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GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

Motion of the Sub-Satellite Point For 24-Hour Orbits

By

Walter H. Stafford,
Carmen R. Catalfamo,
Sam H. Harlin

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FLIGHT OPERATIONS SECTION
ADVANCED FLIGHT SYSTEMS BRANCH
PROPULSION AND VEHICLE ENGINEERING DIVISION

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ABSTRACT

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The effects of small changes in eccentricity, inclination and argument of perigee on the motion of the sub-satellite point for 24-hour orbits were investigated. The eccentricity was varied from 0.0 to 0.1, inclination from 20° to 90°, and argument of perigee from 0° to 90°. Two additional values for the argument of perigees, 135° and 180°, were used in one case for comparison.

The results show graphically the delta longitude and latitude of the sub-satellite point.

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LIST OF SYMBOLS

h	Altitude, km
ϕ	Latitude, deg
λ	Longitude, deg
e	Eccentricity
ω	Argument of perigee, deg
P	Sidereal period, min
r_a	Apogee radius, km
r_p	Perigee radius, km
j	Inclination, deg
Inclination	Angle measured counterclockwise from the equatorial plane to the satellite plane
Sidereal Day	23 hours 56 minutes 04.09 sec of mean solar time
Argument of Perigee	Angle measured in the direction of motion from the ascending node to the perigee point
Ascending Node	The point on the earth's equator where the satellite moves from the southern hemisphere to the northern hemisphere
Sub-Satellite Point	The point of intersection with the earth's surface of a plumb line from the satellite to the center of the earth
Sub-Satellite Path	The path made by the sub-satellite point when the satellite has made one complete revolution
Epoch	An instant of time selected as a point of reference, i.e., the instant for which the orbital parameters were determined

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SUMMARY

The effects of small changes in eccentricity, inclination and argument of perigee on the motion of the sub-satellite point for 24-hour orbits were investigated. The eccentricity was varied from 0.0 to 0.1, inclination from 20° to 90°, and argument of perigee from 0° to 90°. Two additional values for the argument of perigees, 135° and 180°, were used in one case for comparison.

INTRODUCTION

The motion of the sub-satellite point for an orbit shows the true relation between the satellite and the earth's surface. If the sub-satellite paths are available for each of a variety of orbits, they will facilitate the mission planner in selecting the most advantageous orbit or orbits, and their spacing with respect to each other.

The beginning of high-altitude active communications satellite flight tests was the launching of SYNCOM I on February 14, 1963. One of the major decisions to be made in connection with any communications satellite program is the orbit altitude and spacing.

The purpose of this report is to present the results of a study concerning small changes in eccentricity, inclination and argument of perigee for 24-hour orbits and their effect on the sub-satellite paths.

DISCUSSION

A satellite in the so-called "24-hour" circular orbit has a radius of 42164.4 km and a period of one Sidereal Day, which is 23 hours 56 minutes 04.09 seconds of mean solar time. One of the advantages of a 24-hour circular orbit is that its relation to the earth's surface is repeated every day. In other words, it traces out a sub-satellite path during the the first day and, neglecting perturbations, retraces this path on each succeeding day.

When the circular orbit is inclined, the satellite travels such that it traces out a sub-satellite path on the earth's surface resembling a figure "8" as seen in Figure 1. The "mid-point" of the "8" is on the equator with the top symmetrical with the bottom. A vertical line of symmetry can also be drawn from the top to the bottom of the "8" and passing through the mid-point. This line and the symmetry associated with it changes somewhat when the orbit becomes eccentric. Figures 2a through 2e illustrate the effect of changes in the argument of perigee for various inclined orbits with an eccentricity of 0.02. In Figure 2a the perigee point is located at the ascending node. The mid-point of the "8" is located at the equator and the "8" is tilted toward the east. Figure 2b has an argument of perigee of 30 degrees and the "8" is tilted somewhat less than in Figure 2a. However, the mid-point has moved downward.

Figures 3a through 3e illustrate the effect of changes in the argument of perigee for various inclined orbits with an eccentricity of 0.04. These figures show the same trend as Figures 2a through 2e except the tilt is a little greater and the mid-point is farther down.

Figures 4a through 4e were drawn for orbits with an eccentricity of 0.06. These figures continue the trend observed in Figures 2 and 3.

Figures 5a through 5g were drawn for orbits with an eccentricity of 0.1 and arguments of perigee of 0°, 30°, 45°, 60°, 90°, 135° and 180°, respectively. Figure 5a, with an argument of perigee of 0°, is a mirror image of Figure 5g which has an argument of perigee of 180° Also, Figure 5c, with an argument of perigee of 45°, is a mirror image of Figure 5f which has an argument of perigee of 135°.

Figures 6a through 6e were drawn for 30° inclined orbits with arguments of perigee of 0°, 30°, 45°, 60° and 90°, respectively. Each has eccentricity as a parameter.

Figures 7a through 7d were drawn for 30° inclined orbits with eccentricities of 0.02, 0.04, 0.06 and 0.10, respectively. Each has arguments of perigee as a parameter.

Figure 8 gives the apogee and perigee radius versus eccentricity for orbits with a constant period of one Sidereal Day.

Figure 9 shows the latitude of the sub-perigee point versus the argument of perigee with orbit inclination as a parameter.

Figure 10 shows the orbital path of SYNCOM II over the earth's surface on July 31, 1963. During this time the orbital period was greater than one Sidereal Day; hence, the figure "8" was drifting westward.

Figure 11 shows the orbital path desired for SYNCOM II. This can be achieved by initiating an impulse that will reduce the orbital period to exactly one Sidereal Day at the time the orbit is in the desired position.

It should be pointed out that 0° longitude on all graphs refers to the location on the earth's surface of the ascending node, and not to any particular value of the earth's longitude, as measured from the Greenwich Meridian. For proper utilization, one determines the longitude of the ascending node for the orbit of interest, places the 0° longitude point on the pertinent graph at this location, then the orbit is in its true geographical relationship with the earth's surface. Therefore, it is seen that the longitude scale on the graphs are, in reality, the difference in longitude measured from the ascending node to the sub-satellite path at various latitudes.

In this report, all sub-satellite paths are referred to as figure "8" in order to conform with accepted notation; but, it is obvious from the data presented that this is a misnomer when applied to 24-hour orbits of large eccentricities.

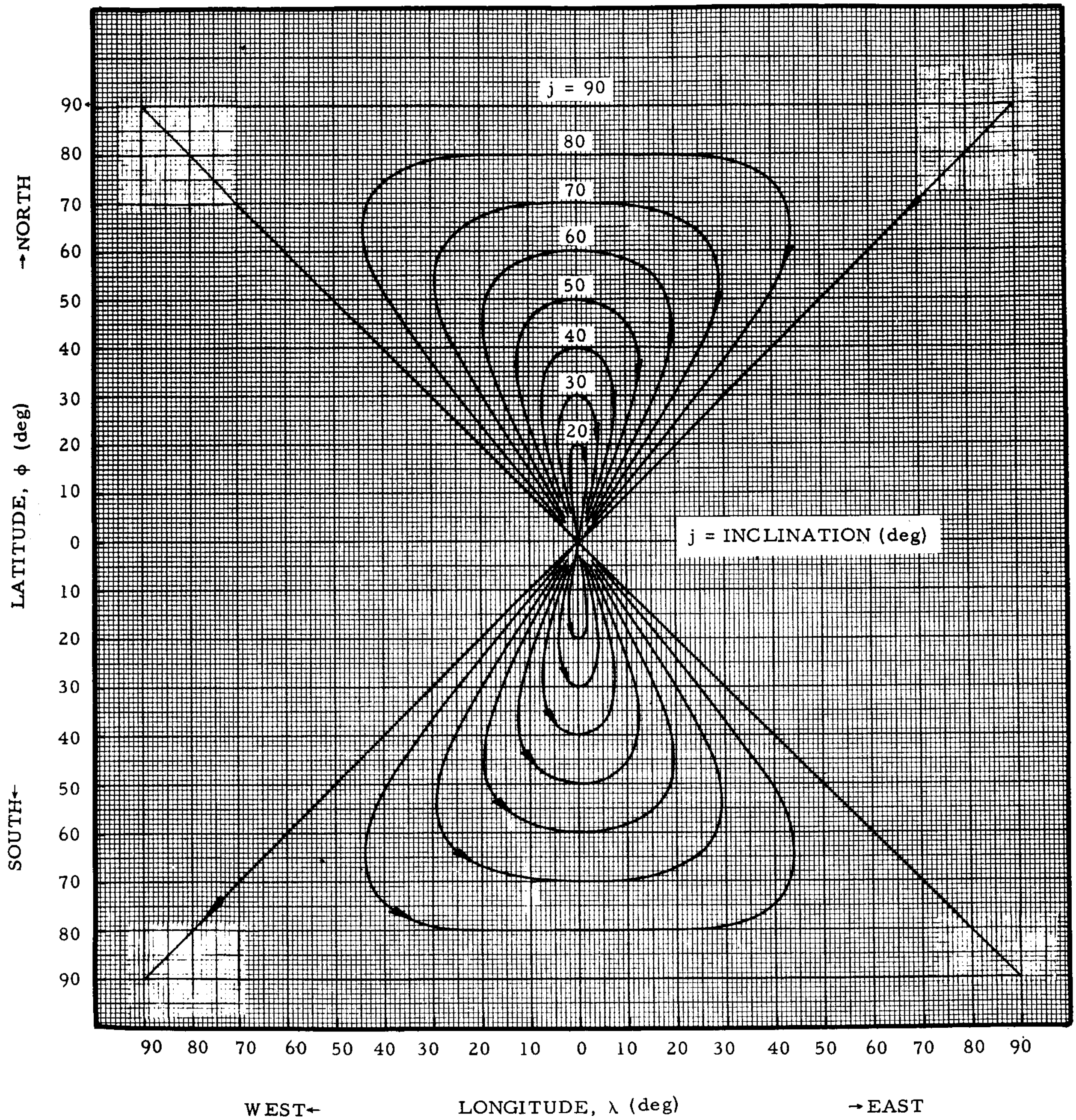

FIGURE 1. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.00.

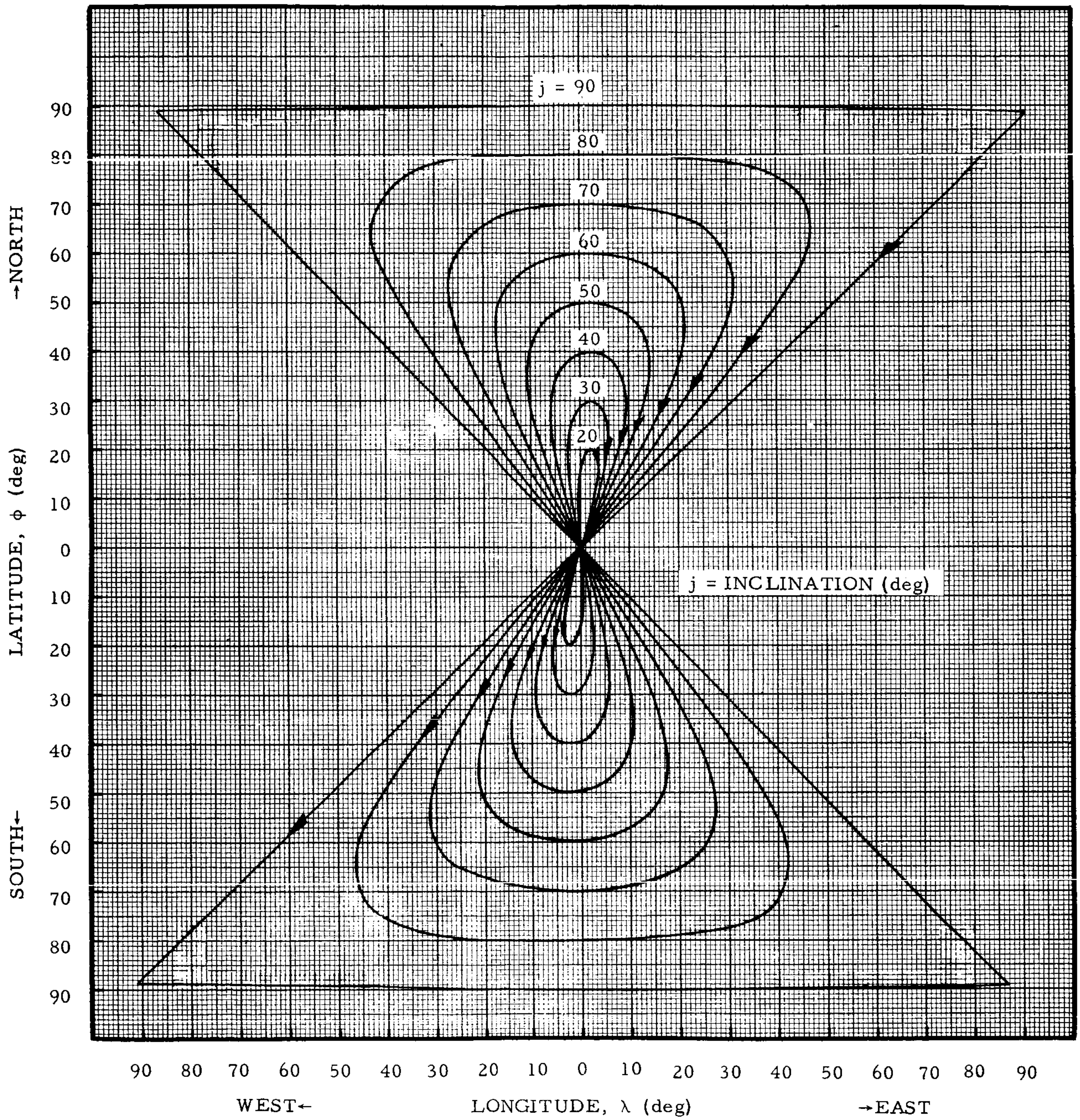

FIGURE 2a. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.02. ARGUMENT OF PERIGEE—0 DEGREES.

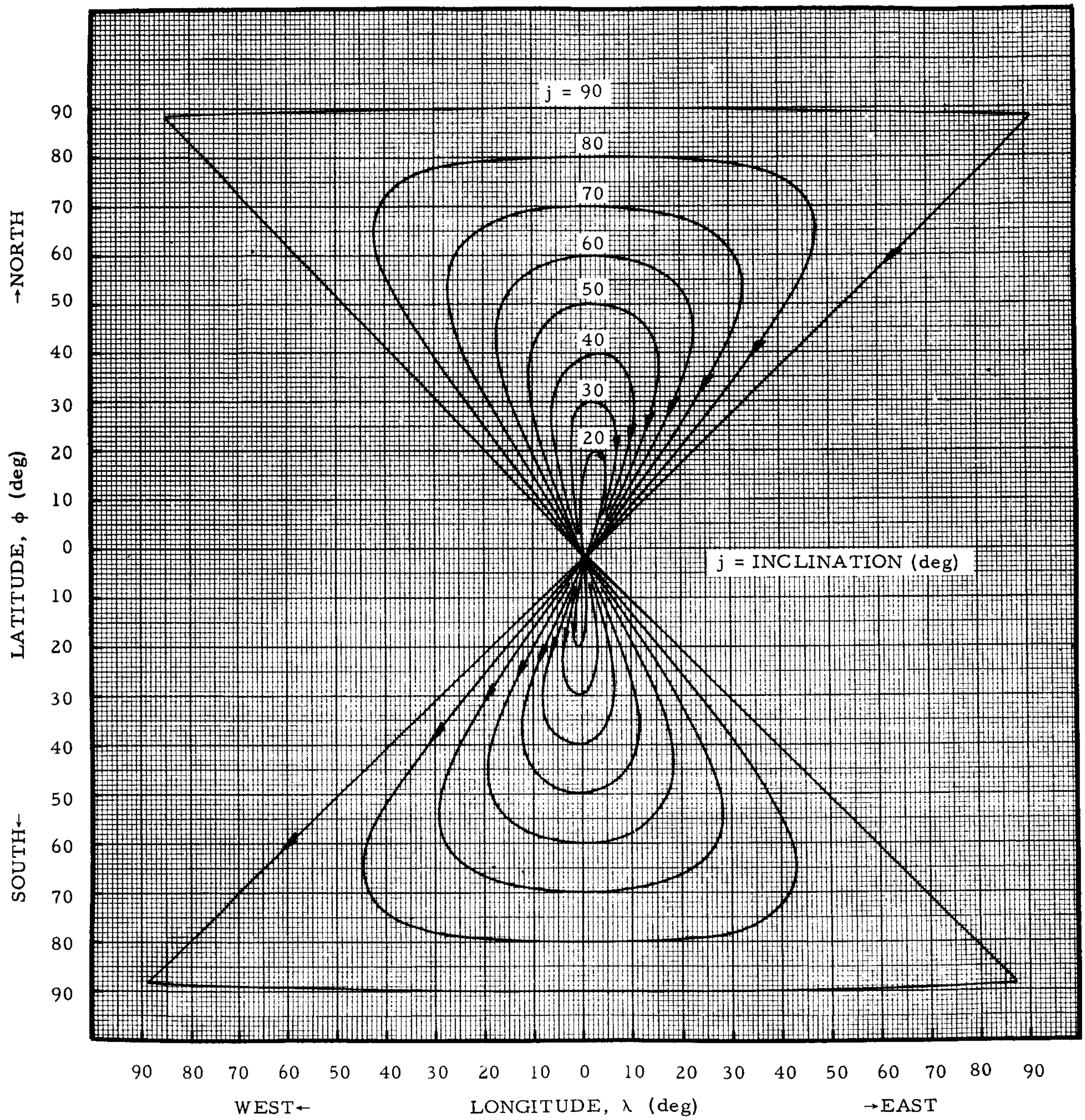

FIGURE 2b. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.02. ARGUMENT OF PERIGEE—30 DEGREES.

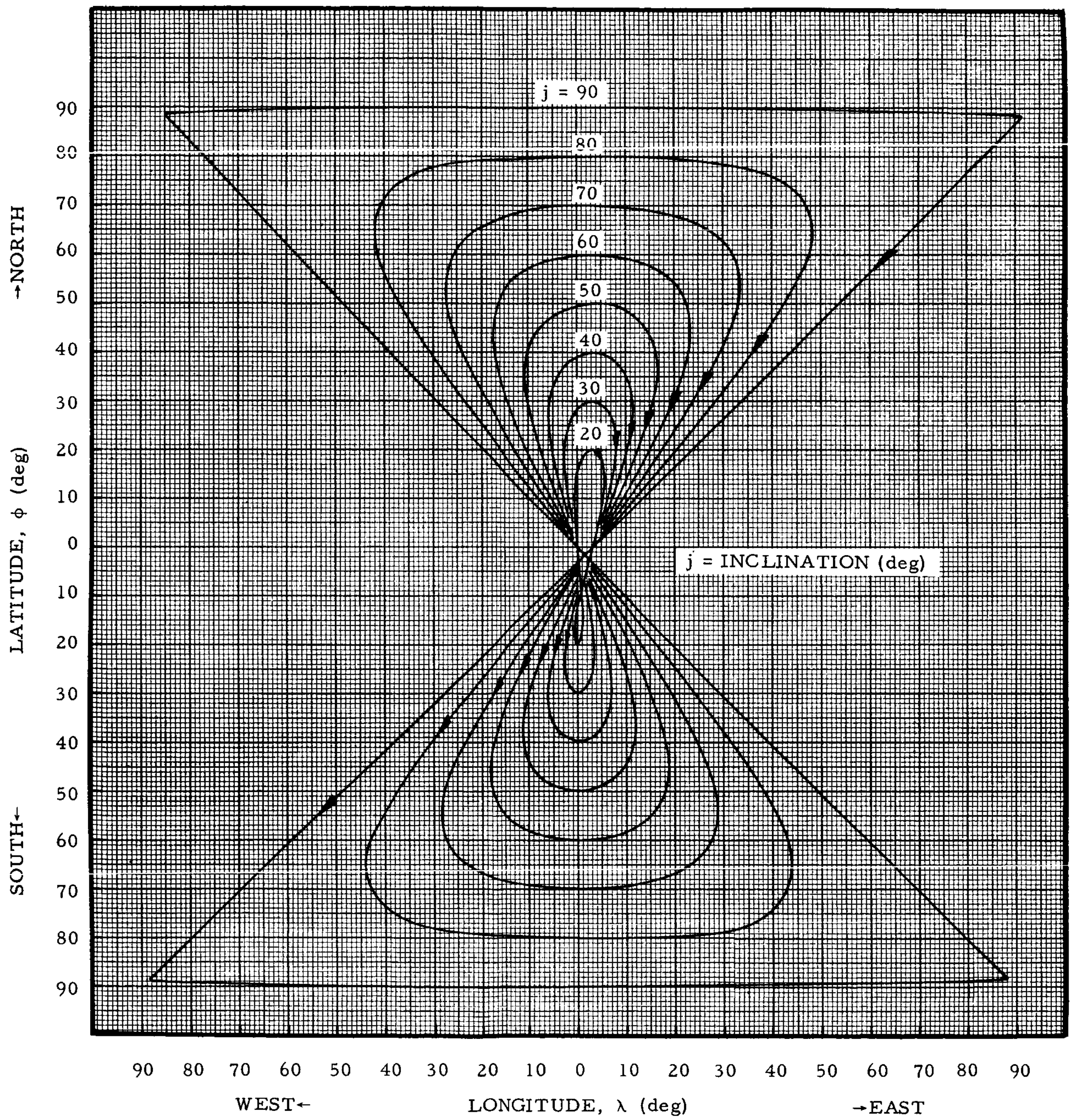

FIGURE 2c. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.02. ARGUMENT OF PERIGEE—45 DEGREES.

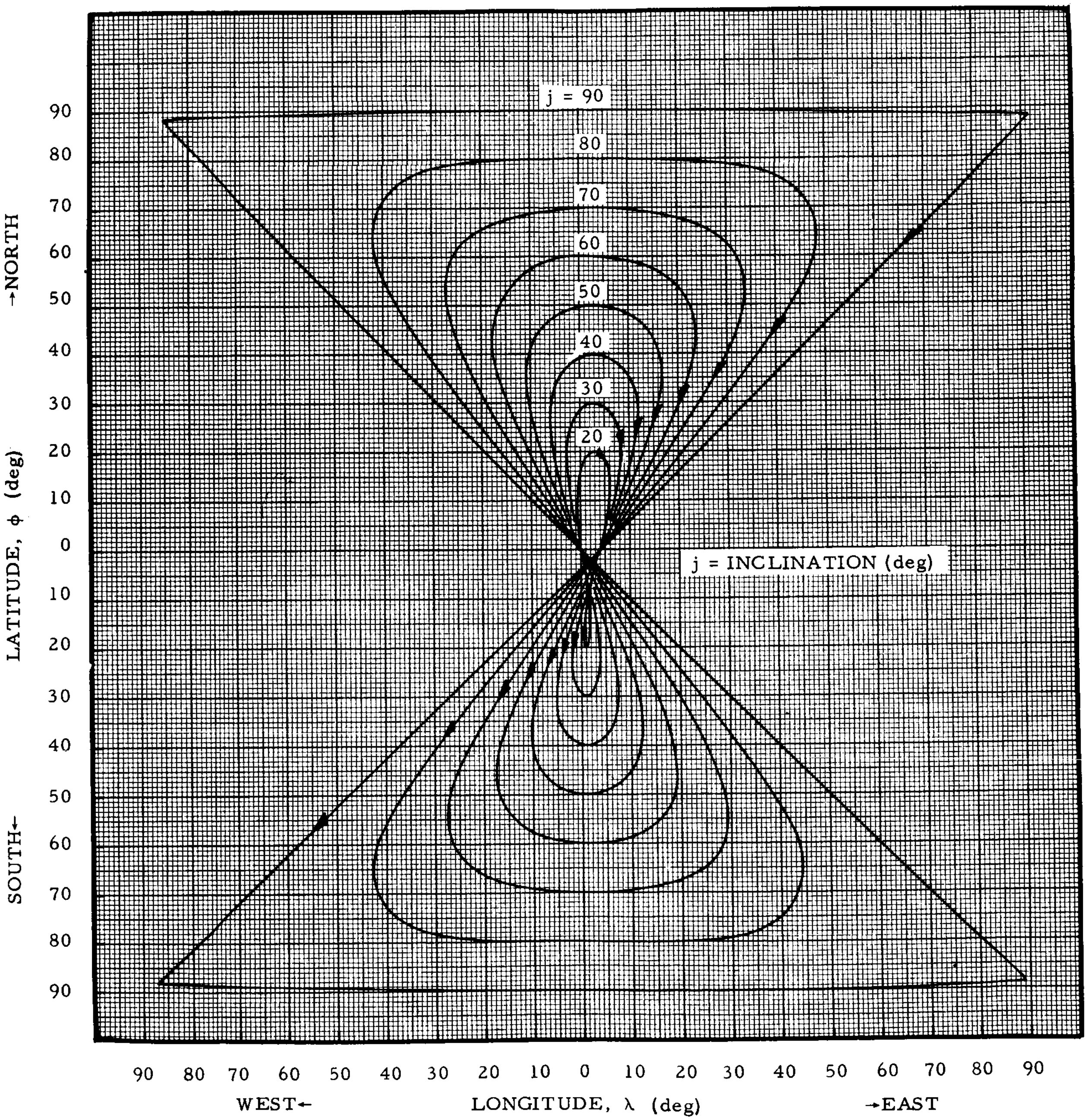

FIGURE 2d. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.02. ARGUMENT OF PERIGEE—60 DEGREES.

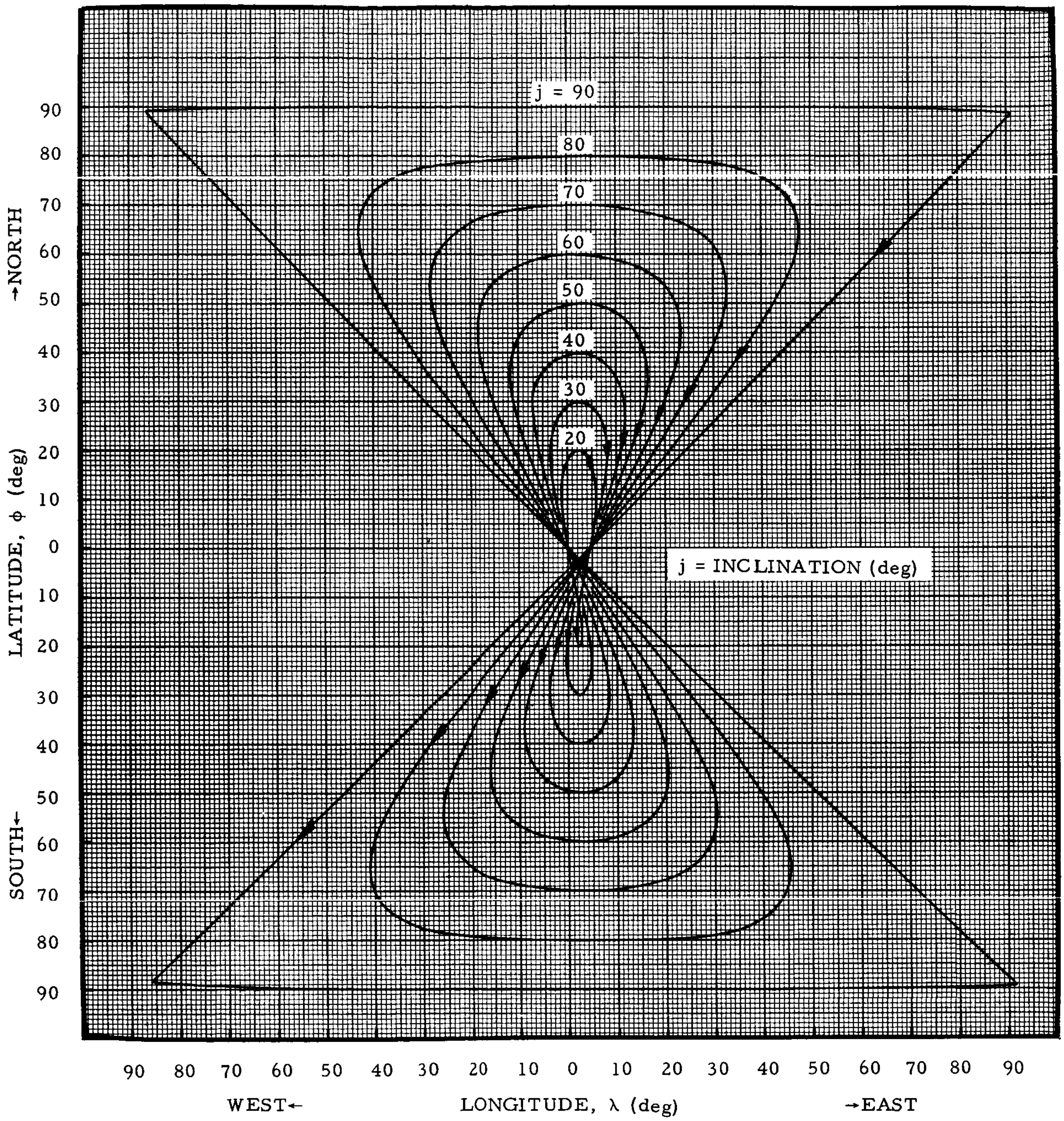

FIGURE 2e. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.02. ARGUMENT OF PERIGEE—90 DEGREES.

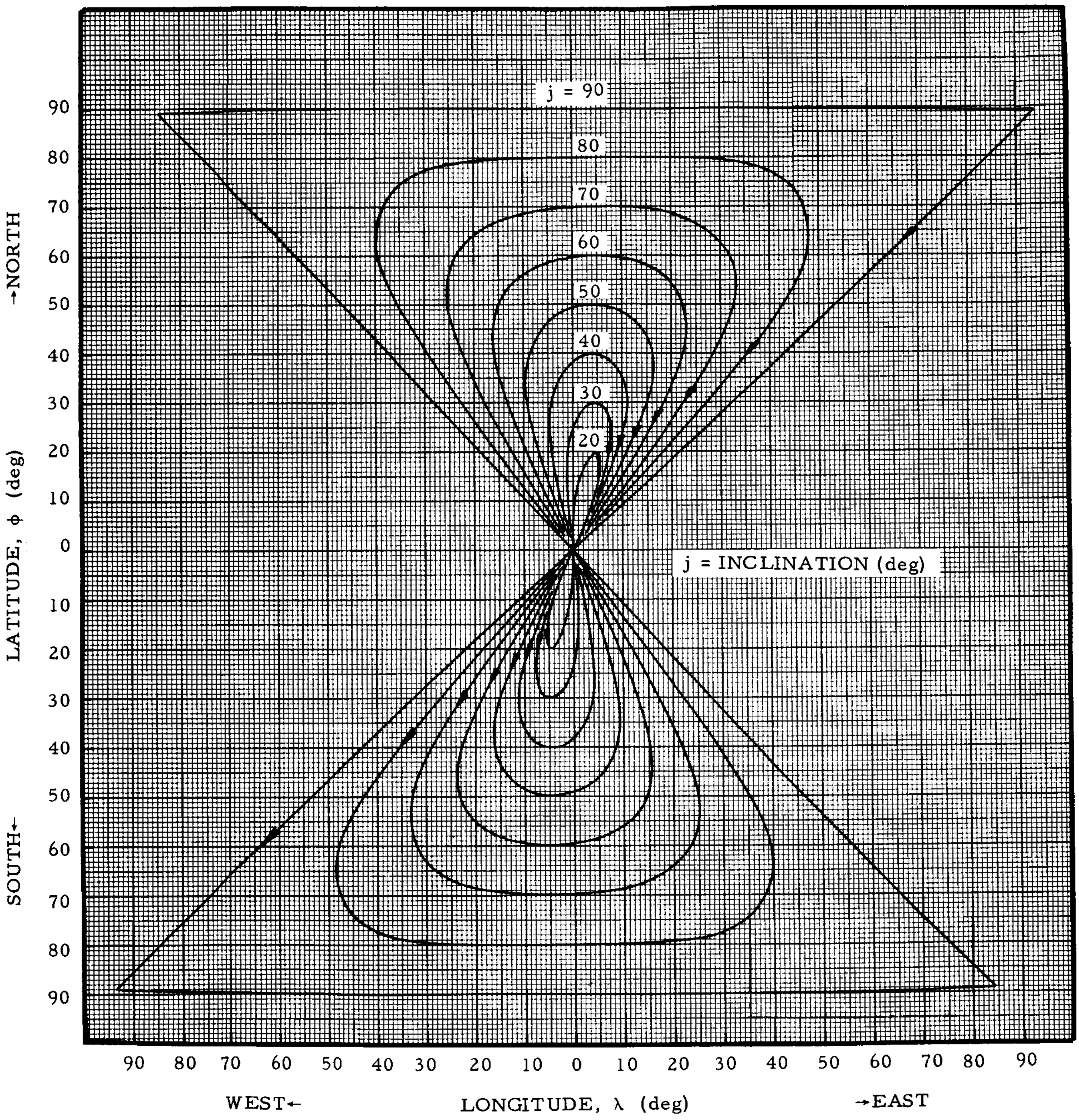

FIGURE 3a. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.04. ARGUMENT OF PERIGEE—0 DEGREES.

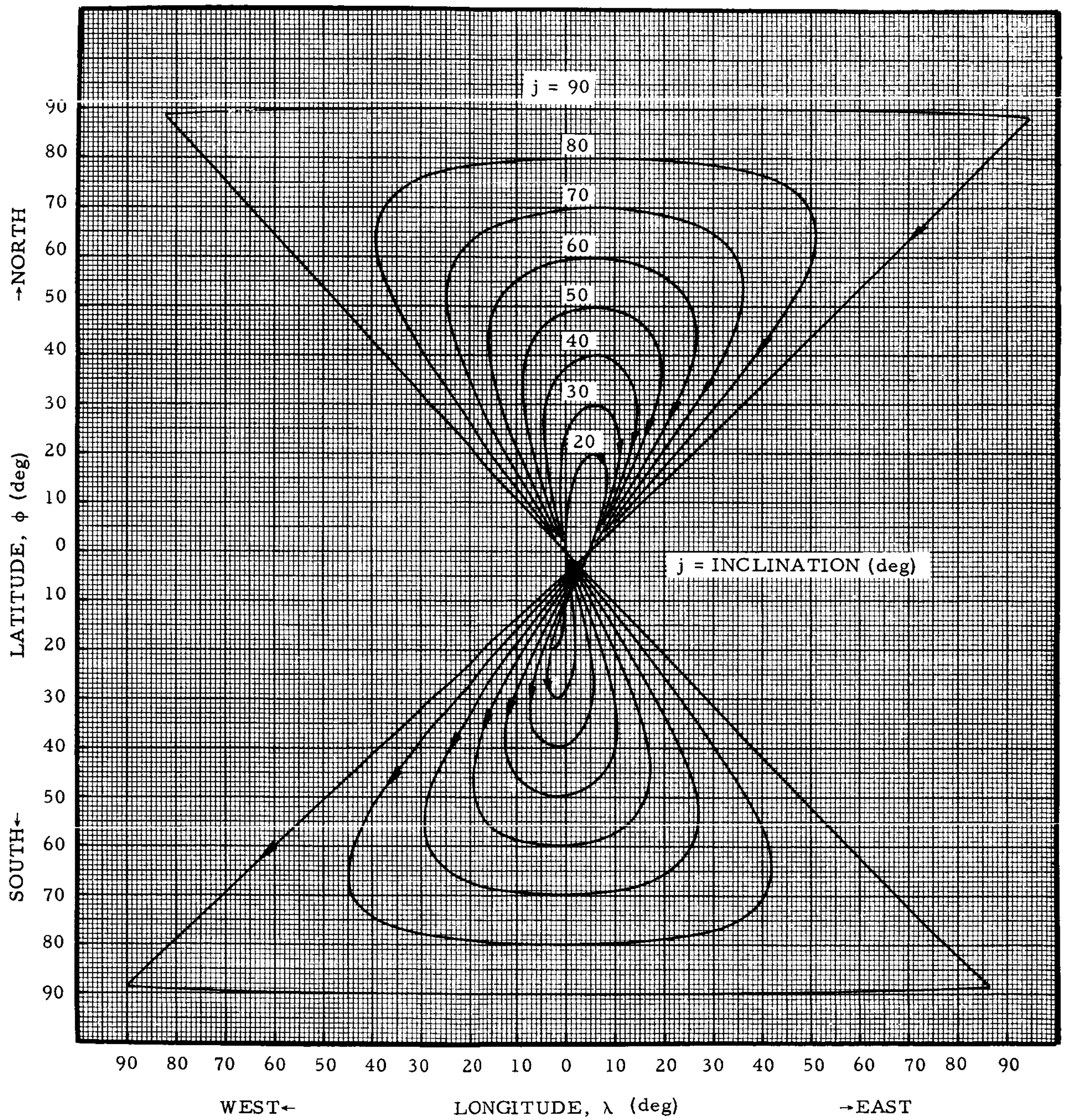

FIGURE 3b. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.04. ARGUMENT OF PERIGEE—30 DEGREES.

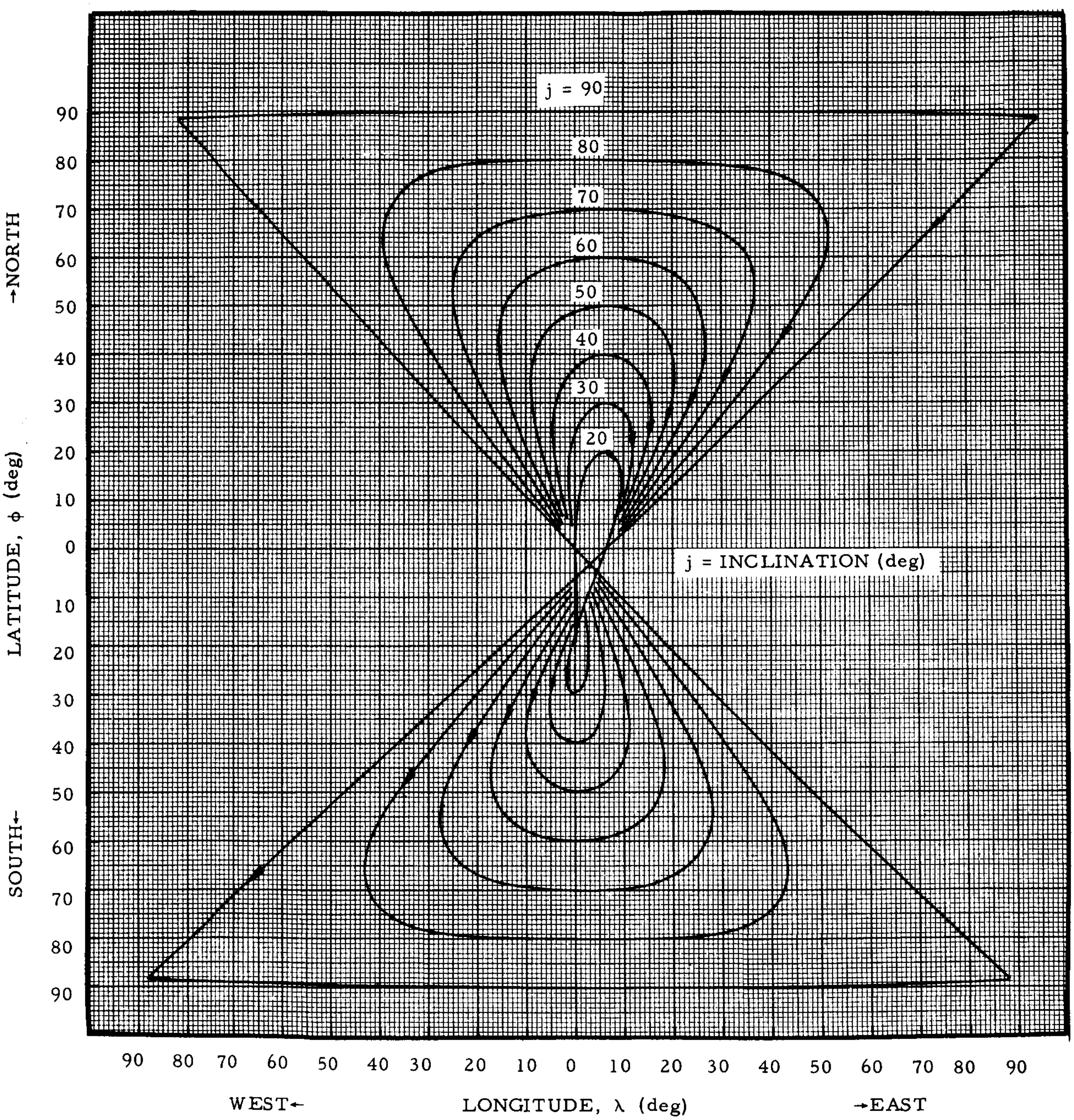

FIGURE 3c. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.04. ARGUMENT OF PERIGEE—45 DEGREES.

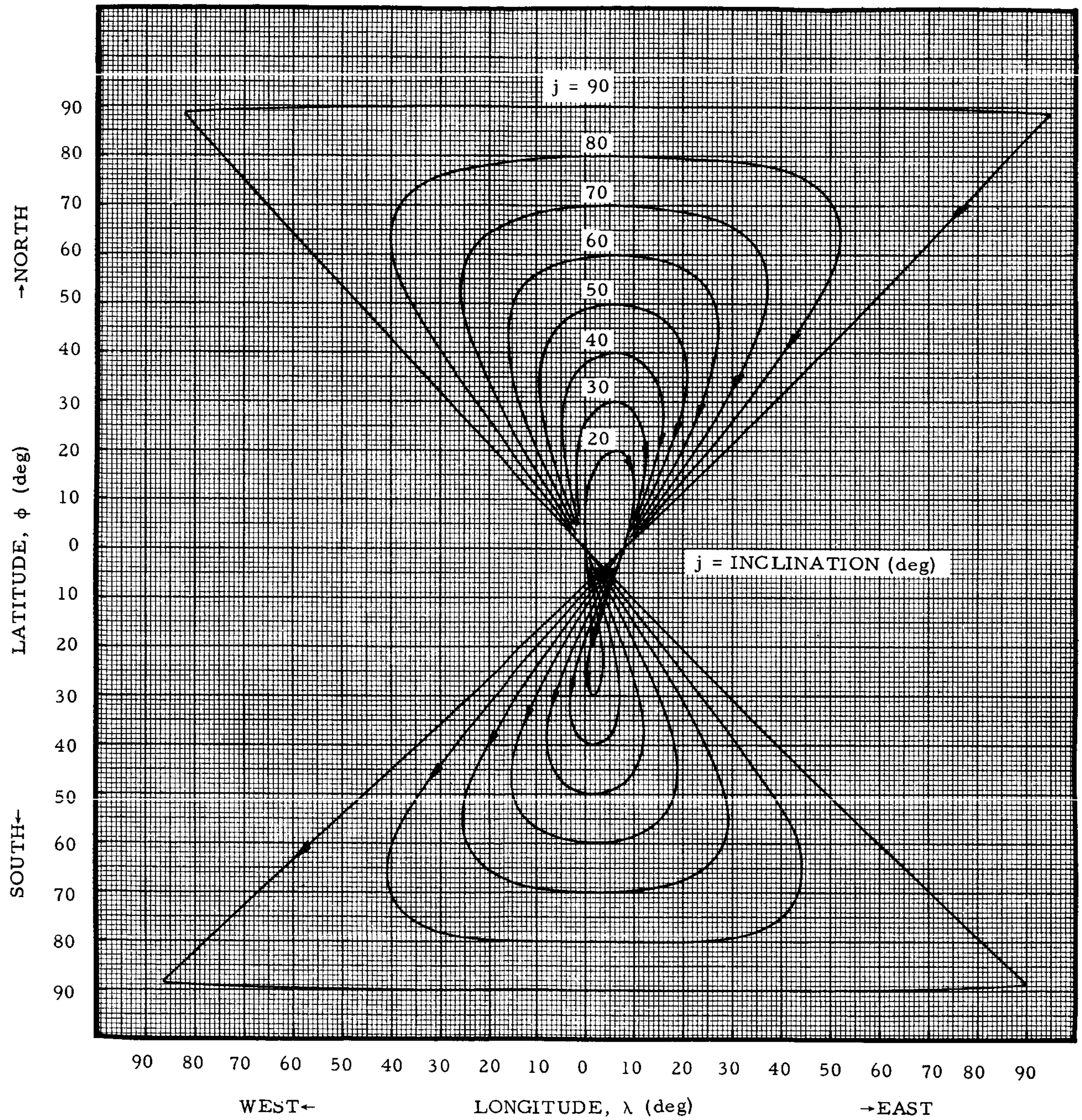

FIGURE 3d. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.04. ARGUMENT OF PERIGEE—60 DEGREES.

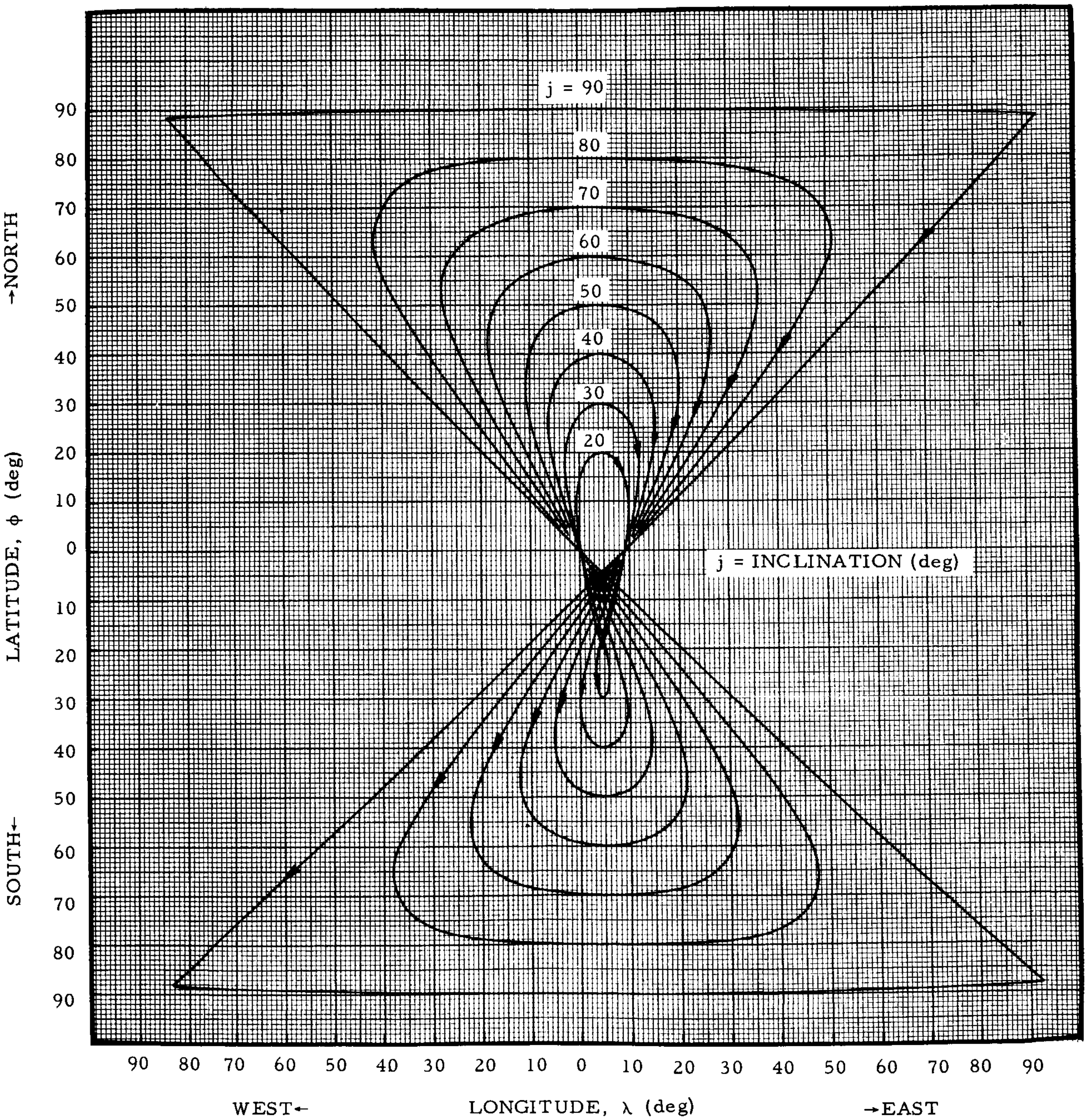

FIGURE 3e. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.04. ARGUMENT OF PERIGEE—90 DEGREES.

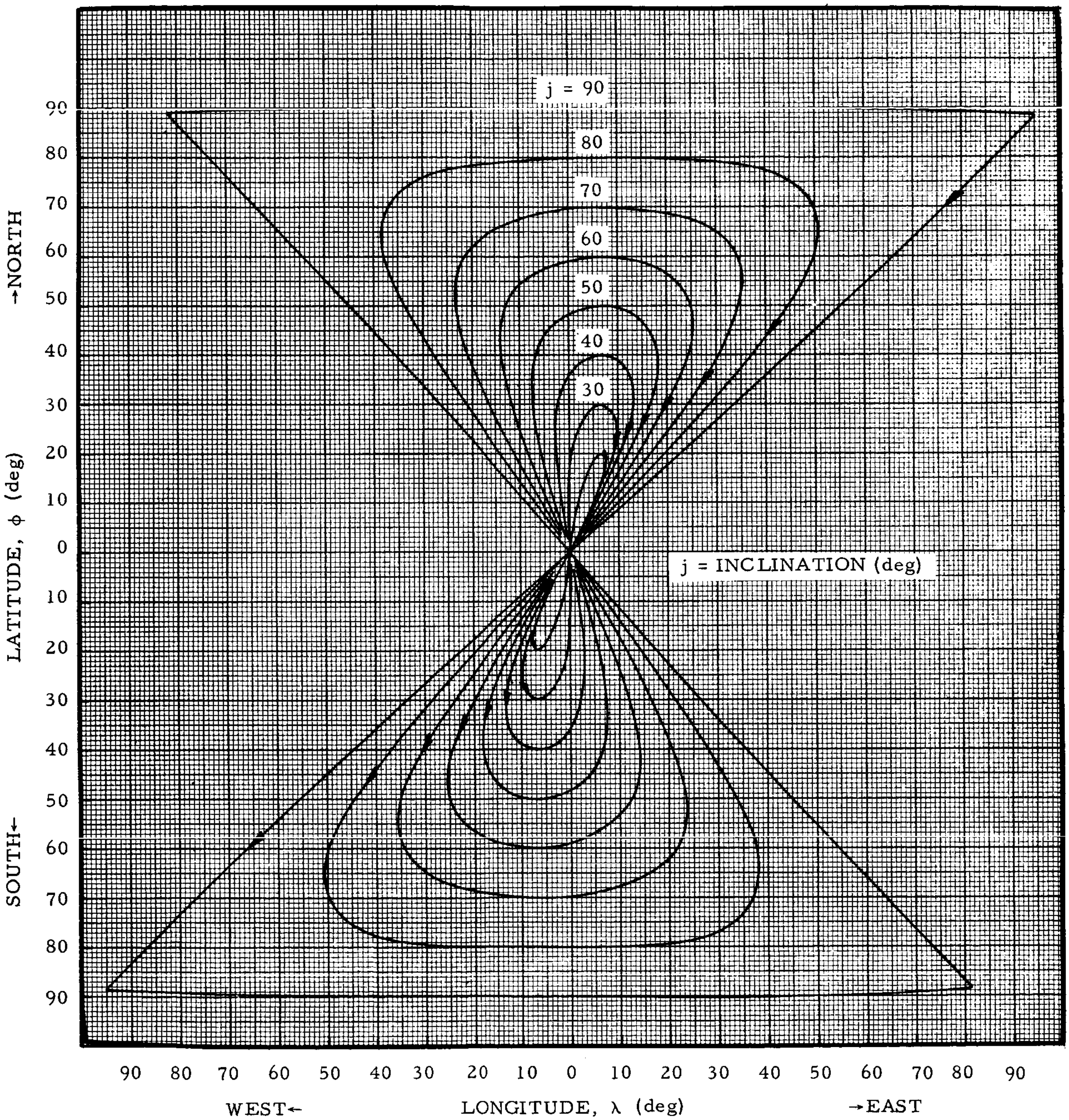

FIGURE 4a. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.06. ARGUMENT OF PERIGEE—0 DEGREES.

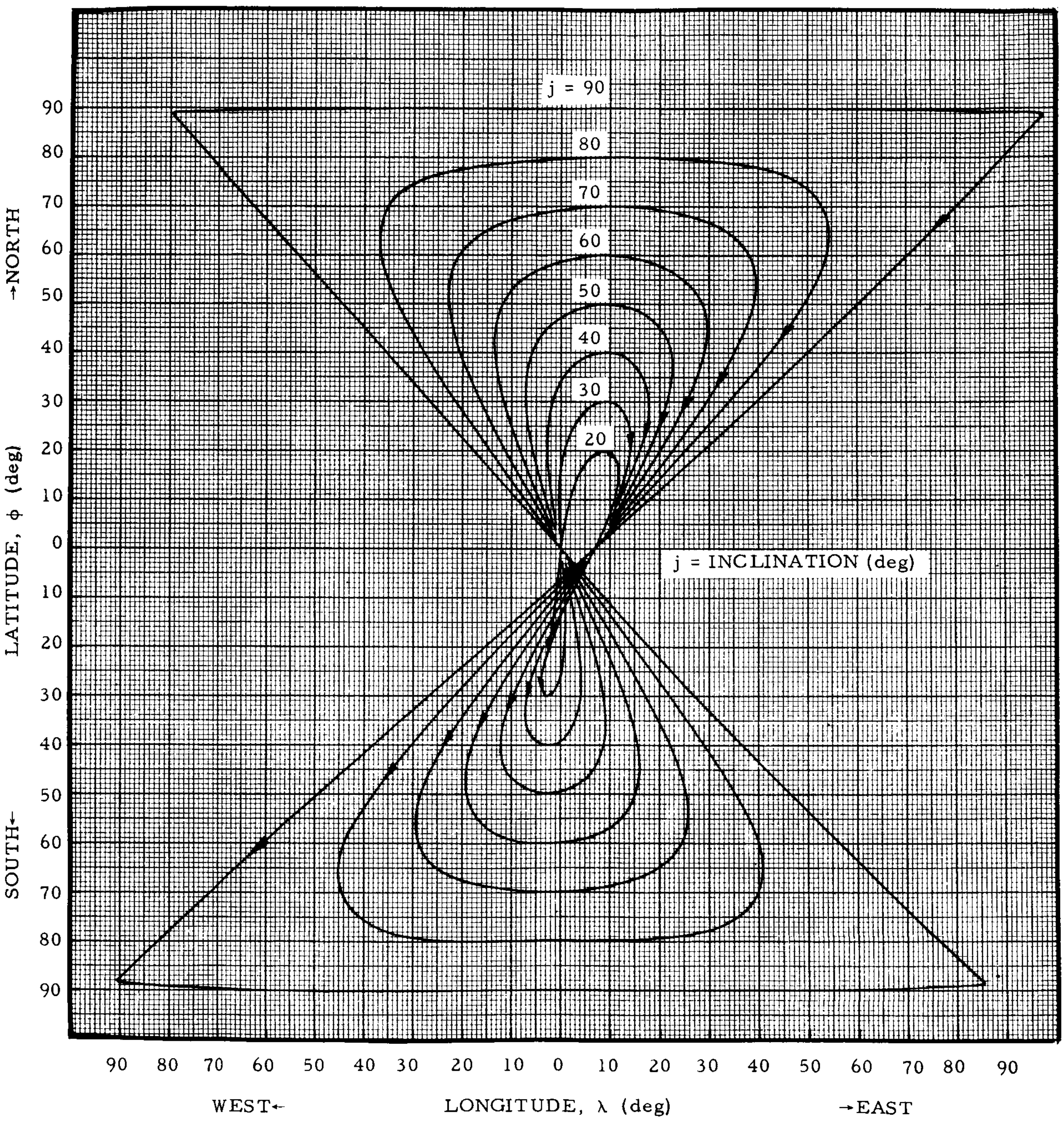

FIGURE 4b. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.06. ARGUMENT OF PERIGEE—30 DEGREES.

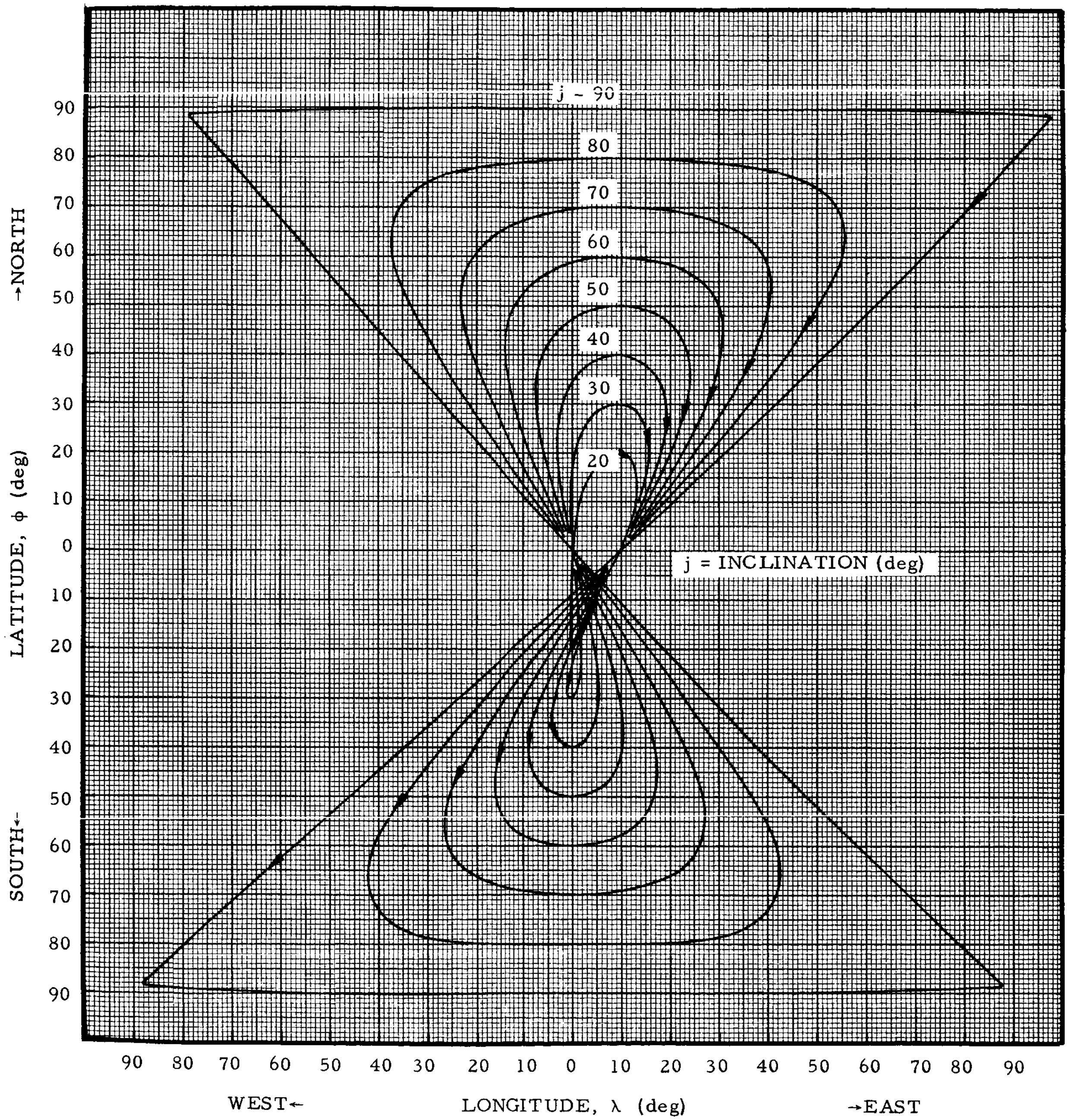

FIGURE 4c. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.06. ARGUMENT OF PERIGEE—45 DEGREES.

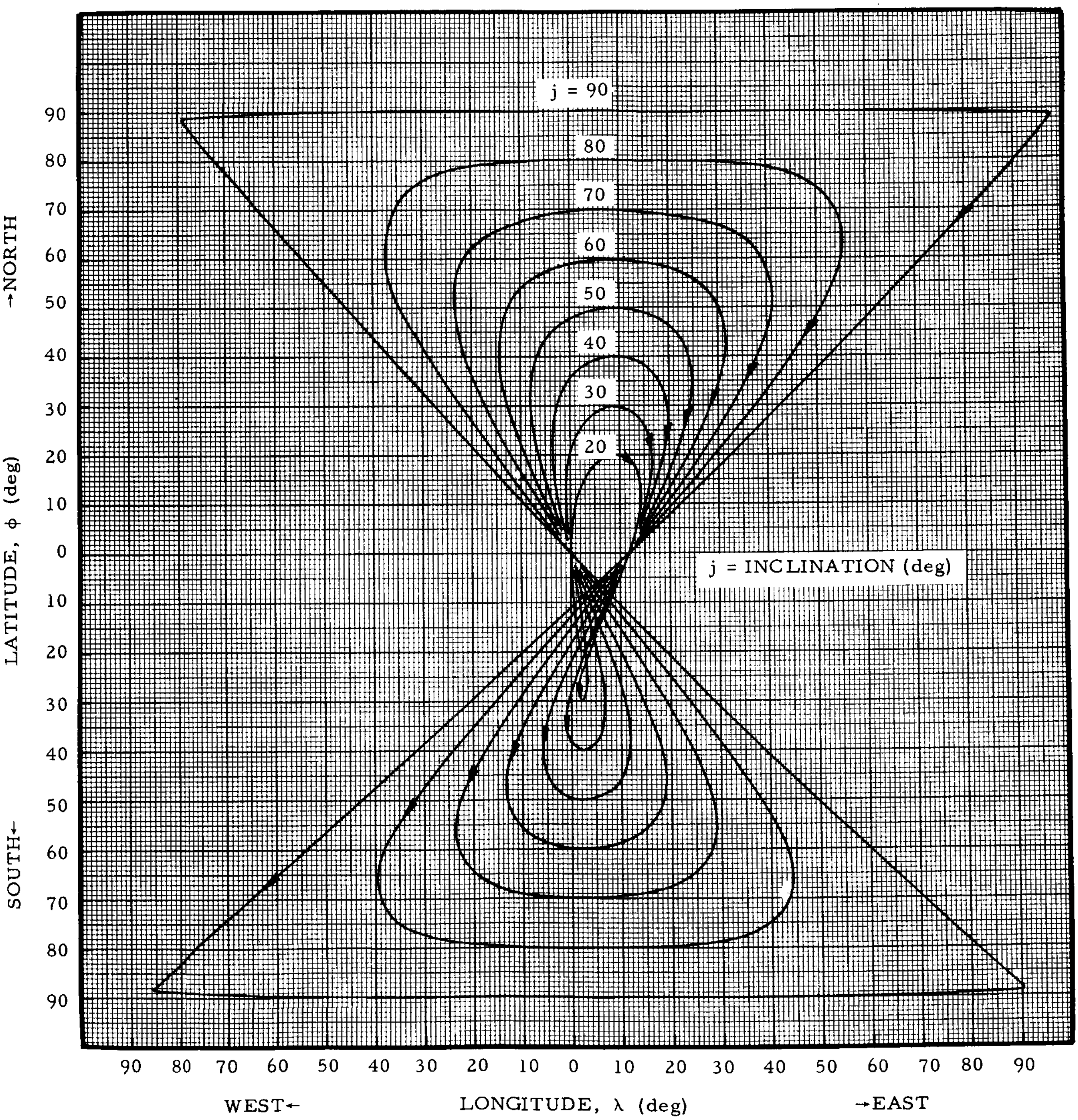

FIGURE 4d. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.06. ARGUMENT OF PERIGEE—60 DEGREES.

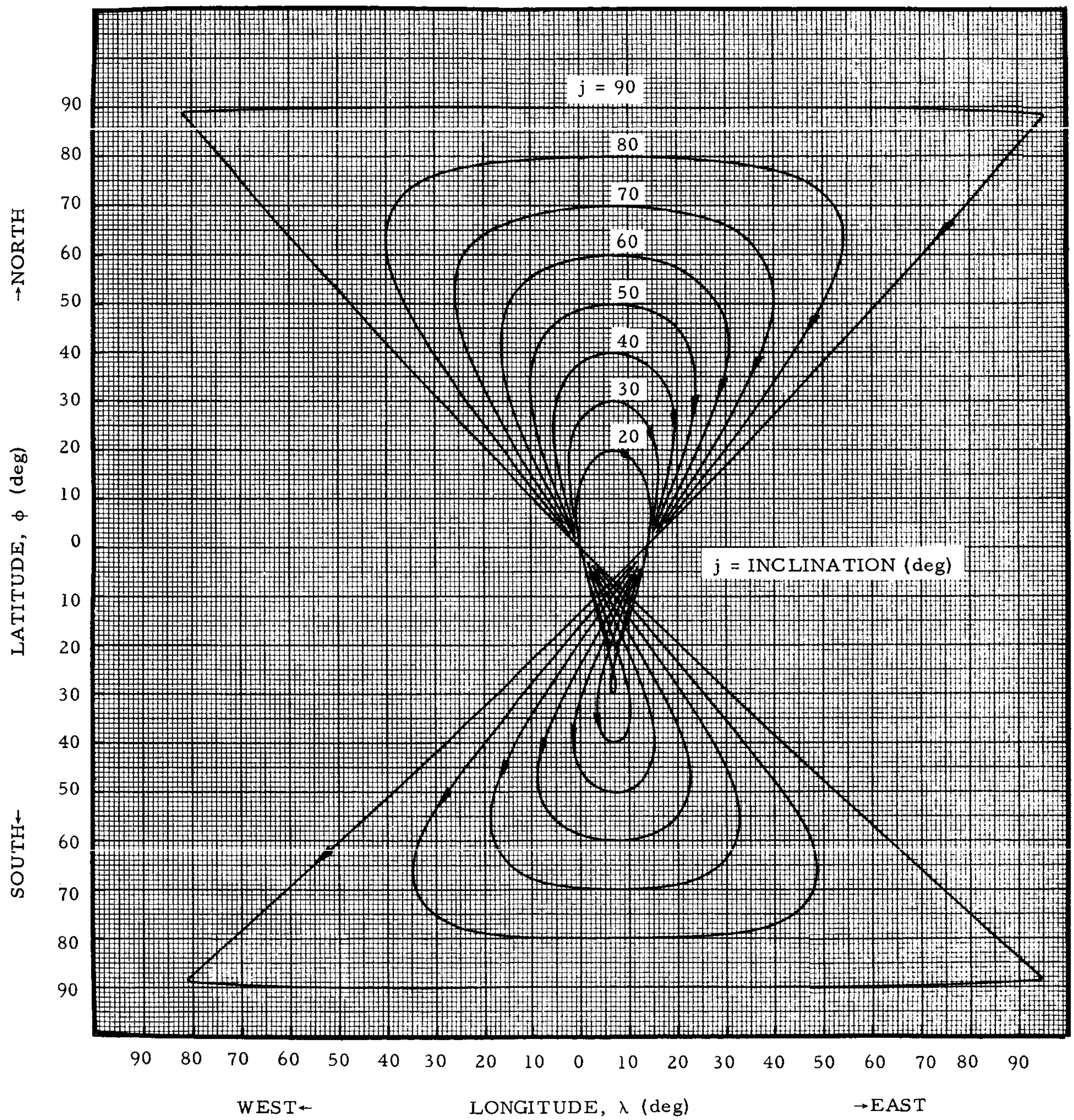

FIGURE 4e. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.06. ARGUMENT OF PERIGEE—90 DEGREES.

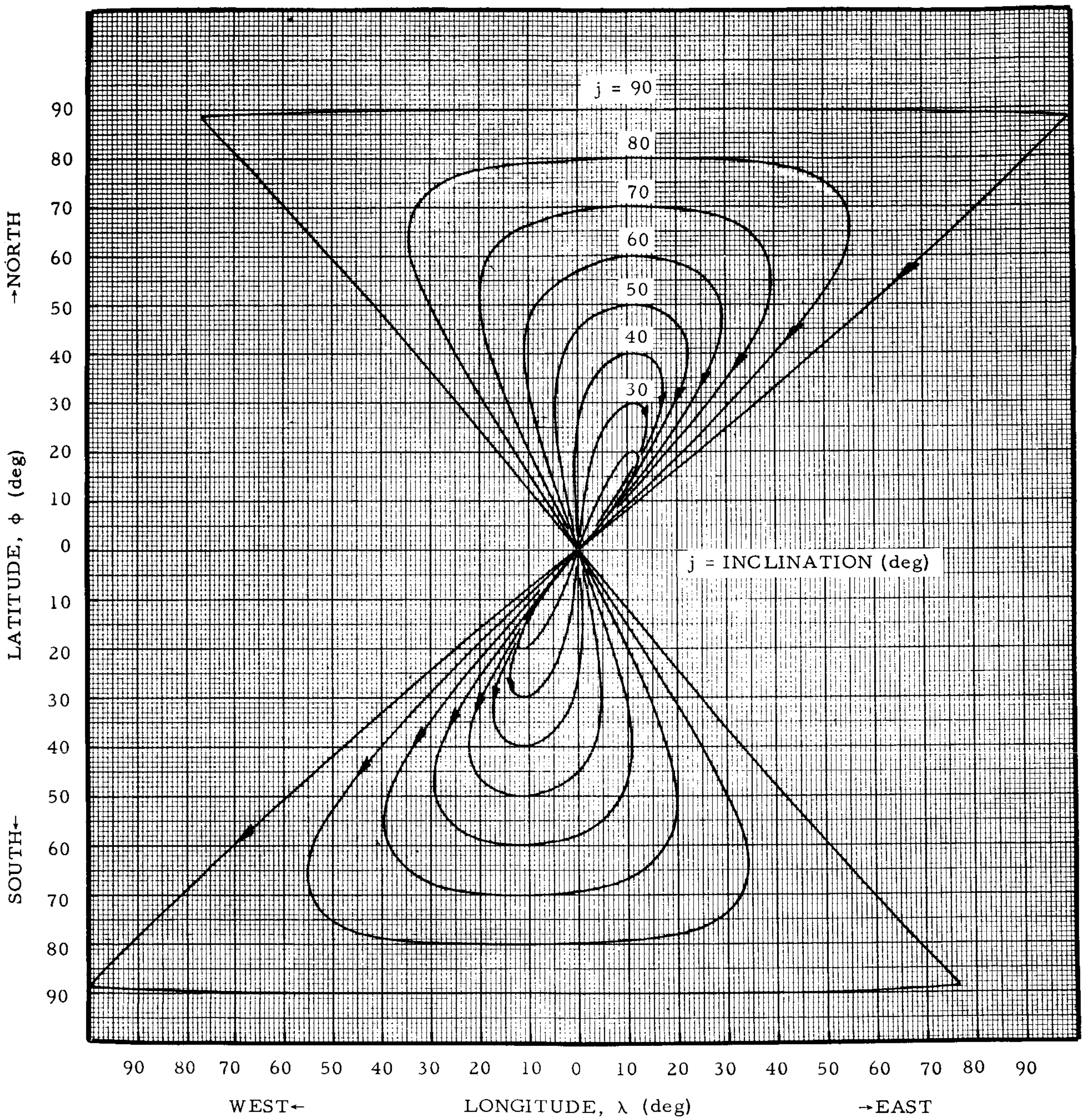

FIGURE 5a. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.1. ARGUMENT OF PERIGEE—0 DEGREES.

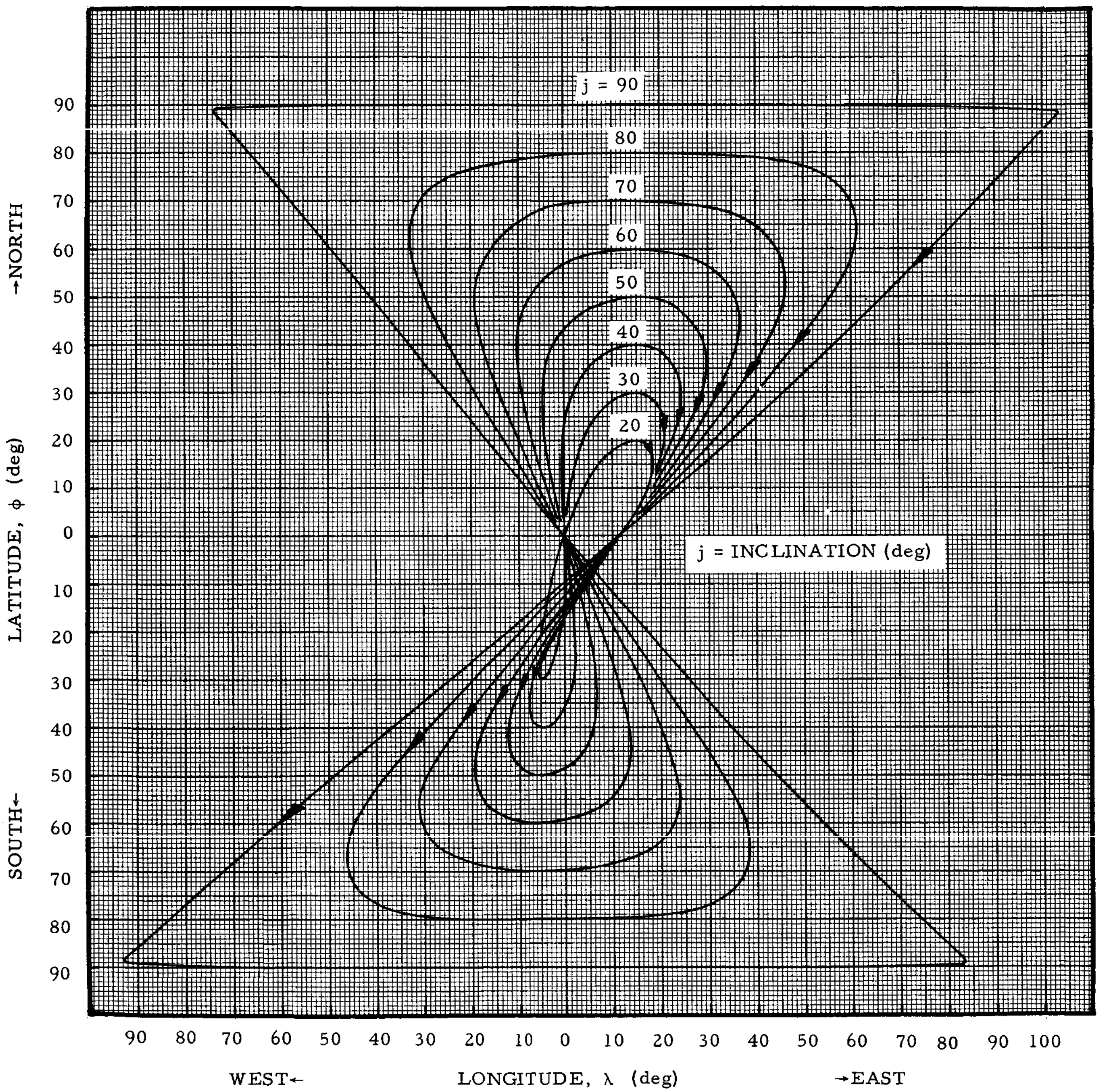

FIGURE 5b. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.1. ARGUMENT OF PERIGEE—30 DEGREES.

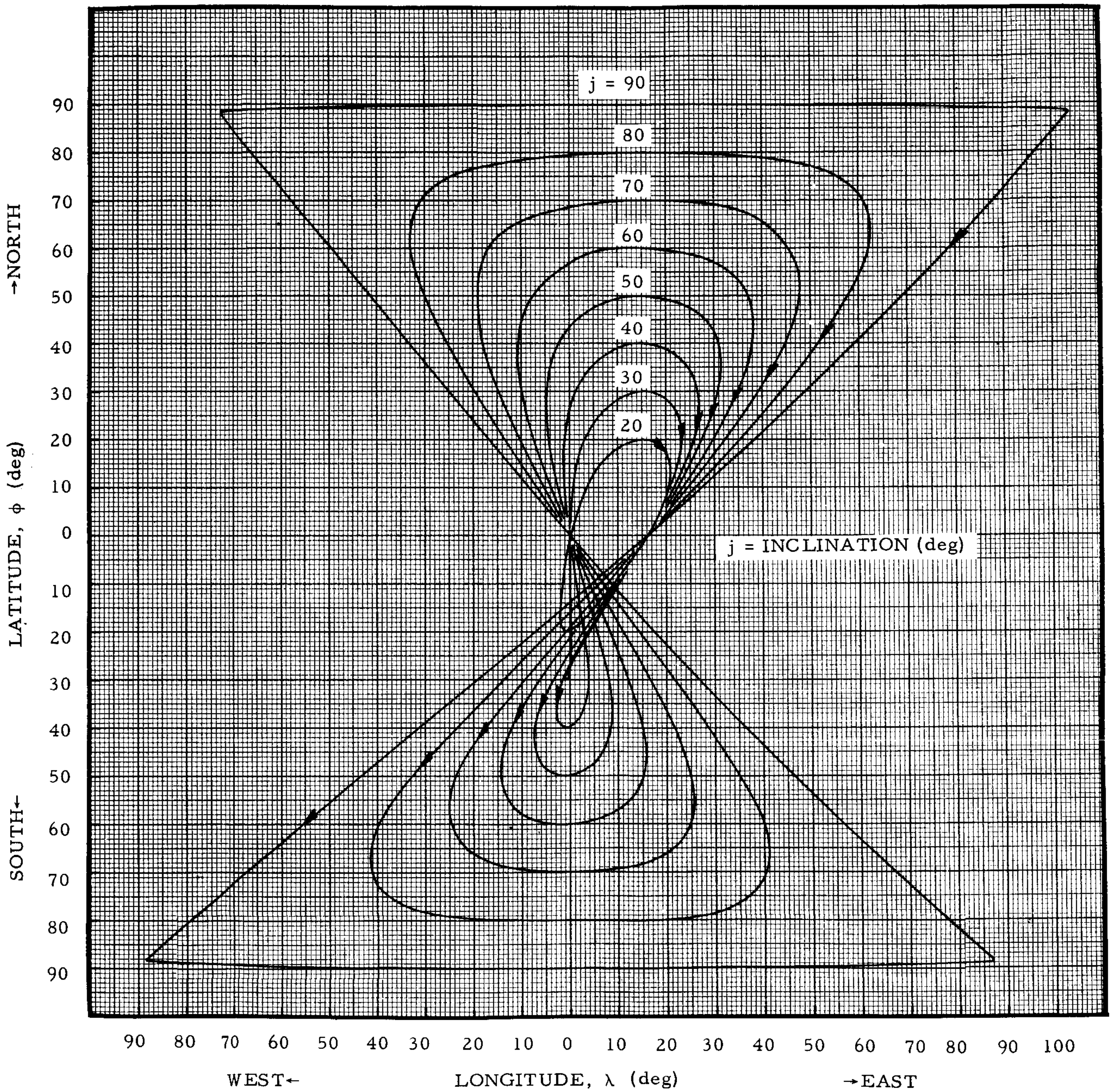

FIGURE 5c. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.1. ARGUMENT OF PERIGEE—45 DEGREES.

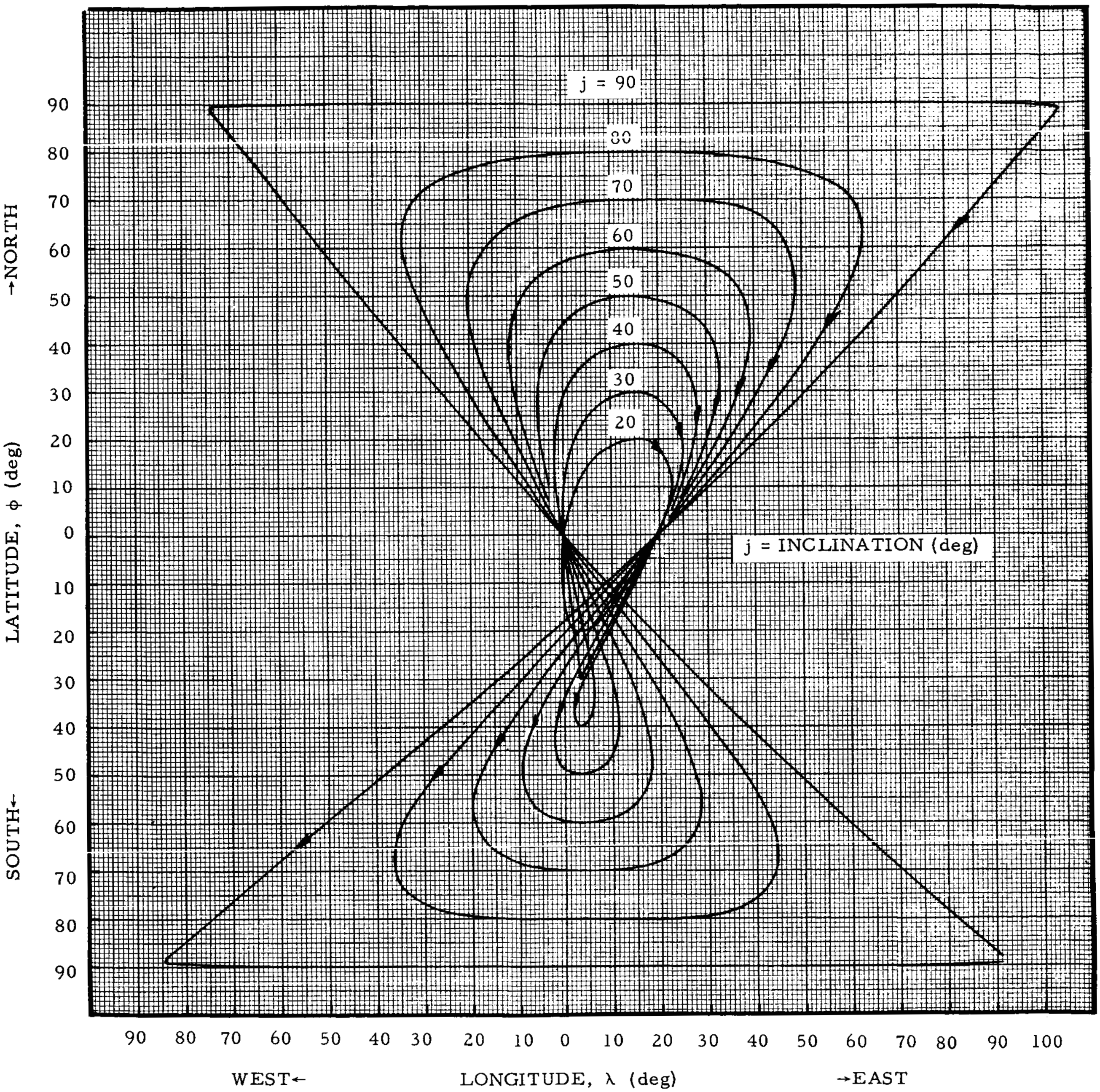

FIGURE 5d. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.1. ARGUMENT OF PERIGEE—60 DEGREES.

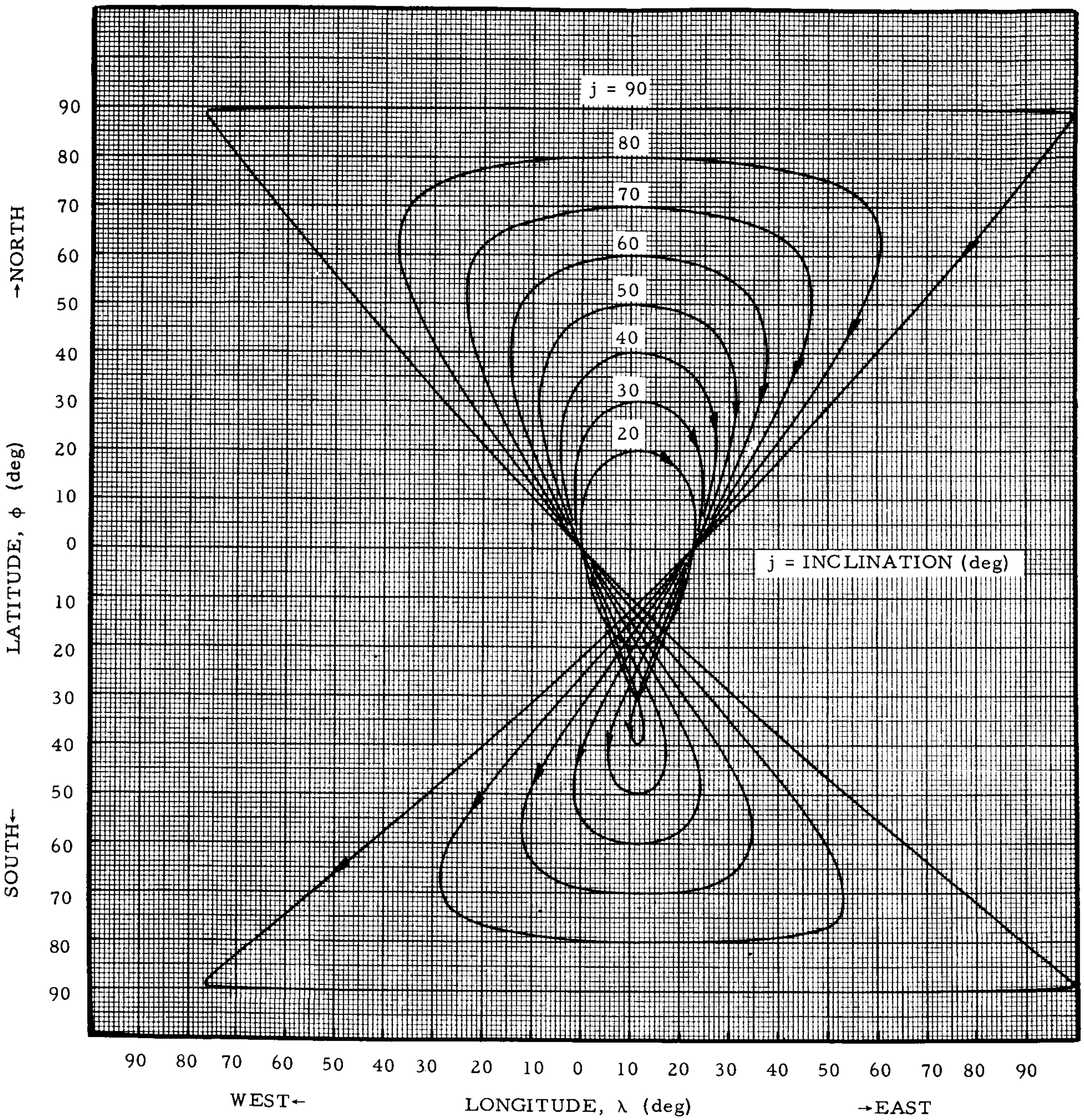

FIGURE 5e. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.1. ARGUMENT OF PERIGEE — 90 DEGREES.

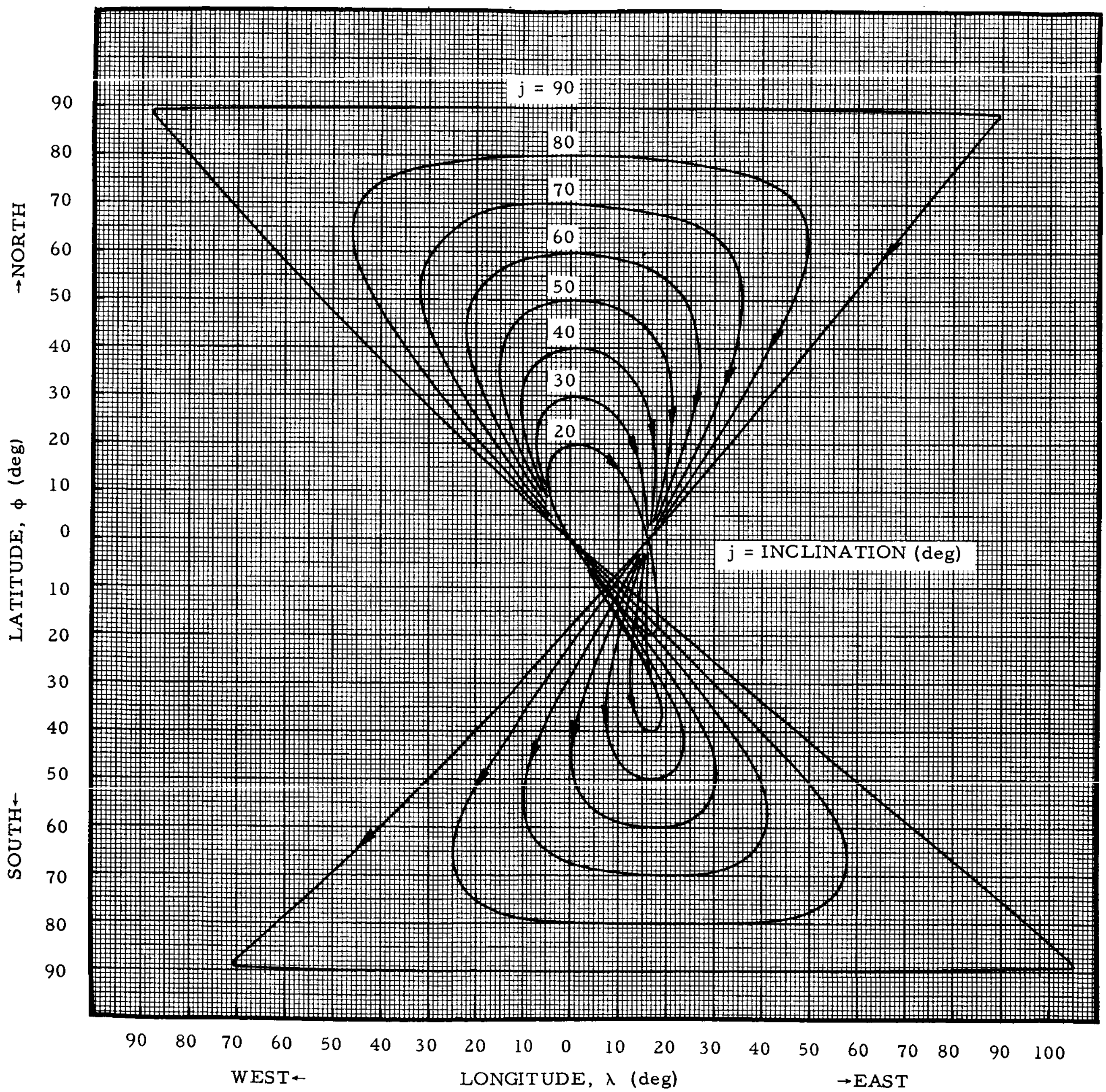

FIGURE 5f. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.1. ARGUMENT OF PERIGEE—135 DEGREES.

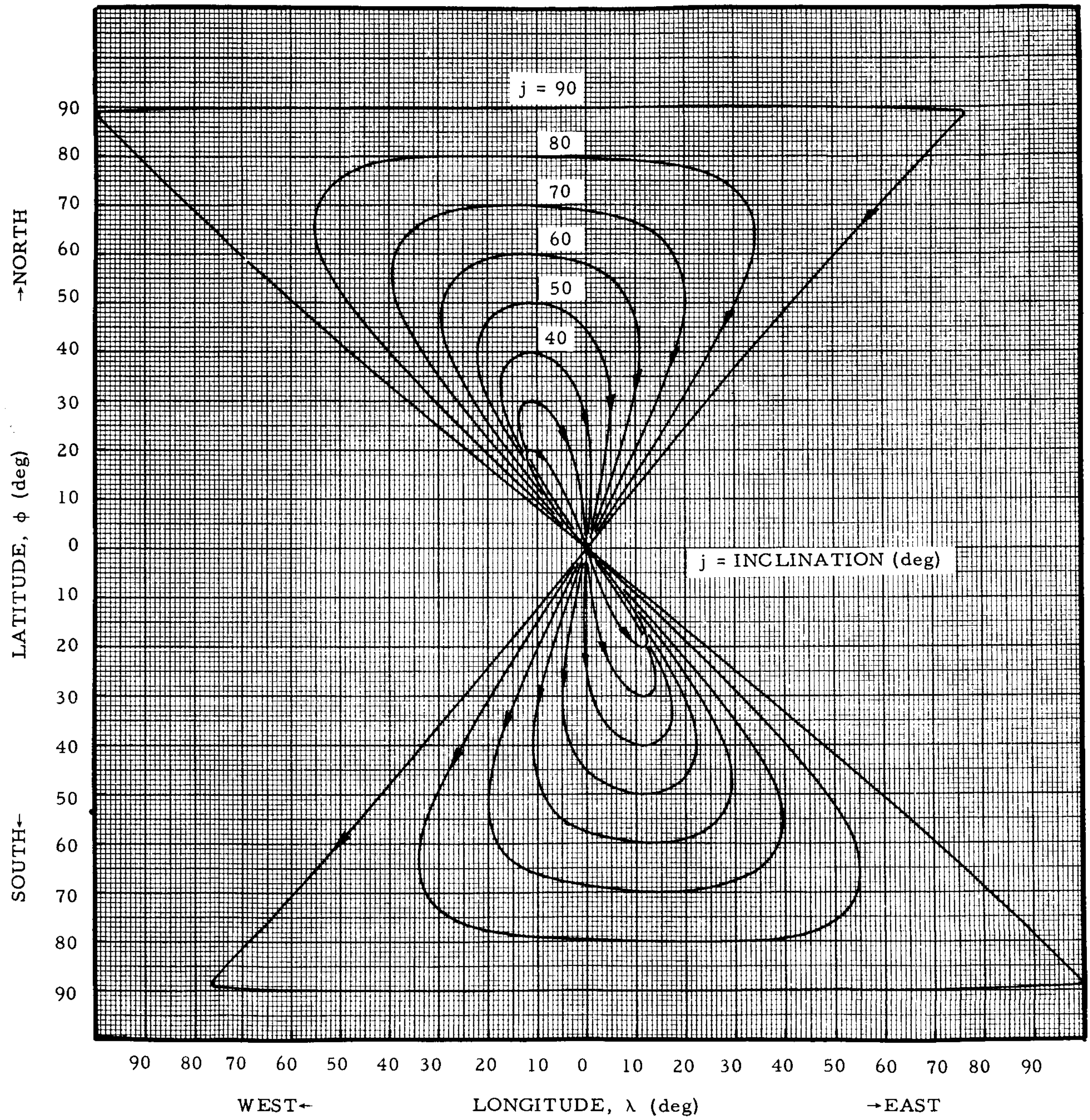

FIGURE 5g. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN ECCENTRICITY OF 0.1. ARGUMENT OF PERIGEE—180 DEGREES.

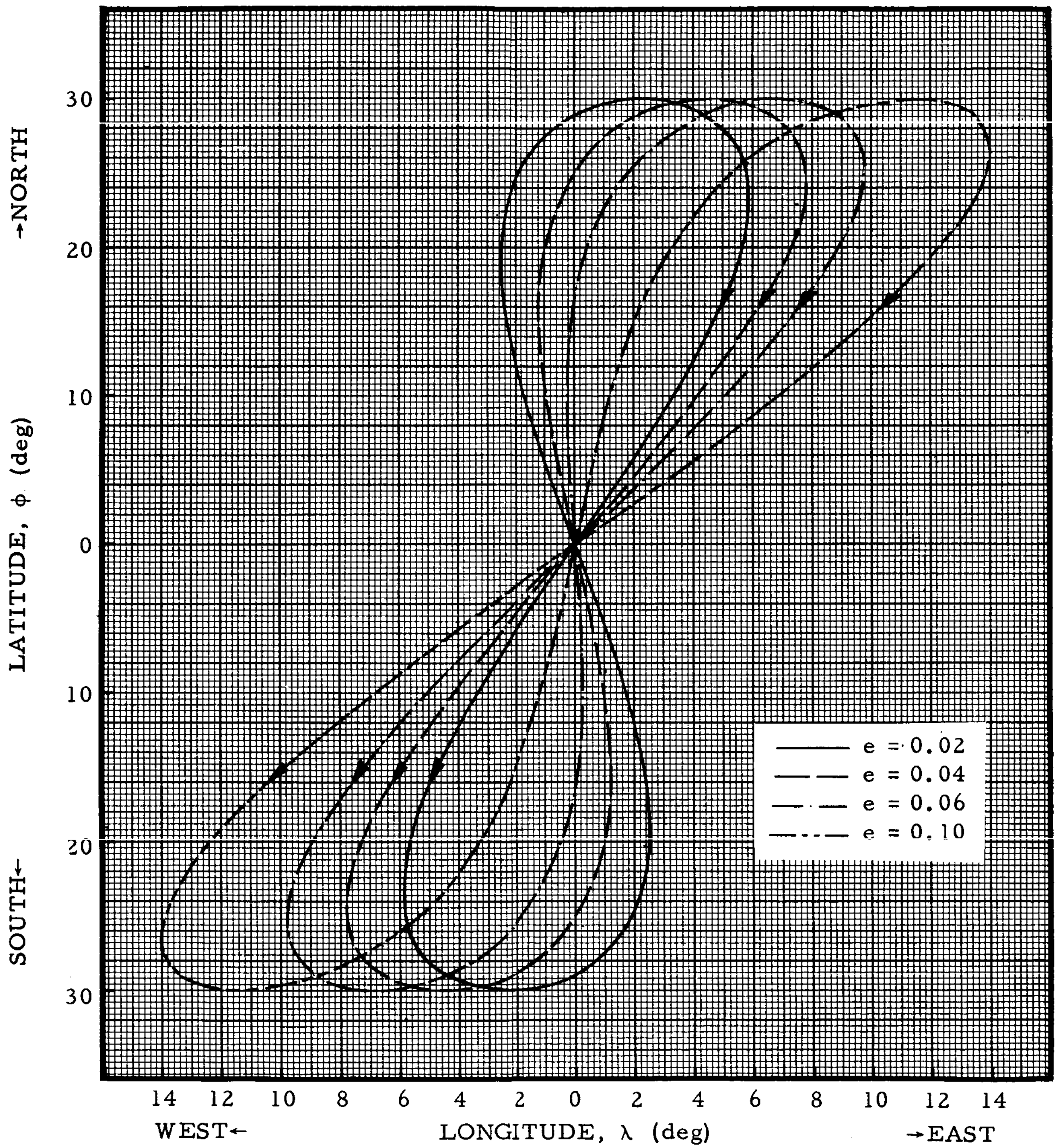

FIGURE 6a. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN INCLINATION OF 30 DEGREES. ARGUMENT OF PERIGEE—0 DEGREES.

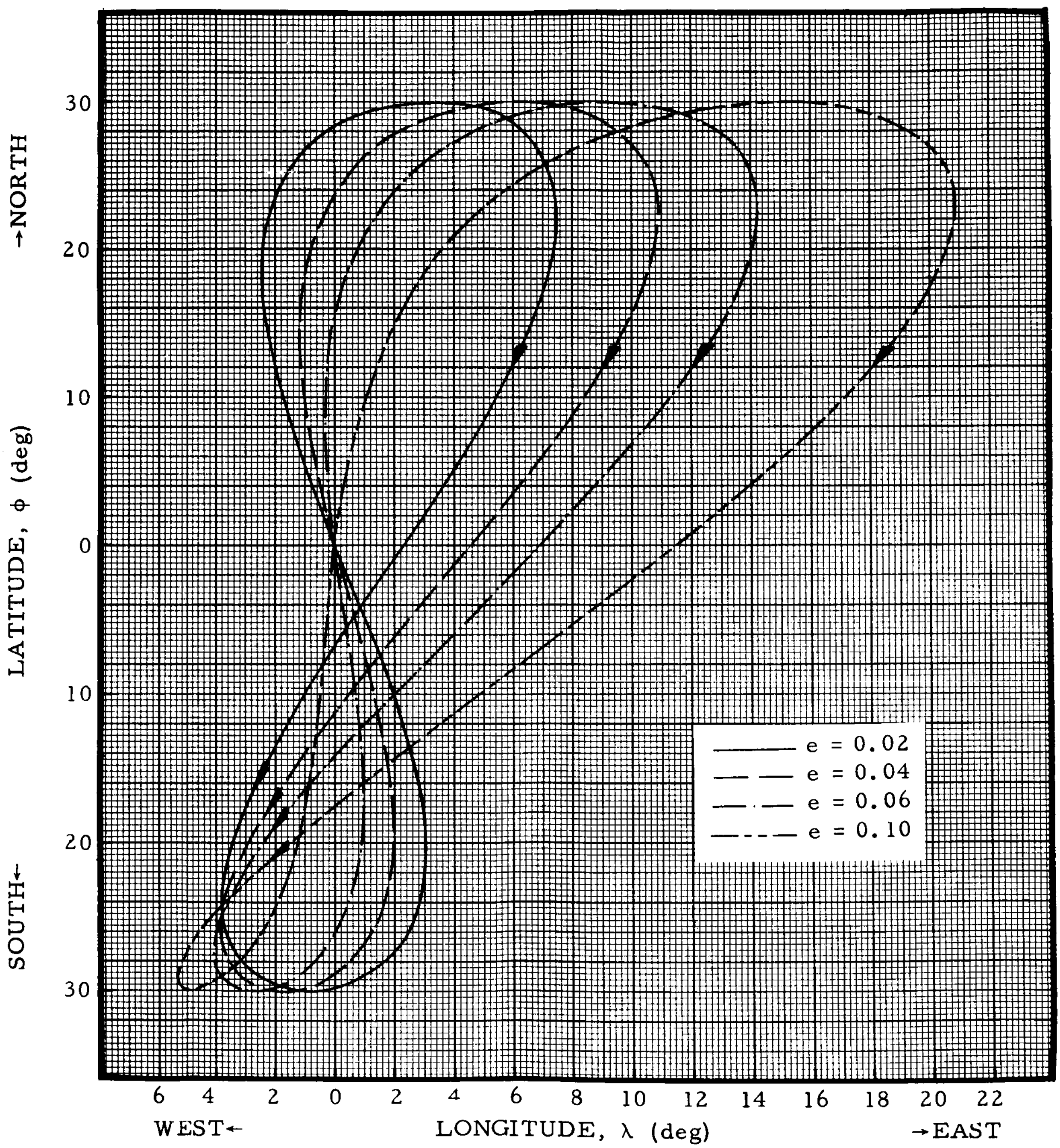

FIGURE 6b. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN INCLINATION OF 30 DEGREES. ARGUMENT OF PERIGEE—30 DEGREES.

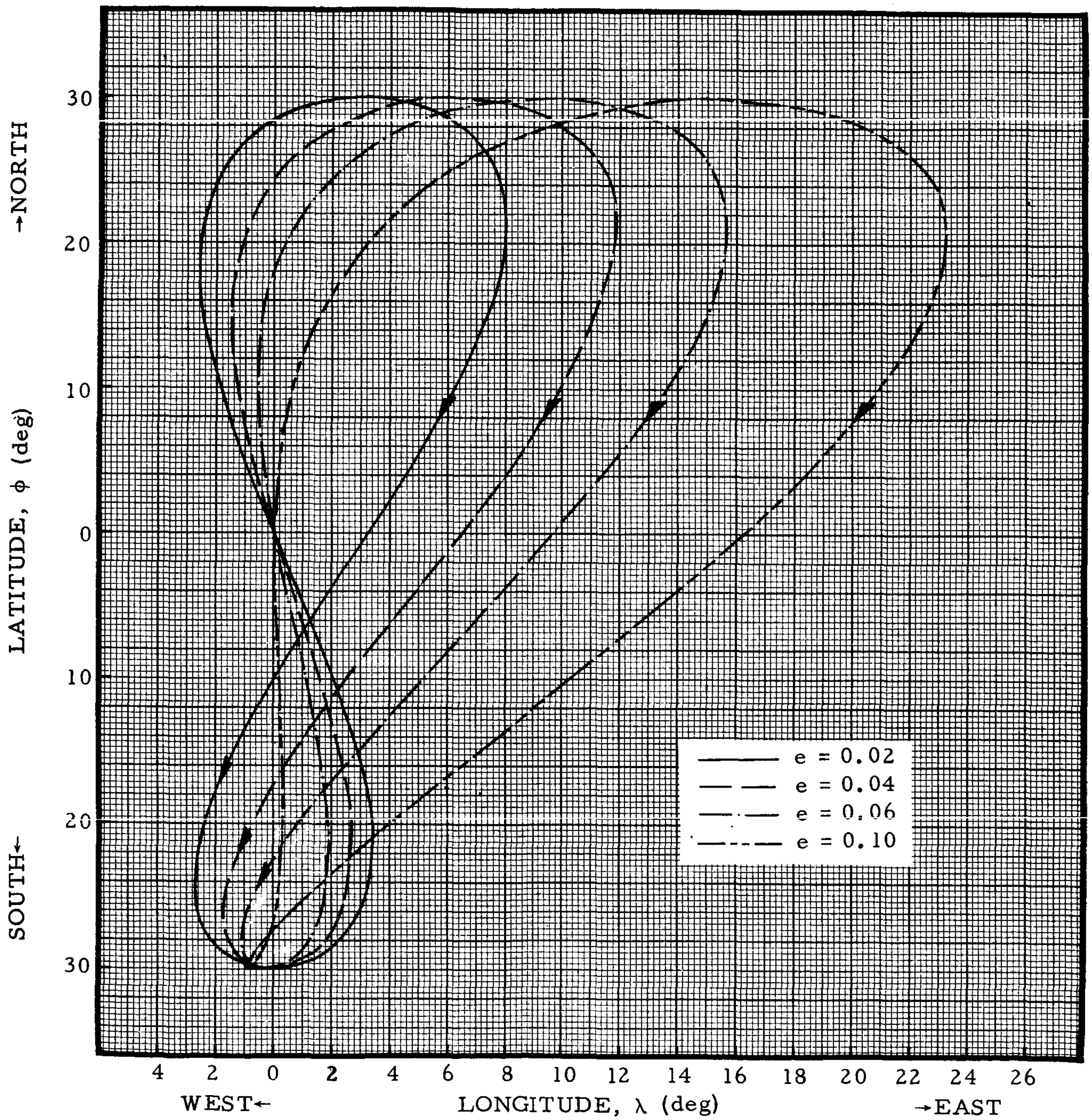

FIGURE 6c. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN INCLINATION OF 30 DEGREES. ARGUMENT OF PERIGEE—45 DEGREES.

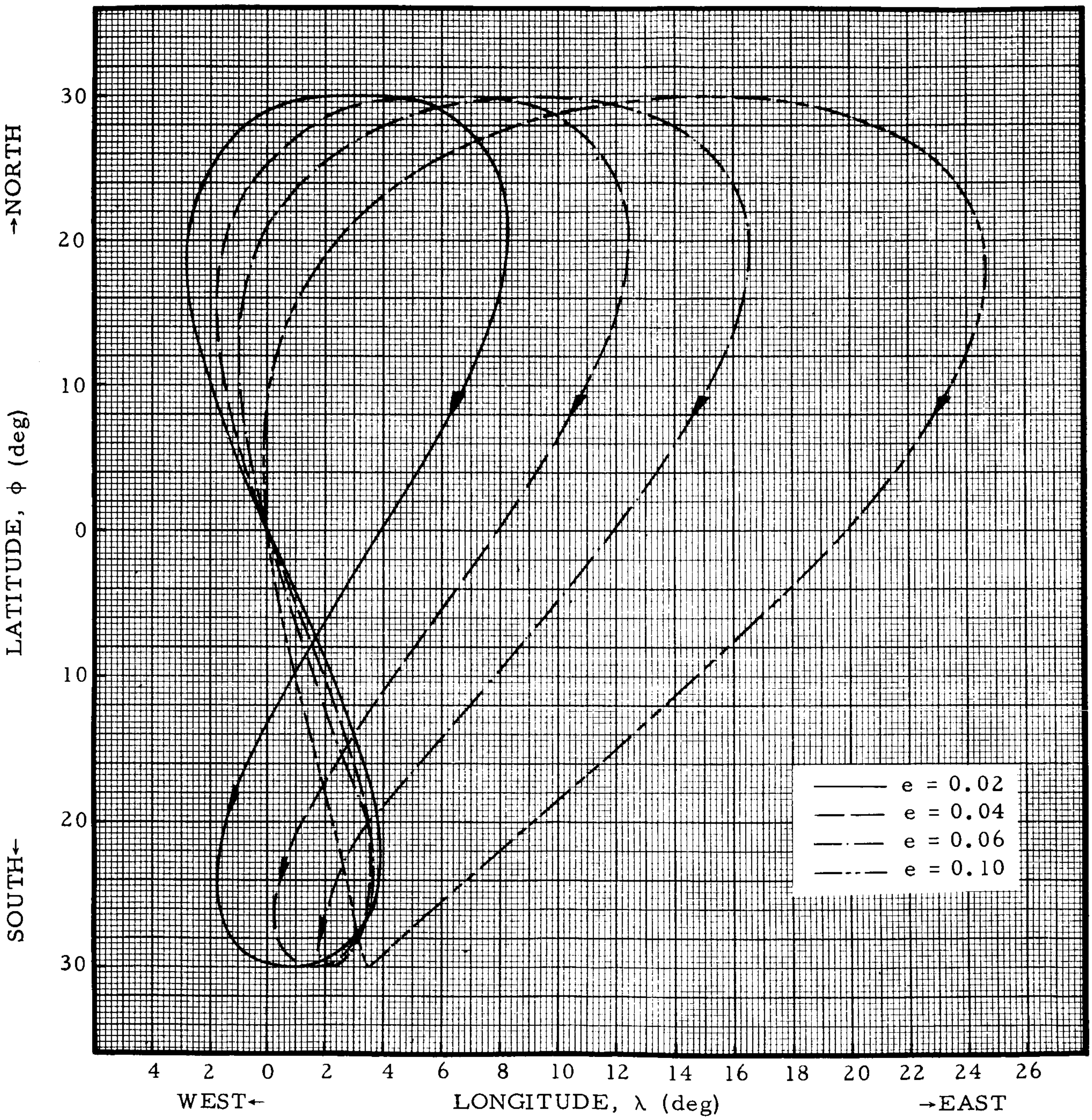

FIGURE 6d. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN INCLINATION OF 30 DEGREES. ARGUMENT OF PERIGEE—60 DEGREES.

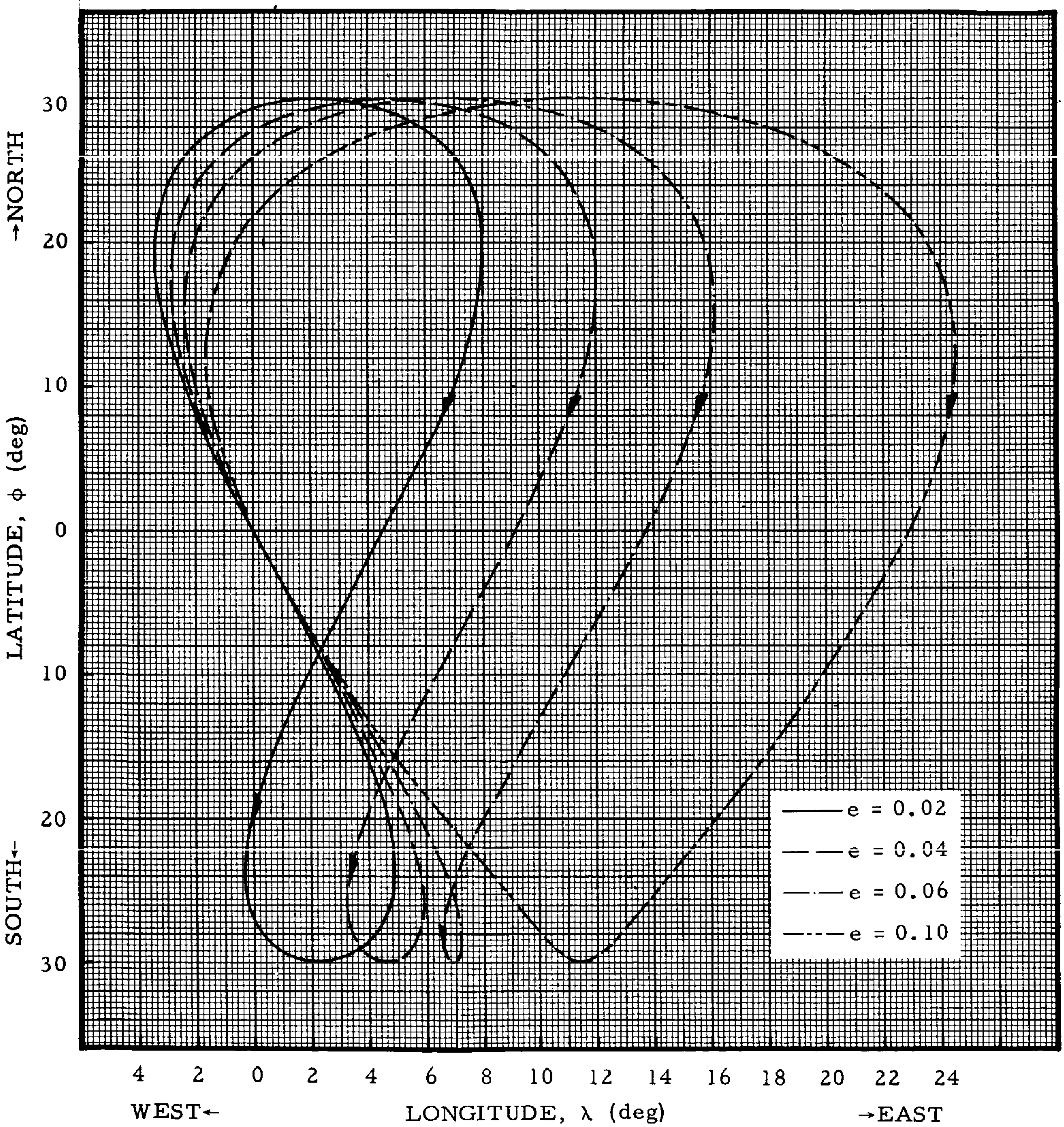

FIGURE 6e. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN INCLINATION OF 30 DEGREES. ARGUMENT OF PERIGEE—90 DEGREES.

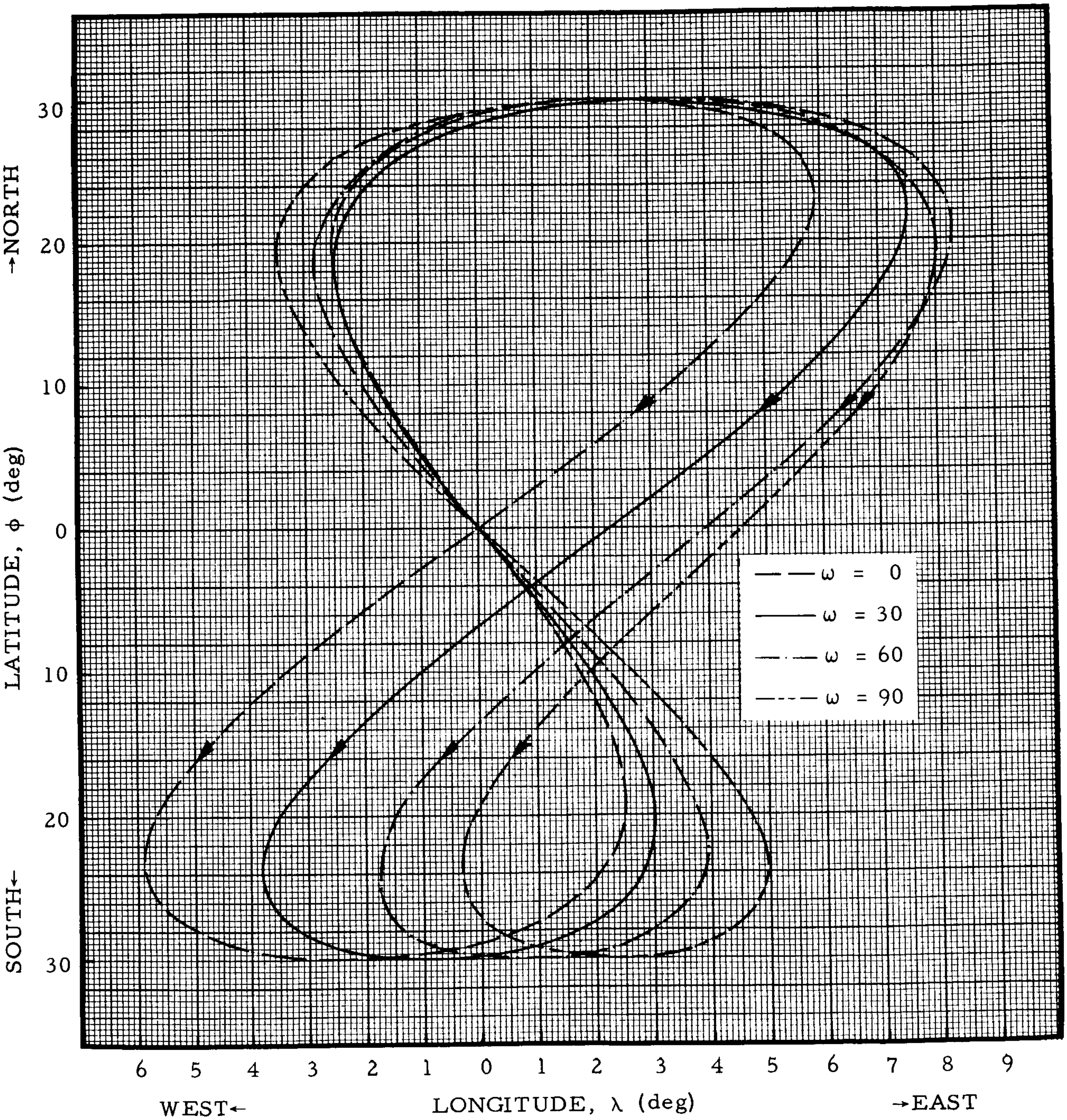

FIGURE 7a. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN INCLINATION OF 30 DEGREES. ECCENTRICITY —— 0.02

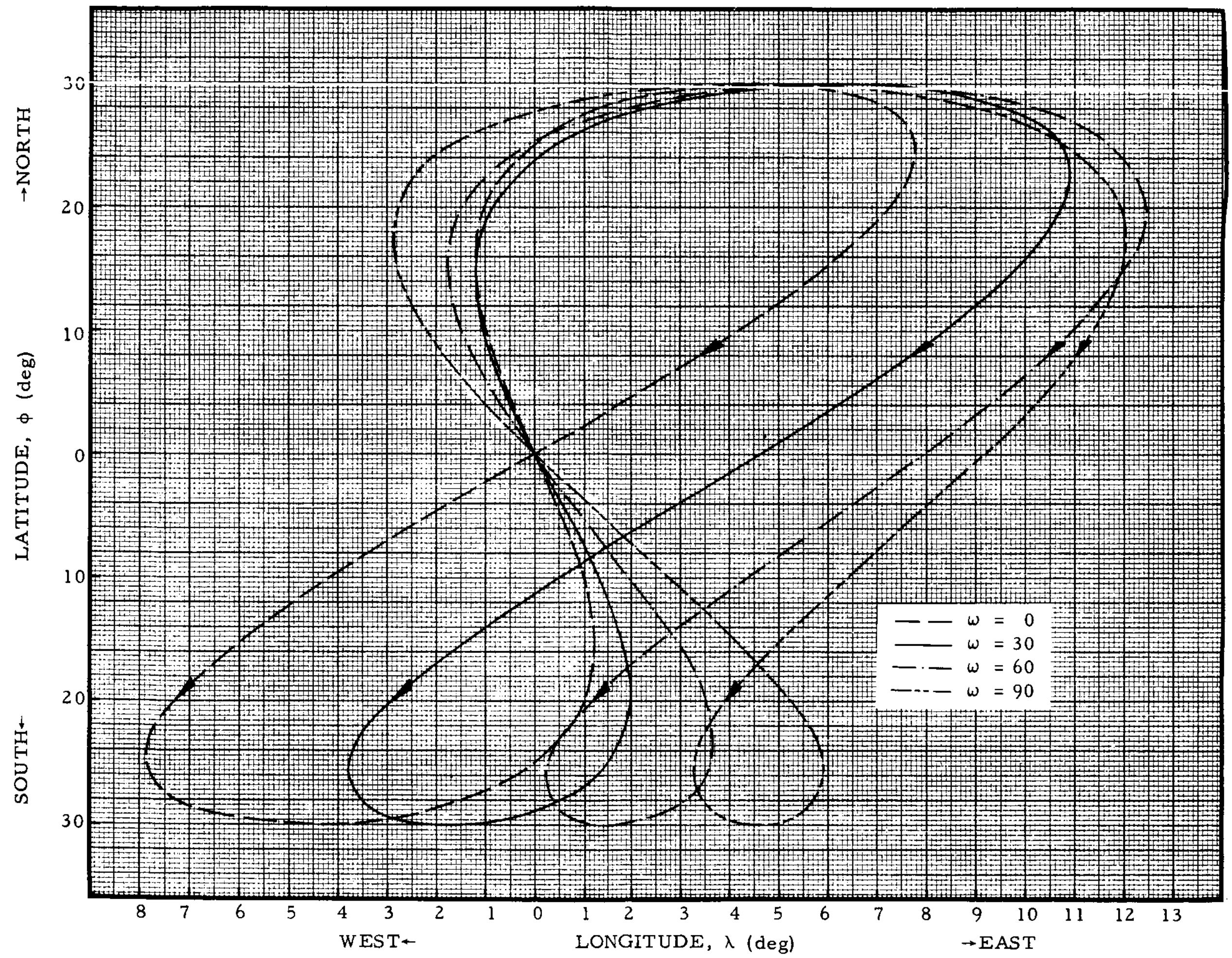

FIGURE 7b. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN INCLINATION OF 30 DEGREES. ECCENTRICITY—0.04.

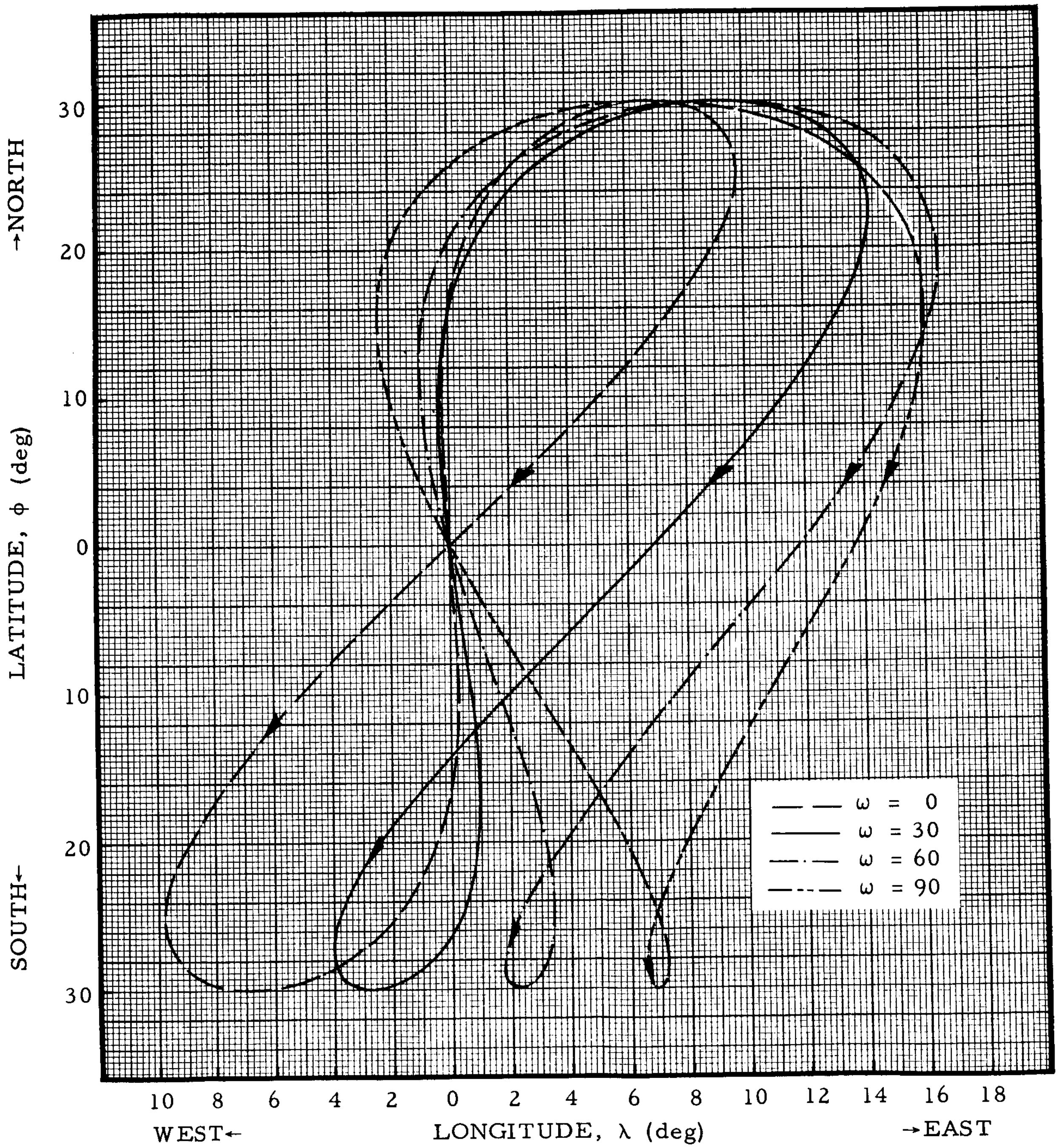

FIGURE 7c. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN INCLINATION OF 30 DEGREES. ECCENTRICITY—0.06.

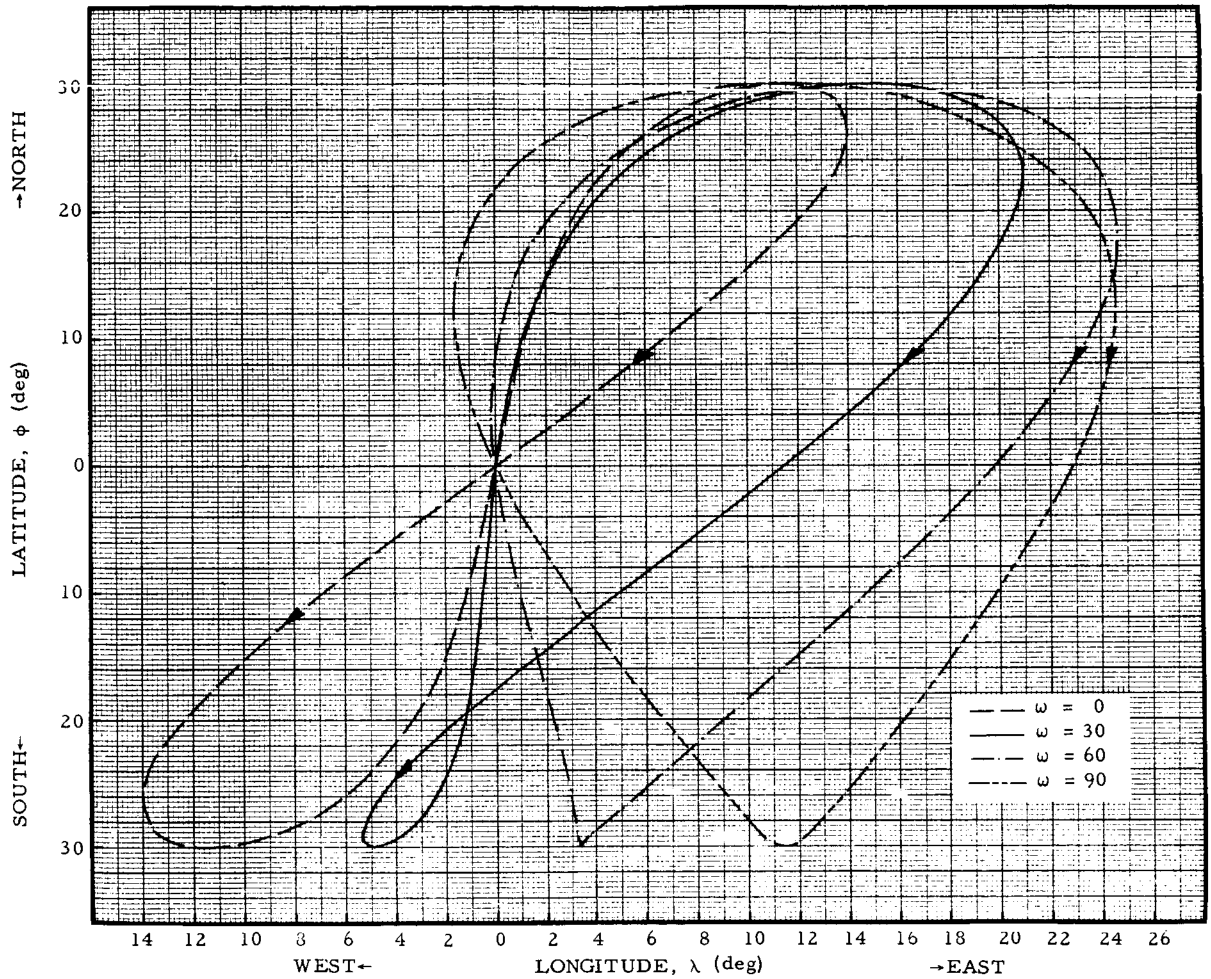

FIGURE 7d. MOTION OF THE SUB-SATELLITE POINT FOR A 24-HOUR ORBIT WITH AN INCLINATION OF 30 DEGREES. ECCENTRICITY—0.10.

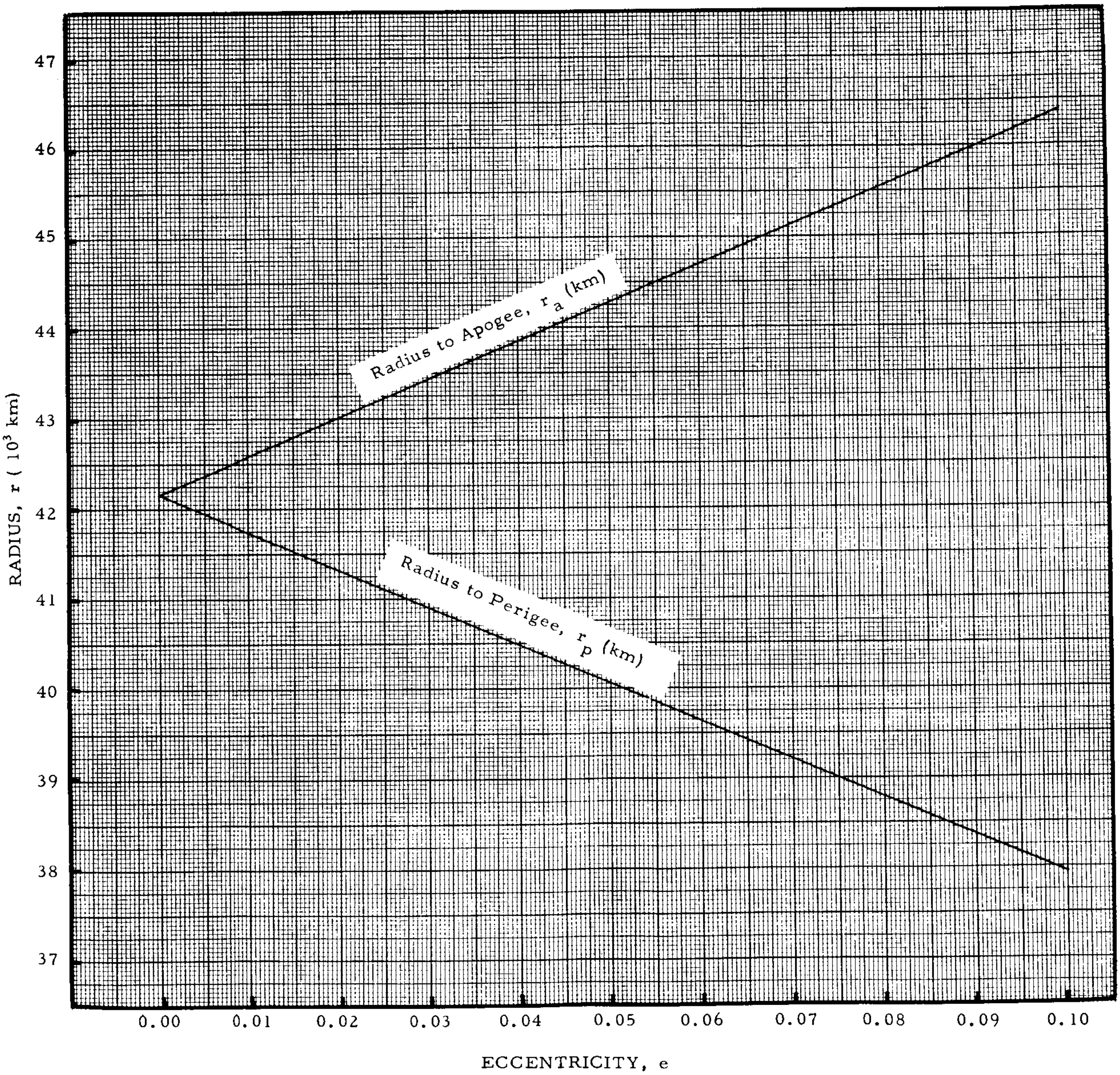

FIGURE 8. APOGEE AND PERIGEE RADIUS AS A FUNCTION OF ECCENTRICITY FOR A 24-HOUR ORBIT

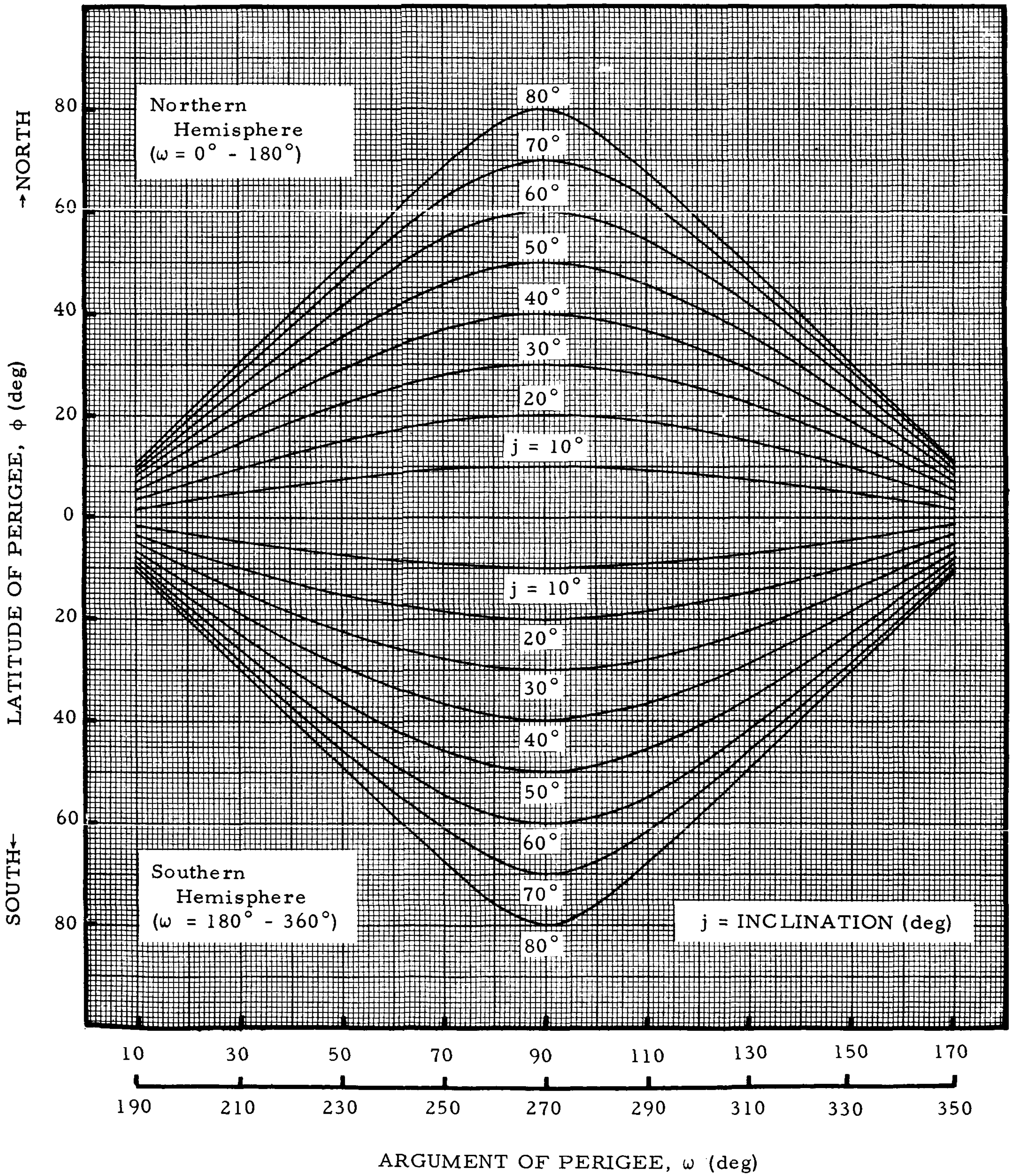

FIGURE 9. LATITUDE OF PERIGEE SUB-SATELLITE POINT VERSUS ARGUMENT OF PERIGEE WITH INCLINATION OF ORBIT PLANE AS A PARAMETER

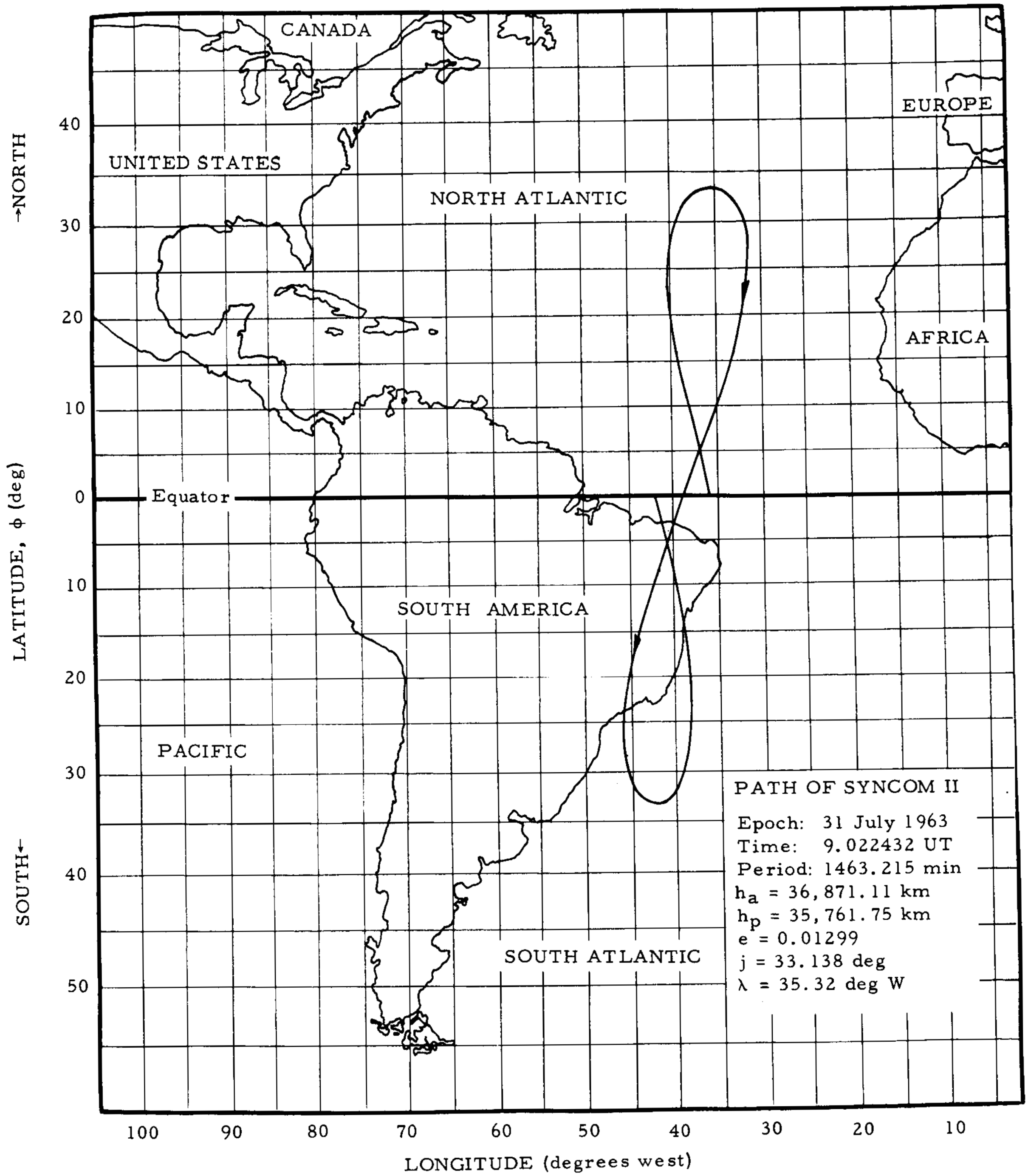

FIGURE 10. THE PATH OF SYNCOM II AS OF JULY 31, 1963

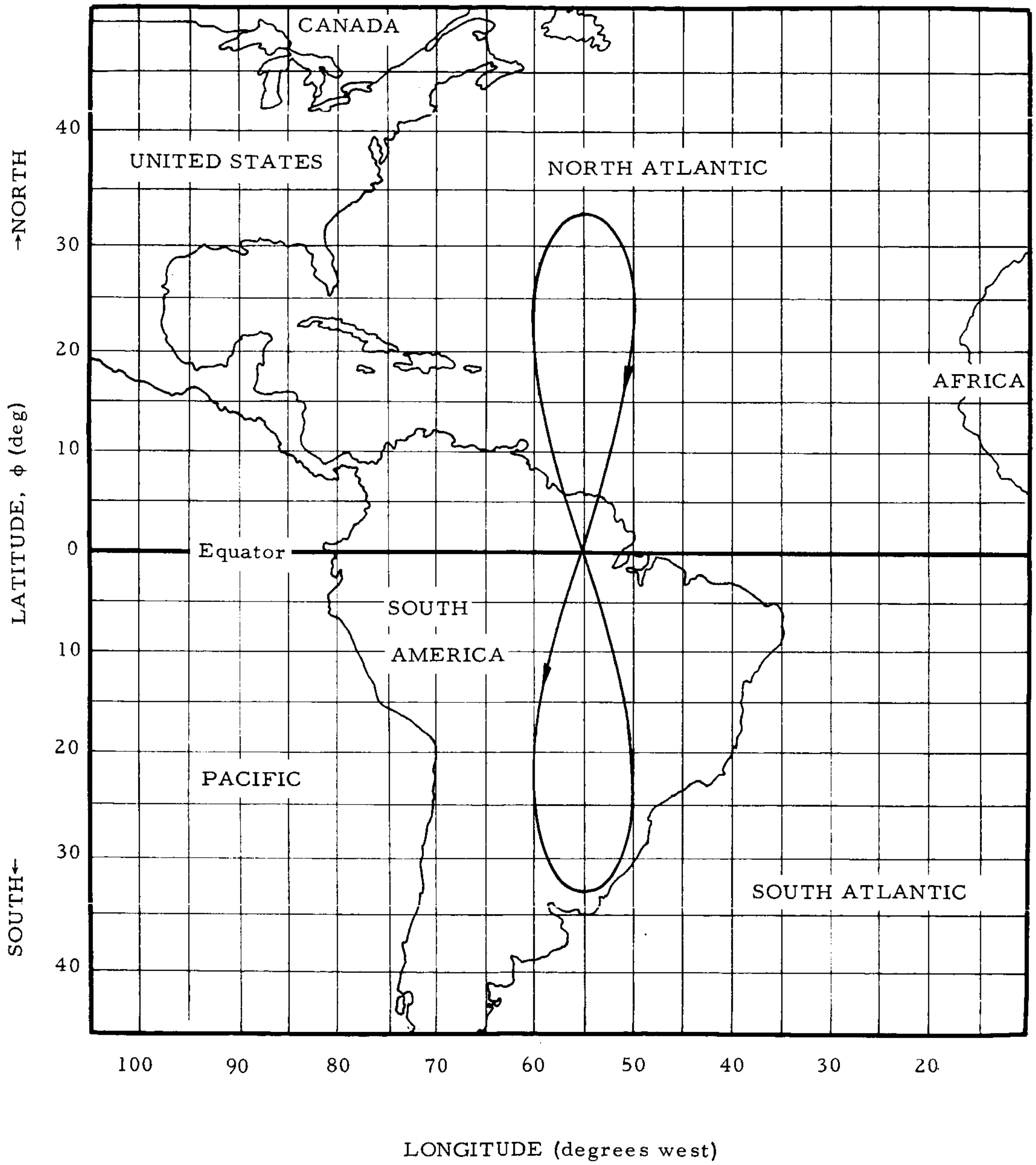

FIGURE 11. THE DESIRED ORBITAL PATH FOR SYNCOM II

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MOTION OF THE SUB-SATELLITE POINT FOR 24-HOUR ORBITS

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