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A STUDY OF CRYOPUMP CONFIGURATIONS IN FREE MOLECULAR FLOW REGIONS

By

J. O. Ballance, W. K. Roberts, and D. W. Tarbell

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ABSTRACT

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In cryopumping systems, the ratio of gas condensed to the area of the condenser should be made as large as possible. By designing an array which would focus the molecules, the condenser area might be reduced, and the ratio thereby increased to a satisfactory value. The present study was initiated to determine if such focusing could be obtained with simple geometric configurations. The kinetics of molecules under free-molecular-flow conditions were examined for circles, ellipses, parabolas, hyperbolas, and triangles. This study consisted of both an exact theoretical analysis and a Monte Carlo computer technique. It was found that the best focusing could be obtained with the circular array, which concentrates 50% of the molecules into 46% of the area. Since this focusing is so slight, an extension of the study is planned to include incident beams which enter the array at an angle rather than directly, as in the present study. Preliminary results from this extension are encouraging.

AUTHOR

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EXPERIMENTAL AERODYNAMICS BRANCH
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SUMMARY

In cryopumping systems, the ratio of gas condensed to the area of the condenser should be made as large as possible. By designing an array which would focus the molecules, the condenser area might be reduced, and the ratio thereby increased to a satisfactory value. The present study was initiated to determine if such focusing could be obtained with simple geometric configurations. The kinetics of molecules under free-molecular-flow conditions were examined for circles, ellipses, parabolas, hyperbolas, and triangles. This study consisted of both an exact theoretical analysis and a Monte Carlo computer technique. It was found that the best focusing could be obtained with the circular array, which concentrates 50% of the molecules into 46% of the area. Since this focusing is so slight, an extension of the study is planned to include incident beams which enter the array at an angle rather than directly, as in the present study. Preliminary results from this extension are encouraging.

I. INTRODUCTION

The cryopump has been established as a necessary component of vacuum systems in which very low densities and relatively high mass flows are required. Since the condensing efficiency of a cryopumping system depends more on the physical arrangement of thermal shields than on the properties of the gases striking the condensing surface, many cryo-array configurations have been developed. The performance of these arrays has been examined both theoretically and experimentally. Such examinations are usually conducted in the free-molecular-flow region, where two basic assumptions can be made: (1) the molecules interact only with the surfaces (not with each other), and (2) the molecules are reflected from the surface diffusely. Although an exact analysis requires knowledge of the accommodation coefficients, sticking coefficients and capture coefficients, some arrays (such as the chevron type) are nearly independent of these parameters. These arrays, however, usually have a very low pumping efficiency; i.e. only a small percentage of the molecules entering the array is condensed.

If there were some way in which the gas molecules could be directed or focused, much better pumping characteristics could be obtained for a given condensing area. The present study was initiated, using the assumptions of free molecular flow, to determine if such focusing could be obtained with simple configurations. It was noted that, even if no focusing were obtained, the results of this study might be useful in determining some of the molecule-surface interaction parameters.

This report includes an outline of the methods used, a summary of the results to date for several simple configurations, a discussion of the accuracy of the data, and mention of a possible experimental application of the results.

II. METHOD

Typical in any study of free-molecular-flow problems are two assumptions: (1) the mean free path of the molecules is much larger than any dimensions involved, and (2) molecules colliding with a surface are reflected according to the Lambert Cosine Law distribution. In this study, it was also assumed that the problem could be reduced to two dimensions by considering the surfaces to be infinitely long cylinders. With this assumption, each array was then studied only in cross section. A study by a group at Arthur D. Little, Inc. has shown that the cosine law can be used in two dimensions without introducing any error.

With these assumptions, two different approaches were used to determine exit distributions of molecules reflected from several simple geometrical configurations. The first was an exact theoretical approach; the second, a Monte Carlo approach, used an IBM 7090 computer. It was possible to obtain an exact theoretical expression for the exit distribution in only one case, viz., molecules reflected after one collision with a circular surface, assuming an incident distribution completely uniform over the surface. This calculation is given in Appendix A. For other configurations and incident distributions, the final results were in the form of integrals which could not be calculated in closed form. However, the theoretical approach did yield other important information: for the case of the circle, distributions over the surface could be calculated as a function of collision number. Expressions for these distributions are given in Appendix B and are plotted in Figure 1 for two cases: (1) incident flux uniform over the surface ("random angle"), and (2) incident flux uniform across the opening ("random y"). From these distributions, the fraction of molecules exiting the array after n collisions, α_n, could be calculated exactly and compared with the computer results. This comparison is given in Table I. In this table are also given some results obtained by graphically integrating the theoretical expression found for the "random y" case.

TABLE I

α_n -- Fraction of Particles Exiting Array After n Collisions---Circles

a. Random Angle Incident Beam

n	A α_n theory	B α_n computer	|A-B|	%
1	0.6366	0.6361	0.0005	0.1
2	0.6259	0.6295	0.0036	0.6
3	0.6279	0.6187	0.0092	1.5
4	0.6281	0.6012	0.0269	4.3

b. Random y Incident Beam

n	A α_n theory	B α_n computer	C α_n graphical	|A-B|	%	|A-C|	%
1	0.6667	0.6608	0.6756	0.0059	0.9	0.0089	1.3
2	0.6186	0.6167	0.6757	0.0019	0.3	0.0571	9.2
3	0.6298	0.6400	0.6759	0.0102	1.6	0.0461	7.3
4	0.6285	0.6538	0.6774	0.0253	4.0	0.0489	7.8

It is seen that the computer results for the fraction α_n are in error by a maximum of 4.3% while those obtained by the graphical method are in error by as much as 9.2%. The graphical method was therefore discarded in favor of the computer method.

The computer method, using the Monte Carlo technique, consisted of the following steps:

(1) A random number, R_1, between 0 and 1, was generated by the computer.

(2) From R_1, a random number $y_1 = 1-2R_1$ (between -1 and +1) was computed and used for the y coordinate of the incident particle.

(3) Using the general equation for the geometry of interest, the x coordinate, x_1, was then calculated.

(4) The equation of the normal to the surface at (x_1, y_1) was then computed.

(5) A new random number, R_2, (also between 0 and 1) was generated and an angle, δ, calculated from $\delta = \sin^{-1}(1-2R_2)$.

(6) The equation for the particle line, at an angle δ with the normal, was then computed; this yields the cosine distribution for the reflected particles.

(7) The intersection of this particle line with the y-axis, "Y", was then calculated. This value, Y, was then tested and placed in the correct group.

(8) If the particle did not fall into any of the 20 groups from Y = -1.0 to Y = +1.0, the intersection of the particle line with the array was calculated. This yielded a new (x_1, y_1) for the particle's second collision and the computer then returned to step (4).

(9) The program was continued until a particle had either been placed in one of the exit groups or had made four collisions (six collisions in the case of the triangle), whereupon it was recorded as "out" and the program started over at step (1).

III. DISCUSSION OF RESULTS

Computer results were printed out in twenty groups, from y = -1.0 to y = +1.0 in intervals of 0.1, with each group broken down into first, second, third and fourth collisions. These distributions, for the five geometries considered, are presented in Figures 2 through 6, with only Figure 2 including the distribution as a function of collision number. It is clear that very little focusing is obtained with any of these arrays. A measure of focusing is given by the percentage area into which the middle 50% of the particles fall. These percentages are given in Table II, for each of the arrays considered.

TABLE II

Percentage Area into which Middle 50% of Particles Fall for Various Surface Geometries

Geometry	% Area
Circle, Random Angle	51.3
" " y	45.8
Ellipse, A = 0.5	47.5
" , A = 1.25	46.9
" , A = 1.50	46.7
" , A = 1.75	46.2
" , A = 2.00	46.5
Parabola, P = 0.5	47.7
" , P = 0.25	46.8
" , P = 0.125	48.4

Geometry	% Area
Parabola, P = 0.0833	47.7
Hyperbola, A = -1	48.3
" , A = -2	46.9
" , A = -3	46.6
" , A = -5	47.5
" , A = -7	47.8
Triangle, 10°	48.9
" , 30°	48.6
" , 45°	47.6
" , 60°	47.7

It can be seen that the best focusing is obtained with the circle with random y incident beam, which "concentrates" 50% of the particles into 45.8% of the area.

Since these results were rather uninteresting, it was then decided to examine cases where the incident beam enters the array at an angle β with the opening. If better results could be obtained under these conditions, then several simple arrays might be combined (in a clover-leaf pattern, for example) to focus the molecules. Preliminary results of this study are presented in Figure 7, which shows the distribution of particles after one collision with a circular array, where the incident beam is at an angle β with the y-axis. These results are somewhat more encouraging, and these studies are being continued at the present time.

IV. DISCUSSION OF ERRORS

Two methods have been used to determine the random errors inherent in the computer approach. The first of these is based on the asymmetry of the results, as follows: Since both the incident beam and the arrays considered were perfectly symmetrical with respect to the x-axis, it was expected that the exit distributions across the y-axis would also exhibit this symmetry. For example, the same number of particles should fall into group 20 as into group 1, the same into group 11 as into group 10, etc. With the finite number of points (10,000) considered by the computer, there was, of course, some asymmetry. Theoretically, the difference between two values obtained should be $0 \pm 2\sigma$ (σ = standard deviation), with 95% confidence, or $0 \pm .6745\sigma$ with 50% confidence, where a standard deviation in this case is given by $\sqrt{2Npq}$ (N is the number of points considered = 10,000; p is the probability of a point falling into a group = $\frac{n}{10,000}$; n is the average number in a group; and q = 1-p). Both the percentage probable error and two standard deviations are plotted in Figure 8a, where the solid curves have been plotted from the above theoretical expressions, and the points have been calculated from the computer data. The figure indicates that the asymmetry obtained was approximately what was expected theoretically.

The second method used to determine the errors in the computer method was based on a comparison with the only case solved exactly by theoretical analysis: the case of the distribution of particles exiting the array after one collision with a circle, where the incident particles are distributed uniformly over the surface. Twenty-four different computer runs were made, varying the number of incident particles. The probable error was then determined from the equation: $\text{P.E.} = .6745 \sqrt{(\Sigma x_i^2)/20}$, where x_i is the difference between the number of particles in the ith group and the average; the index i runs from 1 to 20, since there are 20 groups. The probable error was also calculated theoretically, using P.E. = $.6745\sigma$ and $\sigma = \sqrt{Npq}$ where N is the number of points considered, p = .03183 (calculated from theory) and q = 1-p. The results are shown in Figure 8b. It is seen that the computer errors are within those predicted theoretically, and that the probable error for 10,000 points is approximately 3.7%. Thus the distributions shown in Figures 2 through 6 can be considered accurate within 2 to 5%, depending upon the number of particles in a group (for 250 particles the error is 4.2% or 11 particles; for 600 particles the error is 2.7% or 16 particles).

V. EXPERIMENTAL APPLICATION: MEASUREMENT OF ACCOMMODATION COEFFICIENT

As stated earlier, an exact analysis of an array requires knowledge of the accommodation coefficients, sticking coefficients, etc. At present, this study has not progressed far enough to be concerned with these parameters; however, the results might be very useful in their determination. In particular, an immediate application of the results would be the determination of the accommodation coefficient.

The accommodation coefficient, a, is defined by:

$$a = \frac{T_o - T_n}{T_o - T_w}$$

where T_o, T_n, and T_w are the absolute temperatures of the incident gas, the reflected gas, and the reflecting surface, respectively.

Thus the temperature of N_1 molecules after one collision is $T_1 = T_o - a(T_o - T_w)$. It is easily shown that the temperature of N_n molecules after the <u>n</u>th collision is:

$$T_n = T_w + (T_o - T_w)\ (1 - a)^n \tag{1}$$

For a given experimental array, some molecules will exit the array after one collision, others will collide more than once with the surface of interest, and exit after n collisions. Considering <u>all</u> exiting molecules, the actual measured temperature of the exiting gas will be a weighted average of all these, given by:

$$T_{exit} = \sum_{n=1}^{\infty} N_n\, T_n \Big/ \sum_{n=1}^{\infty} N_n \tag{2}$$

where N_n is the number of molecules exiting the array after n collisions.

The summations are shown here to extend to infinity, but if a cutoff is imposed, this can be replaced by a finite number. For example, if the cutoff is set at 99.9% (i.e., 99.9% of the particles have exited), the upper limit on the summation is a number between 2 and 50, depending upon the geometry.

The fraction of particles exiting the array after n collisions, α_n, has been discussed above (See Section II).

For $\quad n = 1: \quad \alpha_1 \equiv N_1/N_o \quad$ or $\quad N_1 = \alpha_1 N_o$.

Similarly, $\quad N_2 = \alpha_2 (N_o - N_1)$, etc.

In general,

$$N_n = \alpha_n \left(N_o - \sum_{i=1}^{n-1} N_i \right) \tag{3}$$

Substituting (3) and (1) into (2):

$$T_{exit} = \frac{\sum_{n=1}^{\infty} \alpha_n \left(N_o - \sum_{i=1}^{n-1} N_i\right) \left[T_w + (T_o - T_w)\ (1-a)^n\right]}{\sum_{n=1}^{\infty} \alpha_n \left(N_o - \sum_{i=1}^{n-1} N_i\right)} . \qquad (4)$$

Now using the results of Section II above for α_n, assuming a constant incident gas temperature, T_o, and a constant surface temperature, T_w, the exit temperature, T_{exit}, can be plotted as a function of the accommodation coefficient, a. Then, by experimentally measuring T_{exit}, the accommodation coefficient can be found graphically.

Unfortunately, the present study has shown that the distribution of exiting particles across the opening of the array is not uniform and, more importantly, the distribution changes with collision number. Thus, the measured value of T_{exit} will change as the probe is moved along the y-axis (the opening). However, by using the computer data, the various α_n can be determined for any given section of the opening and the above procedure used to yield the accommodation coefficient.

Plans are now in progress to conduct this experiment in the near future.

VI. CONCLUSIONS

This study has proven the great value of the Monte Carlo computer technique in cases where exact theoretical techniques cannot be found. This technique is accurate to better than 4% when 10,000 points are used.

The study has shown that very little focusing can be expected from simple geometric surfaces if the incident beam of molecules is parallel to the axis of symmetry. Some focusing is possible when the incident beam makes a large angle with the axis of symmetry.

An elementary application of the results of the study might be a determination of the accommodation coefficient by a simple experiment.

APPENDIX A

Distribution across Y-Axis of Particles Emitted after One Collision Incident Distribution Uniform over the Reflecting Surface ("Random Angle")

<u>List of Symbols (See Figure A)</u>

y	=	distance along the opening, measured from the center of the semicircle.
P_α	=	point on semicircle at angle α measured from negative y-axis.
α	=	angle measured from negative y-axis to normal at point P_α.
N_o	=	total number of incident particles.
N_α	=	number of particles per radian reflected at point P_α.
φ	=	angle between path of reflected particle and normal to surface at P_α.
$N_{\alpha,\varphi}$	=	number of particles per radian reflected at point P_α which are reflected at an angle φ.
θ	=	angle from normal at P_α to line from P_α to the point $(0,y)$.
$N_{y,\alpha}$	=	number of particles per radian from P_α that pass through a section from $(0,0)$ to $(0,y)$.
N_y	=	total number of particles through the section from $(0,0)$ to $(0,y)$ from <u>all</u> P_α.
γ	=	angle from positive y-axis to normal at $P_\alpha (\gamma = \pi - \alpha)$.
b	=	distance from P_α to point $(0,y)$.

Consider an incident distribution of N_o particles which is uniform over a semicircular surface as shown in Figure A. Assuming that these particles are diffusely reflected according to Lambert's Cosine Law, the distribution of reflected particles across the y-axis from $y = -1$ to $y = +1$ can be calculated as follows:

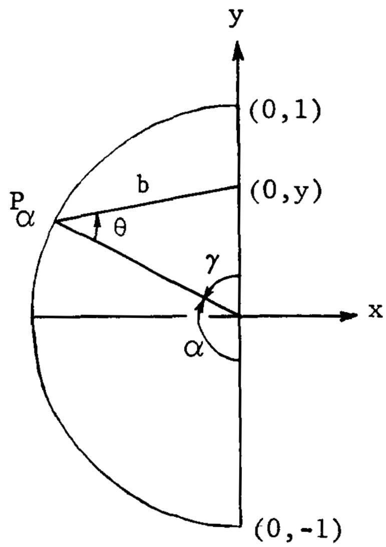

Figure A

Let N_α = total number of particles per radian reflected at point P_α.

Then $N_o = \int_0^\pi N_\alpha \, d\alpha = \pi N_\alpha$.

The number of particles per radian reflected at an angle φ from the normal to the surface at P_α is given by:

$$N_{\alpha,\varphi} = k \, N_\alpha \cos \varphi \qquad \text{(Lambert's Cosine Law)}$$

To find the constant k:

$$N_\alpha = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} N_{\alpha,\varphi} \, d\varphi = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} k \, N_\alpha \cos \varphi \, d\varphi = 2 \, k \, N_\alpha$$

hence $k = \frac{1}{2}$, so that $N_{\alpha,\varphi} = \frac{1}{2} N_\alpha \cos \varphi$.

Now the total number of particles per radian from P_α that passes through a section of the opening from 0 to y is given by:

$$N_{y,\alpha} = \int_0^\theta N_{\alpha,\varphi} \, d\varphi = \int_0^\theta \frac{1}{2} N_\alpha \cos \varphi \, d\varphi = \frac{1}{2} N_\alpha \sin \theta$$

and the total number of particles passing through this section from all P_α is:

$$N_y = \int_0^\pi N_{y,\alpha}\, d\alpha = \int_0^\pi \frac{1}{2}\, N_\alpha \sin\theta\, d\alpha \tag{1}$$

Thus to calculate N_y, $\sin\theta$ must be written in terms of α. Referring to Figure A:

$$\frac{\sin\theta}{y} = \frac{\sin\gamma}{b} \qquad \text{or} \qquad \sin\theta = \frac{y\sin\gamma}{b}$$

but

$$\sin\gamma = \sin(\pi-\alpha) = \sin\alpha$$

and

$$b = \left(1 + y^2 - 2y\cos\gamma\right)^{\frac{1}{2}}$$

Also

$$\cos\gamma = \cos(\pi-\alpha) = -\cos\alpha$$

so that

$$b = \left(1 + y^2 + 2y\cos\alpha\right)^{\frac{1}{2}}$$

Finally

$$\sin\theta = \frac{y\sin\alpha}{\left(1 + y^2 + 2y\cos\alpha\right)^{\frac{1}{2}}}$$

Inserting this expression into equation (1):

$$N_y = \int_0^\pi \frac{1}{2} N_\alpha \frac{y\sin\alpha\, d\alpha}{\left(1 + y^2 + 2y\cos\alpha\right)^{\frac{1}{2}}}$$

This integrates immediately to:

$$N_y = -\frac{N_\alpha}{2}\left(1 + y^2 + 2y\cos\alpha\right)^{\frac{1}{2}}\Big]_0^\pi$$

$$N_y = -\frac{N_\alpha}{2}\left[(1 + y^2 - 2y)^{\frac{1}{2}} - (1 + y^2 + 2y)^{\frac{1}{2}}\right]$$

$$N_y = -\frac{N_\alpha}{2}\left[\pm(1 - y) - \pm(1 + y)\right]$$

Now $(1 + y^2 + 2y \cos \alpha)^{\frac{1}{2}} \equiv b$, which is a positive number (a distance).

Hence, $$N_y = -\frac{N_\alpha}{2}\left[(1 - y) - (1 + y)\right] = N_\alpha y.$$

Thus the total number of particles that pass through any section from 0 to y is simply proportional to y; i.e., the distribution across the y-axis is uniform.

APPENDIX B

Distributions of Particles over a Semicircular Surface as a Function of Collision Number

In general, the number of particles per radian making the nth collision with a semi-circular surface, at an angle α measured from the y-axis, is given by:

$$N_\alpha^{(n)} = - \frac{d}{d\beta} N_\beta^{(n)} \Big]_{\beta=\alpha}$$

where $N_\beta^{(n)} = 2 F^{(n)} (\beta,\beta) - F^{(n)} (0,\beta) - F^{(n)} (\pi,\beta) + N_\beta^{(n-1)}$

and $F^{(n)} (\alpha,\beta) = \frac{1}{2} \int N_\alpha^{(n-1)} \left(\cos \frac{\alpha}{2} \cos \frac{\beta}{2} + \sin \frac{\alpha}{2} \sin \frac{\beta}{2}\right) d\alpha$

can be calculated from the previous expression for N_α, $N_\alpha^{(n-1)}$.

Case I. Incident particles uniform over the semicircular surface (random angle incidence). N_o = total number of incident particles.

$$N_\alpha^{(1)} = N_o/\pi$$

$$N_\alpha^{(2)} = (N_o/2\pi) \left(2-\sin \frac{\alpha}{2} - \cos \frac{\alpha}{2}\right)$$

$$N_\alpha^{(3)} = (N_o/8\pi) \left[8 + \left(\frac{\pi}{2} - 5 - \alpha\right) \sin \frac{\alpha}{2} - \left(\frac{\pi}{2} + 5 - \alpha\right) \cos \frac{\alpha}{2}\right]$$

$$N_\alpha^{(4)} = (N_o/32\pi)\left[32 - \left(22 + \frac{5\pi}{2} - 6\alpha + \frac{\pi\alpha}{2} - \frac{\alpha^2}{2}\right) \cos \frac{\alpha}{2} - \left(22 - \frac{7\pi}{2} + 6\alpha + \frac{\pi\alpha}{2} - \frac{\alpha^2}{2}\right) \sin \frac{\alpha}{2}\right]$$

$$N_\alpha^{(5)} = (N_o/128\pi)\left[128 - \left(93 + \frac{21\pi}{2} - \frac{\pi^2}{4} + \frac{\pi^3}{24} - 29\alpha + 4\pi\alpha - \frac{7\alpha^2}{2} - \frac{\pi\alpha^2}{4} + \frac{\alpha^3}{6}\right)\right.$$

$$\left.\cos\frac{\alpha}{2} + \left(-93 + \frac{37\pi}{2} - \frac{\pi^2}{4} + \frac{\pi^3}{24} - 29\alpha - 3\pi\alpha + \frac{7\alpha^2}{2} - \frac{\pi\alpha^2}{4} + \frac{\alpha^3}{6}\right)\sin\frac{\alpha}{2}\right]$$

Case II. Incident particles uniform over the opening of the array (random y incidence).

$$N_\alpha^{(1)} = (N_o/2)\sin\alpha$$

$$N_\alpha^{(2)} = (N_o/6)\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2} - \sin\alpha\right)$$

$$N_\alpha^{(3)} = (N_o/24)\left[\left(-\frac{1}{3} - \frac{\pi}{2} + \alpha\right)\sin\frac{\alpha}{2} + \left(-\frac{1}{3} + \frac{\pi}{2} - \alpha\right)\cos\frac{\alpha}{2} + \frac{4}{3}\sin\alpha\right]$$

$$N_\alpha^{(4)} = (N_o/96)\left[\left(\frac{22}{9} - \frac{\pi}{6} - \frac{2\alpha}{3} + \frac{\pi\alpha}{2} - \frac{\alpha^2}{2}\right)\cos\frac{\alpha}{2} + \left(\frac{22}{9} - \frac{5\pi}{6} + \frac{2\alpha}{3} + \frac{\pi\alpha}{2} - \frac{\alpha^2}{2}\right)\cdot\right.$$

$$\left.\cdot\sin\frac{\alpha}{2} - \frac{16}{9}\sin\alpha\right]$$

$$N_\alpha^{(5)} = (N_o/48)\left[\left(\frac{47}{216} + \frac{13\pi}{144} - \frac{\pi^2}{32} + \frac{\pi^3}{192} - \frac{37\alpha}{72} + \frac{\pi\alpha}{6} - \frac{5\alpha^2}{48} - \frac{\pi\alpha^2}{32} + \frac{\alpha^3}{48}\right)\right.$$

$$\cos\frac{\alpha}{2} + \left(\frac{47}{216} - \frac{61\pi}{144} + \frac{\pi^2}{32} - \frac{\pi^3}{192} + \frac{37\alpha}{72} + \frac{\pi\alpha}{24} - \frac{5\alpha^2}{48} + \frac{\pi\alpha^2}{32} - \frac{\alpha^3}{48}\right)$$

$$\left.\sin\frac{\alpha}{2} + \frac{8}{27}\sin\alpha\right].$$

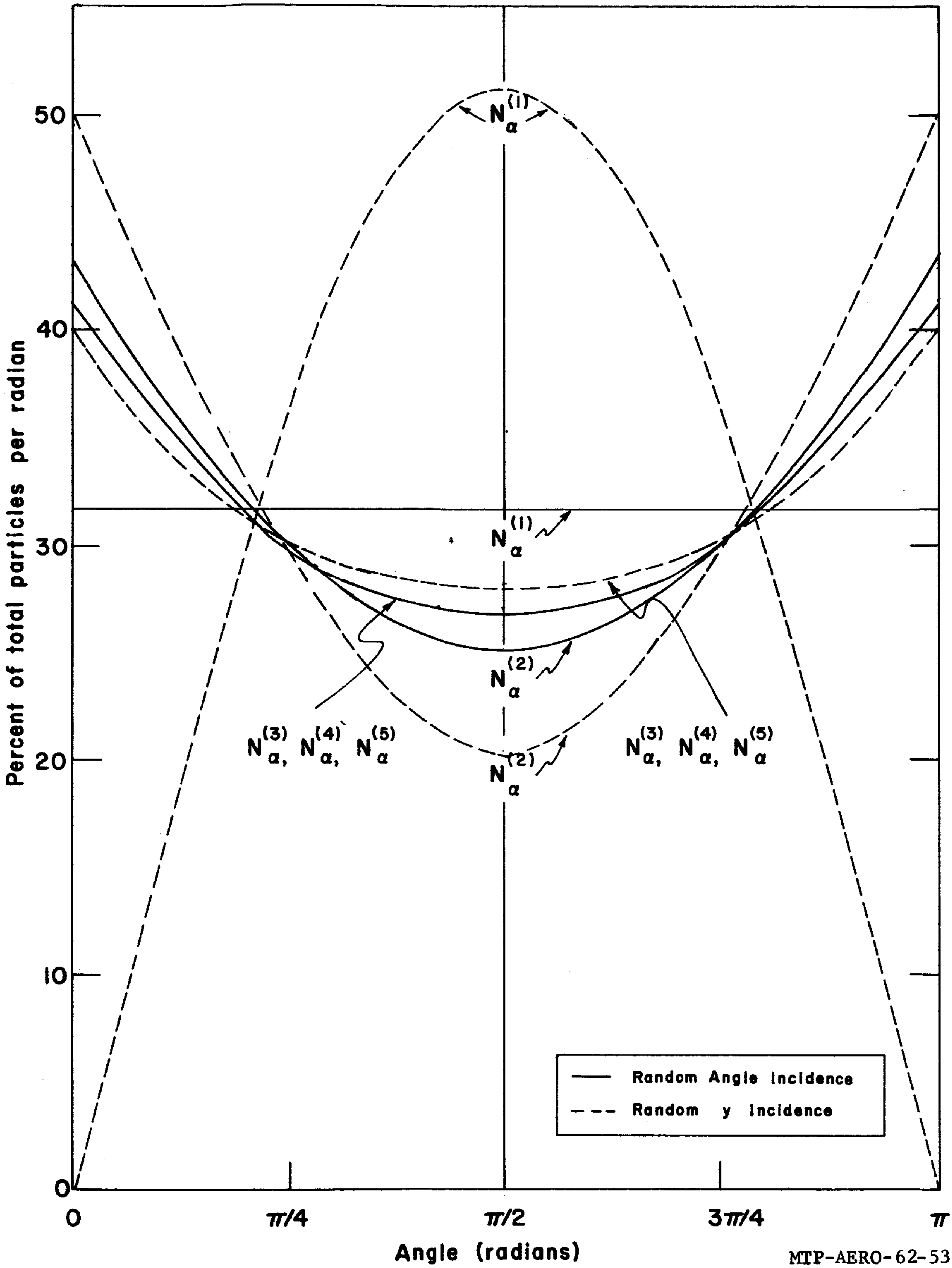

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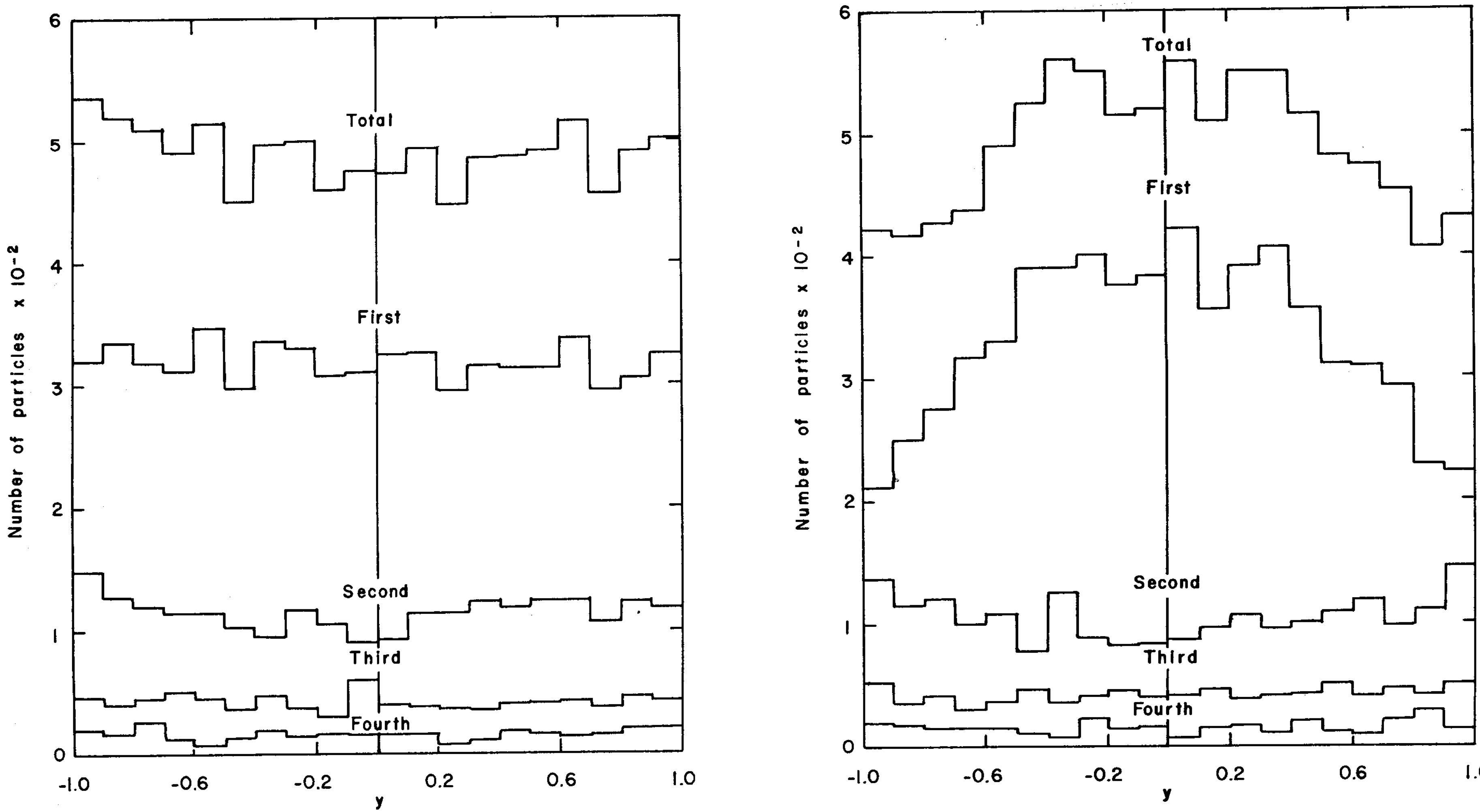
Figure I. Distribution of particles over a semi-circular surface at first five collisions (cosine law of reflection)



a. Random angle incident beam

b. Random y incident beam

Figure 2. Distribution of particles across opening of CIRCLES as a function of collision number (10,000 incident particles)

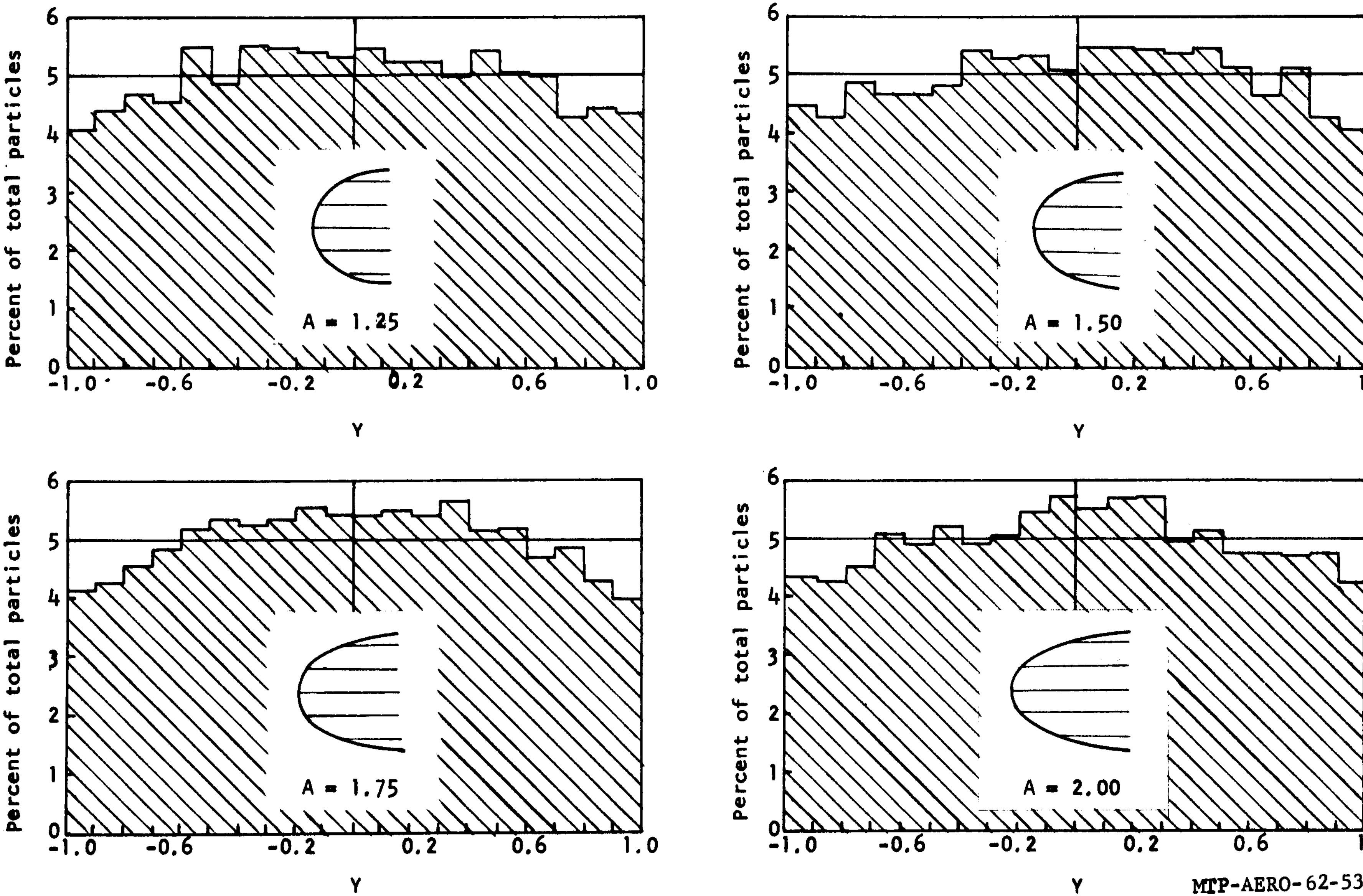


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Figure 3. DISTRIBUTION OF PARTICLES ACROSS OPENINGS OF ELLIPSES $Y^2 = 1 - \frac{X^2}{A^2}$

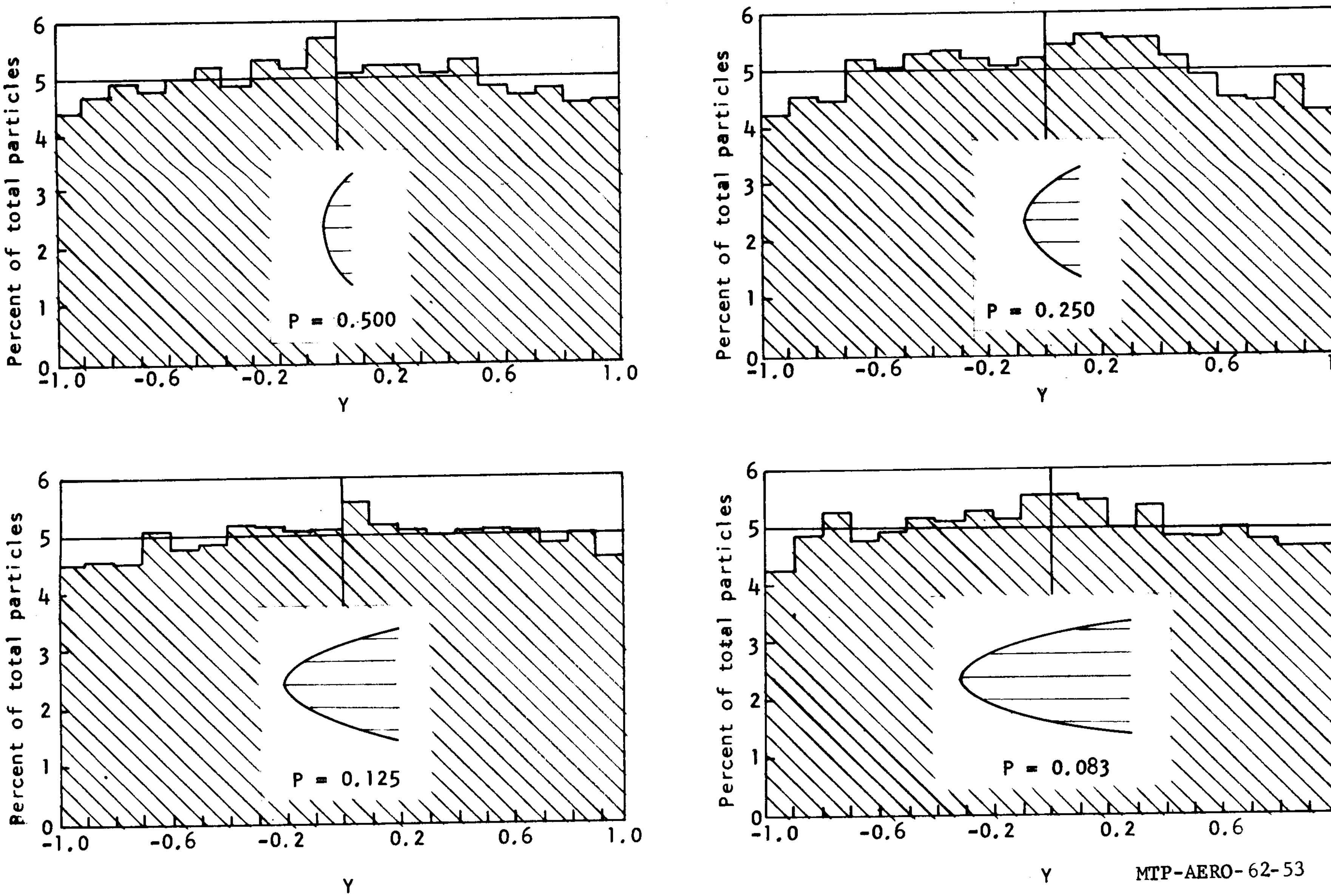

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Figure 4. DISTRIBUTION OF PARTICLES ACROSS OPENINGS OF PARABOLAS $Y^2 = 4PX + 1$

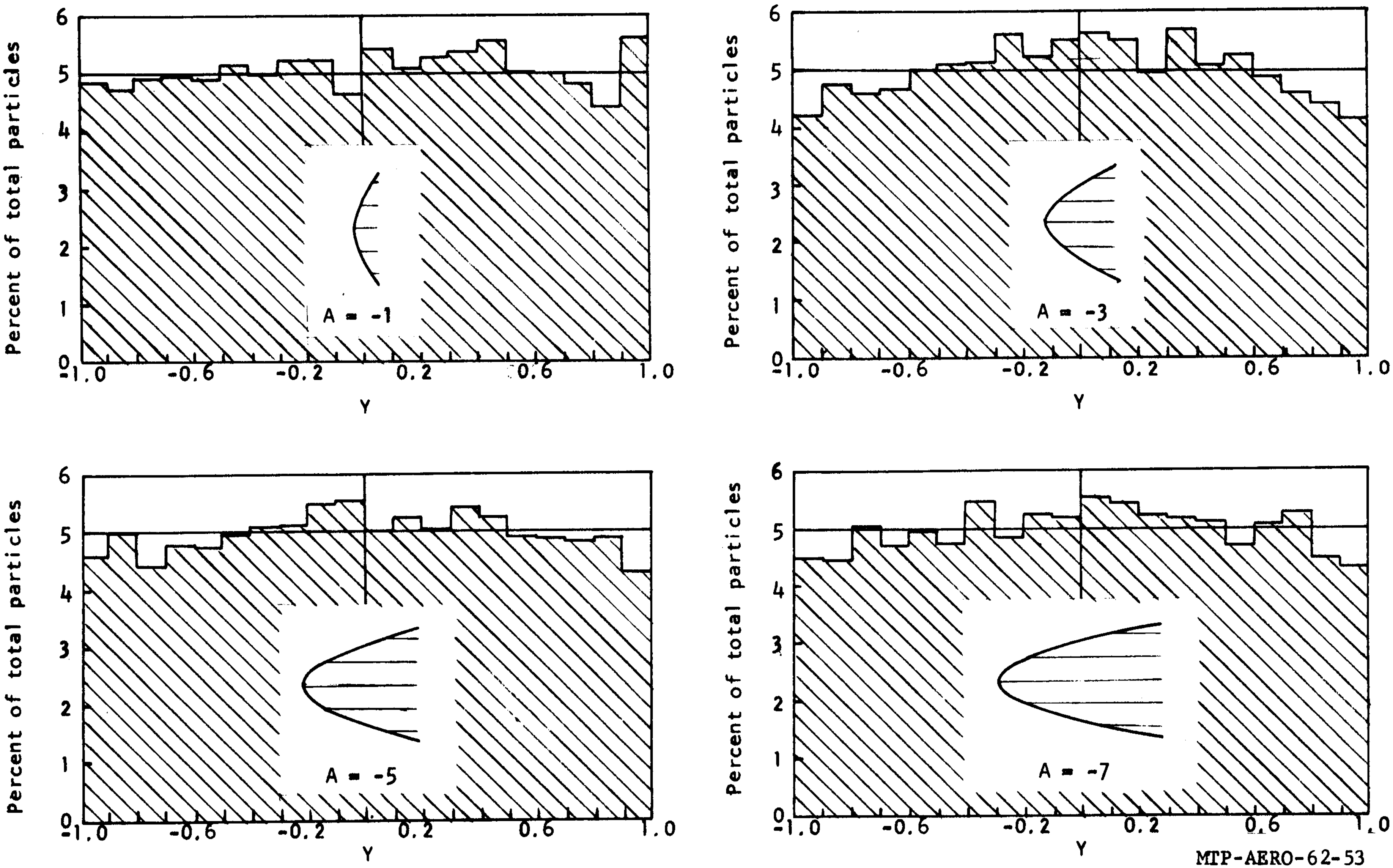

Figure 5. DISTRIBUTION OF PARTICLES ACROSS OPENINGS OF HYPERBOLAS $Y^2 = \frac{(X - A\sqrt{2})^2}{A^2} - 1$

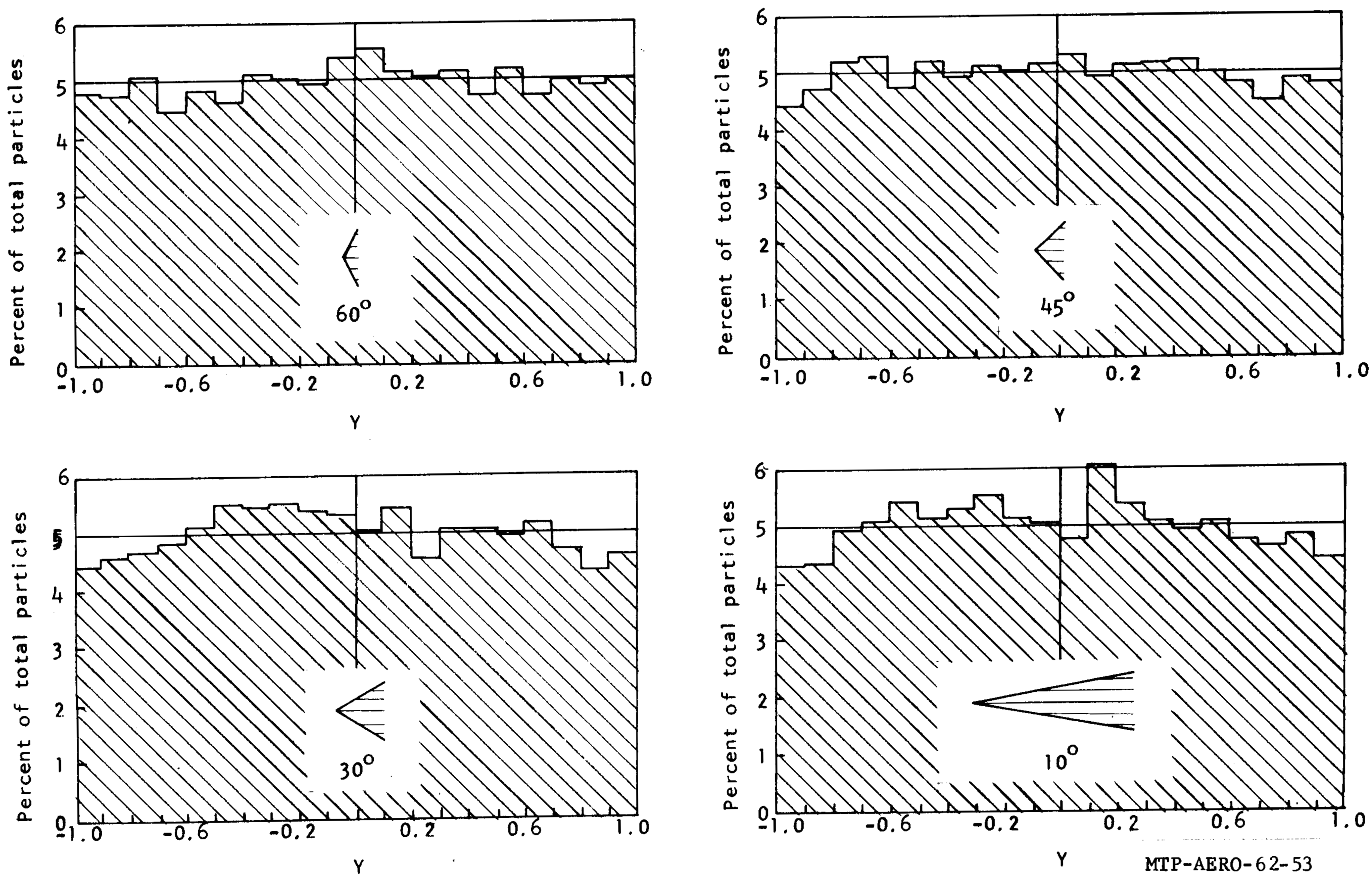

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Figure 6. DISTRIBUTION OF PARTICLES ACROSS OPENINGS OF TRIANGLES

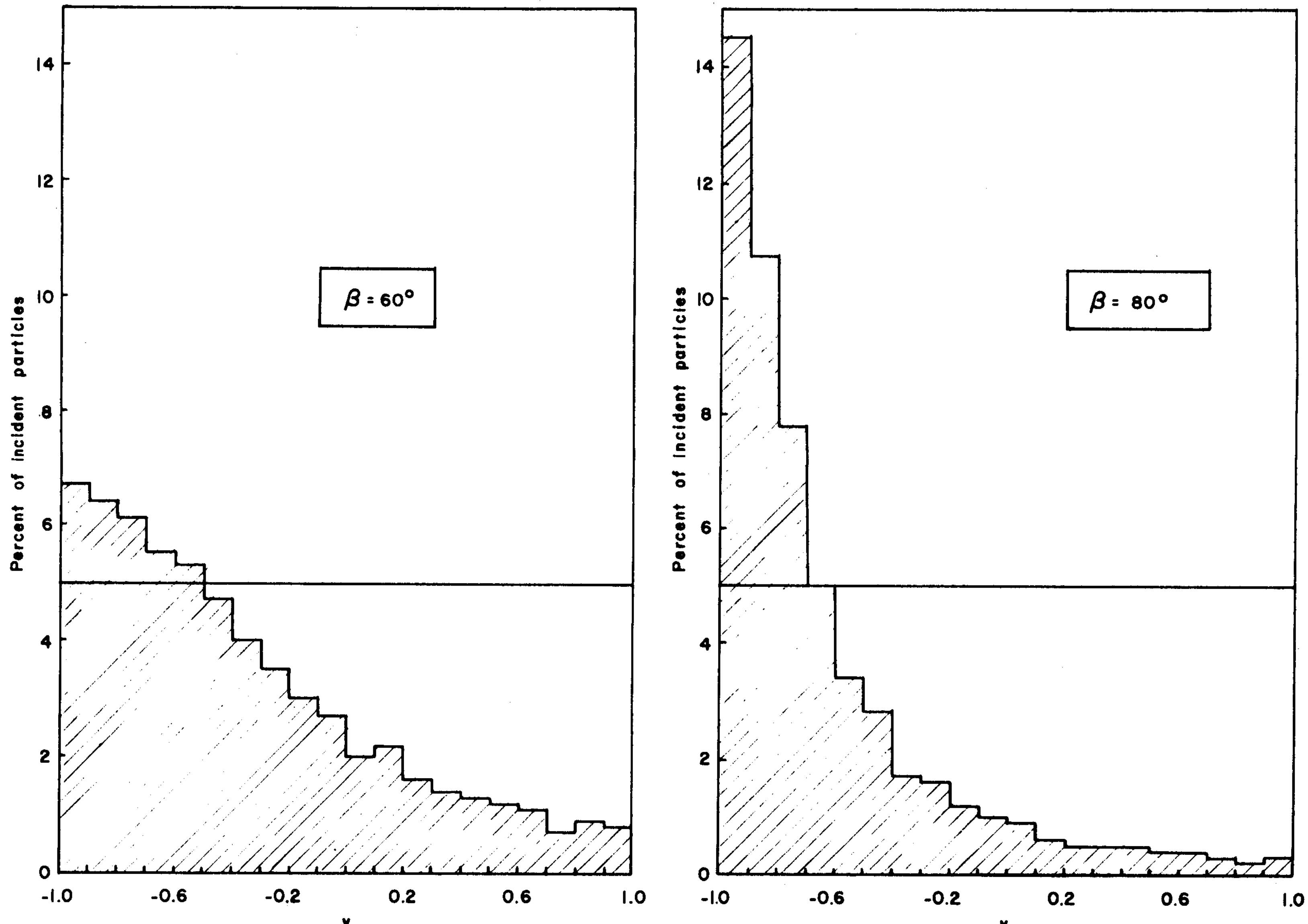

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Figure 7. Distribution of exiting particles across opening of CIRCLE, with incident beam at angle β-- one collision only

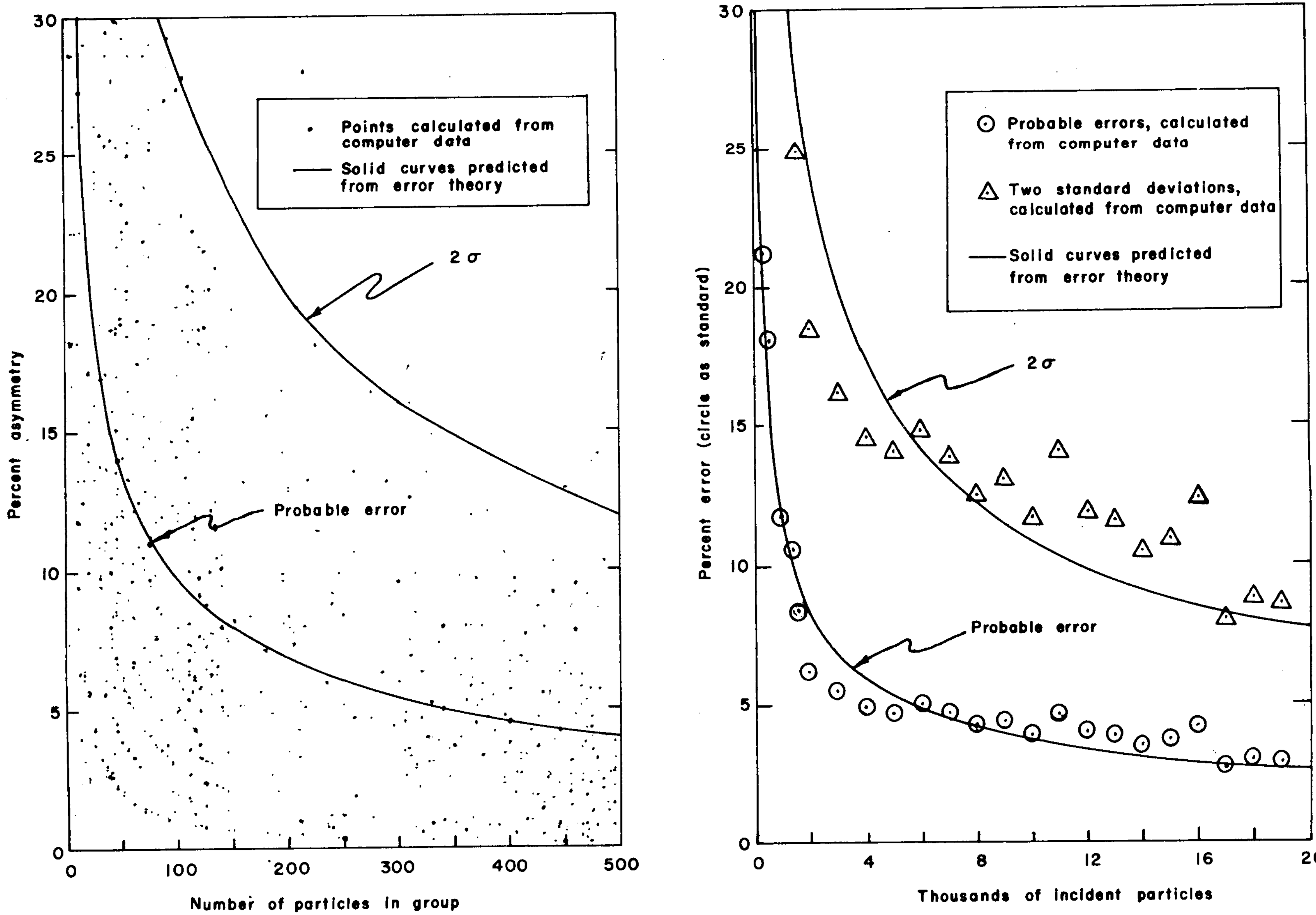

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Figure 8. Error curves

APPROVAL

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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