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|  | FYf $\quad \cdots \quad$ NASA TM X-53 |
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EXPERIMENTAL AND CALCULATED RESULTS OF A FLUTTER
INVESTIGATION OF SOME VERY LOW ASPECT-RATIO
FLAT-PLATE SURFACES AT MACH NUMBERS
FROM 0.62 TO 3.00
By Perry W. Hanson and Gilbert M. Levey
Langley Research Center
Langley Field, Va.

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION


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TECHNICAL MEMORANDUM X-53

# EXPERIMENTAL AND CALCULATED RESULTS OF A FLUTHIER <br> INVESTIGATION OF SOME VERY LOW ASPECT-RATIO 

FTAT-PLATE SURFACES AT MACH NUMBERS

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FROM 0.62 TO 3.00
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## SUMMARY

Some very low aspect-ratio flat-plate surfaces of aluminum alloy were tested for flutter at Mach numbers from 0.62 to 3.00 . Two types of plan forms, a delta and a delta with one-third span cut off, are used in this investigation. Three different panel aspect ratios, 0.728, 0.536 , and 0.353, were tested for each type of plan form. Each model had a l2-inch root chord and was cantilevered from the tunnel wall.

Generally, the clipped-tip-delta plan forms were more susceptible to flutter throughout the Mach number range investigated. The lower aspect-ratio models fluttered at a higher value of the stiffnessaltitude parameter than the higher aspect-ratio models for a given type of plan form and a given Mach number.

Modal-type calculations were made for some supersonic cases by using first-order piston-theory aerodynamic forces. Generally, the theoretical flutter boundaries agreed with the experimental boundaries within 20 percent. The theory was unconservative for the delta plan forms and conservative for the clipped-tip-delta plan forms.

## INIRODUCTION

The use of very low aspect-ratio surfaces is becoming increasingly prevalent in the design of missile and rocket fins, supersonic aircraft, and hypersonic gliders. Although some work has been done in this area of interest (see, for example, refs. 1 to 3), data available for the
flutter characteristics of these types of surfaces at both subsonic and supersonic speeds are meager. It is evident that there is a need for more information of this kind, both to provide trend data for design criteria and to provide a basis for comparison of theory and experiment. Therefore, a systematic investigation was made of the flutter characteristics of some configurations that might be considered representative of those found on these new vehicles.

Some flat-plate semispan models of two different types of plan forms, each with three different panel aspect ratios were tested at Mach numbers from 0.62 to 3.00 . The experimental results were compared with theoretical calculations in the supersonic regime with the use of the method of reference 4 based on the "piston theory" of reference 5. Mode shapes of the models used in the computations were determined by the method of reference 6 .

SYMBOLS

| A | panel aspect ratio (Semispan ${ }^{2 / P a n e l ~ a r e a) ~}$ |
| :---: | :---: |
| a | velocity of sound, $\mathrm{ft} / \mathrm{sec}$ |
| b | semichord at $3 / 4$ semispan, in. |
| $\frac{b u_{a}}{a} \sqrt{\mu}$ | stiffness-altitude parameter |
| c | local chord, in. |
| $\mathrm{f}_{\mathrm{f}}$ | flutter frequency, cps |
| $\mathrm{f}_{\mathrm{n}}$ | natural frequency of nth mode ( $n=1,2,3$, and 4), cps |
| 2 | length of semispan of model, measured normal to stream direction, in. |
| M | Mach number |
| q | dynamic pressure, lb/sq ft |
| t | thickness, in. |
| W | total weight of surface, lb |

X
y
$\delta$
${ }^{\mu}$
$\rho$
$\omega_{\alpha}$
chordwise station, measured parallel to root chord from leading edge, in.
spanwise station, measured perpendicular to root chord from the root
leading- and trailing-edge bevel, measured perpendicular to edges, in.
mass density parameter
air density, slugs/cu ft
wing torsional circular frequency, radians/sec
Subscripts:
ex experimental
th theoretical

MODEL DESCRIPTION

The six model configurations used in the investigation are shown in figure 1. They consisted of two types of plan forms: delta and delta with the outer one-third span cut off. The three delta plan forms were $70^{\circ}, 75^{\circ}$, and $80^{\circ}$ deltas with corresponding panel aspect ratios of $0.728,0.536$, and 0.353 for 12 -inch root chords. The three clipped-tip-delta plan forms also had l2-inch root chords, and the dimensions of these plan forms were chosen to give the same aspect ratios as the delta plan forms.

All the models were made from 2024-T3 aluminum-alloy sheets with the thicknesses and leading- and trailing-edge bevels as indicated in figure 1 . The models were mounted in the wind-tunnel side wall and clamped between two $1 / 2$-inch-thick steel plates over the entire root chord. These plates were made to hold the models $1 / 2$ inch out from the wind-tunnel wall in a triangular shaped body. The method of mounting is illustrated in figure 2.

TEST PROCEDURE

The tests were conducted in the Langley 9- by 18-inch supersonic aeroelasticity tunnel. This tunnel is of the intermittent blowdown
type with fixed nozzle blocks and operates from a high-pressu $?$ source to a vacuum. The transonic tests of the delta plan forms ere made with the use of a slotted-test-section nozzle with a choking device employed in the diffuser to obtain the desired Mach number in the test section.

The tests were made at constant Mach number with the dynamic pressure being increased until flutter was encountered or until the tunnel limits were reached. During each test, continuous records of wind-tunnel conditions and model behavior were recorded on an oscillograph.

Generally, the models were not damaged during flutter tests and could be used for succeeding tests. When models were damaged and new ones were made, it was found that the models could be duplicated very easily and that the natural frequencies and node lines of the new models were virtually the same as those of the previous models. The variations in natural frequencies listed in table I were probably the result of small differences in tightness of the root mount. Resistance wire straingage bridges mounted at the root of the model at about 70 percent of the chord were used to record natural frequencies listed in table I. Mode shapes of the models were obtained by the method of reference 6 for use in the piston-theory analysis and are presented in table II along with typical natural vibration node lines of the first four modes.

## RESULTS AND DISCUSSION

The experimental and theoretical results are listed in table $I$ and are shown in figure 3 in which both an experimental and a theoretical stiffness-altitude parameter $\frac{b \omega_{\alpha}}{a} \sqrt{\mu}$ required for flutter are plotted as a function of Mach number. The $\omega_{\infty}$ is the second natural frequency $\mathrm{f}_{2}$ which is predominantly torsional for all models. The mass-density parameter $\mu$ is the ratio of the mass of the wing to the mass of a volume of air enclosing the wing. For the delta plan forms, the volume is that of a cone with the base diameter parallel to the airstream and equal to the root chord. For the cllpped-tip-delta plan forms, the volume is that of a truncated cone with the two ends parallel to the airstream with diameters equal to the root and tip chords. The air density $\rho$, which is used in the computation of $\mu$, is the test-section density at flutter. In figure 3 constant-density (altitude) lines are horizontal and density decreases as $\frac{b u_{a}}{a} \sqrt{\mu}$ increases. Constant dynamic pressure lines are radial from the origin and increase clockwise. The flutter region is below the curves and the no-flutter region is above the curves.

When figures $3(a), 3(b)$, and $3(c)$ are compared, several general observations can be made. The flutter boundaries for the delta-planform models showed little change with aspect ratio except for the lowest aspect-ratio model at the higher Mach numbers. The clipped-tip-delta-plan-form models, however, exhibited a considerable change in the flutter boundaries with aspect ratio. (See figs. 3(d), 3(e), and 3(f).) As the aspect ratio decreased, the flutter boundary was raised. For a given aspect ratio, the clipped-tip-delta plan forms fluttered at a higher value of the stiffness-altitude parameter than the deltas at all Mach numbers.

The theoretical flutter boundaries shown in figure 3 were calculated with the use of aerodynamic forces obtained from first-order piston theory and using the first three (experimentally determined) naturalvibration modes. When the theoretical and experimental flutter boundaries are compared, it is seen that the shape of the boundaries agrees very well for all the cases considered except for the lowest aspectratio delta (fig. 3(c)). The agreement between the experimental and theoretical flutter boundaries is poor at all Mach numbers for the lowest aspect-ratio model of the clipped-tip-delta models. Generally, the theoretical flutter boundaries were conservative with respect to the experimental boundaries for the clipped-tip-delta plan forms; that is, a greater density was required to flutter the models than was predicted by theory. For the delta plan forms, however, the theory was unconservative.

Figure 4 shows the variation of the ratio of theoretical flutter frequency to experimental flutter frequency with Mach number. In all cases, the theoretical flutter frequency was greater than the experimental flutter frequency. For the delta-plan-form models, the agreement between the theoretical and experimental flutter frequencies was best for the largest aspect-ratio model and became worse as the aspect ratio decreased, whereas the opposite was true for the clipped-tip-delta-plan-form models.

## CONCLUDING REMARKS

An investigation conducted in the Langley 9- by 18-inch supersonic aeroelasticity tunnel of very low aspect-ratio flat-plate models with two types of plan forms and three aspect ratios for each type of plan form indicate that the clipped-tip-delta plan forms were more susceptible to flutter than the delta plan forms throughout the Mach number range investigated. For a given Mach number and a given type of plan form, the lower aspect-ratio models fluttered at a higher value of the stiffness-altitude parameter than the higher aspect-ratio models. The agreement between the experimental flutter boundaries and the theoretical
flutter boundaries (as computed from first-order piston theory) was generally good. The theory was conservative for the clipped-tip deltas and unconservative for the deltas. The agreement was poorest for the lowest aspect-ratio models of both types of plan forms.

Langley Research Center,
National Aeronautics and Space Administration, Langley Field, Va., May 12, 1959.

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TABLE I.- EXPERTMENTAL AND THEORETICAL RESULTS

| Frequencles, cps |  |  |  |  | M | Flutter conditions |  |  |  | $\frac{b u^{2}}{a} \sqrt{\mu}$ |  | $\frac{f_{f, t h}}{f_{f, e x}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}_{1}$ | $\mathrm{f}_{2}$ | $\mathrm{f}_{3}$ | $\mathrm{f}_{4}$ | $\mathrm{f}_{\mathrm{f}, \mathrm{ex}}$ |  | slug/cu ft | $\begin{aligned} & \mathrm{a}, \\ & \mathrm{f}_{\mathrm{ps}} \end{aligned}$ | $\mathrm{lb} / \mathrm{sq} \mathrm{f}^{\mathrm{ft}}$ | $\mu$ | Exp. | Theory |  |
| Model 1A |  |  |  |  |  |  |  |  |  |  |  |  |
| 78 | 183 | 325 | 395 | 166 | 0.63 | 0.001505 | 1,102 | 363 | 17.17 | 0.54 | ---- | ---- |
| 72 | 171 | 320 | 367 | 150 | . 64 | . 001442 | 1,093 | 354 | 17.93 | . 52 | ---- | ---- |
| 78 | 186 | 320 | 398 | 157 | . 75 | . 001307 | 1,089 | 436 | 19.80 | . 60 | ---- | ---- |
| 79 | 193 | 350 | 396 | 150 | . 79 | . 001471 | 1,077 | 532 | 17.58 | . 59 | ---- | ---- |
| 78 | 186 | 322 | 398 | 140 | . 88 | . 001346 | 1,070 | 598 | 19.20 | . 60 | ---- | ---- |
| 79 | 186 | 331 | 398 | 140 | . 96 | . 001281 | 1,051 | 652 | 20.18 | . 63 | ---- | ---- |
| 79 | 192 | 350 | 402 | 142 | . 96 | . 001339 | 1,053 | 685 | 19.30 | . 63 | ---- | ---- |
| 78 | 178 | 342 | 388 | 148 | 1.01 | . 001006 | 1,039 | 555 | 25.67 | . 68 | ---- | ---- |
| 77 | 181 | 320 | 367 | 133 | 1.19 | . 000760 | 1,013 | 553 | 34.00 | . 82 | ---- | --- |
| 75 | 170 | 305 | 367 | 150 | 1.30 | . 000786 | 980 | 643 | 32.98 | . 78 | 0.62 | 1.15 |
| 76 | 173 | 318 | 372 | 160 | 1.64 | . 000736 | 915 | 829 | 35.12 | . 88 | . 72 | 1.07 |
| 75 | 173 | 320 | 379 | 153 | 2.00 | . 000592 | 850 | 846 | 43.67 | 1.06 | . 79 | 1.12 |
| 76 | 173 | 320 | 375 | 160 | 2.55 | . 000651 | 770 | 1,264 | 39.70 | 1.11 | . 90 | 1.08 |
| 75 | 174 | 325 | 383 | 161 | 3.00 | . 000736 | 721 | 1,723 | 35.10 | 1.12 | . 98 | 1.06 |
| Model 1B |  |  |  |  |  |  |  |  |  |  |  |  |
| 127 | 277 | 457 | 640 | 222 | 0.62 | 0.003993 | 1,107 | 943 | 6.34 | 0.50 | ---- | ---- |
| 127 | 275 | 457 | 627 | 225 | . 75 | . 003213 | 1,086 | 1,065 | 7.88 | . 56 | ---- | ---- |
| 128 | 277 | 460 | 642 | 214 | . 86 | . 002666 | 1,071 | 1,130 | 9.50 | . 63 | ---- | ---- |
| 129 | 275 | 467 | 646 | 210 | 1.14 | . 002404 | 1,024 | 1,641 | 10.54 | .69 | 0.53 | 1.79 |
| 128 | 273 | 460 | 644 | 264 | 1.25 | . 002570 | 1,012 | 2,058 | 9.86 | .67 | . 57 | 1.39 |
| 126 | 271 | 458 | 635 | 245 | 1.26 | . 002364 | 1,007 | 1,906 | 10.71 | . 69 | . 57 | 1.50 |
| 127 | 275 | 457 | 640 | 250 | 1.30 | . 002380 | 988 | 1,963 | 10.64 | . 71 | . 60 | 1.49 |
| 130 | 283 | 462 | 650 | 300 | 1.64 | . 002453 | 949 | 2,915 | 10.32 | .76 | . 70 | 1.26 |
| 123 | 267 | 454 | 600 | 238 | 2.00 | . 001510 | 870 | 2,278 | 16.36 | . 99 | . 84 | 1.18 |
| 127 | 269 | 460 | 600 | 250 | 2.55 | -.001210 | 796 | 2,490 | 13.96 | . 99 | . 78 | 1.09 |
| 125 | 273 | 450 | 625 | (a) | 3.00 | ${ }^{\text {a }} .001017$ | ${ }^{8} 731$ | $\mathrm{a}_{2,448}$ | 24.90 | 1.47 | . 85 | ---- |
| Model IC |  |  |  |  |  |  |  |  |  |  |  |  |
| 213 | 386 | 580 | 738 | 314 | 0.63 | 0.004195 | 1,109 | 1,025 | 5.16 | 0.62 | ---- | ---- |
| 217 | 383 | 575 | 750 | 316 | . 75 | . 003132 | 1,086 | 1,038 | 6.91 | . 73 | ---- | ---- |
| 215 | 389 | 580 | 744 | 300 | . 90 | . 002911 | 1,066 | 1,340 | 7.43 | . 78 | --- | ---7 |
| 212 | 387 | 580 | 738 | 306 | 1.16 | . 002401 | 1,020 | 1,681 | 9.01 | . 90 | 0.85 | 1.72 |
| 213 | 375 | 567 | 720 | 300 | 1.24 | . 002188 | 997 | 1,675 | 9.89 | . 93 | . 89 | 1.74 |
| 217 | 388 | 554 | 725 | 350 | 1.30 | . 002488 | 990 | 2,058 | 8.70 | . 91 | . 99 | 1.45 |
| 216 | 400 | 585 | 775 | 360 | 1.64 | . 002104 | 928 | 2,435 | 10.27 | 1.09 | 1.08 | 1.48 |
| 210 | 467 | 560 | 786 | 313 | 2.00 | .001468 | 859 | 2,092 | 15.25 | 1.31 | 1.11 | 1.61 |
| 213 | 400 | 600 | 833 | 300 | 3.00 | . 000948 | 748 | 2,392 | 22.82 | 2.00 | 1.42 | 1.81 |
| Model 2A |  |  |  |  |  |  |  |  |  |  |  |  |
| 35 | 95 | 183 | 209 | 93 | 1.30 | 0.000628 | 979 | 509 | 38.67 | 0.95 | 0.97 | 1.75 |
| 35 | 100 | 188 | 208 | 102 | 1.64 | . 000581 | 918 | 660 | 41.79 | 1.11 | 1.18 | 1.64 |
| 36 | 105 | 197 | 233 | 94 | 2.00 | . 000724 | 847 | 1,039 | 33.55 | 1.13 | 1.20 | 1.88 |
| 35 | 110 | 196 | 232 | 100 | 2.55 | . 000799 | 777 | 1,580 | 30.40 | 1.23 | 1.43 | 1.76 |
| 34 | 109 | 193 | 233 | 105 | 3.00 | . 000655 | 720 | 1,533 | 37.08 | 1.45 | 1.57 | 1.86 |
| Model 28 |  |  |  |  |  |  |  |  |  |  |  |  |
| 60 | 122 | 225 | 325 | 115 | 1.30 | 0.000618 | 977 | 498 | 39.32 | 1.24 | 1.36 | 1.70 |
| 59 | 114 | 213 | 331 | 108 | 1.64 | . 000397 | 916 | 449 | 61.18 | 1.53 | 1.50 | 1.71 |
| 60 | 125 | 229 | 332 | 117 | 2.00 | . 000495 | 847 | 710 | 49.11 | 1.63 | 1.71 | 1.70 |
| 59 | 117 | 207 | 318 | 109 | 3.00 | . 000363 | 693 | 784 | 66.97 | 2.18 | 2.21 | 1.66 |
| Model 2 C |  |  |  |  |  |  |  |  |  |  |  |  |
| 126 | 216 | 350 | 507 | 175 | 1.30 | 0.001305 | 982 | 1,062 | 18.61 | 1.49 | $2 . \stackrel{1}{2}$ | 1.21 |
| 125 | 213 | 358 | 500 | 183 | 1.64 | . 001120 | 924 | 1,286 | 21.69 | 1.69 | 2.55 | 1.13 |
| 122 | 204 | 342 | 487 | 165 | 2.00 | . 000754 | 858 | 1,110 | 32.18 | 2.12 | 2.87 | 1.20 |
| 130 | 218 | 368 | 540 | 183 | 2.55 | . 000829 | 790 | 1,677 | 29.30 | 2.34 | 3.18 | 1.14 |
| 118 | 197 | 323 | 485 | 170 | 3.00 | . 000642 | 714 | 1,472 | 37.81 | 2.67 | 3.51 | 1.12 |

$\mathrm{a}_{\text {No }}$ flutter - maximum tunnel conditions.

TABLE II．－REPRESENTATIVE MODE SHAPES AND NODE LINES OF MODELS
［Deflections normalized on maximum deflection，considered positive when deflected wing is above static position］
（a）Model LA

| $x / c$ | Normalized deflection at $\mathrm{y} / \mathrm{l}=$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
| $\mathrm{f}_{1}=76 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.004 | 0.012 | 0.019 | 0.033 | 0.060 | 0.130 | 0.331 | 0.550 | 0.775 | 1.000 |
| ． 25 | ． 009 | ． 018 | ． 033 | ． 062 | ． 124 | ． 275 | ． 443 | ． 625 | ． 810 | 1.000 |
| ． 50 | ． 014 | ． 034 | ． 061 | ． 120 | ． 233 | ． 363 | ． 514 | ． 670 | ． 830 | 1.000 |
| ． 75 | ． 023 | ． 082 | ． 152 | ． 235 | ． 353 | ． 465 | ． 587 | ． 715 | ． 850 | 1.000 |
| 1.00 | ． 058 | ． 131 | ． 216 | ． 306 | ． 418 | ． 525 | ． 639 | ． 750 | ． 875 | 1.000 |
| $\mathrm{f}_{2}=165 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | －0．019 | －0．045 | －0．116 | －0．345 | －0．719 | －0．900 | －0．836 | －0．415 | 0.190 | 1.000 |
| ． 25 | －． 042 | －． 135 | －． 348 | －． 600 | －． 741 | －． 800 | －． 676 | －． 255 | ． 330 | 1.000 |
| ． 50 | －． 100 | －． 225 | －． 443 | －． 560 | －． 555 | －． 600 | －． 353 | ． 100 | ． 550 | 1.000 |
| ． 75 | －． 023 | －． 060 | －． 124 | －． 160 | －． 084 | ． 115 | ． 321 | ． 550 | ． 770 | 1.000 |
| 1.00 | ． 096 | ． 310 | ． 487 | ． 590 | ． 683 | .745 | ． 815 | ． 875 | ． 935 | 1.000 |
| $\mathrm{f}_{3}=291 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | －0．024 | －0．170 | －0．533 | －0．780 | －0．783 | －0．592 | 1.000 | 1.000 | 0.294 | －0．864 |
| ． 25 | －． 155 | －． 400 | －． 729 | －． 745 | －． 352 | ． 553 | 1.000 | ． 990 | ． 095 | －． 864 |
| ． 50 | －． 209 | －． 230 | －． 108 | ． 150 | ． 486 | ． 587 | ． 525 | ． 380 | －． 466 | －． 864 |
| ． 75 | ． 105 | ． 190 | ． 280 | ． 360 | ． 416 | ． 256 | －． 228 | －． 500 | －． 722 | －． 864 |
| 1.00 | －． 256 | －． 600 | －． 844 | －． 930 | －． 950 | －． 938 | －． 850 | －． 710 | －． 729 | －． 864 |
| $\mathrm{f}_{4}=383 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.007 | 0.030 | 0.062 | 0.130 | 0.117 | －0．050 | 0.060 | 0.360 | 0.940 | 1.000 |
| ． 25 | ． 032 | ． 060 | ． 071 | ． 035 | －． 038 | －． 025 | ． 109 | ． 380 | ． 771 | 1.000 |
| ． 50 | －． 007 | －． 020 | －． 037 | －． 040 | －． 044 | －． 045 | ． 111 | ． 380 | ． 677 | 1.000 |
| ． 75 | －． 034 | －． 075 | －． 111 | －． 150 | －． 161 | －． 150 | ． 060 | ． 340 | ． 618 | 1.000 |
| 1.00 | －． 052 | －． 315 | －． 860 | －． 930 | －． 909 | －． 800 | －． 538 | ． 250 | ． 600 | 1.000 |



| Mode | Node line |
| :---: | :---: |
| 1 | At root |
| 2 | ----- |
| 3 | $--\infty$ |
| 4 | - |

TABLE II.- REPRESENTATIVE MODE SHAPES AND NODE LINES OF MODELS - Continued
(b) Model IB

| $x / c$ | Normalized deflection at $\mathrm{y} / \imath=$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
| $\mathrm{f}_{1}=124 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.001 | 0.002 | 0.007 | 0.015 | 0.034 | 0.095 | 0.200 | 0.327 | 0.561 | 1.000 |
| . 25 | . 001 | . 004 | . 013 | . 028 | . 078 | . 159 | . 267 | . 389 | . 615 | 1.000 |
| . 50 | . 004 | . 016 | . 041 | . 089 | . 162 | . 244 | . 348 | . 481 | . 684 | 1.000 |
| . 75 | . 013 | . 045 | .103 | . 171 | . 250 | . 343 | . 456 | . 582 | .743 | 1.000 |
| 1.00 | . 030 | . 085 | . 152 | . 223 | . 305 | . 401 | .512 | . 624 | . 757 | 1.000 |
| $\mathrm{f}_{2}=278 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.002 | 0.012 | 0.043 | 0.132 | 0.330 | 0.754 | 1.000 | 0.633 | -0.022 | -0.968 |
| . 25 | . 011 | . 045 | . 139 | . 274 | . 553 | . 877 | . 598 | . 231 | -. 242 | -. 968 |
| . 50 | . 044 | . 135 | . 280 | . 382 | . 404 | . 352 | .193 | -. 058 | -. 417 | -. 968 |
| . 75 | . 046 | . 099 | . 118 | . 104 | . 060 | -. 037 | -. 181 | -. 378 | -. 630 | -. 968 |
| 1.00 | -. 072 | -. 153 | -. 239 | -. 331 | -. 432 | -. 533 | -. 643 | -. 747 | -. 857 | -. 968 |
| $\mathrm{f}_{3}=457 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.014 | 0.068 | 0.291 | 0.709 | 1.000 | 0.926 | -0.058 | -0.560 | -0.311 | 0.719 |
| . 25 | . 058 | .254 | . 612 | . 813 | . 512 | . 038 | -. 532 | -. 544 | . 053 | . 719 |
| . 50 | . 141 | . 214 | . 211 | . 128 | -. 270 | -. 515 | -. 536 | -. 262 | . 302 | - 719 |
| . 75 | -. 031 | -. 183 | -. 250 | -. 270 | -. 237 | -. 119 | . 066 | . 271 | . 500 | . 719 |
| 1.00 | . 166 | . 304 | .387 | . 454 | . 515 | . 572 | .572 | . 658 | . 692 | . 719 |
| $\mathrm{f}_{4}=630 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | -0.007 | -0.052 | -0.562 | -0.708 | -0.458 | 0.225 | 0.472 | -0.406 | -0.815 | -0.524 |
| . 25 | -. 180 | -. 3.340 | -. 319 | -. 140 | . 320 | . 412 | -. 108 | -. 680 | -. 729 | -. 524 |
| . 50 | -. 005 | . 036 | . 164 | . 166 | . 131 | -. 044 | -. 342 | -. 539 | -. 585 | -. 524 |
| . 75 | . 029 | . 088 | . 123 | . 133 | . 130 | . 100 | -. 064 | -. 195 | -. 385 | -. 524 |
| 1.00 | . 288 | . 791 | . 974 | 1.000 | . 954 | . 791 | . 495 | . 171 | -. 216 | -. 524 |



| Mode | Node line |
| :---: | :---: |
| 1 | At root |
| 2 | ----- |
| 3 | ------------ |

table it.- Representative mode shapes and node lines of models - Continued
(c) Model 1 C

| $\mathrm{x} / \mathrm{c}$ | Normallized deflection at $\mathrm{y} / \mathrm{l}=$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
| $\mathrm{f}_{1}=242 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.003 | 0.006 | 0.012 | 0.019 | 0.029 | 0.047 | 0.135 | 0.365 | 0.635 | 1.000 |
| . 25 | . 004 | . 010 | . 017 | . 029 | . 047 | . 117 | . 295 | . 505 | . 700 | 1.000 |
| . 50 | . 006 | . 017 | . 032 | . 065 | . 135 | . 282 | . 425 | . 585 | . 760 | 1.000 |
| . 75 | . 010 | . 050 | . 117 | . 220 | . 320 | . 435 | . 545 | . 675 | . 820 | 1.000 |
| 1.00 | . 034 | . 084 | . 167 | . 286 | . 400 | . 524 | . 636 | . 757 | . 873 | 1.000 |
| $\mathrm{f}_{2}=440 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.002 | 0.009 | 0.013 | 0.036 | 0.142 | 0.463 | 0.894 | 0.768 | -0.005 | -1.000 |
| . 25 | . 005 | . 018 | . 048 | . 169 | . 412 | . 753 | . 756 | . 458 | -. 320 | -1.000 |
| . 50 | . 017 | . 077 | . 212 | . 379 | . 474 | . 505 | . 323 | -. 045 | -. 548 | -1.000 |
| . 75 | . 050 | . 112 | . 133 | . 130 | . 080 | -. 021 | -. 198 | -. 507 | -. 759 | -1.000 |
| 1.00 | -. 082 | -. 236 | -. 370 | -. 479 | -. 605 | -. 699 | -. 791 | -. 871 | -. 932 | -1.000 |
| $\mathrm{f}_{3}=650 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.009 | 0.018 | 0.039 | 0.079 | 0.373 | 0.608 | 0.374 | -0.610 | -0.472 | 1.000 |
| . 25 | . 015 | . 047 | . 125 | . 465 | . 499 | . 265 | -. 257 | -. 727 | -. 109 | 1.000 |
| . 50 | . 078 | . 242 | . 255 | .169 | -. 041 | -. 370 | -. 618 | -. 434 | . 183 | 1.000 |
| . 75 | -. 027 | -. 077 | -. 178 | -. 269 | -. 373 | -. 293 | -. 066 | . 214 | . 512 | 1.000 |
| 1.00 | . 055 | . 154 | . 265 | . 378 | . 499 | . 612 | . 727 | . 835 | . 906 | 1.000 |



TABLE II.- REPRESENIATIVE MODE SHAPES AND NODE LINES OF MODELS - Continued
(d) Model 2 A

| $\mathrm{x} / \mathrm{c}$ | Normalized deflection at $\mathrm{y} / 2=$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
| $\mathrm{f}_{1}=36 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.013 | 0.032 | 0.064 | 0.125 | 0.215 | 0.310 | 0.404 | 0.515 | 0.657 | 0.904 |
| . 25 | . 018 | . 050 | . 097 | . 162 | . 261 | . 350 | . 457 | . 575 | .741 | . 955 |
| . 50 | . 027 | . 068 | . 130 | . 200 | . 301 | . 392 | . 508 | . 630 | . 790 | . 973 |
| . 75 | . 039 | . 095 | . 171 | . 256 | . 358 | . 462 | . 569 | . 680 | . 814 | . 994 |
| 1.00 | . 046 | . 123 | . 217 | . 315 | . 428 | . 520 | . 611 | . 720 | . 837 | 1.000 |
| $\mathrm{f}_{2}=96 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | -0.031 | -0.115 | -0.226 | -0.550 | -0.785 | -0.800 | -0.757 | -0.700 | -0.636 | -0.556 |
| . 25 | -. 073 | - . 250 | -. 495 | - . 620 | -. 664 | -. 660 | -. 636 | -. 550 | -. 432 | -. 138 |
| . 50 | -. 062 | -. 180 | -. 331 | -. 380 | -. 367 | -. 300 | -. 191 | . 040 | . 274 | . 547 |
| . 75 | -. .006 | -. 010 | . 017 | . 060 | . 135 | . 400 | . 386 | . 565 | . 710 | . 790 |
| 1.00 | . 080 | . 200 | . 380 | . 580 | . 772 | . 915 | . 916 | . 950 | . 982 | 1.000 |
| $\mathrm{f}_{3}=188 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | -0.042 | -0.135 | -0.266 | -0.410 | -0.491 | -0.430 | -0.222 | 0.300 | 0.715 | 1.000 |
| . 25 | -. 114 | -. 210 | -. 301 | -. 330 | -. 291 | - . 130 | . 127 | . 370 | . 635 | . 900 |
| . 50 | -. 073 | -. 170 | -. 251 | -. 280 | -. 223 | -. 060 | . 161 | . 410 | . 620 | . 830 |
| . 75 | -. 085 | -. 200 | -. 314 | -. 410 | -. 458 | -. 440 | -. 337 | 0 | . 330 | . 670 |
| 1.00 | -. 169 | -. 400 | -. 641 | -. 710 | -. 709 | -. 690 | -. 642 | -. 530 | -. 218 | . 450 |
| $\mathrm{f}_{\mathrm{f}}=204 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.055 | 0.220 | 0.424 | 0.530 | 0.557 | 0.520 | 0.268 | 0.100 | 0.069 | 0.118 |
| . 25 | . 087 | . 250 | . 320 | . 290 | . 151 | -. 040 | -. 131 | -. 150 | -. 125 | . 158 |
| . 50 | -. 049 | -. 100 | -. 161 | -. 270 | -. 388 | -. 410 | -. 371 | -. 190 | . 043 | . 279 |
| . 75 | -. 079 | -. 200 | -. 342 | -. 410 | -. 410 | -. 350 | -. 235 | 0 | . 284 | . 442 |
| 1.00 | -. 120 | -. 200 | -. 271 | -. 270 | -. 199 | 0 | . 327 | . 570 | . 789 | 1.000 |



TABLE II.- REPRESENTATIVE MODE SHAPES AND NODE LINES OF MODELS - Continued
(e) Model $2 B$

| $x / \mathrm{c}$ | Normalized deflection at $\mathrm{y} / 2=$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
| $\mathrm{f}_{1}=60 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.005 | 0.018 | 0.039 | 0.069 | 0.114 | 0.177 | 0.277 | 0.407 | 0.572 | 0.730 |
| . 25 | . 009 | . 032 | . 068 | . 116 | . 185 | . 270 | . 384 | . 510 | . 678 | . 833 |
| . 50 | . 014 | . 048 | . 094 | . 156 | . 236 | . 347 | . 474 | . 602 | . 750 | . 904 |
| . 75 | . 028 | . 058 | . 144 | . 223 | . 317 | . 424 | . 553 | . 680 | . 823 | . 960 |
| 1.00 | . 037 | . 097 | . 169 | . 266 | . 368 | . 479 | . 608 | . 729 | . 867 | 1.000 |
| $\mathrm{f}_{2}=123 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | -0.010 | -0.034 | -0.074 | -0.164 | -0.320 | -0.473 | -0.506 | -0.513 | -0.491 | -0.432 |
| . 25 | -. 024 | -. 079 | -. 161 | -. 290 | -. 385 | -. 420 | -. 417 | -. 387 | -. 318 | -. 156 |
| . 50 | -. 016 | -. 060 | -. 122 | -. 192 | -. 223 | -. 208 | -. 158 | -. 067 | . 067 | . 223 |
| . 75 | -. 007 | -. 009 | . 004 | . 043 | . 095 | . 168 | . 253 | . 351 | . 461 | . 586 |
| 1.00 | . 049 | . 149 | . 250 | . 348 | . 458 | . 565 | . 674 | . 784 | . 897 | 1.000 |
| $\mathrm{f}_{3}=222 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 0.756 | 0.935 | 0.990 | 1.000 | 0.973 | 0.854 | -0.735 | -0.919 |
| . 25 | . 397 | . 752 | . 812 | . 827 | . 808 | . 727 | -. 349 | -. 814 | -. 950 | -. 981 |
| . 50 | . 029 | . 058 | . 052 | -. 167 | -. 449 | -. 685 | -. 804 | -. 858 | -. 885 | -. 881 |
| . 75 | -. 157 | -. 213 | -. 236 | -. 244 | -. 244 | -. 244 | -. 244 | -. 244 | -. 244 | -. 244 |
| 1.00 | . 365 | . 187 | . 831 | . 908 | . 948 | . 969 | . 973 | . 960 | . 939 | . 904 |



| Mode | Node line |
| :---: | :---: |
| 1 | At root |
| 2 | ----- |
| 3 | - |
| 4 | - |

TABLE II.- REPRESENTATIVE MODE SHAPES AND NODE LINES OF MODELS - Concluded
(f) Model 2 C

| $x / c$ | Normalized deflection at $\mathrm{y} / 2=$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
| $\mathrm{f}_{1}=122 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.003 | 0.010 | 0.019 | 0.032 | 0.048 | 0.080 | 0.122 | 0.199 | 0.324 | 0.487 |
| . 25 | . 006 | . 016 | . 029 | . 054 | . 093 | . 144 | . 215 | .324 | . 455 | . 599 |
| . 50 | . 010 | . 032 | . 061 | . 099 | . 157 | . 234 | . 330 | . 449 | . 593 | . 737 |
| . 75 | . 029 | . 067 | . 115 | . 179 | . 253 | . 356 | . 462 | . 587 | . 728 | . 875 |
| 1.00 | . 035 | . 093 | . 163 | . 247 | . 337 | . 439 | . 551 | . 696 | . 849 | 1.000 |
| $\mathrm{f}_{2}=214 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.006 | 0.020 | 0.060 | 0.123 | 0.210 | 0.375 | 0.702 | 0.875 | 0.954 | 1.000 |
| . 25 | . 032 | . 092 | . 167 | . 268 | . 403 | . 705 | . 787 | . 792 | . 772 | . 716 |
| . 50 | . 053 | . 056 | . 243 | . 324 | . 404 | . 450 | . 432 | . 307 | . 115 | -. 182 |
| . 75 | . 003 | . 008 | . 005 | -. 022 | -. 070 | -. 140 | -. 233 | -. 375 | -. 565 | -. 770 |
| 1.00 | -. 049 | -. 132 | -. 222 | -. 332 | -. 487 | -. 653 | -. 760 | -. 850 | -. 929 | -1.000 |
| $\mathrm{f}_{3}=343 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | 0.006 | 0.039 | 0.130 | 0.461 | 0.729 | 0.850 | 0.851 | 0.808 | 0.385 | -0.515 |
| . 25 | . 158 | .380 | . 531 | . 634 | . 702 | . 708 | . 475 | -. 278 | -. 617 | -. 818 |
| . 50 | . 030 | . 108 | . 115 | . 056 | -. 163 | -. 410 | -. 567 | -. 647 | -. 669 | -. 568 |
| . 75 | - . 061 | -. 141 | -. 224 | -. 261 | -. 252 | -. 199 | -. 129 | -. 020 | . 168 | . 416 |
| 1.00 | . 098 | . 250 | . 429 | . 592 | . 714 | . 807 | . 872 | . 926 | . 966 | 1.000 |
| $\mathrm{f}_{4}=518 \mathrm{cps}$ |  |  |  |  |  |  |  |  |  |  |
| 0 | -0.017 | -0.054 | -0.153 | -0.453 | -0.629 | -0.695 | -0.330 | 0.429 | 0.650 | 0.601 |
| . 25 | -. 165 | -. 274 | -. 300 | -. 246 | -. 067 | -. 300 | . 478 | . 472 | -. 049 | -. 455 |
| . 50 | . 009 | . 081 | . 136 | . 149 | . 137 | . 084 | -. 217 | -. 398 | -. 562 | -. 707 |
| . 75 | -. 024 | -. 071 | -. 130 | -. 191 | -. 252 | -. 306 | -. 319 | -. 290 | -. 219 | -. 118 |
| 1.00 | . 099 | . 303 | . 453 | . 586 | . 693 | . 785 | . 856 | . 915 | . 967 | 1.000 |



| Mode | Node line |
| :---: | :---: |
| 1 | At root |
| 2 | - - - - |
| 3 | -- - - |
| 4 | - |


$A=0.536$
$W=0.0573 \mathrm{lb}$
$t=0.032 \mathrm{in}$.
$\delta=3 / 32$ in
in.

Delta plan form

$A=0.728$
$W=0.143 \mathrm{lb}$
$t=0.032 \mathrm{in}$.
$\delta=3 / 32$ in.

$A=0.536$
$W=0.106 \mathrm{lb}$
$t=0.032 \mathrm{in}$.
$\delta=3 / 32$ in.

$A=0.353$
$\mathrm{W}=0.0694 \mathrm{lb}$
$t=0.032 \mathrm{in}$.
$\delta=3 / 32 \mathrm{in}$.

Clipped-tip delta plan form
Figure 1.- Model geometry.

Side view
Figure 2.- Method of mounting models.

Mach number
Figure 3.- Experimental and theoretical variation of stiffness-altitude parameter with
Mach number.


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Mach number
Figure 3.- Continued.

22.



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