each agent has at most* $2^m$ *actions;*
3. $GRID_m \wedge tr_2(\varphi)$ *is* CP*-satisfiable.*

*Proof.* $\boxed{1 \Rightarrow 2}$ Consider a STIT formula $\varphi$ of the individual STIT fragment. If $\varphi$ is STIT-satisfiable and $m$ is the length of $\varphi$, then $\varphi$ is STIT-satisfiable in a model where there are at most $2^m$ worlds (see Lemma 7 in Balbiani et al., 2008b). This implies that there are at most $2^m$ actions in that model. The implications $2 \Rightarrow 3$ and $3 \Rightarrow 1$ come from Theorem 3. □

Thanks to Theorem 4, we reduce the NEXPTIME-complete satisfiability problem of individual STIT (Balbiani et al., 2008b) to the CP-satisfiability problem. As the reduction is polynomial, we obtain the following lower bound complexity result for the CP-satisfiability problem.

**Corollary 2.** *The* CP*-satisfiability problem is NEXPTIME-hard.*

### 3.4 Group STIT where Coalitions Are Nested

In this subsection we address the satisfiability problem of the fragment of CP consisting of formulae $\varphi$ of $\mathcal{L}_{\mathsf{CP}}$ such that the sets of atomic propositions that appear in any operator $[X]$ occurring in $\varphi$ form a linear set of sets of atomic propositions. More formally, if $[X]$ and $[X']$ are two operators occurring in $\varphi$ then either $X \subseteq X'$ or $X' \subseteq X$. For instance, the formula $[\{p,q\}](\psi \wedge [\{p\}][\{p,q,r,s\}]\varphi)$ belongs to the fragment because $\{p\} \subseteq \{p,q\} \subseteq \{p,q,r,s\}$. On the contrary, the formula $[\{p\}]p \wedge [\{q\}]p$ is not an element of this fragment of CP.

We call the satisfiability problem of this fragment of CP the CP-nested satisfiability problem. Due to the embedding proposed in Theorem 3 of STIT into CP, we provide the following lower bound complexity result for the CP-nested satisfiability problem. The proof is given in Appendix D.

**Theorem 5.** *The* CP*-nested satisfiability problem is PSPACE-hard.*

11. Some authors (e.g., Broersen, 2008; Wansing, 2006) use the term 'multi-agent STIT' to designate the logic where operators are of the form $[\{j\} : stit]$. Here we prefer to use the more explicit term 'individual STIT' as in Herzig and Schwarzentruber's work (2008).

The following theorem provides an upper bound complexity result for this fragment of CP. The proof is given in Appendix E.

**Theorem 6.** *The* CP*-nested satisfiability problem is in PSPACE.*

This concludes our analysis of STIT logics via CP. In the next section we move to normal coalition logic.

### 3.5 Normal Coalition Logic

We conclude this section on STIT by briefly mentioning a related system, normal coalition logic. Normal coalition logic NCL was introduced by Broersen et al. (2007) to provide an embedding in a normal modal logic of the influential—and non-normal—coalition logic (Pauly, 2002). The embedding was based on a general simulation technique developed by Gasquet and Herzig (1994) and showed for the first time how coalition logic—which had already been recognized as the fragment of ATL containing only the 'next' operator (Goranko, 2001)—could actually be interpreted on very traditional structures such as Kripke frames based on equivalence relations. NCL was further studied by Balbiani et al. (2008a). Also NCL has a known atemporal variant, introduced and studied by Balbiani et al. (2008a) and Lorini and Schwarzentruber (2011).

Two results on atemporal NCL from the literature are worth mentioning in this context. First, Balbiani et al. (2008a, Thm. 38) show that the satisfiability problem for atemporal NCL (when $|AGT| \geq 2$) is NEXPTIME-complete, like CP; second, Lorini and Schwarzentruber (2011, Prop. 1) show that when the $|AGT| \leq 2$, then atemporal STIT is embeddable in atemporal NCL and vice versa, and that an embedding (in both directions) in the general case is possible only by considerably restricting the syntax of $\mathcal{L}_{\mathsf{STIT}}$.

## 4. The *Ceteris Paribus* Structure of Coalition Logic of Propositional Control

In this section we study the relationships between CP, atemporal 'bounded' group STIT, and another well-known game logic, the logic CL−PC (*coalition logic of propositional control*).[12] Specifically, we show that CL−PC can be embedded, preserving satisfiability, into atemporal 'bounded' group STIT and, by the fact that atemporal 'bounded' group STIT can be embedded into CP (Section 3.1), we indirectly show that CL−PC can be embedded into CP. To complete the picture we also provide a direct embedding from CL−PC to CP. This latter embedding is of particular interest to highlight the striking similarities between the models of CP and of CL−PC.

CL−PC was introduced by van der Hoek and Wooldridge (2005) as a formal language for reasoning about capabilities of agents and coalitions in multiagent environments. In this logic the notion of capability is modeled by means of the concept of *control*. In particular, it is assumed that each agent $i$ is associated with a specific finite subset $\mathbf{P}_i$ of the finite set of all propositions $\mathbf{P}$. $\mathbf{P}_i$ is the set of propositions *controlled* by the agent $i$. That is, the agent $i$ has the ability to assign a (truth) value to each proposition $\mathbf{P}_i$ but cannot affect the truth

12. In Gerbrandy's work (2006) generalizations of some of the assumptions underlying CL−PC have been studied. Here we only consider the original version of CL−PC proposed by van der Hoek and Wooldridge.

values of the propositions in $\mathbf{P} \setminus \mathbf{P}_i$. In the variant of $\mathsf{CL-PC}$ studied by van der Hoek and Wooldridge (2005) it is also assumed that control over propositions is exclusive, that is, two agents cannot control the same proposition (i.e., if $i \neq j$ then $\mathbf{P}_i \cap \mathbf{P}_j = \emptyset$). Moreover, it is assumed that control over propositions is complete, that is, every proposition is controlled by at least one agent (i.e., for every $p \in \mathbf{P}$ there exists an agent $i$ such that $p \in \mathbf{P}_i$).

The preceding concepts and assumptions are precisely formulated in the following section, which illustrates the syntax and the formal semantics of $\mathsf{CL-PC}$.

### 4.1 Syntax and Semantics of $\mathsf{CL-PC}$

The *language of* $\mathsf{CL-PC}$ is built from a *finite* set of atomic propositions $\mathbf{P}$ and a finite set of agents $AGT = \{1, \ldots, n\}$, and is defined by the following BNF:

$$\mathcal{L}_{\mathsf{CL-PC}}(\mathbf{P}, AGT) : \varphi \ ::= \ p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Diamond_J \varphi$$

where $p$ ranges over $\mathbf{P}$ and $J$ ranges over $2^{AGT}$. Operator $\Diamond_J$ is called *cooperation modality*, and the construction $\Diamond_J \varphi$ means that "group $J$ has the contigent ability to achieve $\varphi$".

**Definition 5** ($\mathsf{CL-PC}$ model)**.** *A model for* $\mathsf{CL-PC}$ *is a tuple* $\mathcal{M} = (\mathbf{P}_1, \ldots, \mathbf{P}_n, X)$ *where:*

- $\mathbf{P}_1, \ldots, \mathbf{P}_n$ *is a partition of* $\mathbf{P}$ *among the agents in* $AGT$*;*
- $X \subseteq \mathbf{P}$ *is the set of propositions which are true in the initial state.*

For every group of agents $J \subseteq AGT$, let $\mathbf{P}_J = \bigcup_{i \in J} \mathbf{P}_i$ be the set of atomic propositions controlled by the group $J$. Moreover, for every group $J \subseteq AGT$ and for every set of atomic propositions $X \subseteq \mathbf{P}$, let $X_J = X \cap \mathbf{P}_J$ be the set of atomic propositions in $X$ controlled by the group $J$. Sets $X_J$ are called $J$-valuations.

Given a $\mathsf{CL-PC}$ model $\mathcal{M} = (\mathbf{P}_1, \ldots, \mathbf{P}_n, X)$, the truth conditions of $\mathsf{CL-PC}$ formulae are the following:

$$\begin{aligned}
\mathcal{M} \models_{\mathsf{CL-PC}} p &\iff p \in X; \\
\mathcal{M} \models_{\mathsf{CL-PC}} \neg\varphi &\iff \mathcal{M} \not\models_{\mathsf{CL-PC}} \varphi; \\
\mathcal{M} \models_{\mathsf{CL-PC}} \varphi \wedge \psi &\iff \mathcal{M} \models_{\mathsf{CL-PC}} \varphi \text{ AND } \mathcal{M} \models_{\mathsf{CL-PC}} \psi; \\
\mathcal{M} \models_{\mathsf{CL-PC}} \Diamond_J \varphi &\iff \exists X'_J \subseteq \mathbf{P}_J : \mathcal{M} \bigoplus X'_J \models_{\mathsf{CL-PC}} \varphi
\end{aligned}$$

where $\mathcal{M} \bigoplus X'_J$ is the $\mathsf{CL-PC}$ model $(\mathbf{P}_1, \ldots, \mathbf{P}_n, X'')$ such that:

$$\begin{aligned}
X''_{AGT \setminus J} &= X_{AGT \setminus J} \\
X''_J &= X'_J
\end{aligned}$$

That is, $\Diamond_J \varphi$ is true at a given model $\mathcal{M}$ if and only if, the coalition $J$ can change the truth values of the atoms that it controls in such a way that $\varphi$ will be true afterwards (i.e., given the actual truth-value combination of the atoms which are not controlled by $J$, there exists a truth-value combination of the atoms controlled by $J$ which ensures $\varphi$).

Let us illustrate the $\mathsf{CL-PC}$ semantics with an example.

**Example 6.** *Let* $AGT = \{1, 2, 3\}$*,* $\mathbf{P} = \{p, q, r\}$*,* $\mathbf{P}_1 = \{p\}$*,* $\mathbf{P}_2 = \{q\}$ *and* $\mathbf{P}_3 = \{r\}$*.*

*Consider the* CL−PC *model* $\mathcal{M} = (\mathbf{P}_1, \mathbf{P}_2, , \mathbf{P}_3, \{r\})$*. We have that:*

$$\mathcal{M} \models_{\mathsf{CL-PC}} \Diamond_{\{1,2\}}((p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r)).$$

*Indeed, there exists a set of atoms* $X'_{\{1,2\}} \subseteq \mathbf{P}_{\{1,2\}}$ *controlled by* $\{1, 2\}$ *such that* $\mathcal{M} \bigoplus X'_{\{1,2\}} \models_{\mathsf{CL-PC}} ((p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r))$*. An example is* $X'_{\{1,2\}} = \{p\} \subseteq \mathbf{P}_{\{1,2\}}$*, from which,* $(\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \{p, r\}) \models_{\mathsf{CL-PC}} ((p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r))$ *where* $(\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \{p, r\}) = \mathcal{M} \bigoplus \{p\}$*.*

### 4.2 Embedding CL−PC into STIT

The aim of this section is to provide an embedding of CL−PC into the variant of atemporal group STIT with bounded choices (atemporal 'bounded' group STIT) that have been presented in Section 3.1.

Let us provide the following STIT formulae which catpure four basic assumptions of CL−PC:

$$EXC^{+} \stackrel{\text{def}}{=} \bigwedge_{p \in \mathbf{P}} \bigwedge_{i,j \in AGT : i \neq j} (\langle \emptyset : stit \rangle [\{i\} : stit] p \rightarrow \neg \langle \emptyset : stit \rangle [\{j\} : stit] p) \quad (5)$$

$$EXC^{-} \stackrel{\text{def}}{=} \bigwedge_{p \in \mathbf{P}} \bigwedge_{i,j \in AGT : i \neq j} (\langle \emptyset : stit \rangle [\{i\} : stit] p \rightarrow \neg \langle \emptyset : stit \rangle [\{j\} : stit] \neg p) \quad (6)$$

$$COMPL \stackrel{\text{def}}{=} \bigwedge_{p \in \mathbf{P}} \bigvee_{i \in AGT} [\emptyset : stit]([\{i\} : stit] p \vee [\{i\} : stit] \neg p) \quad (7)$$

$$GRID^{*} \stackrel{\text{def}}{=} \bigwedge_{X \subseteq \mathbf{P}} \langle \emptyset : stit \rangle \left( \bigwedge_{p \in X} p \wedge \bigwedge_{p \in \mathbf{P} \setminus X} \neg p \right) \quad (8)$$

Formulae $EXC^{+}$ and $EXC^{-}$ mean that control over atomic propositions in $\mathbf{P}$ is exclusive (i.e., there is no proposition in $\mathbf{P}$ which can be forced to be true or false by more than one agent), whereas formula *COMPL* means that exercise of control over atomic propositions in $\mathbf{P}$ is complete (i.e., for every proposition in $\mathbf{P}$ there exists at least one agent who either forces it to be true or forces it to be false). Finally, formula $GRID^{*}$ means that all the possible truth-value combinations of the atomic propositions in $\mathbf{P}$ are possible. Note that $EXC^{+}$, $EXC^{-}$, *COMPL* and $GRID^{*}$ are well-formed STIT formulae because of the assumption that the set $\mathbf{P}$ is finite.[13]

We define the following translation from $\mathcal{L}_{\mathsf{CL-PC}}(\mathbf{P}, AGT)$ to $\mathcal{L}_{\mathsf{STIT}}(\mathbf{P}, AGT)$:

$$\begin{aligned} tr_3(p) &= p \quad \text{FOR } p \in \mathbf{P} \\ tr_3(\neg\varphi) &= \neg tr_3(\varphi) \\ tr_3(\varphi \wedge \psi) &= tr_3(\varphi) \wedge tr_3(\psi) \\ tr_3(\Diamond_J \varphi) &= \langle AGT \setminus J : stit \rangle tr_3(\varphi) \end{aligned}$$

The following theorem highlights that 'bounded' group STIT embeds CL−PC. The proof is given in Appendix F.

13. This assumption is also made by van der Hoek and Wooldridge (2005).

**Theorem 7.** *Let $m = |\mathbf{P}|$. Then, a* CL−PC *formula $\varphi$ is* CL−PC*-satisfiable if and only if $(EXC^+ \wedge EXC^- \wedge COMPL \wedge GRID^*) \wedge tr_3(\varphi)$ is satisfiable in a* STIT *model where each agent has at most $2^m$ actions.*

As CP embeds atemporal 'bounded' group STIT (Theorem 3 in Section 3.1), from Theorem 7 it follows that CP also embeds CL−PC. Indeed, given a CL−PC-satisfiable formula $\varphi$, one can use the translation $tr_2$ given in Section 3.1 in order to find a corresponding STIT formula which is STIT-satisfiable. Then, one uses the preceding translation $tr_3$ in order to find a corresponding CP formula which is CP-satisfiable.

**Corollary 3.** *Let $m = |\mathbf{P}|$. Then, a* CL−PC *formula $\varphi$ is* CL−PC*-satisfiable if and only if $GRID_m \wedge tr_2((EXC^+ \wedge EXC^- \wedge COMPL \wedge GRID^*) \wedge tr_3(\varphi))$ is* CP*-satisfiable.*

### 4.3 Directly Embedding CL−PC into CP

To complete the picture, we study here a direct embedding of CL−PC into CP.

**Definition 6** (From CL−PC to CP models)**.** *Let $\mathcal{M} = (\mathbf{P}_1, \ldots, \mathbf{P}_n, X)$ be a* CL−PC*-model. Define $\mathcal{M}_{\mathsf{CP}} = (W, V)$ as follows:*

- *$W = 2^{\mathbf{P}}$;*
- *$V$ is such that $V(p) = \{w \mid p \in w\}$ for all $p \in \mathbf{P}$.*

Intuitively, $V$ is such that any truth-assignment on $\mathbf{P}$ is witnessed by exactly one $w \in W$ and $w_X$ is the witness of the truth assignment represented by $X$ (i.e., makes all atoms in $X$ true and the rest false). So $\mathcal{M}_{\mathsf{CP}}$ is a non-redundant universal CP model and $\mathcal{M}_{\mathsf{CP}}, X$ is a pointed CP-model (Definition 2). Now define the following translation from $\mathcal{L}_{\mathsf{CL-PC}}(\mathbf{P}, AGT)$, and a partition $\mathbf{P}_1, \ldots, \mathbf{P}_n$ of $\mathbf{P}$, to $\mathcal{L}_{\mathsf{CP}}(\mathbf{P})$:

$$\begin{aligned} tr_4(p) &= p \quad \text{FOR } p \in \mathbf{P} \\ tr_4(\neg\varphi) &= \neg tr_4(\varphi) \\ tr_4(\varphi \wedge \psi) &= tr_4(\varphi) \wedge tr_4(\psi) \\ tr_4(\Diamond_J \varphi) &= \langle Y_J \rangle tr_4(\varphi) \end{aligned}$$

where $Y_J = -\bigcup_{j \in AGT} \mathbf{P}_j$ (i.e., the atoms that are not controlled by anybody in $J$).

**Theorem 8.** *Let $\mathcal{M} = (\mathbf{P}_1, \ldots, \mathbf{P}_n, X)$ be a* CL−PC*-model and $\varphi \in \mathcal{L}_{\mathsf{CL-PC}}(\mathbf{P}, AGT)$:*

$$\mathcal{M} \models_{\mathsf{CL-PC}} \varphi \iff \mathcal{M}_{\mathsf{CP}}, X \models_{\mathsf{CP}} tr_4(\varphi)$$

*Proof.* We proceed by induction on the syntax of $\varphi$. Base Trivial by the construction of $\mathcal{M}_{\mathsf{CP}}$ (Definition 6). Step The cases for the Boolean connectives are straightforward. We focus on the modal case:

$$\mathcal{M} \models_{\mathsf{CL-PC}} \Diamond_J \varphi \iff \mathcal{M}_{\mathsf{CP}}, X \models_{\mathsf{CP}} \langle Y_J \rangle tr_4(\varphi)$$

where $Y_J = -\bigcup_{j \in N} \mathbf{P}_j$. The case is proven by the following series of equivalences:

$$\begin{aligned}
\mathcal{M} \models_{\mathsf{CL-PC}} \Diamond_J \varphi \iff & \exists X'_J \subseteq \mathbf{P}_J : \mathcal{M} \bigoplus X'_J \models_{\mathsf{CL-PC}} \varphi \\
& \text{Semantics of } \Diamond_J \\
\iff & \exists X'_J \subseteq \mathbf{P}_J : (\mathbf{P}_1, \ldots, \mathbf{P}_n, X'_J \cup X_{AGT \setminus J}) \models_{\mathsf{CL-PC}} \varphi \\
& \text{Definition of } \bigoplus \\
\iff & \exists X'_J \subseteq \mathbf{P}_J : \mathcal{M}_{\mathsf{CP}}, X'_J \cup X_{AGT \setminus J} \models_{\mathsf{CP}} tr_4(\varphi) \\
& \text{Definition 6 and IH} \\
\iff & \exists Y \sim_{Y_J} X \text{ AND } \mathcal{M}_{\mathsf{CP}}, Y \models_{\mathsf{CP}} tr_4(\varphi) \\
& \text{Definition 1} \\
\iff & \mathcal{M}_{\mathsf{CP}}, X \models_{\mathsf{CP}} \langle Y_J \rangle tr_4(\varphi) \\
& \text{Definition 3}
\end{aligned}$$

This completes the proof. □

## 5. The *Ceteris Paribus* Structure of Dynamic Logic of Propositional Assignments

The dynamic logic of propositional assignments (DL−PA) is the *concrete* variant of propositional dynamic logic (PDL) (Harel et al., 2000) in which atomic programs are assignments of propositional variables to true or to false.[14] The complexities of the model checking and of the satisfiability problem for DL−PA have been recently studied by Balbiani et al. (2013). The *starless* version of DL−PA was previously studied by van Eijck (2000) and recently put to use by Herzig et al. (2011), who have shown that it embeds CL−PC. In the next section we study the relationship between CP and DL−PA. Specifically, we provide a truth-preserving embedding of *starless* DL−PA into CP as well as a truth-preserving embedding of CP into DL−PA.

DL−PA, it has been argued by Herzig et al. (2011), represents a very general and—because of its direct link with PDL—natural formalism for reasoning about agency. The results in this section, we argue, point to a similar status for CL−PC, modulo the use of the Kleene star about which we will comment in Section 7.

14. Programs in standard PDL are abstract as they are just letters $a, b, \ldots$ from some alphabet.

### 5.1 Syntax and Semantics of DL−PA

The *language of* DL−PA is built from a *finite* set of atomic propositions **P** and is defined by the following BNF:

$$\begin{array}{rcl} \pi & ::= & +p \mid -p \mid \pi;\pi \mid \pi \cup \pi \mid \pi^* \mid \varphi? \\ \mathcal{L}_{\mathsf{DL-PA}}(\mathbf{P}) : \varphi & ::= & p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle\pi\rangle\varphi \end{array}$$

We will use $\pm p$ to denote $(+p \cup -p)$.

**Definition 7** (DL−PA model)**.** *A* DL−PA*-model is a set* $X \subseteq \mathbf{P}$*.*

That is, a DL−PA model is a propositional valuation.
The semantics is given by induction as follows:

- $[\![+p]\!] = \{(X, X') \mid X' = X \cup \{p\}\}$;
- $[\![-p]\!] = \{(X, X') \mid X' = X - \{p\}\}$;
- $[\![\pi;\pi']\!] = [\![\pi]\!] \circ [\![\pi']\!]$;
- $[\![\pi \cup \pi']\!] = [\![\pi]\!] \cup [\![\pi']\!]$;
- $[\![\pi^*]\!] = \bigcup_{k\in\mathbb{N}} [\![\pi]\!]^k$;
- $[\![\varphi?]\!] = \{(X, X) \mid X \in [\![\varphi]\!]\}$;
- $[\![p]\!] = \{X \mid p \in X\}$;
- $[\![\neg\varphi]\!] = 2^{\mathbf{P}} - [\![\varphi]\!]$;
- $[\![\varphi \vee \psi]\!] = [\![\varphi]\!] \cup [\![\psi]\!]$;
- $[\![\langle\pi\rangle\varphi]\!] = \{X \mid \text{ there exists } X' \text{ s.th. } (X, X') \in [\![\pi]\!] \text{ and } X' \in [\![\varphi]\!]\}$.

We write $X \models_{\mathsf{DL-PA}} \varphi$ for $X \in [\![\varphi]\!]$. We will refer to the fragment of DL−PA without $*$ operator as *starless* DL−PA.

### 5.2 Some Properties of DL−PA

Like in PDL, program constructors ; , ∪ and ? are eliminable:

**Fact 2.** *The following are* DL−PA *validities:*

$$\begin{array}{rcl} \langle\pi;\pi'\rangle\varphi & \leftrightarrow & \langle\pi\rangle\langle\pi'\rangle\varphi \\ \langle\pi \cup \pi'\rangle\varphi & \leftrightarrow & \langle\pi\rangle\varphi \vee \langle\pi'\rangle\varphi \\ \langle\psi?\rangle\varphi & \leftrightarrow & \psi \wedge \varphi \end{array}$$

However, unlike in PDL, the $*$ operator is also eliminable in DL−PA:

**Fact 3** (Balbiani et al., 2013)**.** *For every* $\varphi \in \mathcal{L}_{\mathsf{CL-PC}}(\mathbf{P})$ *there exists* $\varphi' \in \mathcal{L}_{\mathsf{CL-PC}}(\mathbf{P})$ *such that* $\varphi \leftrightarrow \varphi'$ *is* DL−PA *valid.*

### 5.3 Embedding Starless DL−PA into CP

By a direct adaptation of Definition 6 above, a DL−PA model $X$ can be translated to the pointed CP model $\mathcal{M}_{\mathsf{CP}}, X$ defined in Definition 6. Now fix the following translation:[15]

$$\begin{aligned} tr_5(p) &= p \quad \text{FOR } p \in \mathbf{P} \\ tr_5(\neg\varphi) &= \neg tr_5(\varphi) \\ tr_5(\varphi \wedge \psi) &= tr_5(\varphi) \wedge tr_5(\psi) \\ tr_5(\langle +p \rangle \varphi) &= \langle \mathbf{P} \setminus \{p\} \rangle (p \wedge tr_5(\varphi)) \\ tr_5(\langle -p \rangle \varphi) &= \langle \mathbf{P} \setminus \{p\} \rangle (\neg p \wedge tr_5(\varphi)) \end{aligned}$$

Notice that the translation from *starless* DL−PA to CP does not need to include the cases for sequential composition (;), nondeterministic choice ($\cup$) and test ($\varphi$?) since they are eliminable in DL−PA. Therefore, it guarantees that in CP we could do the same kind of reasoning as in *starless* DL−PA:

**Theorem 9.** *Let $X$ be a* DL−PA *model and $\varphi$ belong to the language of starless* DL−PA*:*

$$X \models_{\mathsf{DL-PA}} \varphi \iff \mathcal{M}_{\mathsf{CP}}, X \models_{\mathsf{CP}} tr_5(\varphi)$$

*Proof.* We proceed by induction on the syntax of $\varphi$. [Base] Trivial by the construction of $\mathcal{M}_{\mathsf{CP}}$ (Definition 6). [Step] The cases for the Boolean connectives are straightforward. We focus on the modal case:

$$X \models_{\mathsf{DL-PA}} \langle +p \rangle \varphi \iff \mathcal{M}_{\mathsf{CP}}, X \models_{\mathsf{CP}} \langle \mathbf{P} \setminus \{p\} \rangle tr_5(\varphi)$$

The case is proven by the following series of equivalences:

$$\begin{aligned} X \models_{\mathsf{DL-PA}} \langle +p \rangle \varphi &\iff X \cup \{p\} \models_{\mathsf{DL-PA}} \varphi && \text{Semantics of } \langle +p \rangle \\ &\iff \exists Y \sim_{\mathbf{P}-\{p\}} X : \mathcal{M}_{\mathsf{CP}}, Y \models_{\mathsf{CP}} tr_5(\varphi) && \text{Definition 1 and IH} \\ &\iff \mathcal{M}_{\mathsf{CP}}, X \models_{\mathsf{CP}} \langle \mathbf{P} \setminus \{p\} \rangle tr_5(\varphi) && \text{Definition 3} \end{aligned}$$

The case for $\langle -p \rangle$ is identical. □

### 5.4 Embedding of CP into Starless DL−PA

The above subsection has shown that the semantics of CP and DL−PA are closely related. However, DL−PA has a built-in assumption to the effect that any valuation (i.e., set of atoms) is feasible. From the point of view of CP this means that DL−PA actually works with *universal* models (cf. Definition 2). Here, we establish an embedding from CP interpreted on universal models,[16] to starless DL−PA. Consider the following translation from $\mathcal{L}_{\mathsf{CP}}(\mathbf{P})$

15. It must be observed that for this translation to work $\mathbf{P}$ should be finite. If not, then $-\{p\}$ is co-finite and $\Diamond_{-\{p\}}$ would not belong to $\mathcal{L}_{\mathsf{CP}}$.
16. This is the class of models one can axiomatize by extending the axiom system of Figure 2 with axioms of form $\langle \emptyset \rangle \delta$ where $\delta$ ranges over propositional formulae encoding one single valuation.

into $\mathcal{L}_{\mathsf{DL-PA}}(\mathbf{P})$:

$$\begin{array}{rcl} tr_7(p) & = & p \quad \text{FOR } p \in \mathbf{P} \\ tr_7(\neg\varphi) & = & \neg tr_7(\varphi) \\ tr_7(\varphi \wedge \psi) & = & tr_7(\varphi) \wedge tr_7(\psi) \\ tr_7(\langle X \rangle \varphi) & = & \langle \pm p_1 \rangle \dots \langle \pm p_n \rangle tr_7(\varphi) \end{array}$$

where $p_1, \dots, p_n$ is an enumeration of the atoms in $\mathbf{P} \setminus X$.[17] We have the following result:

**Theorem 10.** *Let $\mathcal{M}$ be an* CP*-model and $\varphi \in \mathcal{L}_{\mathsf{CP}}(\mathbf{P})$:*

$$\mathcal{M}, w \models_{\mathsf{CP}} \varphi \iff w \models_{\mathsf{DL-PA}} tr_7(\varphi)$$

*Proof.* We proceed by induction on the syntax of $\varphi$. [Base] Trivial by the construction of $\mathcal{M}_{\mathsf{CP}}$ (Definition 6). [Step] The cases for the Boolean connectives are straightforward. We focus on the modal case:

$$\mathcal{M}, w \models_{\mathsf{CP}} \langle X \rangle \varphi \iff w \models_{\mathsf{DL-PA}} \langle \pm p_1 \rangle \dots \langle \pm p_n \rangle tr_7(\varphi)$$

where $p_1, \dots, p_n$ is an enumeration of the atoms in $\mathbf{P} \setminus X$. The case is proven by the following series of equivalences:

$$\begin{array}{rcll} \mathcal{M}, w \models_{\mathsf{CP}} \langle X \rangle \varphi & \iff & \exists w' \sim_X w \text{ S.T. } \mathcal{M}, w' \models_{\mathsf{CP}} \varphi & \text{Definition 1} \\ & \iff & \exists w' \sim_X w \text{ S.T. } w' \models_{\mathsf{DL-PA}} tr_7(\varphi) & \text{IH} \\ & \iff & \exists w' \text{ S.T. } w' = (\dots(w \bigstar \{p_1\}) \bigstar \dots) \bigstar \{p_n\} & \\ & & \qquad \text{AND } w' \models_{\mathsf{DL-PA}} tr_7(\varphi) & \text{Definition 1} \\ & \iff & w \models_{\mathsf{DL-PA}} \langle \pm p_1 \rangle \dots \langle \pm p_n \rangle tr_7(\varphi) & \text{Semantics of DL-PA} \end{array}$$

where $\bigstar \in \{\cup, -\}$, and $p_1, \dots, p_n$ is an enumeration of the atoms in $\mathbf{P} - X$. □

From Theorem 10 we can obtain as corollary a satisfiability-preserving embedding of CP in DL−PA. Fix the formula[18]

$$GRID^{**} \stackrel{\text{def}}{=} \bigwedge_{X \subseteq \mathbf{P}} \langle \emptyset \rangle \left( \bigwedge_{p \in X} p \wedge \bigwedge_{p \in \mathbf{P} \setminus X} \neg p \right) \tag{9}$$

which, it is easy to see, forces a CP-model to contain all propositional valuations from $\mathbf{P}$. We then have:

**Corollary 4.** *Let $\varphi \in \mathcal{L}_{\mathsf{CP}}(\mathbf{P})$. Then, $tr_7(\varphi)$ is* DL−PA *satisfiable iff $GRID^{**} \wedge \varphi$ is* CP *satisfiable.*

This concludes the presentation of the embeddings of STIT, CL−PC and DL−PA into CP (Figure 1). In the following section we take stock commenting on the technical results presented and drawing links with related work.

17. Again, it is crucial that $\mathbf{P}$ be finite.
18. Cf. Formula (8).

## 6. Discussion and Related Work

In this section we provide a summary of the results presented and discuss their implications. We also position our work with respect to existing contributions in the literature on logic and games.

### 6.1 Discussion

The paper has introduced a modal logic that arises naturally by interpreting modal operators on the equivalence relations induced by finite sets of propositional atoms. This logic, called $\mathsf{CP}$, has been axiomatized and embedded (exponentially) into $\mathsf{S5}$. $\mathsf{CP}$ has then been used as a tool to compare three logics of one-shot strategic interaction—atemporal $\mathsf{STIT}$, the coalitional logic of propositional control $\mathsf{CL{-}PC}$ and the dynamic logic of propositional assignments $\mathsf{DL{-}PA}$. All these logics have been embedded into $\mathsf{CP}$.

These embeddings (recall Figure 1) put us in the position to to draw the following general remarks.

- It appears to be justified to talk about a common *ceteris paribus* structure underpinning several of the main logics of game forms as they are all embeddable into $\mathsf{CP}$. This illustrates a striking uniformity in the logical tools needed for expressing choice and effectivity of games in logical languages, and $\mathsf{CP}$ appears to offer a well-suited abstraction for systematizing existing formalisms.

- Furthermore, all these logics are embeddable in $\mathsf{S5}$ (either directly or via $\mathsf{CP}$), highlighting the fact that in order to reason about choice and effectivity in games one essentially reasons over suitably defined partitions of the state space.

- New interesting and so far unexplored embeddings are obtainable as corollaries. In particular, it follows from our results that atemporal $\mathsf{STIT}$ on bounded models can be embedded into starless $\mathsf{DL{-}PA}$ via $\mathsf{CP}$.

- Via logic $\mathsf{S5}$, one can easily show that embeddings in the other directions are also possible (albeit at exponential cost), so that all arrows in Figure 1 may actually be made symmetric. $\mathsf{S5}$ embeds $\mathsf{CP}$, but it is also directly embeddable in all the mentioned logic, as they all contain the universal modality, in the following forms: $\langle\emptyset\rangle$ in $\mathsf{CP}$, $\langle AGT \setminus \emptyset : stit\rangle$ in atemporal $\mathsf{STIT}$, $\Diamond_{AGT}$ in $\mathsf{CL{-}PC}$ and $\langle \pm p_1\rangle \ldots \langle \pm p_n\rangle$ in $\mathsf{DL{-}PA}$ (where $p_1, \ldots, p_n$ is an enumeration of $\mathbf{P}$).

All in all our results unveil a striking—and to some extent unexpected—uniformity underpinning all the formalisms we considered.

### 6.2 Related Work

We review two sets of related contributions.

#### 6.2.1 CP and Modal Ceteris Paribus Logics

There are two logics in the modal logic literature that are strictly related to $\mathsf{CP}$: release logic, and the logic of ceteris paribus preference.

Release logic is a relatively less known formalism in the landscape of modal logics for artificial intelligence. It has been introduced and studied by Krabbendam and Meyer (2003, 2000) in order to provide a modal logic characterization of a general notion of irrelevancy. Modal operators in release logic are $\mathsf{S5}$ operators indexed by subsets of a finite set $\mathtt{Iss}$ of abstract elements denoting the issues that are taken to be irrelevant, or that can be *released*, while evaluating the formula in the scope of the operator. A release model is therefore a tuple $(W, \{\sim^r_X\}_{X\subseteq\mathtt{Iss}}, V)$ where all $\sim^r_X$ are equivalence relations with the additional constraint that if $X \subseteq Y$ then $\sim^r_X \subseteq \sim^r_Y$, that is, by releasing more issues one obtains a more coarse equivalence relation. Formally, this is the semantics of release operators:

$$\mathcal{M}, w \models \Diamond_X \varphi \iff \exists w' \in W : w \sim^r_X w' \text{ AND } \mathcal{M}, w' \models \varphi$$

where $X \subseteq \mathtt{Iss}$ and $\mathcal{M} = (W, \{\sim^r_X\}_{X\subseteq\mathtt{Iss}}, V)$.

One can easily observe that, by Fact 1 (clause *(ii)*), $\mathsf{CP}$ models are release models where $\mathtt{Iss} = \mathbf{P}$ and where the release relation $\sim^r_X = \sim_{-X}$. Vice versa, for $\mathtt{Iss} = \mathbf{P}$, not all release models are $\mathsf{CP}$ models. As a consequence, the logic of $\langle -X \rangle$ operators in $\mathsf{CP}$ is a conservative extension of the logic of $\Diamond_X$ release operators.

Preference logic has also long been concerned with so-called ceteris paribus preferences, that is, preferences incorporating an "all other things being equal" condition. A first logical analysis of such preferences dates back to Von Wright's work (1963), where dyadic modal operators are studied representing statements like '$\varphi$ is preferred to $\psi$, ceteris paribus'. More recently, van Benthem et al. (2009) studied a modal logic of ceteris paribus preferences based on standard unary modal operators. Leaving the preferential component of such logic aside, its ceteris paribus fragment concerns sentences of the form $\langle \Gamma \rangle \varphi$ whose intuitive meaning is "there exists a state which is equivalent to the current (evaluation) state with respect to all the formulae in the (finite) set $\Gamma$ and which satisfies $\varphi$", where the formulae in $\Gamma$ are not only atoms but formulae from the full language. Logic $\mathsf{CP}$ is, therefore, a fragment of the ceteris paribus logic studied by van Benthem et al. where $\Gamma$ is allowed to consist only of a finite set of atoms.

### 6.2.2 Other Contributions to a Systematization of Game Logics

Despite the wealth of approaches that can be found in the literature on game logics, only very few papers have attempted some form of comparison spanning across several formalisms, and attempting some kind of systematization. Two in particular are worth mentioning here.

The most recent one is Herzig's work (2014), which provides a very comprehensive analysis of the field by classifying the existing logics depending on what aspects of agency (e.g., whether they capture strategic interaction or not, whether they handle uncertainty and epistemic attitudes) they capture in their languages. The logics we considered in this paper, for instance, would fall into the *strategic* and *no uncertainty* categories according to the terminology used by Herzig. This analysis is conceptual and predominantly driven by syntactic features of the logics, that is, by the theorems about agency that they enable.

An earlier work which is methodologically closer to ours in its focus on semantics is Goranko and Jamroga's work (2004). That paper compares $\mathsf{ATL}$, its epistemic variant $\mathsf{ATEL}$ (*epistemic* $\mathsf{ATL}$, van der Hoek & Wooldridge, 2003) and $\mathsf{ECL}$ (*extended* $\mathsf{CL}$,[19] Pauly, 2001)

19. This is $\mathsf{CL}$ extended with a Kleene star operator.

providing constructive transformations between their models and establishing, in particular, that ATL subsumes ECL and that ATEL can be embedded into ATL preserving satisfiability.

## 7. Conclusions and Future Work

The paper has provided a unification of the, to date, most influential logics for the representation of one-shot strategic interaction—atemporal STIT, CL−PC and starless DL−PA—under the *ceteris paribus* abstraction formalized in logic CP.

One natural future research direction presents itself, which consists in extending logic CP with a Kleene star operator, in analogy with DL−PA. We conjecture DL−PA to be embeddable into CP with Kleene star and it remains to be investigated whether such new logic could play the same unifying role for logics of extensive form games, that we show it plays in the atemporal case. This will complete the systematization program initiated by the current paper.

Related to the above question, but in a somewhat more technical vein, we have shown in this paper that CP and atemporal individual STIT have the same high complexity of the satisfiability problem when we consider the whole languages. The study of efficient syntactic fragments is then important and we intend to pursue this study in parallel both for CP and for atemporal individual STIT. We expect that several complexity results about fragments of atemporal STIT may be transferred to fragments of CP and viceversa.

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## Appendix A. Proof of Claim (†) in Theorem 1

*Proof.* One can show that REP is derivable for every operator $[X]$ as follows: first one shows that each $[X]$ operator satisfies the Axiom K and the rule of necessitation N.

We provide here below the syntactic proofs of these two claims. For notational convenience we use the following abbreviation:

$$\widehat{Y} \stackrel{\texttt{def}}{=} \left( \bigwedge_{p \in Y} p \wedge \bigwedge_{p \in X \setminus Y} \neg p \right)$$

Derivation of K for $[X]$:

1. $\vdash_{\mathsf{CP}} [X](\varphi \to \psi) \leftrightarrow \bigwedge_{Y \subseteq X} \left(\widehat{Y} \to [\emptyset]\left(\widehat{Y} \to (\varphi \to \psi)\right)\right)$
by Reduce
2. $\vdash_{\mathsf{CP}} \left(\widehat{Y} \to (\varphi \to \psi)\right) \to \left((\widehat{Y} \to \varphi) \to (\widehat{Y} \to \psi)\right)$
by P
3. $\vdash_{\mathsf{CP}} \bigwedge_{Y \subseteq X} \left(\widehat{Y} \to [\emptyset]\left(\widehat{Y} \to (\varphi \to \psi)\right)\right) \to \bigwedge_{Y \subseteq X} \left(\widehat{Y} \to [\emptyset]\left((\widehat{Y} \to \varphi) \to (\widehat{Y} \to \psi)\right)\right)$
by P, 2 and rule RM for $[\emptyset]$ (i.e., *if* $\vdash \varphi \to \psi$ *then* $\vdash [\emptyset]\varphi \to [\emptyset]\psi$)
4. $\vdash_{\mathsf{CP}} \bigwedge_{Y \subseteq X} \left(\widehat{Y} \to [\emptyset]\left((\widehat{Y} \to \varphi) \to (\widehat{Y} \to \psi)\right)\right) \to \bigwedge_{Y \subseteq X} \left(\widehat{Y} \to \left([\emptyset](\widehat{Y} \to \varphi) \to [\emptyset](\widehat{Y} \to \psi)\right)\right)$
by K and P
5. $\vdash_{\mathsf{CP}} \bigwedge_{Y \subseteq X} \left(\widehat{Y} \to \left([\emptyset](\widehat{Y} \to \varphi) \to [\emptyset](\widehat{Y} \to \psi)\right)\right) \to \left(\bigwedge_{Y \subseteq X} \left(\widehat{Y} \to [\emptyset]\left(\widehat{Y} \to \varphi\right)\right) \to \bigwedge_{Y \subseteq X} \left(\widehat{Y} \to [\emptyset]\left(\widehat{Y} \to \psi\right)\right)\right)$
by P
6. $\vdash_{\mathsf{CP}} (\bigwedge_{Y \subseteq X} \left(\widehat{Y} \to [\emptyset]\left(\widehat{Y} \to \varphi\right)\right) \to \bigwedge_{Y \subseteq X} \left(\widehat{Y} \to [\emptyset]\left(\widehat{Y} \to \psi\right)\right)) \leftrightarrow ([X]\varphi \to [X]\psi)$
by Reduce
7. $\vdash_{\mathsf{CP}} [X](\varphi \to \psi) \to ([X]\varphi \to [X]\psi)$
from 1 and 3-6

Derivation of N for $[X]$:

1. $\vdash_{\mathsf{CP}} \varphi$
hypothesis
2. $\vdash_{\mathsf{CP}} [\emptyset]\varphi$
from 1 by N for $[\emptyset]$
3. $\vdash_{\mathsf{CP}} \bigwedge_{Y \subseteq X} [\emptyset]\left(\widehat{Y} \to \varphi\right)$
from 2 by the S5 theorem $[\emptyset]\varphi \to [\emptyset](\psi \to \varphi)$
4. $\vdash_{\mathsf{CP}} \bigwedge_{Y \subseteq X} \left(\widehat{Y} \to [\emptyset]\left(\widehat{Y} \to \varphi\right)\right)$
from 3 by P
5. $\vdash_{\mathsf{CP}} [X]\varphi$
from 4 by Reduce and MP

Then one proves that REP is derivable by an induction routine analogous to the one used by Chellas (1980, Thm. 4.7). □

## Appendix B. Proof of Theorem 2

Let $\varphi_0$ be a CP-formula. We have equivalence between $\varphi_0$ is CP-satisfiable and $tr(\varphi_0)$ is S5-satisfiable.

*Proof.* $\boxed{\Rightarrow}$ Suppose that there exists a CP-model $\mathcal{M} = (W, V)$ and a world $w \in W$ such that $\mathcal{M}, w \models_{\mathsf{CP}} \varphi_0$. Let $V'$ be the valuation $V$ modified such that $p_\varphi$ is true in exactly all worlds $u$ such that $\mathcal{M}, u \models_{\mathsf{CP}} \varphi$. Let $\mathcal{M}'$ be the S5-model defined as $(W, V')$. A standard induction provides that $\mathcal{M}', w \models_{\mathsf{S5}} tr(\varphi_0)$. More precisely, let us prove by induction that for all $\varphi \in SF(\varphi_0)$, we have $\mathcal{M}, u \models_{\mathsf{CP}} \varphi$ iff $\mathcal{M}', u \models_{\mathsf{S5}} tr_1(\varphi)$ for all $u \in W$.

- Propositional case: for all atomic propositions $p$, we have $\mathcal{M}, u \models_{\mathsf{CP}} p$ iff $u \in V(p)$ iff $u \in V'(p)$ iff $\mathcal{M}', u \models_{\mathsf{S5}} tr_1(p)$.
- Negation: $\mathcal{M}, u \models_{\mathsf{CP}} \neg\varphi$ iff $\mathcal{M}, u \not\models_{\mathsf{CP}} \varphi$ iff $\mathcal{M}', u \not\models_{\mathsf{S5}} tr_1(\varphi)$ iff $\mathcal{M}', u \models_{\mathsf{S5}} tr_1(\neg\varphi)$.
- Conjunction: $\mathcal{M}, u \models_{\mathsf{CP}} \varphi \wedge \psi$ iff $\mathcal{M}, u \models_{\mathsf{CP}} \varphi$ and $\mathcal{M}, u \models_{\mathsf{CP}} \psi$ iff $\mathcal{M}', u \models_{\mathsf{S5}} tr_1(\varphi)$ and $\mathcal{M}', u \models_{\mathsf{S5}} tr_1(\psi)$ iff $\mathcal{M}', u \models_{\mathsf{S5}} tr_1(\varphi \wedge \psi)$.
- Case of a formula of the form $[X]\varphi$:

  $\mathcal{M}, u \models_{\mathsf{CP}} [X]\varphi$

  iff for all $v \in W$, $u \sim_X^V v$ implies $\mathcal{M}, v \models_{\mathsf{CP}} \varphi$

  iff for all $v \in W$, $u \sim_X^V v$ implies $\mathcal{M}', v \models_{\mathsf{S5}} p_\varphi$

  (by construction of $V'$)

  iff $\mathcal{M}', u \models_{\mathsf{S5}} tr_1([X])\varphi$

By construction of $V'$, we have $\mathcal{M}', w \models_{\mathsf{S5}} \bigwedge_{\varphi \in SF(\varphi_0)} \Box(p_\varphi \leftrightarrow tr_1(\varphi))$. As $\mathcal{M}, w \models_{\mathsf{CP}} \varphi_0$ we have $\mathcal{M}', w \models_{\mathsf{S5}} tr_1(\varphi_0)$ thus $\mathcal{M}', w \models_{\mathsf{S5}} p_{\varphi_0}$ by construction of $V'$. As a result, $\mathcal{M}', w \models_{\mathsf{S5}} tr(\varphi_0)$.

$\boxed{\Leftarrow}$ Suppose that there exists a S5 model $\mathcal{M}' = (W, V)$ and a world $w \in W$ such that $\mathcal{M}', w \models_{\mathsf{S5}} tr(\varphi_0)$. We define the relations $\sim_X$ where $X \subseteq \mathbf{P}$ as in the Definition 1. Let $\mathcal{M}$ be the CP-model equal to $(W, V)$. A standard induction provides that $\mathcal{M}, w \models_{\mathsf{CP}} \varphi_0$. More precisely, let us prove by induction that for all $\varphi \in SF(\varphi_0)$, we have $\mathcal{M}, u \models_{\mathsf{CP}} \varphi$ iff $\mathcal{M}', u \models_{\mathsf{S5}} tr_1(\varphi)$ for all $u \in W$.

- Propositional case: for all atomic propositions $p$, we have $\mathcal{M}, u \models_{\mathsf{CP}} p$ iff $u \in V(p)$ iff $u \in V'(p)$ iff $\mathcal{M}', u \models_{\mathsf{S5}} tr_1(p)$.
- Negation: $\mathcal{M}, u \models_{\mathsf{CP}} \neg\varphi$ iff $\mathcal{M}, u \not\models_{\mathsf{CP}} \varphi$ iff $\mathcal{M}', u \not\models_{\mathsf{S5}} \varphi$ iff $\mathcal{M}', u \models_{\mathsf{S5}} \neg\varphi$.
- Conjunction: $\mathcal{M}, u \models_{\mathsf{CP}} \varphi \wedge \psi$ iff $\mathcal{M}, u \models_{\mathsf{CP}} \varphi$ and $\mathcal{M}, u \models_{\mathsf{CP}} \psi$ iff $\mathcal{M}', u \models_{\mathsf{S5}} tr_1(\varphi)$ and $\mathcal{M}', u \models_{\mathsf{S5}} tr_1(\psi)$ iff $\mathcal{M}', u \models_{\mathsf{S5}} tr_1(\varphi \wedge \psi)$.

- Case of a formula of the form $[X]\varphi$:

$\mathcal{M}, u \models_{\mathsf{CP}} [X]\varphi$

iff for all $v \in W$, $u \sim_X^V v$ implies $\mathcal{M}, v \models_{\mathsf{CP}} \varphi$

iff for all $v \in W$, $u \sim_X^V v$ implies $\mathcal{M}', v \models_{\mathsf{S5}} tr_1(\varphi)$

(by induction)

iff for all $v \in W$, $u \sim_X^V v$ implies $\mathcal{M}', v \models_{\mathsf{S5}} p_\varphi$

(because, as $\mathcal{M}', w \models_{\mathsf{S5}} tr(\varphi_0)$ we have that

for all $v \in W$, $\mathcal{M}', v \models_{\mathsf{S5}} (p_\varphi \leftrightarrow tr_1(\varphi)))$

iff $\mathcal{M}', u \models_{\mathsf{S5}} tr_1([X])\varphi$

As $\mathcal{M}', w \models_{\mathsf{S5}} tr(\varphi_0)$, we have that $\mathcal{M}', w \models_{\mathsf{S5}} (p_{\varphi_0} \leftrightarrow tr_1(\varphi_0))$ and $\mathcal{M}', w \models_{\mathsf{S5}} p_{\varphi_0}$. Thus, $\mathcal{M}', w \models_{\mathsf{S5}} tr_1(\varphi_0)$. Hence $\mathcal{M}, w \models_{\mathsf{CP}} \varphi_0$. □

## Appendix C. Proof of Theorem 3

Let us consider a group STIT formula $\varphi$. Let $m$ be an integer. Then the following items are equivalent:

1. $\varphi$ is satisfiable in a model where each agent has at most $2^m$ actions;
2. $\varphi$ is satisfiable in a model where each agent has exactly $2^m$ actions;
3. $GRID_m \wedge tr_2(\varphi)$ is CP-satisfiable.

*Proof.* $\boxed{1 \Rightarrow 2}$ Let $\mathcal{M}^0 = (W^0, \{R_J^0\}_{J \subseteq AGT}, V^0)$ be a STIT-model with at most $2^m$ actions per agent and $w \in W^0$ such that $\mathcal{M}^0, w \models_{\mathsf{STIT}} \varphi$. We construct a sequence of models $\mathcal{M}^j = (W^j, \{R_J^j\}_{J \subseteq AGT}, V^j)$ such that all agents $j' \in \{1, \ldots, j\}$ have exactly $2^m$ actions in $\mathcal{M}_j$ and such that $\mathcal{M}^j$ is bisimilar to $\mathcal{M}^{j-1}$. We construct $\mathcal{M}^j$ from $\mathcal{M}^{j-1}$ as follows. Let $R_{\{j\}}^{j-1}(w_1), \ldots, R_{\{j\}}^{j-1}(w_k)$ be an enumeration of $R_{\{j\}}^{j-1}$- classes (that is, actions for agents $j$), where $k \leq 2^m$. Let $(Copy_\ell)_{\ell \in \{k+1, \ldots, 2^m\}}$ be a family of disjoint copies of $R_{\{j\}}^{j-1}(w_1)$. We write $u\mathbf{C}v$ to say that $u = v$ or $v$ is a copy of $u$ or $u$ is a copy of $v$. The model $\mathcal{M}^j = (W^j, \{R_J^j\}_{J \subseteq AGT}, V^j)$ is defined as follows:

- $W^j = W^{j-1} \cup \bigcup_{\ell \in \{k+1, \ldots, 2^m\}} Copy_\ell$;
- $R_{\{j\}}^j = R_{\{j\}}^{j-1} \cup \bigcup_{\ell \in \{k+1, \ldots, 2^m\}} \{(u, v) \mid u, v \in Copy_\ell\}$
- $R_{\{j'\}}^j = \mathbf{C} \circ R_{\{j'\}}^{j-1} \circ \mathbf{C}$ for all $j' \neq j$;
- $V^j(p) = \{v \in W^j \mid v\mathbf{C}u \text{ and } u \in V^{j-1}(p)\}$.

This construction makes that $\mathcal{M}^j$ and $\mathcal{M}^{j-1}$ are bisimilar and by induction we have that all agents $j' \in \{1, \dots, j\}$ have exactly $2^m$ actions in $\mathcal{M}_j$. Finally, we have $\mathcal{M}^n, w \models_{\mathsf{STIT}} \varphi$ and each agent has exactly $2^m$ actions in $\mathcal{M}^n$.

$\boxed{2 \Rightarrow 3}$ Let us consider a STIT model $\mathcal{M} = (W, \{R_J\}_{J \subseteq AGT}, V)$ in which each agent has exactly $2^m$ actions. Let $w \in W$ be such that $\mathcal{M}, w \models_{\mathsf{STIT}} \varphi$. For all $j \in AGT$, let $R_{\{j\}}(w_{j,1}), \dots, R_{\{j\}}(w_{j,2^m})$ be an enumeration of all $R_{\{j\}}$-classes in $\mathcal{M}$. Let us extend $V$ such that in all worlds of $R_{\{j\}}(w_{j,i})$ the valuations of the atomic propositions in $\mathcal{R}^j$ correspond to the binary digits in the binary representation of $i$. For all $d \in \{1, \dots, m\}$:

$$V(rep_d^j) = \bigcup_{i=1..2^m \mid \text{ the } d^{th} \text{ digit of } i \text{ is } 1} R_{\{j\}}(w_{j,i}) \tag{10}$$

Independence of agents in $\mathcal{M}$ ensures that $\mathcal{M}, w \models_{\mathsf{CP}} GRID_m$. We prove that $\mathcal{M}, u \models_{\mathsf{CP}} tr_2(\psi)$ iff $\mathcal{M}, u \models_{\mathsf{STIT}} \psi$ by induction over all subformulae $\psi$ of $\varphi$.

$\boxed{3 \Rightarrow 1}$ Let $\mathcal{M} = (W, V)$ be a CP-model and $w \in W$ such that $\mathcal{M}, w \models_{\mathsf{CP}} GRID_m \wedge tr_2(\varphi)$. We define $R_J = \sim_{\bigcup_{j \in J} \mathcal{R}^j}$. The resulting Kripke-model $\mathcal{M}' = (W, \{R_J\}_{J \subseteq AGT}, V)$ is a STIT-model where each agent has exactly $2^m$ actions. In particular, it satisfies the independence of agents because $\mathcal{M}, w \models_{\mathsf{CP}} GRID_m$. We prove that $\mathcal{M}, u \models_{\mathsf{CP}} tr_2(\psi)$ iff $\mathcal{M}', u \models_{\mathsf{STIT}} \psi$ by induction over all subformulae $\psi$ of $\varphi$. $\square$

## Appendix D. Proof of Theorem 5

The CP-nested satisfiability problem is PSPACE-hard.

*Proof.* We reduce the satisfiability problem of STIT-formulae where coalitions are taken from a linear set of coalitions, which is PSPACE-complete (Schwarzentruber, 2012) to the CP-nested satisfiability problem: we use the translation $tr_2$ of Subsection 3.1. Let $\varphi$ be a STIT-formula. We have $\varphi$ is STIT-satisfiable iff $tr_2(\varphi)$ is CP-satisfiable.

$\boxed{\Rightarrow}$ As it stated by Schwarzentruber (2012), the STIT where coalitions are taken from a linear set of coalitions has the exponential model property. So the result of Theorem 3 is true. Hence if $\varphi$ is STIT-satisfiable then $GRID_m \wedge tr_2(\varphi)$ is CP-satisfiable (where $m$ is the length of $\varphi$). Hence $tr_2(\varphi)$ is CP-satisfiable.

$\boxed{\Leftarrow}$ Suppose that there exists a CP-model $\mathcal{M} = (W, V)$ and $w \in W$ such that $\mathcal{M}, w \models tr_2(\varphi)$. We define $R_J = \sim_{\bigcup_{j \in J}}$. Then the STIT model $\mathcal{M}' = (W, (R_J)_{J \in \varphi}, V)$ is such that $\mathcal{M}', w \models \varphi$. Remark that we do not need to specify all the relations $R_J$ for all $J$. As long as $R_J$ is specified for all coalitions $J$ that appear in $\varphi$ and that $R_J \subseteq R_{J'}$ if $J' \subseteq J$, we can extend the Kripke model $\mathcal{M}'$ to a completely specified STIT-model also satisfying $\varphi$.[20] $\square$

## Appendix E. Proof of Theorem 6

The CP-nested satisfiability problem is in PSPACE.

*Proof.* We reduce the CP-nested satisfiability problem to the satisfiability problem of STIT where coalitions are taken from a linear set of coalitions. We define $A_X = \{j_p \text{ such that } p \in X\}$

20. See Schwarzentruber's paper (2012) for more details about this construction.

where $j_p$ is a fresh agent corresponding to the atomic proposition $p$. Let us define the following translation:

- $tr(p) = p$;
- $tr(\neg\varphi) = \neg tr(\varphi)$;
- $tr(\varphi \wedge \psi) = tr(\varphi) \wedge tr'(\psi)$;
- $tr([X]\varphi) = [A_X : stit]tr(\varphi)$.

Let us consider a fixed CP-formula $\varphi$. We recall that a signature $X$ appears in $\varphi$ if there exists a formula $\psi$ such that $\langle X\rangle\psi \in SF(\varphi)$. We have also to define the following formula

$$CONTROL = [\emptyset : stit] \bigwedge_{X \text{ appearing in } \varphi} \bigwedge_{p\in X}(p \leftrightarrow [A_X : stit]p) \wedge (\neg p \leftrightarrow [A_X : stit]\neg p).$$

$tr(\varphi) \wedge CONTROL$ is a STIT-formula which is computable in polynomial time and which satisfies the condition of nesting over groups (i.e., for any two operators $[J : stit]$ and $[J' : stit]$ occurring in the formula either $J \subseteq J'$ or $J' \subseteq J$). We also have that $\varphi$ is CP-satisfiable iff $tr(\varphi) \wedge CONTROL$ is satisfiable in a STIT-model.

$\boxed{\Rightarrow}$ Suppose that there exists an CP-model $\mathcal{M} = (W, V)$ and $w \in W$ such that $\mathcal{M}, w \models_{\mathsf{CP}} \varphi$. We define $R_{A_X} = \sim_X$. Then the STIT model $\mathcal{M}' = (W, (R_{A_X})_{X\in\varphi}, V)$ is such that $\mathcal{M}', w \models_{\mathsf{STIT}} tr(\varphi) \wedge CONTROL$. Remark that we do not need to specify all the relations $R_J$ for all $J$. As long as $R_J$ is specified for all coalitions $J$ that appear in $tr(\varphi) \wedge CONTROL$ and that $R_J \subseteq R_{J'}$ if $J' \subseteq J$, we can extend the Kripke model $\mathcal{M}'$ to a completely specified STIT-model also satisfying $tr(\varphi) \wedge CONTROL$.[21]

$\boxed{\Leftarrow}$ Suppose that there exists a STIT-model $\mathcal{M}' = (W, (R_{A_X})_{X\in\varphi}, V)$ and a world $w \in W$ such that $\mathcal{M}', w \models_{\mathsf{STIT}} tr(\varphi) \wedge CONTROL$. As $\mathcal{M}', w \models CONTROL$, we have $\sim_X = R_{A_X}$. This is the reason why we define $\mathcal{M} = (W, \{\sim_X\}_{X\in 2^{\mathbf{P}}}, V)$. Consequently, we have $\mathcal{M}, w \models_{\mathsf{CP}} \varphi$. □

## Appendix F. Proof of Theorem 7

Let $m = |\mathbf{P}|$. Then, a CL−PC formula $\varphi$ is CL−PC satisfiable if and only if $(EXC^+ \wedge EXC^- \wedge COMPL \wedge GRID^*) \wedge tr_3(\varphi)$ is satisfiable in a STIT model where each agent has at most $2^m$ actions.

*Proof.* Let us suppose $|\mathbf{P}| = m$.

$\boxed{\Rightarrow}$ Let $\mathcal{M}^* = (\mathbf{P}_1, \ldots, \mathbf{P}_n, X^*)$ be a CL−PC model such that $\mathcal{M}^* \models_{\mathsf{CL-PC}} \varphi$, where $\mathbf{P}_1, \ldots, \mathbf{P}_n$ is a partition of $\mathbf{P}$ among the agents in $AGT$. We build the STIT model $\mathcal{M} = (W, \{R_J\}_{J\subseteq AGT}, V)$ as follows:

- $W = \{X : X \subseteq \mathbf{P}\}$,
- for all $J \subseteq AGT$ and for all $X, X' \in W$, $(X, X') \in R'_J$ if and only if $X_J = X'_J$,

21. Again see Schwarzentruber's paper for more details about this construction.

- for all $p \in \mathbf{P}$ and for all $X \in W$, $X \in V(p)$ if and only if $p \in X$,

where for any $X \subseteq \mathbf{P}$ and for any $J \subseteq AGT$, $X_J = X \cap \mathbf{P}_J$ (with $\mathbf{P}_J = \bigcup_{i \in J} \mathbf{P}_i$). The size of $\mathcal{M}$ is $2^m$. It follows that the number of $R_{AGT}$-equivalence classes (*alias* joint actions) is equal or lower than $2^m$. Consequently, the number of actions for every agent is bounded by $2^m$.

It is straightforward to prove that for all $X \in W$ we have $\mathcal{M}, X \models_{\mathsf{STIT}} EXC^+ \wedge EXC^- \wedge COMPL \wedge GRID^*$. Moreover, by induction on the structure of $\varphi$, we prove that $\mathcal{M}, X^* \models_{\mathsf{STIT}} tr_3(\varphi)$. The only interesting case is $\varphi = \Diamond_J \psi$:

$$\begin{aligned} \mathcal{M}^* \models_{\mathsf{CL-PC}} \Diamond_J \psi \text{ iff } & \text{there exists } X_J \subseteq \mathbf{P}_J \text{ s.t. } \mathcal{M}^* \bigoplus X_J \models_{\mathsf{CL-PC}} \psi \\ \text{iff } & \text{there exists } X_J \subseteq \mathbf{P}_J \text{ s.t.} \\ & \quad \mathcal{M}, X_J \cup X^*_{AGT \setminus J} \models_{\mathsf{STIT}} tr_3(\psi) \text{ (by I.H.)} \\ \text{iff } & \mathcal{M}, X^* \models_{\mathsf{STIT}} \langle AGT \setminus J : stit \rangle tr_3(\psi) \end{aligned}$$

$\boxed{\Leftarrow}$ Let $\mathcal{M} = (W, \{R_J\}_{J \subseteq AGT}, V)$ be a $\mathsf{STIT}$ model where the number of actions for every agent is bounded by $2^m$ and $w_0 \in W$ such that $\mathcal{M}, w_0 \models_{\mathsf{STIT}} (EXC^+ \wedge EXC^- \wedge COMPL \wedge GRID^*) \wedge tr_3(\varphi)$.

For any $i \in AGT$, let

$$Ctrl_i = \left\{ p \in \mathbf{P} : \ \forall v \in W, \begin{array}{c} \mathcal{M}, v \models [\{i\} : stit]p \text{ or} \\ \mathcal{M}, v \models [\{i\} : stit]\neg p \end{array} \right\}$$

be the set of atoms in $\mathbf{P}$ controlled by agent $i$. For any $J \subseteq AGT$, let $Ctrl_J = \bigcup_{i \in J} Ctrl_i$.

**Lemma 3.** *For all $J \subseteq AGT$, $X \subseteq \mathbf{P}$, $\pi_X \subseteq X$ and $w \in W$ we have:*

*(i) if $Ctrl_J = X$ then $Ctrl_{AGT \setminus J} = \mathbf{P} \setminus X$,*

*(ii) if $\mathcal{M}, w \models_{\mathsf{STIT}} \bigwedge_{p \in \pi^+_X} p \wedge \bigwedge_{p \in \pi^-_X} \neg p$ and $Ctrl_J = X$ then, for all $v \in R_J(w)$, we have $\mathcal{M}, v \models_{\mathsf{STIT}} \bigwedge_{p \in \pi^+_X} p \wedge \bigwedge_{p \in \pi^-_X} \neg p$,*

*(iii) if $Ctrl_J = X$ then, for all $\pi'_{\mathbf{P} \setminus X} \subseteq \mathbf{P} \setminus X$, there exists $v \in R_J(w)$ such that $\mathcal{M}, v \models_{\mathsf{STIT}} \bigwedge_{p \in \pi'^+_{\mathbf{P} \setminus X}} p \wedge \bigwedge_{p \in \pi'^-_{\mathbf{P} \setminus X}} \neg p$.*

*where for any $X \subseteq \mathbf{P}$ and for any $\pi_X \subseteq X$, $\pi^+_X = \pi_X$ and $\pi^-_X = X \setminus \pi_X$.*

*Proof.* $\boxed{\text{(i)}}$ Let us suppose that $p \notin Ctrl_J$. We are going to prove that $p \in Ctrl_{AGT \setminus J}$. From $p \notin Ctrl_J$ it follows that for all $w \in W$ we have $\mathcal{M}, v \models_{\mathsf{STIT}} \neg p$ for some $v \in R_J(w)$. This implies that for all $i \in J$ and for all $w \in W$ we have $\mathcal{M}, w \models_{\mathsf{STIT}} \neg[\{i\} : stit]p \wedge \neg[\{i\} : stit]\neg p$. From $\mathcal{M}, w_0 \models_{\mathsf{STIT}} COMPL$ it follows that there is $i \in AGT \setminus J$ such that $\mathcal{M}, w \models_{\mathsf{STIT}} [\{i\} : stit]p \vee [\{i\} : stit]\neg p$ for all $w \in W$. The latter implies that $p \in Ctrl_{AGT \setminus J}$. The other direction (i.e., $p \in Ctrl_J$ implies $p \notin Ctrl_{AGT \setminus J}$) follows from $\mathcal{M}, w_0 \models_{\mathsf{STIT}} EXC^+ \wedge EXC^-$.

(ii) Let us suppose that $\mathcal{M}, w \models \bigwedge_{p \in \pi_X^+} p \wedge \bigwedge_{p \in \pi_X^-} \neg p$ and $Ctrl_J = X$. By the fact that relations $R_J$ are reflexive, it follows that, for all $p \in \pi_X^+$, there exists $i \in J$ such that $\mathcal{M}, w \models_{\mathsf{STIT}} [\{i\} : stit]p$ and for all $p \in \pi_X^-$ there exists $i \in J$ such that $\mathcal{M}, v \models_{\mathsf{STIT}} [\{i\} : stit]\neg p$. From the latter it follows that for all $p \in \pi_X^+$ we have $\mathcal{M}, w \models_{\mathsf{STIT}} [J : stit]p$ and for all $p \in \pi_X^-$ we have $\mathcal{M}, v \models_{\mathsf{STIT}} [J : stit]\neg p$. Therefore, for all $v \in R_J(w)$, we have $\mathcal{M}, v \models_{\mathsf{STIT}} \bigwedge_{p \in \pi_X^+} p \wedge \bigwedge_{p \in \pi_X^-} \neg p$.

(iii) Let us suppose that $Ctrl_J = X$ and let us consider an arbitrary $\pi'_{\mathbf{P} \setminus X} \subseteq \mathbf{P} \setminus X$ and $w \in W$. From $\mathcal{M}, w_0 \models_{\mathsf{STIT}} GRID^*$ it follows that there exists $v \in W$ such that $\mathcal{M}, v \models_{\mathsf{STIT}} \bigwedge_{p \in \pi'^+_{\mathbf{P} \setminus X}} p \wedge \bigwedge_{p \in \pi'^-_{\mathbf{P} \setminus X}} \neg p$. By item (ii), the latter implies that there exists $v \in W$ such that $\mathcal{M}, v \models_{\mathsf{STIT}} [AGT \setminus J : stit](\bigwedge_{p \in \pi'^+_{\mathbf{P} \setminus X}} p \wedge \bigwedge_{p \in \pi'^-_{\mathbf{P} \setminus X}} \neg p)$. From the constraint of *independence of agents* it follows that there exists $v \in R_J(w)$ such that $\mathcal{M}, v \models_{\mathsf{STIT}} \bigwedge_{p \in \pi'^+_{\mathbf{P} \setminus X}} p \wedge \bigwedge_{p \in \pi'^-_{\mathbf{P} \setminus X}} \neg p$. □

We transform the STIT model $\mathcal{M}$ in a CL−PC model $\mathcal{M}^* = (\mathbf{P}_1, \ldots, \mathbf{P}_n, X^*)$ as follows:

- for all $p \in \mathbf{P}$, $p \in X^*$ if and only if $w_0 \in V(p)$,
- for all $p \in \mathbf{P}$ and for all $i \in AGT$, $p \in \mathbf{P}_i$ if and only if $p \in Ctrl_i$.

By the item (i) of Lemma 3 it is easy to check that $\mathcal{M}^*$ is indeed a CL−PC model. In particular, $\mathbf{P}_1, \ldots, \mathbf{P}_n$ is a partition of $\mathbf{P}$ among the agents in $AGT$.

By induction on the structure of $\varphi$ and by using Lemma 3 it is straightforward to prove that $\mathcal{M}^* \models_{\mathsf{CP}} \varphi$. The only interesting case is $\varphi = \Diamond_J \psi$:

$$
\begin{aligned}
&\mathcal{M}, w_0 \models_{\mathsf{STIT}} \langle AGT \setminus J : stit \rangle tr_3(\psi) \\
&\quad \text{iff } \mathcal{M}, v \models_{\mathsf{STIT}} tr_3(\psi) \text{ for some } v \in R_{AGT \setminus J}(w_0) \\
&\quad \text{iff there exists } X_J \subseteq \mathbf{P}_J \text{ s.t.} \\
&\qquad (\mathbf{P}_1, \ldots, \mathbf{P}_n, X_J \cup X^*_{AGT \setminus J}) \models_{\mathsf{CP}} \psi \\
&\qquad \text{(by I.H., and items (ii) and (iii)} \\
&\qquad \text{of Lemma 3)} \\
&\quad \text{iff } \mathcal{M}^* \models_{\mathsf{CP}} \Diamond_J \psi
\end{aligned}
$$

This completes the proof. □

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