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THE EFFECT OF SOLAR RADIATION PRESSURE ON THE SPIN OF EXPLORER XII

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SUMMARY

An equation of motion about a satellite spin axis is derived that includes solar radiation torques produced by a solar paddle array. The equation is applied to the Explorer XII and solved to explain the observed increase in the satellite spin. Also, an approximate equation is developed which permits rapid calculation of the effect of solar torques on satellite spin.

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INTRODUCTION

Recent Goddard Space Flight Center satellites have had relatively large solar cell paddles to obtain power from the sun. An interesting effect of this has been the marked change of satellite spin. For the Explorer XII (launched August 16, 1961), the spin increased approximately 20 percent from its initial value in four months. It is the purpose of this note to report an analysis which shows that the increase of the Explorer XII spin can be attributed to solar radiation pressure.

ANALYSIS

It is a well established fact that sunlight is electro-magnetic energy. From Einstein's equation $E = mc^2$, where E is energy, m is mass, and c is the velocity of light, a mass and thus a momentum can be associated with sunlight. Whenever light impinges on a surface (for example, solar paddles on a satellite) there is a change of momentum and hence a force (torque) developed which in the case of spinning satellite may effect the rotation. Although this torque is usually small, it can under certain conditions produce significant results.

In the interest of reducing algebraic complexity, improving clarity, and to develop equations for engineering application, a restricted point of view will be taken in the subsequent analysis. That is, motion and torques only about the satellite spin axis will be considered. For the Explorer XII, this is found to be adequate to describe spin changes. A more general and abstract approach to solar torques acting on a satellite is given in Reference 1 and 2.

Once the satellite is injected into orbit and any nutation angle has been damped out due to energy dissipation, the satellite can essentially be considered as a one degree of freedom system (A favorable moment of inertia ratio is assumed). The effect of forces other than around the spin axis usually can be neglected if satellite spin is relatively high (20 to 30 rpm with a spin moment of inertia of about 3 to 4 slug feet^2). Moments tending to affect the spin axis are mainly magnetic, solar radiation, and aerodynamic. Because of the highly eccentric orbit of the Explorer XII and relatively high perigee (after one month in orbit: apogee 77, 199 km, perigee 448 km), the aerodynamic moments are small and can be neglected. The Explorer XII was also designed to minimize magnetic torques and the high apogee tends to reduce magnetic effects even further. Hence, magnetic torques will also be neglected. Thus, only solar radiation torques are left to be considered.

Figure 1 shows the general configuration of the satellite. Only direct radiation torques produced by the solar paddles will be considered; radiation torques produced by the main satellite body geometry about the spin axis are considered negligible. The equation of motion around the spin axis is simply

$$\sum \text{TORQUES} = I \frac{d\omega}{dt} \tag{1}$$

where $\omega = d\theta/dt$, θ is the angular coordinate around the spin axis, and I is the spin axis moment of inertia.

In developing the moment expression it will be assumed that there are two distinct pairs of paddles. The force acting on a paddle is the product of the radiation pressure factor, P (lbs/ft^2), and the projected area perpendicular to the sun's rays, A (ft^2):

$$\text{Incident light force} = P\,A\ .$$

For the reflected light, a reflectivity coefficient, ρ, must be included

$$\text{Reflected light force} = \rho P\,A\ .$$

In general P varies with the earth-sun distance and this is discussed later. The reflected light from a solar paddle is a rather complicated phenomenon being dependent upon the optical properties of the reflecting surface and the impinging light. It is assumed here that the reflection is spectral, that is, angle of incidence equals angle of reflection and that the reflectivity is independent of the incident angle. Figure 2 shows the geometry for the torque equation. The partial component of torque about the spin axis for the incident light is

$$PAh\,|\cos\theta|\sin\phi\ ,$$

where h is the distance from the spin axis to the centroid of the paddle, and ϕ is the sunline-spin axis angle. For the reflected light it is

$$\rho PA\,|h\cos\theta|\sin(\phi - 2ns)\ ,$$

where ns is the angle between the normal to the paddle and the direction of the sun. Absolute signs are used on the cosine term since the moment arm is always positive. Summing the torques for each paddle and considering counterclockwise positive, we obtain

$$
\begin{aligned}
\sum \text{TORQUES} = & - PA_1 h_1 \left|\cos\theta\right| \left(\sin\phi + \rho \sin\left(\phi - 2ns_1\right)\right) \\
& + PA_2 h_2 \left|\sin\theta\right| \left(\sin\phi + \rho \sin\left(\phi - 2ns_2\right)\right) \\
& + PA_3 h_1 \left|\cos\theta\right| \left(\sin\phi + \rho \sin\left(\phi - 2ns_3\right)\right) \\
& - PA_4 h_2 \left|\sin\theta\right| \left(\sin\phi + \rho \sin\left(\phi - 2ns_4\right)\right)
\end{aligned}
\tag{2}
$$

Equation 2 is the basic torque expression for direct solar radiation effects about the spin axis due to the solar paddles.

The projected area of a paddle as a function of ϕ, θ, and the geometry of the paddle is developed in Appendix A. Using the basic projected area equation, the projected area of the four paddles can be expressed in the following way

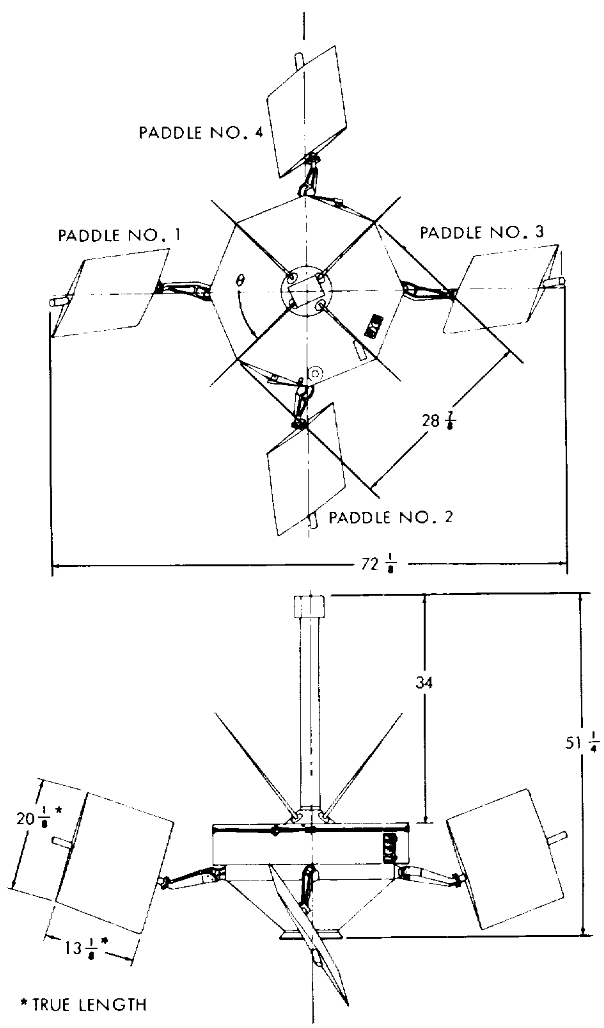

Figure 1—Explorer XII Energetic Particle Satellite.

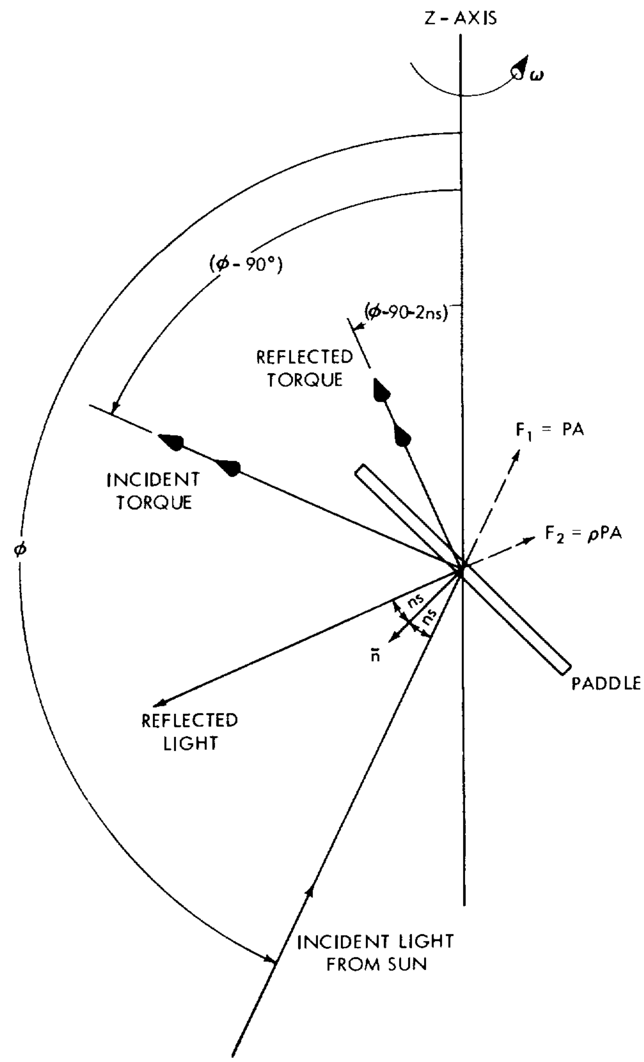

Figure 2—Geometry for solar radiation torques produced by a solar paddle.

$$
\left.\begin{aligned}
A_1 &= A_0\,|\cos\theta\sin nz_1\sin\phi + \cos nz_1\cos\phi| ,\\
A_2 &= A_0\,|-\sin\theta\sin nz_2\sin\phi + \cos nz_2\cos\phi| ,\\
A_3 &= A_0\,|-\cos\theta\sin nz_1\sin\phi + \cos nz_1\cos\phi| ,\\
A_4 &= A_0\,|\sin\theta\sin nz_2\sin\phi + \cos nz_2\cos\phi| ,
\end{aligned}\right\} \tag{3}
$$

where nz_1, nz_2 are the angles from the normal to the paddle to the spin axis and A_0 is the geometric area of the paddle. Absolute signs are used since a minus area means that the back side of a paddle is illuminated. Substituting Equation 3 into 2, Equation 2 into 1, and making the abbreviation

$$
\begin{aligned}
M &= \sin nz\sin\phi ,\\
N &= \cos nz\cos\phi ,
\end{aligned}
$$

the equation of motion becomes

$$
\begin{aligned}
I\frac{d\omega}{dt} = PA_0\,K(\phi,\theta)\Big\{ &-\Big|M_1\,|\cos\theta| + N_1\Big|\,h_1\,|\cos\theta|\,\Big(\sin\phi + \rho\sin(\phi - 2ns_1)\Big)\\
&+\Big|-M_2\,|\sin\theta| + N_2\Big|\,h_2\,|\sin\theta|\,\Big(\sin\phi + \rho\sin(\phi - 2ns_2)\Big)\\
&+\Big|-M_1\,|\cos\theta| + N_1\Big|\,h_1\,|\cos\theta|\,\Big(\sin\phi + \rho\sin(\phi - 2ns_3)\Big)\\
&-\Big|M_2\,|\sin\theta| + N_2\Big|\,h_2\,|\sin\theta|\,\Big(\sin\phi + \rho\sin(\phi - 2ns_4)\Big)\Big\} .
\end{aligned} \tag{4}
$$

Again absolute signs are used inside the parenthesis because there is 180 degree symmetry with the two paddles. That is one paddle presents the same area as the other paddle 180 degrees later. A shadow factor $K(\phi,\theta)$ is included to account for paddle on paddle shadowing and shadowing by the satellite structure.

Strictly speaking Equation 4 should contain a torque term due to albedo radiation from the earth. Because of the complicated mathematical description and the fact that it contributes very little in this application (it decreases as $1/\sigma^2$ where σ is the ratio of the satellite distance to earth radius, Reference 3), the albedo radiation term will be neglected.

As was noted earlier, the solar radiation factor, P, in Equation 4 is variable; it varies inversely as the square of the earth-sun distance. By considering the earth's orbit around the sun as a perturbed circular orbit, it can be shown that the distance between the sun and the earth as a function of time can be expressed as

$$
r(t) = r_0\left(1 + e\cos 2\pi t/T\right)^2 , \tag{5}
$$

where

e = eccentricity of the earth's orbit = .01674,
T = 365 days,
r_0 = mean earth-sun distances.

Time zero is at the maximum earth-sun distance of approximately June 21st. P can then be expressed as

$$P(t) = \frac{p_0}{(1 + e \cos 2\pi t/T)^2} \tag{6}$$

where p_0 is the mean solar constant at the mean solar-earth distance. $P(t)$ is considered zero when the satellite is eclipsed by the earth.

The change of satellite spin is obtained by integrating Equation 4; this is indicated symbolically in two ways below:

$$\omega - \omega_0 = \frac{A_0}{I} \int_0^t P(t)\, K(\phi,\theta)\{\quad\}dt \,, \tag{7}$$

or

$$\omega^2 - \omega_0{}^2 = \frac{2A_0}{I} \int_0^\theta P(t)\, K(\phi,\theta)\{\quad\}d\theta \,, \tag{8}$$

where the ns angles are obtained from the following equations

$$\begin{aligned} \cos ns_1 &= |M_1| \cos\theta| + N_1| \\ \cos ns_2 &= |-M_2| \sin\theta| + N_2| \\ \cos ns_3 &= |-M_1| \cos\theta| + N_1| \\ \cos ns_4 &= |M_2| \sin\theta| + N_2| \end{aligned} \tag{8a}$$

and the quantity inside the brace is the same as that given in Equation 4. To carry out the integration indicated by Equation 7 or 8 is impossible by analytical means and very messy and tedious to do numerically; a digital computer solution is required. This has been done and the results are discussed below. It is of value though to develop an approximate equation (and solution) so that "ball park" answers can be determined readily. Within certain limits, it has been found that the approximate solution gives answers comparable to the digital solution.

APPROXIMATE METHODS

An approximate expression will now be developed for the torque equation. Directing attention to Equation 4 and noting that if ns, ns_2, ns_3 and ns_4 are replaced by an average angle, $\langle ns \rangle$, Equation 4 can be put into the following form:

$$I\frac{d\omega}{dt} = PA_0\ K(\phi,\theta)\Big\{\Big(\big|-M_1|\cos\theta|+N_1\big|-\big|M_1|\cos\theta|+N_1\big|\Big)h_1|\cos\theta|$$
$$+\Big(\big|-M_2|\sin\theta|+N_2\big|-\big|M_2|\sin\theta|+N_2\big|\Big)h_2|\sin\theta|\Big\}\big(\sin\phi+\rho\sin(\phi-2\langle ns\rangle)\big). \quad (9)$$

For θ near zero and $M > N$ (this restricts ϕ around 90°), the quantity in the brace in Equation 9 has the approximate value

$$-2N_1h_1|\cos\theta|$$

and for θ near $\pi/2$ the brace has the approximate value

$$-2N_2h_2|\sin\theta|$$

For an approximate solution, it does not seem unreasonable to replace the complicated expression inside the brace by the sum of these two quantities

$$-2N_1h_1|\cos\theta|-2N_2h_2|\sin\theta|$$

throughout the paddle array rotation. Therefore, the following approximation can be made to Equation 7.

$$\omega-\omega_0 = -2\frac{A_0}{I}\int_0^t P(t)\ K(\phi,\theta)\Big\{\cos nz_1\ h_1|\cos\theta|+\cos nz_2\ h_2|\sin\theta|\Big\}$$
$$\big(\sin\phi+\rho\sin(\phi-2\langle ns\rangle)\big)\cos\phi\ dt\ . \quad (10)$$

The same result can be obtained by squaring a typical quantity $|M\cos\theta+N|$, neglecting N^2, then taking the square root, expand in a power series and retain the first two terms.

The average angle $\langle ns\rangle$ must now be evaluated. In an approximate equation it is desirable to have the equation as simple as possible and still retain the essence of the phenomena it is to describe. An expression can be developed for the average angle but it is complicated and unwieldy. The simplest value for $\langle ns\rangle$ would be zero. At first glance, this may appear to be an unwarranted simplification. Subsequent comparison of numerical results from the theoretically correct equation will show that this is justified. Essentially, when $\langle ns\rangle$ is given a value of zero, it is tacitly assumed that the reflected light reinforces the direct light effect.

Using the method of Kryloff and Bogoliuboff (Reference 4), that is, averaging over a cycle while considering $d\omega/dt$ constant during the averaging period, Equation 10 can be simplified to

$$\omega-\omega_0 = -\beta\int_0^t R(t)\ K(\phi)\ \sin 2\phi\ dt\ , \quad (11)$$

where

$$R(t) = \frac{1}{(1 + e \cos 2\pi t/T)^2}$$

$$\beta = 2(1 + \rho) \frac{A_0 p_0}{\pi I} \left(h_1 \cos nz_1 + h_2 \cos nz_2\right) .$$

Equation 11 still cannot be integrated analytically since $K(\phi)$ and ϕ are usually known only graphically. Using the trapezoidal rule, the integration can be replaced by a finite summation

$$\omega_N - \omega_0 = -\beta \sum_{i=0}^{N} \delta_i \, R(t_i) \, K(\phi_i) \sin 2\phi_i \, \Delta t , \tag{12}$$

where $\delta_0 = \delta_N = 1/2$, and the other δ's are equal to 1. If ϕ is known day by day, the increment of integration, Δt, can be a day without any appreciable loss of accuracy. For a quick estimate of solar radiation effect on the spin, $K(\phi)$ can be set equal to unity.

DISCUSSION

The exact spin equation (Equation 8) was applied to the Explorer XII satellite with the following parameters:

$A_0 = 1.75 \text{ ft}^2$,	$I = 3.53 \text{ slug ft}^2$,
$h_1 = h_2 = 2.35 \text{ ft}$,	$p_0 = 9.50 \times 10^{-8} \text{ lb/ft}^2$,
$nz_1 = nz_2 = 59° \, 47'$,	$\rho \approx 0.10$.
$M_1 = M_2$, $N_1 = N_2$,	

The reflectivity coefficient for a solar paddle is a rather uncertain quantity; the above value for ρ was estimated from tests conducted on plain solar cells. A plot of the telemetry optical aspect sunline-spin axis angle used in the calculations is shown in Figure 3. The shadowing factor, K vs ϕ, is shown in Figure 4. Since Explorer XII was in the sunlight more than 98 percent of the time, no eclipse factor was used. Equation 8 was solved on a 7090 digital computer in an iterative procedure by calculating the change of spin over one rotation and considering this change of spin applied for 200 revolutions. The elapsed time was calculated and the sunline-spin axis angle was changed accordingly; the calculation was then repeated. The above method resulted in reasonable computer times and still gave, it was felt, accurate results. Computer results are compared with actual sattellite spin data in Figure 5. Calculations were started two days after launch to allow the nutation angle to dampen out since after yo-yo operation (a spin reduction device), Explorer XII had a five degree half cone angle nutation. The optical aspect sensor indicated that after the initial coning

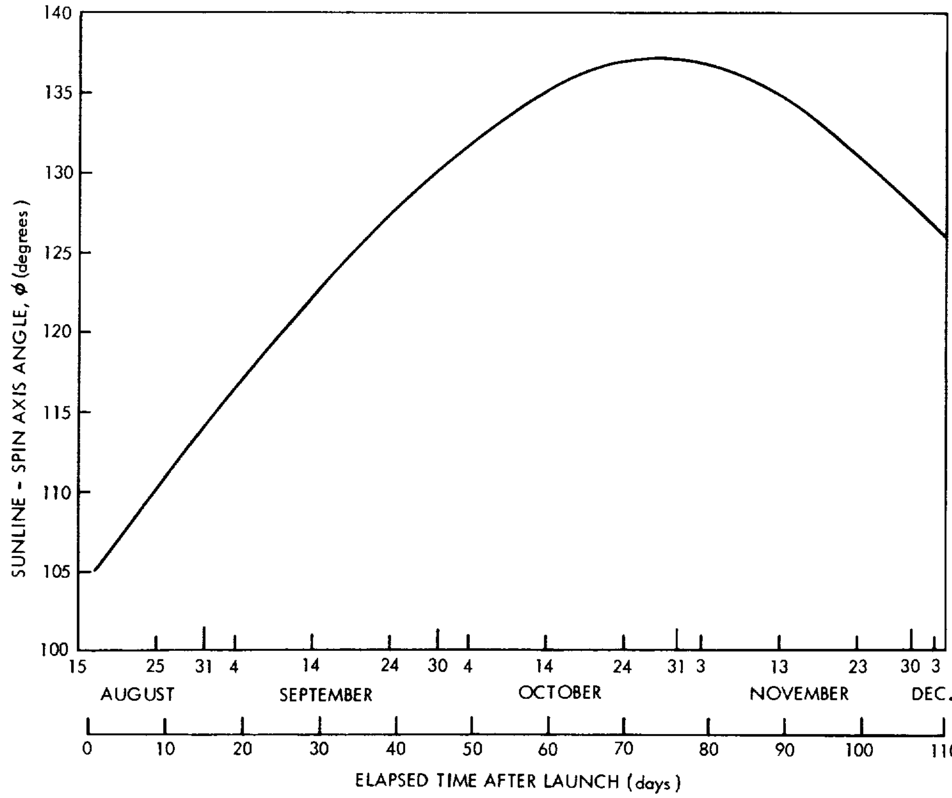

Figure 3—Sunline-spin axis angle vs time.

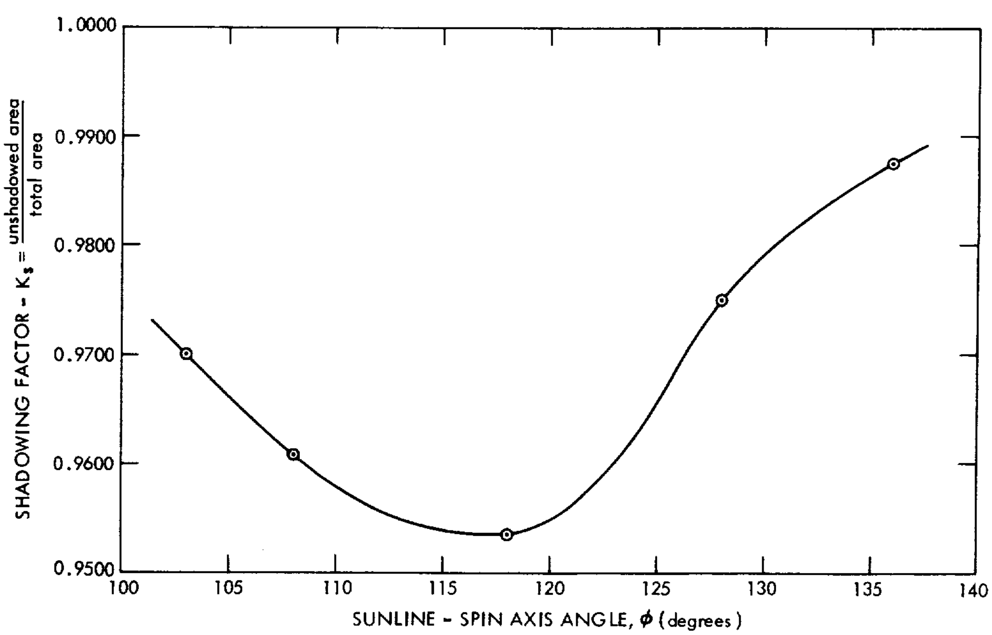

Figure 4—Shadowing factor vs sunline-spin axis angle.

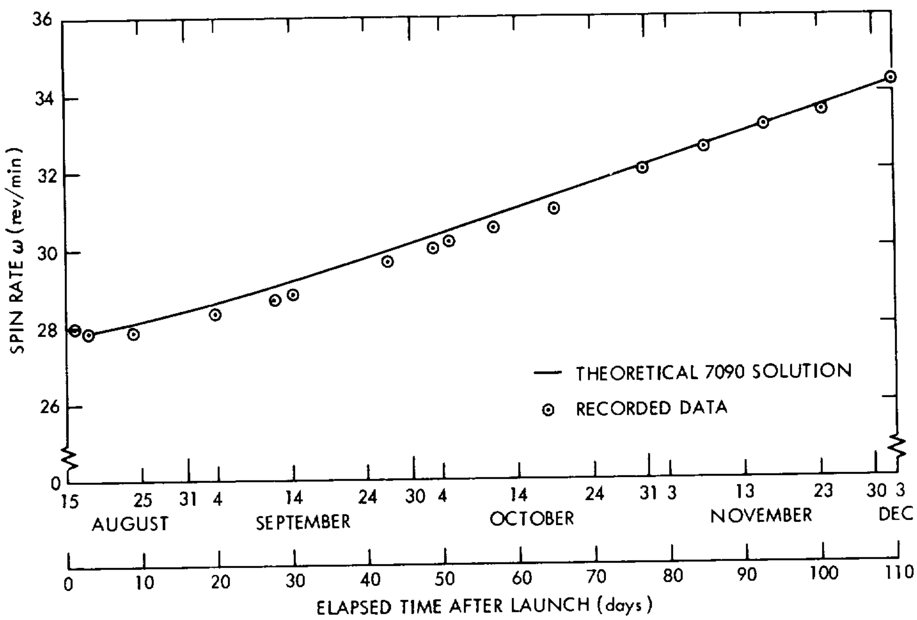

Figure 5—Spin rate of Explorer XII vs time.

angle damped out, there was virtually no nutation angle during the active life of the satellite. It will be noticed from the figure that there is good agreement between the theoretical curve and actual spin data. Maximum deviation from the theoretical curve is about one percent. Considering the uncertainty of the reflectivity coefficient and the small but finite perturbations of the albedo, magnetic, and aerodynamic torques, the agreement is satisfactory.

The approximate equation given by Equation 12 was also applied with the Explorer XII parameters (Δt was taken to be one day) and results are compared with the computer solution. This is shown in Figure 6. It will be noted that there is very little difference between the computer solution and the approximate solution. For most practical purposes the curves are essentially the same. Because of the good agreement it is inferred that the approximate equation gives satisfactory results for ϕ between 45° and 135° and $\cos nz < 1/2$.

It is of value to note that in general solar radiation torques about the spin axis occur because of the different tilt (projected area) of the paddles. This is clearly seen in Equation 2; if the projected areas are equal, (ns angles are also equal) there are no solar torques about the spin axis. For small reflectivities, it can also be stated that if the sun shines from below the equator of the satellite ($\phi > 90°$), the satellite will spin up; if the sun shines from above the equator ($\phi < 90°$), the satellite will spin down. This is easily seen from the approximate Equations 10 or 11. This effect has been observed in satellites launched after Explorer XII. It is interesting to note that the maximum value of the solar torque around the spin axis is about 2×10^{-7} ft. lbs. This is a very small torque, indeed. As was noted though, a small but persistent torque acting on a satellite for a long period of time does have an appreciable effect.

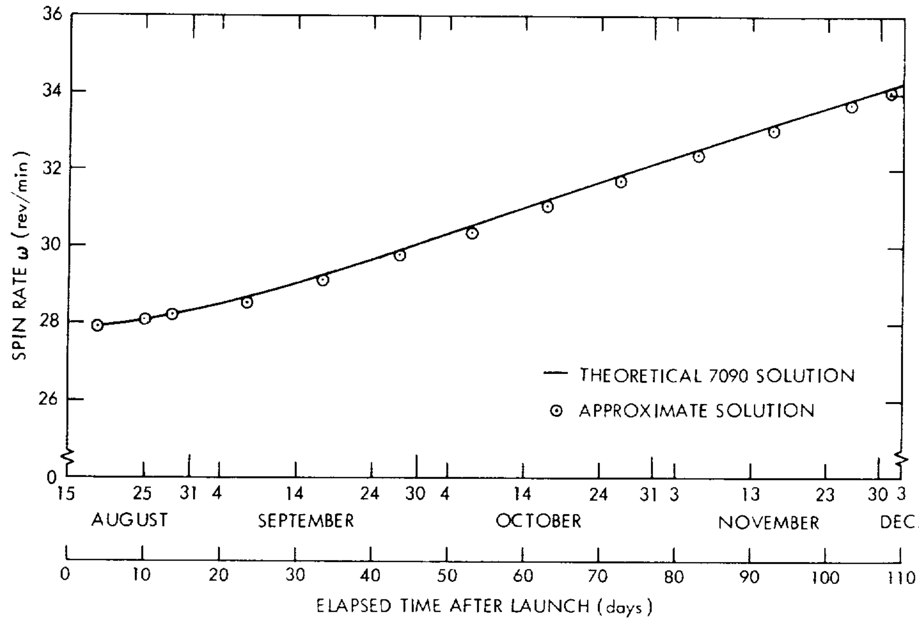

Figure 6—Explorer XII spin rate vs time: comparison of computer calculation with approximate solution (Equation 12).

RESUMÉ

An equation of motion about a satellite spin axis has been developed that includes spectral solar radiation torques. The equation was applied to the Explorer XII to account for the observed increase in spin. Calculated spin values agree within about one percent of the observed satellite spin values. It is concluded from the analysis and calculations that Explorer XII spin-up was due to solar radiation torques. Also, an approximate spin equation was developed which permits rapid calculations of the effect of solar radiation on satellite spin.

ACKNOWLEDGMENT

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REFERENCES

1. McElvain, R. J., "Effects of Solar Radiation Pressure upon Satellite Attitude Control," in: Progress in Astronautics and Rocketry edited by Roberson, R. E., and Farrior, J. S., New York: Academic Press, 1962.

2. Clancy, T. F., and Mitchell, T. P., "Effects of Radiation Forces upon the Attitude of an Artificial Earth Satellite," Center for Radiophysics and Space Research Report #129, September 1962, Cornell University, Ithaca, New York.

3. Levin, E., "Reflected Radiation Received by an Earth Satellite", ARS Journal 32(9):1328-1331, September 1962.

4. Kryloff, N., and Bogoliuboff, N., "Introduction to Non-Linear Mechanics" (translation), Princeton: Princeton University Press, 1947, chapters 10-12.

5. Curtiss, D. R., and Moulton, E. J., "Essentials of Trigonometry", New York: D. C. Heath and Company, 1942.

Appendix A

Projected Areas of a Solar Paddle

The effective projected area of a solar paddle will now be determined. We wish to derive an expression involving the angle between the spin axis of the satellite and the sunline, ϕ, the rotation angle θ, and geometrical properties of the solar paddles. Initially the paddle will be considered stationary and any shadowing by the satellite will be neglected.

Let $\bar{n}$ be the unit normal to the paddle and $\bar{s}$ the unit vector in the direction of the sun (Figure A1). The projected area perpendicular to $\bar{s}$ is

$$A_p = A_0 \cos ns \,, \qquad \text{(A1)}$$

where

A_p = effective paddle area,

A_0 = geometric paddle area,

ns = angle between vector $\bar{n}$ and $\bar{s}$.

Now $\cos ns$ can be obtained from the vector dot product

$$\cos ns = \bar{n} \cdot \bar{s} \,. \qquad \text{(A2)}$$

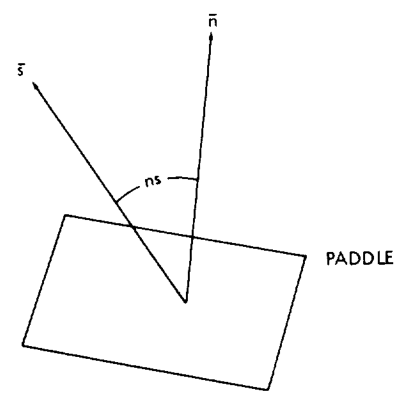

Figure A1

If we choose the coordinate system shown in Figure A2, where i, j, k are unit vectors, the $\bar{n}$ and $\bar{s}$ vectors have the following components:

$\bar{n}$ components - $\cos nx, \cos ny, \cos nz$,

$\bar{s}$ components - $\sin \phi, 0, \cos \phi$.

Carrying out the dot product results in

$$\bar{n} \cdot \bar{s} = \cos nx \sin \phi + \cos nz \cos \phi \,. \qquad \text{(A3)}$$

Hence, Equation A1 can be written as

$$A_p = A_0(\cos nx \sin \phi + \cos nz \cos \phi) \,. \qquad \text{(A4)}$$

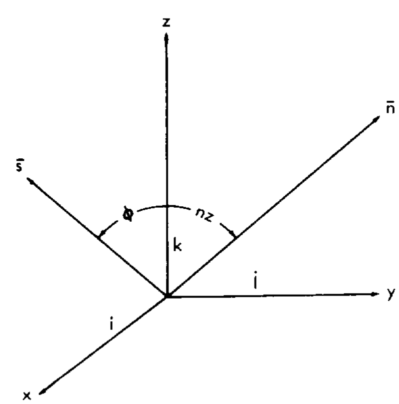

Figure A2

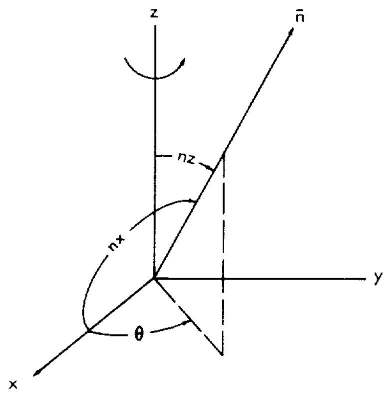

Figure A3

Equation A4 is for the static case (no spin). When spin is present we have the situation shown in Figure A3. Though $\bar{s}$ is not shown in the figure for clarity, it is still pointing in the direction of the sun. The z axis is considered to be the spin axis and therefore the angle nx will vary with time (see Equation A4). From solid trigonometry we can write the following formula (Reference 5, pp. 112-113)

$$\cos nx = \cos\theta \cos(\pi/2 - nz) = \cos\theta \sin nz \, . \qquad \text{(A5)}$$

Substituting Equation A5 into Equation A4 results in

$$A_p = A_0 \left| \sin nz \sin\phi \cos\theta + \cos nz \cos\phi \right| . \qquad \text{(A6)}$$

Absolute signs are used since a negative area means that the back side of a paddle is illuminated. Equation A6 is the basic equation for the projected area of a solar paddle.

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