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LUNAR MISSION PERFORMANCE EVALUATION PROCEDURE FOR ORBIT-LAUNCHED NUCLEAR VEHICLES

by Robert E. Austin and George B. Kearns

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Huntsville, Alabama

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LIST OF SYMBOLS

Symbol	Definition
A	Specific weight parameter proportional to thrust
B	Specific weight parameter proportional to propellant loading
C	Specific weight parameter proportional to initial gross weight in orbit
F	Thrust
F/W_o	Initial thrust-to-weight ratio based on sea level weight, or initial acceleration measured in multiples of g_n
g	Acceleration of gravity
g_n	Mean apparent gravity at sea level; international standard for weight-mass conversion; 9.81992 m/sec^2
h	Altitude, $r - r_\oplus$
h_b	Burnout altitude, $r_b - r_\oplus$
h_o	Altitude of initial orbit, $r_o - r_\oplus$
I_{sp}	Specific impulse
m	Mass
r	Radius from center of earth to vehicle
r_o	Radius of initial orbit
r_b	Radius at burnout
$r_\oplus$	Radius of mean spherical earth
t_b	Burning time
V	Inertial velocity
x	Range

LIST OF SYMBOLS (Continued)

Symbol	Definition
T_{tr}	Lunar transfer time
ΔV_{id}	Characteristic velocity
W_o	Initial weight in orbit
W_N	Effective net structural weight
W_L	Gross payload weight
W_p	Useable propellant weight
β	Thrust vector orientation angle measured from the velocity vector to the thrust vector (positive down)
θ	Flight path angle, measured from the local vertical (positive down)
ψ	Central angle measured from local radius vector at ignition
ζ	Propellant ratio, $\frac{W_p}{W_o}$
λ	Gross payload ratio, $\frac{W_L}{W_o}$
ϵ_N	Net structural weight ratio, $\frac{W_N}{W_o}$
$\mu_\oplus$	Gravitational constant of earth

Subscripts

o	Initial
b	Burnout
$\oplus$	Earth

LIST OF SYMBOLS (Concluded)

Subscripts	Definition
ref	Denotes reference conditions
185.2	Denotes a change in a parameter due to a 185.2 km (100 N.M.) change in h_o
A, B, T_{tr}, h_o	Indicate that a parameter is calculated for arbitrary values of A, B, T_{tr}, or h_o, respectively

Abbreviations

rad	Radians
km	Kilometers
m	Meters
N.M.	Nautical miles
s	Seconds
deg	Degrees

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SUMMARY

An estimation procedure, based primarily on the analysis of Reference 1, is described and reference data are presented by which the influence of certain design parameters and lunar mission velocity requirements can be determined for orbit-launched nuclear vehicles. The use of this procedure allows a rapid, unsophisticated method of calculating the performance of nuclear propulsion systems for specific lunar mission profiles. The performance parameters considered are as follow: payload ratio, characteristic velocity requirement, propellant ratio, flight path angle, central angle, burning time, and injection altitude. The effects of initial thrust-to-weight ratio, specific impulse, lunar transfer time, and initial orbital altitude are considered. The range of specific impulse assumed is restricted to that considered practical for nuclear heat exchanger propulsion systems.

Application of this estimation procedure is demonstrated by numerical examples included in Appendix A. The results of these and other examples are compared with exact computer solutions and are shown to be well within the accuracy usually required in preliminary design studies.

SECTION I. INTRODUCTION

With increasing interest in the application of nuclear rocket space systems to lunar transportation, it becomes important to have available a rapid, practical method for relating nuclear vehicle design and performance parameters to trajectory characteristics, once preliminary mission profiles and conceptual vehicle designs are established. A comprehensive procedure of this nature can provide a useful tool for conducting feasibility studies of contemplated missions. The estimation procedure used must have sufficient accuracy for preliminary design studies.

The purpose of this report is to present reference data and a procedure by which the effect of certain design parameters, upon the basic performance of orbit-launched nuclear vehicles, can be determined. Lunar transportation systems will require large initial gross weights in earth orbit. Conventional chemical vehicles are comparatively inefficient for the orbit-launch boost phase of such missions. Hence, the range of specific impulse assumed in this report is restricted to that considered practical for nuclear heat-exchanger propulsion systems, namely 700 to 1000 seconds. While the reference data presented are restricted to nuclear rocket systems, the general method of performance estimation can be applied with equivalent success to any orbit-launched system.

The method of calculation of the parametric data presented in this report is primarily based on the analysis of Reference 2; and, the performance evaluation procedure is based on that of Reference 1. Since the scope of Reference 1 is limited to escape and planetary missions, this report extends that work to include a range of lunar missions with transfer times of 49.75 hours (earth escape) through 120 hours. Although the work done in Reference 2 covers lunar missions, it is concerned primarily with the influence of structural weight parameters on the optimum thrust-to-weight ratio of orbit-launched vehicles. Hence, the method of calculation is briefly presented in this report for ease of reference.

The reference transfer time assumed in this investigation is 70 hours, because it is representative of current lunar mission studies. Only transfer times between 49.75 hours (earth escape) and 120 hours have been considered. Performance evaluation procedures for transfer times less than 49.75 hours can be found in Reference 1, where the missions correspond to earth escape and hyperbolic excess speeds.

It is pointed out in Reference 2 that tangential thrust vector orientation is near optimum for near escape, escape, and particularly for hyperbolic excess speeds; hence, tangential thrust is the only mode considered in this report. A single stage with constant thrust is assumed from earth orbit to final injection. Initial thrust-to-weight ratio, F/W_o, is varied from 0.09 to 1.0, with both limits being arbitrarily selected. Initial orbit altitudes are assumed to lie between 185.2 and 740.8 km (100 and 400 N.M.).

Reference curves for each of the parameters are presented for the specific impulse range considered, along with correction curves for variations in transfer time and initial orbit altitude. Variations in vehicle structural and engine weights can be accounted for in one algebraic equation.

Calculations were made by numerical integration on the IBM 7094 digital computer. The equations of motion are related to a spherical earth. The analysis is limited to planar trajectories with all aerodynamic and perturbative forces being neglected.

SECTION II. ASSUMPTIONS

The basic assumptions made in the analysis are summarized as follows:

1. Acceleration of a single stage out of a circular orbit, with constant tangential thrust.

2. Range of independent performance parameters:

I_{sp} = 700 - 1000 sec

T_{tr} = 49.75 - 120 hr

h_o = 185.2 - 740.8 km (100 - 400 N.M.)

F/W_o = 0.09 - 1.00

3. Reference conditions:

T_{tr} = 70 hr

h_o = 555.6 km (300 N. M.)

A = B = 0.1

C = 0

4. Mean spherical earth model with:

$\mu_\oplus = 398,613 \frac{km^3}{s^2}$

$r_\oplus$ = 6371.2 km (3440.2 N. M.)

SECTION III. ANALYSIS

The impulsive velocity requirements, for injection into a transfer orbit having 80-km periselenum, were computed using Figure 1. The velocities in Figure 1 were calculated assuming horizontal injection ($\theta = 90°$), whereas the flight path angle at injection in this analysis was not restricted to this condition. It is pointed out in Reference 2 that the errors introduced by this approach are negligible. The method of calculation is based on the sphere-of-influence concept, reducing the n-body problem to two 2-body problems.

Since the thrust is assumed to be tangential ($\beta = 0°$), no attitude restriction can be placed on the resulting trajectory. Hence, the attainment of the desired transfer time injection velocity is assumed to be a sufficient end condition.

From Reference 1, the general equations of motion for a vehicle leaving circular orbit are:

$$\dot{V} = \frac{F \cos \beta}{m} - g \cos \theta \qquad (1)$$

$$\dot{\theta} = \frac{F \sin \beta}{mV} + \left(\frac{g}{V} - \frac{V}{r}\right) \sin \theta \qquad (2)$$

Numerical integration of the equations of motion determine the velocity and flight path angle. The range and altitude are then calculated by:

$$x = \int \frac{r_\oplus}{r} V \sin \theta \, dt$$

$$h = h_o + \int V \cos \theta \, dt$$

The central angle is determined by:

$$\psi = \int \frac{\dot{x}}{r_\oplus} \, dt$$

It should be noted that the initial thrust-to-weight ratio is hidden in Equation 1, since

$$\frac{F}{m_o} = \left(\frac{F}{W_o}\right) g_n$$

In Reference 1 it is pointed out that the weight characteristics of a vehicle must be defined before an evaluation of payload performance can be made. Defining the weight characteristics, as in Reference 1, the first group, A, is composed of the engine, propulsion system hardware, and structural members which may be assumed proportional to thrust. The second group, B, is composed of propellant tankage and any propellant residuals which may be assumed proportional to propellant loading; and the third group, C, contains interstage structure, astrionic gear, and various miscellaneous equipment which may be assumed proportional to the initial weight of the vehicle. The net effective structural weight is expressed as

$$W_N = AF + BW_p + CW_o$$

Dividing the above equation by W_o to nondimensionalize, gives

$$\epsilon_N = A\,(F/W_o) + B\,\zeta + C \qquad (3)$$

The relation between stage payload, propellant loading and net structural weight is

$$W_L = W_o - W_p - W_N$$

Nondimensionalizing,

$$\lambda = 1 - \zeta - \epsilon_N \tag{4}$$

Rearranging Equations 3 and 4,

$$\lambda = 1 - A\,(F/W_o) - (B + 1)\,\zeta - C \tag{5}$$

where: $\lambda = \dot{W}_L / W_o$

$\zeta = W_p / W_o$

The propellant ratio is defined for constant thrust and specific impulse by

$$\zeta = \left(\frac{F}{W_o}\right) \frac{t_b}{I_{sp}} \tag{6}$$

All reference curves shown for payload are for the specific weight parameter C equal to zero. The effects of non-zero values of C are shown to be accounted for by a single algebraic equation later in this report.

Reference values for initial orbit altitude, transfer time, and specific weight parameters were assumed arbitrarily, but should be reasonable since they only provide a basis for the estimation procedure. The reference values chosen for this report are:

1. $T_{tr} = 70$ hr
2. $h_o = 555.6$ km (300 N. M.)
3. $A = B = 0.1$
4. $C = 0$

Reference curves are shown in Figures 2 through 8 for the various parameters. For preliminary design purposes, payload and characteristic velocity are perhaps the most important of all the performance parameters. Hence, the curves of payload ratio and characteristic velocity shown in Figure 2 and 3, respectively, are plotted on a highly expanded scale.

Once reference data are delineated for the individual parameters, consideration must be given to variations in vehicle specific weights, transfer time, and initial orbit altitude. The effects of such variations and the means of estimating each of the parameters will now be discussed.

A. PAYLOAD RATIO

Of all the parameters considered, only λ is affected by changes in A and B. For reference conditions, Equation 5 can be written

$$\lambda_{ref} = 1 - A_{ref} (F/W_o) - (B_{ref} + 1) \zeta_{ref} \tag{7}$$

where both λ_{ref} and ζ_{ref} are functions of F/W_o and I_{sp}, as illustrated in Figures 2 and 4, respectively. Arbitrary values of A and B can be expressed in the form,

$$\begin{aligned} A &= A_{ref} + \Delta A \\ &= 0.1 + \Delta A \end{aligned} \tag{8}$$

$$\begin{aligned} B &= B_{ref} + \Delta B \\ &= 0.1 + \Delta B \end{aligned} \tag{9}$$

Hence, for arbitrary values of A and B, Equation 7 becomes

$$(\lambda_{ref})_{A, B} = 1 - A (F/W_o) - (B + 1) \zeta_{ref}$$

where the terminology is adopted that subscripts outside parentheses, or brackets, enclosing one of the parameters indicate non-reference conditions. Hence $(\lambda_{ref})_{A, B}$ denotes λ at reference conditions with the exception of A and B which are non-reference. It follows that the change in payload ratio from reference conditions due to a change in A and B is

$$\Delta\lambda_{A, B} = (\lambda_{ref})_{A, B} - \lambda_{ref} = - [\Delta A(F/W_o) + \Delta B (\zeta_{ref})] \tag{10}$$

Changes in λ due to variations in T_{tr}, for a given I_{sp}, can be expressed as

$$(\Delta\lambda)_{T_{tr}} = (\lambda_{ref})_{T_{tr}} - \lambda_{ref} \tag{11}$$

Calculations were made varying T_{tr} in increments of 5, 10, and 20 hours over the range considered. The changes in payload ratio were computed using Equation 11 and are shown in Figure 9. Linear interpolation between T_{tr} curves is sufficiently accurate for preliminary design purposes.

The values of $\Delta\lambda_{T_{tr}}$ are independent of the vehicle specific weight parameters assumed, but are slightly dependent upon initial orbit altitude. However, it was found that within the altitude range considered, this dependency can be neglected.

Changes in the initial orbit altitude, other reference conditions remaining constant, also must be accounted for in estimating payload ratio. It was discovered that the change in payload ratio for an incremental change in altitude is approximately the same over the range of altitude considered, regardless of the value of I_{sp}. Consequently, altitude and transfer time corrections to the reference payload ratio can be made independently. The same applies to the other parameters.

Generally initial orbit altitudes will be lower than 555.6 km (300 N. M.); therefore the corrections for initial altitudes will be based upon a 185.2-km (100-N. M.) incremental decrease. The change in reference payload ratio for an incremental decrease of 185.2 km (100 N. M.) is shown in Figure 10. Linear interpolation between curves presented for the assumed value of I_{sp} is sufficient, since altitude corrections are generally small. For arbitrary initial altitude, h_o, the change in payload is given by

$$(\Delta\lambda)_{h_o} = \frac{(h_o)_{ref} - h_o}{185.2} (\Delta\lambda_{h_o})_{185.2} = \frac{555.6 - h_o}{185.2} (\Delta\lambda_{h_o})_{185.2} \qquad (12)$$

where $(\Delta\lambda_{h_o})_{185.2}$ is the correction shown in Figure 10, $(h_o)_{ref}$ and h_o are measured in km.

Combining the corrections given by Equations 10, 11, and 12, the payload ratio for any assumed set of conditions can be found by

$$\lambda_{A, B, T_{tr}, h_o} = \lambda_{ref} + \Delta\lambda_{A, B} + \Delta\lambda_{T_{tr}} + \Delta\lambda_{h_o}$$

The effect of non-zero values of C can now be shown. Since C is a function of W_o only, it can be subtracted directly from $(\lambda)_{C = 0}$. Hence

$$\lambda_{A, B, C, T_{tr}, h_o} = \lambda_{A, B, T_{tr}, h_o} - C$$

or

$$\lambda_{A, B, C, T_{tr}, h_o} = \lambda_{ref} + \Delta\lambda_{A, B} + \Delta\lambda_{T_{tr}} + \Delta\lambda_{h_o} - C \tag{13}$$

Using this relation, the payload ratio for a particular vehicle with a given mission can be computed.

B. CHARACTERISTIC VELOCITY

The characteristic velocity, ΔV_{id}, representing a combination of the inertial velocity requirement and gravity losses, is computed in a manner simular to that described for estimating payload ratio. Reference curves for ΔV_{id} are presented in Figure 3. Correction curves for change in transfer time requirement and initial orbit altitude are presented in Figure 11 and 12, respectively. The corrections for T_{tr} were computed from the relation,

$$\Delta(\Delta V_{id})_{T_{tr}} = [(\Delta V_{id})_{ref}]_{T_{tr}} - (\Delta V_{id})_{ref} \tag{14}$$

which is analogous to Equation 11.

The change in reference characteristic velocity for an incremental altitude decrease of 185.2 km (100 N. M.) is shown in Figure 11 and corrections for arbitrary initial altitudes are given by,

$$(\Delta V_{id})_{h_o} = \frac{(h_o)_{ref} - h_o}{185.2} \left[\Delta(\Delta V_{id})_{h_o}\right]_{185.2}$$

$$= \frac{555.6 - h_o}{185.2} \left[\Delta(\Delta V_{id})_{h_o}\right]_{185.2} \tag{15}$$

with $\left[\Delta(\Delta V_{id})_{h_o}\right]_{185.2}$ determined from Figure 12.

The characteristic velocity for arbitrary T_{tr} and h_o is then given by

$$(\Delta V_{id})_{T_{tr}, h_o} = (\Delta V_{id})_{ref} + \Delta(\Delta V_{id})_{T_{tr}} + \Delta(\Delta V_{id})_{h_o} \tag{16}$$

C. PROPELLANT RATIO

With the value of ΔV_{id} corrected to the desired conditions, the propellant ratio may be determined using the familiar relation

$$\Delta V_{id} = g_n \, I_{sp} \ln \frac{1}{1 - \zeta}$$

rearranging,

$$(\zeta)_{T_{tr}, h_o} = 1 - \exp\left[\frac{-(\Delta V_{id})_{T_{tr}, h_o}}{g_n \, I_{sp}}\right] \tag{17}$$

The reference curves for ζ, shown in Figure 4, are used to determine payload ratio, λ, in Equation 10.

D. BURNING TIME

As in the calculation of ζ, burning time is readily determined from Equation 6. Hence,

$$(t_b)_{T_{tr}, h_o} = \frac{I_{sp} (\zeta)_{T_{tr}, h_o}}{(F/W_o)} \qquad (18)$$

The reference burning time curves in Figure 5 are shown only for the purpose of illustrating the typical variation with F/W_o and I_{sp}.

E. FLIGHT PATH ANGLE AND CENTRAL ANGLE

The procedures for estimating the flight path angle and the central angle are identical to the procedure outlined for calculating ΔV_{id} and they are discussed together. Thus, analogous to Equations 14 and 15, the corrections for θ and ψ with T_{tr} are given by

$$(\Delta\theta)_{T_{tr}} = (\theta_{ref})_{T_{tr}} - \theta_{ref} \qquad (19)$$

and

$$(\Delta\psi)_{T_{tr}} = (\psi_{ref})_{T_{tr}} - \psi_{ref} \qquad (20)$$

These corrections are shown in Figures 13 and 15, respectively.

The corrections for θ and ψ for arbitrary h_o are determined from the relations,

$$(\Delta\theta)_{h_o} = \frac{555.6 - h_o}{185.2} (\Delta\theta_{h_o})_{185.2} \qquad (21)$$

and

$$(\Delta\psi)_{h_o} = \frac{555.6 - h_o}{185.2} (\Delta\psi_{h_o})_{185.2} \qquad (22)$$

where $(\Delta\theta_{h_o})_{185.2}$ and $(\Delta\psi_{h_o})_{185.2}$, the corrections for a 185.2-km (100-N. M.) incremental decrease in initial orbit altitude, are given in Figures 14 and 16. It was discovered that the initial orbit altitude corrections for these parameters were insensitive to specific impulse variations. Hence, the resulting corrections are presented by only one curve for each parameter.

It follows that, for arbitrary conditions,

$$(\theta)_{T_{tr}, h_o} = \theta_{ref} + (\Delta\theta)_{T_{tr}} + (\Delta\theta)_{h_o} \tag{23}$$

and

$$(\psi)_{T_{tr}, h_o} = \psi_{ref} + (\Delta\psi)_{T_{tr}} + (\Delta\psi)_{h_o} \tag{24}$$

F. ALTITUDE AT INJECTION

The procedure for estimating the altitude at injection is the same as for the previous parameters. Thus, the correction for T_{tr} is given by

$$(\Delta h_b)_{T_{tr}} = [(h_b)_{ref}]_{T_{tr}} - (h_b)_{ref} \tag{25}$$

where $(\Delta h_b)_{T_{tr}}$ is shown in Figure 17.

The correction for h_b for arbitrary h_o is determined from the relation,

$$(\Delta h_b)_{h_o} = \frac{555.6 - h_o}{185.2} [(\Delta h_b)_{h_o}]_{185.2} \tag{26}$$

where $[(\Delta h_b)_{h_o}]_{185.2}$, the correction for 185.2-km incremental decrease of initial altitude, is given in Figure 18.

Then for arbitrary conditions,

$$(h_b)_{T_{tr}, h_o} = (h_b)_{ref} + (\Delta h_b)_{T_{tr}} + (\Delta h_b)_{h_o} \tag{27}$$

This completes the estimation procedures for all the parameters considered.

SECTION IV. ACCURACY EVALUATION

Results of several numerical examples are compared with exact computer solutions and the associated errors for each parameter are included in Table 2. Two of the numerical examples are presented in detail in Appendix A for the purpose of illustrating the performance estimation procedure. One example is included, in which the initial orbit altitude is outside the range assumed, to determine the accuracy with which the data can be extrapolated.

The assumed data for the numerical examples were deliberately chosen so that interpolation was necessary in most cases. Hence, the results should be indicative of the maximum errors which can be expected in practical applications of the procedure. Since the examples are not numerous enough to represent a statistical sampling, the errors shown should not be construed as being the most probable.

SECTION V. CONCLUSIONS

1. Several numerical examples were worked to test the procedure for both speed and accuracy. From the experience gained by working these examples it is concluded that the procedure is rapid and has sufficient accuracy for preliminary design studies.

2. From the results of the numerical examples and their associated accuracy, presented in Table 2, it is concluded that the average errors that can be expected in a particular application will be less than 1 per cent for all the parameters considered. The maximum errors associated with the various parameters that can be expected with careful use of the procedure are as follows:

λ, ΔV_{id}, ζ, t_b, θ, ψ - - - - $< 1\%$

h_b - - - - - - - - - - - - - - - - $< 2\%$

The burnout altitude estimation exceeds 1 per cent only for the cases involving low thrust-to-weight ratios; however, the accuracy is still within preliminary design requirements.

3. Performance estimations can be made for any nuclear vehicle and any lunar mission profile, within the range assumed for the associated parameters. Also, this method of performance evaluation should have the capability of being extended to any orbit-launched systems with equivalent success.

APPENDIX A

NUMERICAL EXAMPLES

The purpose of these numerical examples is to illustrate the methods utilized in the estimation procedure. Five numerical examples were solved and the results are presented in Table 2; however, only two of these are presented here.

A. NUMERICAL EXAMPLE 1

The first step in the estimation procedure is the determination of reference values, since these are the basis for the procedure. The arbitrarily assumed conditions for Example 1 are presented below. At the values of $F/W_o = 0.2$ and $I_{sp} = 735$ seconds, the necessary reference values can be read from Figures 2, 3, 4, 6, and 7 as follows:

Assumed Conditions	Reference Values
$F/W_o = 0.2$	$\lambda_{ref} = 0.5863$
$I_{sp} = 735$ s	$\zeta_{ref} = 0.3579$
$A = 0.2$	$(\Delta V_{id})_{ref} = 3197$ m/s
$B = 0.15$	$\psi_{ref} = 1.5275$ rad $(87.5°)$
$C = 0.0$	$\theta_{ref} = 1.1572$ rad $(66.3°)$
$T_{tr} = 85$ hr	$(h_b)_{ref} = 2300$ km
$h_o = 277.8$ km	

The first performance parameter discussed, λ, is probably the most important one. Corrections of λ_{ref} are necessary for changes in A, B, C, T_{tr}, and h_o. For the assumed values of A and B, Equations 8 and 9 yield:

$$\Delta A = A - 0.10$$

$$= 0.20 - 0.10$$

$$= +0.10$$

$$\Delta B = B - 0.10$$

$$= 0.15 - 0.10$$

$$= +0.05$$

From Equation 10 the change in λ_{ref} due to non-reference values of A and B is

$$\Delta\lambda_{A,B} = -[\Delta A(F/W_o) + \Delta B(\zeta_{ref})]$$

$$= -[0.10\,(0.20) + 0.05\,(0.3579)]$$

$$= -0.0379$$

From Figure 9 the change in λ_{ref} due to non-reference T_{tr} is

$$\Delta\lambda_{T_{tr}} = +0.0017$$

From Figure 10 and Equation 12 the change due to non-reference h_o is

$$(\Delta\lambda_{h_o})_{185.2} = -0.0056$$

from Equation 12

$$\Delta\lambda_{h_o} = \frac{555.6 - h_o}{185.2}(\Delta\lambda_{h_o})_{185.2}$$

$$\Delta\lambda_{h_o} = \frac{555.6 - 277.8}{185.2}(-0.0056)$$

$$= -0.0084$$

Therefore the corrected value of λ is

$$\lambda = \lambda_{ref} + \Delta\lambda_{A,B} + \Delta\lambda_{T_{tr}} + \Delta\lambda_{h_o} - C$$

$$= 0.5863 - 0.0379 + 0.0017 - 0.0084 - 0$$

$$= +0.5417$$

The characteristic velocity is determined in a similar manner. The T_{tr} correction is found in Figure 11 to be

$$\Delta(\Delta V_{id})_{T_{tr}} = -16 \text{ m/s}$$

From Figure 12 and Equation 15 the correction in $(\Delta V_{id})_{ref}$ for non-reference h_o is

$$[\Delta(\Delta V_{id})_{h_o}]_{185.2} = +55.8 \text{ m/s}$$

from Equation 15

$$\Delta(\Delta V_{id})_{h_o} = \frac{555.6 - h_o}{185.2}[\Delta(\Delta V_{id})_{h_o}]_{185.2}$$

$$= \frac{555.6 - 277.8}{185.2}(55.8)$$

$$= +83.7 \text{ m/s}$$

Therefore from Equation 16,

$$\Delta V_{id} = (\Delta V_{id})_{ref} + \Delta(\Delta V_{id})_{T_{tr}} + \Delta(\Delta V_{id})_{h_o}$$

$$= 3197 - 16 + 83.7$$

$$= +3265 \text{ m/s}$$

The propellant ratio is determined from Equation 17 as follows,

$$\zeta = 1 - \exp\left[\frac{-(\Delta V_{id})_{T_{tr}, h_o}}{g_n I_{sp}}\right]$$

$$= 1 - \exp\left[\frac{-3265}{9.82\ (735)}\right]$$

$$= +0.3638$$

The burning time is found from Equation 18,

$$t_b = \frac{I_{sp}\ (\zeta)_{T_{tr}, h_o}}{F/W_o}$$

$$= \frac{735\ (0.3638)}{0.20}$$

$$= +1337 \text{ sec}$$

The T_{tr} correction for θ_{ref} is found in Figure 13 to be,

$$\Delta\theta_{T_{tr}} = +0.0019 \text{ rad}$$

From Figure 14 and Equation 21 the change in θ_{ref} is,

$$(\Delta\theta_{h_o})_{185.2} = -0.0158 \text{ rad}$$

and from Equation 21,

$$\Delta\theta_{h_o} = \frac{555.6 - h_o}{185.2}\ (\Delta\theta_{h_o})_{185.2}$$

$$= \frac{555.6 - 277.8}{185.2}\ (-0.0158)$$

$$= -0.0236 \text{ rad}$$

The corrected value of θ, from Equation 23, is then

$$\theta = \theta_{ref} + \Delta\theta_{T_{tr}} + \Delta\theta_{h_o}$$

$$= 1.5272 + 0.0019 - 0.0236$$

$$= +1.1355 \text{ rad } (65.06^\circ)$$

The central angle at injection is determined by the same method. The T_{tr} correction read from Figure 15 is:

$$\Delta\psi_{T_{tr}} = -0.0063 \text{ rad}$$

From Figure 16 and Equation 22 the change in ψ_{ref} for non-reference h_o is:

$$(\Delta\psi_{h_o})_{185.2} = +0.0705 \text{ rad}$$

and from Equation 22,

$$\Delta\psi_{h_o} = \frac{555.6 - h_o}{185.2} (\Delta\psi_{h_o})_{185.2}$$

$$= \frac{555.6 - 277.8}{185.2} (+0.0705)$$

$$= +0.1058 \text{ rad}$$

The corrected value of ψ, from Equation 24, is

$$\psi = \psi_{ref} + \Delta\psi_{T_{tr}} + \Delta\psi_{h_o}$$

$$= 1.5272 - 0.0063 + 0.1058$$

$$= +1.6267 \text{ rad } (93.20^\circ)$$

The burnout altitude is determined in a similar way. The burnout altitude correction for non-reference transfer time is found in Figure 17 to be

$$(\Delta h_b)_{T_{tr}} = -21 \text{ km}$$

The correction for the non-reference h_o is determined by using Figure 18 and Equation 26. From Figure 18

$$[(\Delta h_b)_{h_o}]_{185.2} = -68 \text{ km}$$

and from Equation 26,

$$(\Delta h_b)_{h_o} = \frac{555.6 - h_o}{185.2} [(\Delta h_b)_{h_o}]_{185.2}$$

$$= \frac{555.6 - 277.8}{185.2} (-68)$$

$$= -102 \text{ km}$$

Therefore from Equation 27 the corrected value of h_b is

$$h_b = (h_b)_{ref} + (\Delta h_b)_{T_{tr}} + (\Delta h_b)_{h_o}$$

$$= 2300 - 21 - 102$$

$$= +2177 \text{ km}$$

B. NUMERICAL EXAMPLE 2

The arbitrarily assumed conditions for Example 2 are presented below. At the values of $F/W_o = 0.4$ and $I_{sp} = 980$ seconds, the necessary reference values can be read from Figures 2, 3, 4, 6, and 7 as follows:

Assumed Conditions	Reference Values
$F/W_o = 0.4$	$\lambda_{ref} = 0.6565$
$I_{sp} = 980$ s	$\zeta_{ref} = 0.2756$
$A = 0.09$	$(\Delta V_{id})_{ref} = 3107$ m/s
$B = 0.15$	$\psi_{ref} = 0.8465$ rad (48.5°)
$C = 0.02$	$\theta_{ref} = 1.3308$ rad (76.2°)
$T_{tr} = 78$ hr	$(h_b)_{ref} = 1090$ km
$h_o = 463.0$ km	

Corrections for λ_{ref} are necessary for changes in A, B, C, T_{tr}, and h_o. For the assumed values of A and B, Equations 8 and 9 yield:

$$\Delta A = A - 0.10$$

$$= 0.09 - 0.10$$

$$= -0.01$$

$$\Delta B = B - 0.10$$

$$= 0.15 - 0.10$$

$$= +0.05$$

From Equation 10 the change due to A and B is

$$\Delta\lambda_{A,B} = -[\Delta A\,(F/W_o) + \Delta B\,(\zeta_{ref})]$$

$$= -[-0.01\,(0.4) + 0.05\,(0.2756)]$$

$$= -0.0098$$

From Figure 9 the T_{tr} correction is

$$\Delta\lambda_{T_{tr}} = +0.0008$$

From Figure 10 and Equation 12 the h_o correction is

$$(\Delta\lambda_{h_o})_{185.2} = -0.0041$$

and from Equation 12,

$$\Delta\lambda_{h_o} = \frac{555.6 - h_o}{185.2} (\Delta\lambda_{h_o})_{185.2}$$

$$= \frac{555.6 - 463.0}{185.2} (-0.0041)$$

$$= -0.0021$$

Therefore, the corrected value of λ is

$$\lambda = \lambda_{ref} + \Delta\lambda_{A, B} + \Delta\lambda_{T_{tr}} + \Delta\lambda_{h_o} - C$$

$$= 0.6565 - 0.0098 + 0.0008 - 0.0021 - 0.0200$$

$$= +0.6254$$

The characteristic velocity is determined in a similar manner. From Figure 11 the T_{tr} correction is

$$\Delta(\Delta V_{id})_{T_{tr}} = -8.8 \text{ m/s}$$

From Figure 12 and Equation 15 the h_o correction is

$$[\Delta(\Delta V_{id})_{h_o}]_{185.2} = +45.5 \text{ m/s}$$

and from Equation 15,

$$\Delta(\Delta V_{id})_{h_o} = \frac{555.6 - h_o}{185.2} \left[\Delta(\Delta V_{id})_{h_o}\right]_{185.2}$$

$$= \frac{555.6 - 463.0}{185.2} (45.5)$$

$$= +22.8 \text{ m/s}$$

Therefore from Equation 16 the corrected value of ΔV_{id} is

$$\Delta V_{id} = (\Delta V_{id})_{ref} + \Delta(\Delta V_{id})_{T_{tr}} + \Delta(\Delta V_{id})_{h_o}$$

$$= 3107 - 8.8 + 22.8$$

$$= +3121 \text{ m/s}$$

The propellant ratio is determined from Equation 17 as follows:

$$\zeta = 1 - \exp\left[\frac{-(\Delta V_{id})_{T_{tr}, h_o}}{g_n \, I_{sp}}\right]$$

$$= 1 - \exp\left[\frac{-3121}{(9.82)(980)}\right]$$

$$= +0.2775$$

From Equation 18 the burning time is

$$t_b = \frac{I_{sp} (\zeta)_{T_{tr}, h_o}}{F/W_o}$$

$$= \frac{980 (0.2775)}{0.4}$$

$$= +679.9 \text{ sec}$$

In Figure 13 the T_{tr} correction for θ is

$$\Delta\theta_{T_{tr}} = +0.0061 \text{ rad}$$

From Figure 14 and Equation 21 the change in θ_{ref} is

$$(\Delta\theta_{h_o})_{185.2} = -0.0105 \text{ rad}$$

and from Equation 21,

$$\Delta\theta_{h_o} = \frac{555.6 - h_o}{185.2} (\Delta\theta_{h_o})_{185.2}$$

$$= \frac{555.6 - 463.0}{185.2} (-0.0105)$$

$$= -0.0052 \text{ rad}$$

The corrected value of θ is then,

$$\theta = \theta_{ref} + \Delta\theta_{T_{tr}} + \Delta\theta_{h_o}$$

$$= 1.3328 + 0.0061 - 0.0052$$

$$= +1.3263 \text{ rad } (76.0^\circ)$$

The central angle at injection is determined by a similar method. From Figure 15 the T_{tr} correction is

$$\Delta\psi_{T_{tr}} = -0.0028 \text{ rad}$$

From Figure 16 and Equation 22 the h_o correction is

$$(\Delta\psi_{h_o})_{185.2} = +0.0422 \text{ rad}$$

and from Equation 22,

$$\Delta\psi_{h_o} = \frac{555.6 - h_o}{185.2} (\Delta\psi_{h_o})_{185.2}$$

$$= \frac{555.6 - 463.0}{185.2} (0.0422)$$

$$= +0.0211 \text{ rad}$$

The corrected value of ψ is then,

$$\psi = \psi_{ref} + \Delta\psi_{T_{tr}} + \Delta\psi_{h_o}$$

$$= 0.8465 - 0.0028 + 0.0211$$

$$= +0.8648 \text{ rad } (49.55^\circ)$$

The burnout altitude is determined in a similar manner. The transfer time correction from Figure 17 is

$$(\Delta h_b)_{T_{tr}} = -5.0 \text{ km}$$

The correction for arbitrary initial altitude, from Figure 18 and Equation 26 is

$$(\Delta h_b)_{h_o \; 185.2} = -144 \text{ km}$$

and from Equation 26

$$(\Delta h_b)_{h_o} = \frac{555.6 - h_o}{185.2} [(\Delta h_b)_{h_o}]_{185.2}$$

$$= \frac{555.6 - 463.0}{185.2} (-144)$$

$$= -72 \text{ km}$$

The corrected value of h_b from Equation 27 is then

$$h_b = (h_b)_{ref} + (\Delta h_b)_{T_{tr}} + (\Delta h_b)_{h_o}$$

$$= 1090 - 5.0 - 72$$

$$= +1013 \text{ km}$$

The results of these examples, along with three others are compared with exact computer runs and the per cent errors associated with each of the parameters are presented in Table 2.

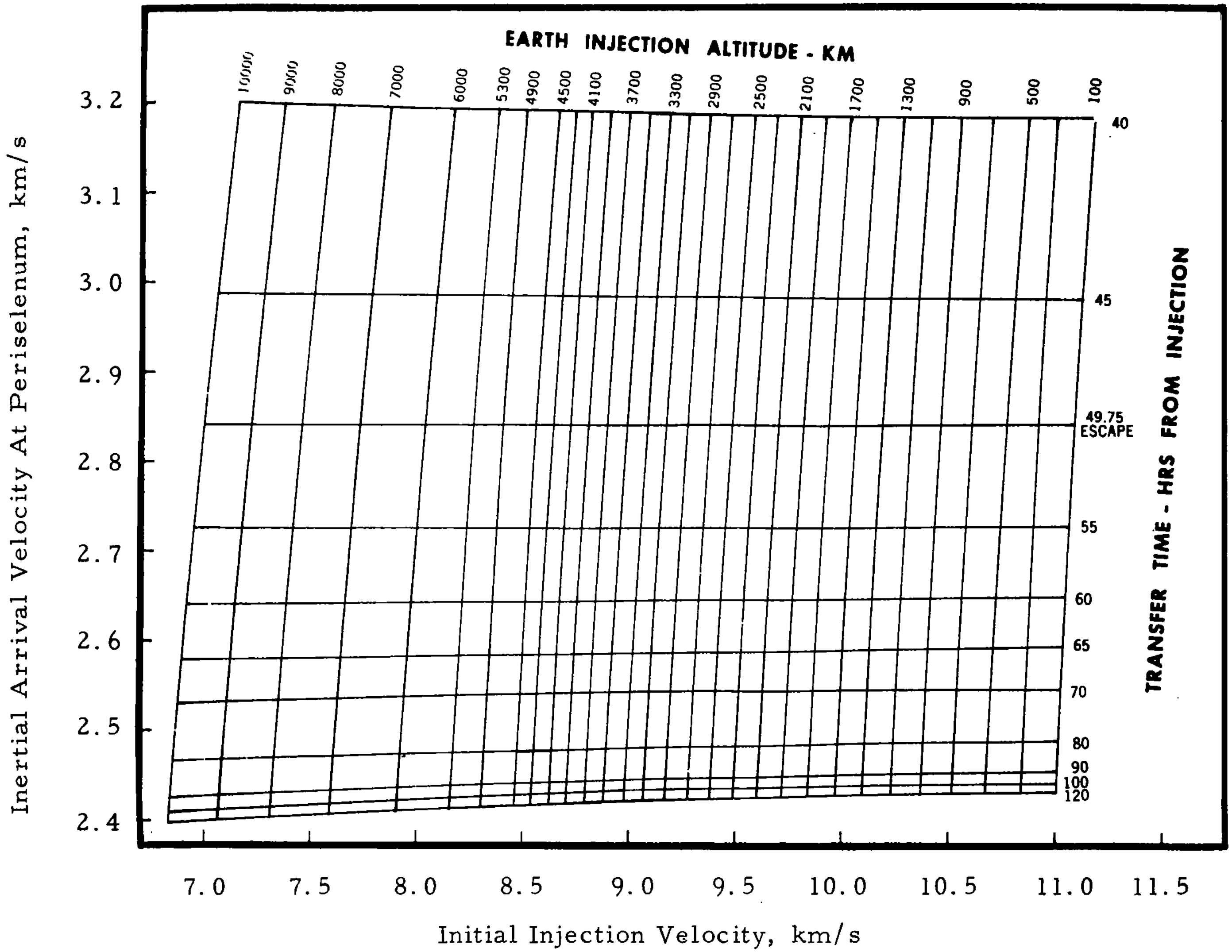

FIGURE 1. LUNAR MISSION ENERGY REQUIREMENTS
80-KM PERISELENUM

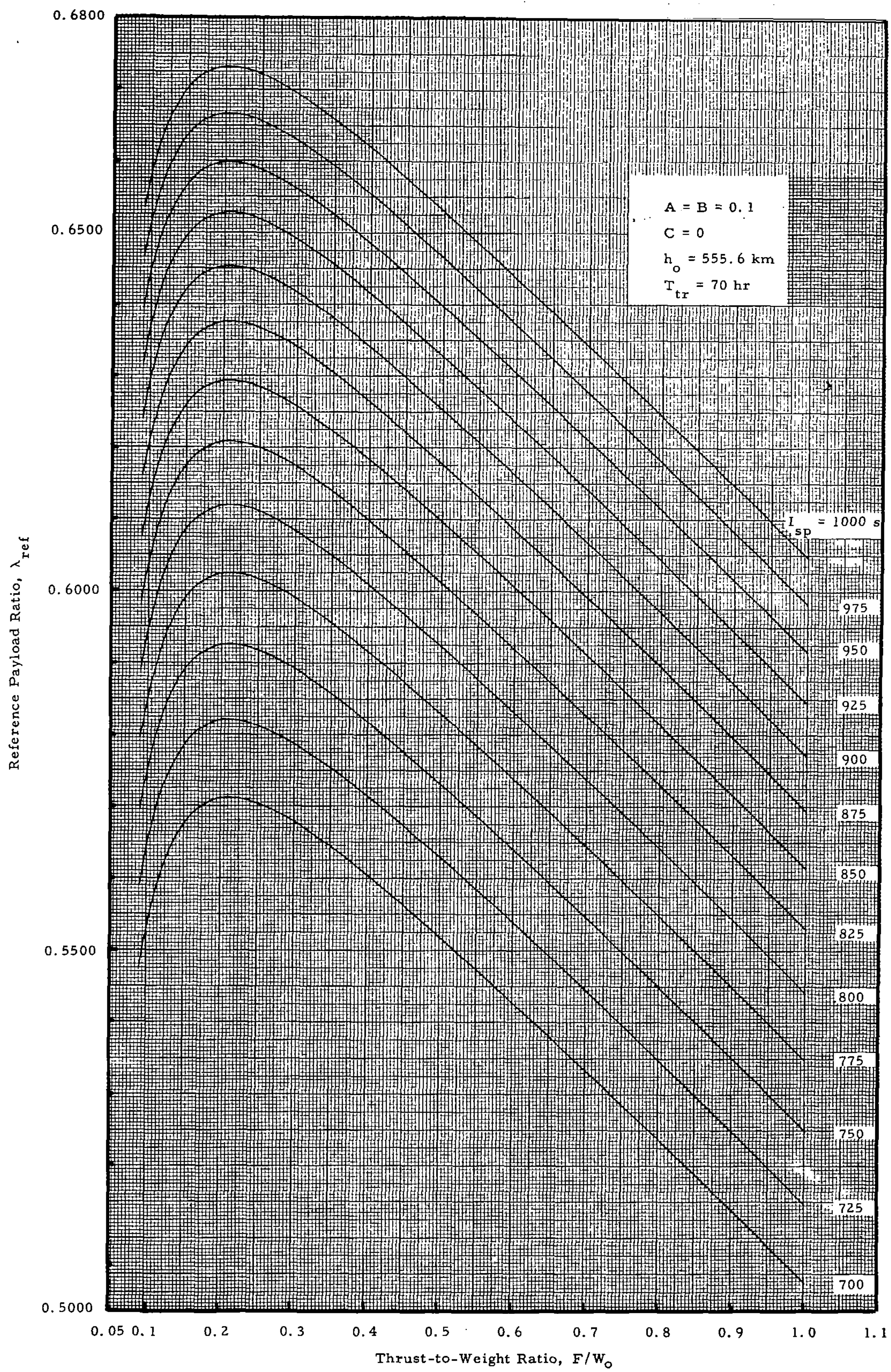

FIGURE 2. REFERENCE PAYLOAD RATIO

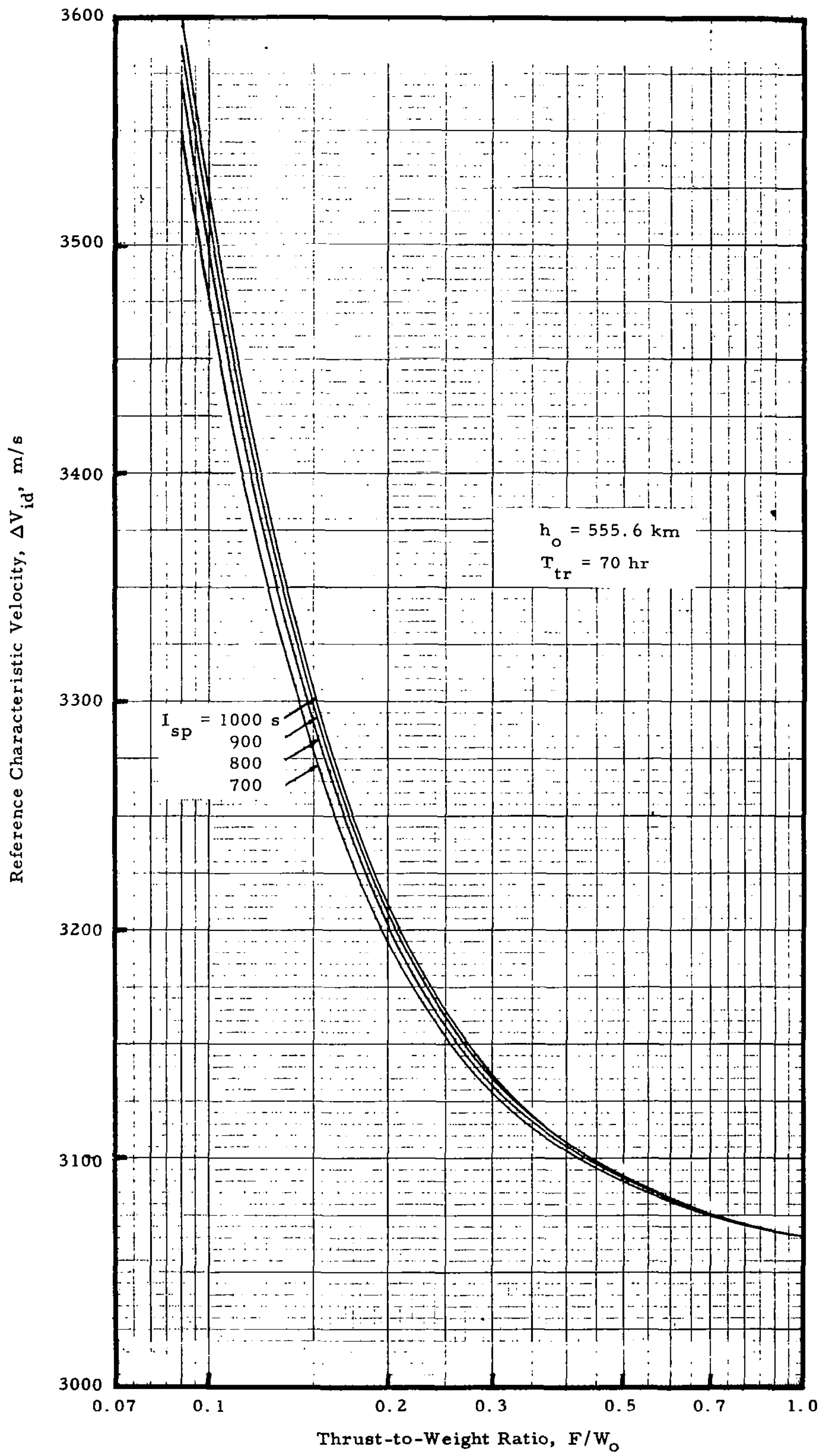

FIGURE 3. REFERENCE CHARACTERISTIC VELOCITY

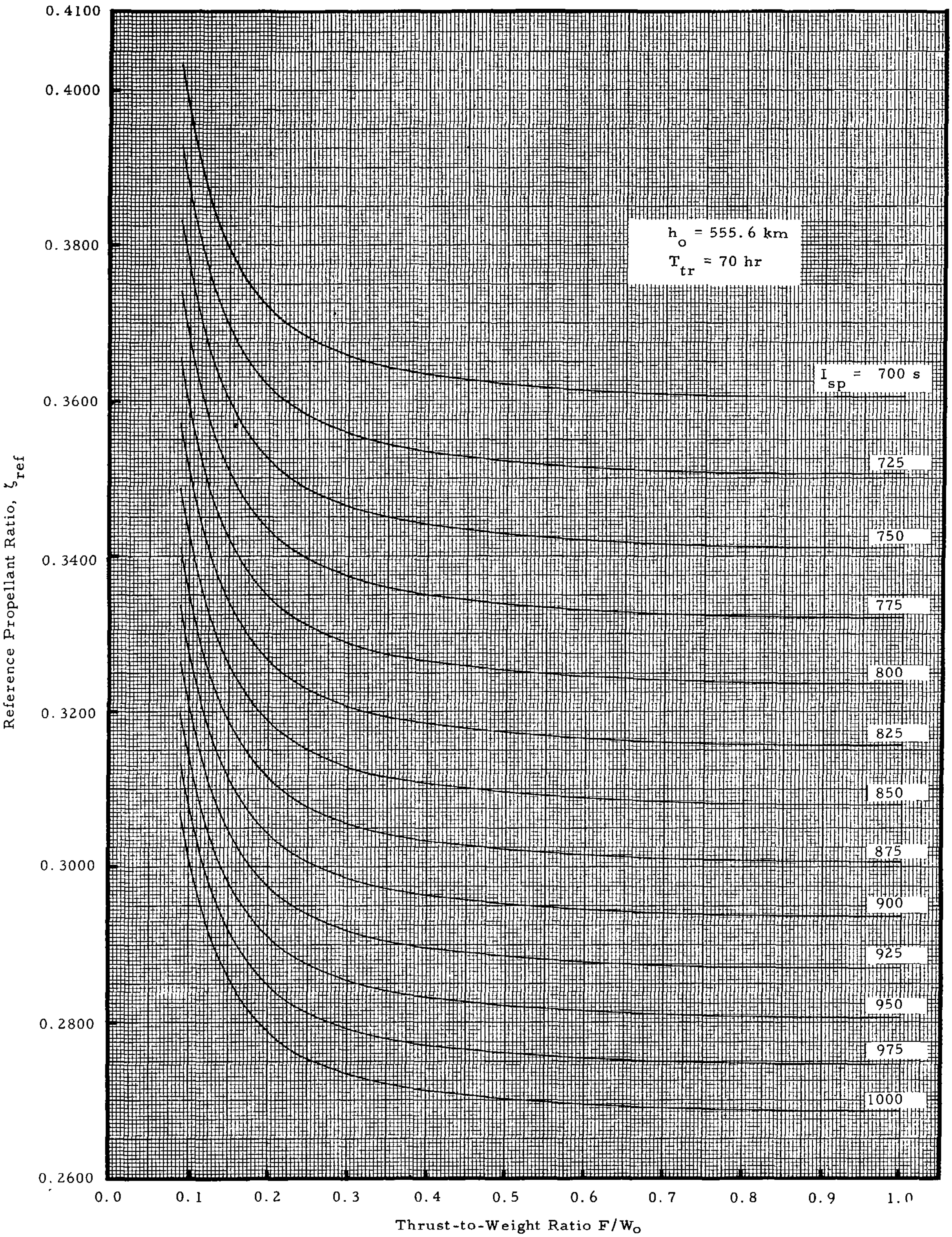

FIGURE 4. REFERENCE PROPELLANT RATIO

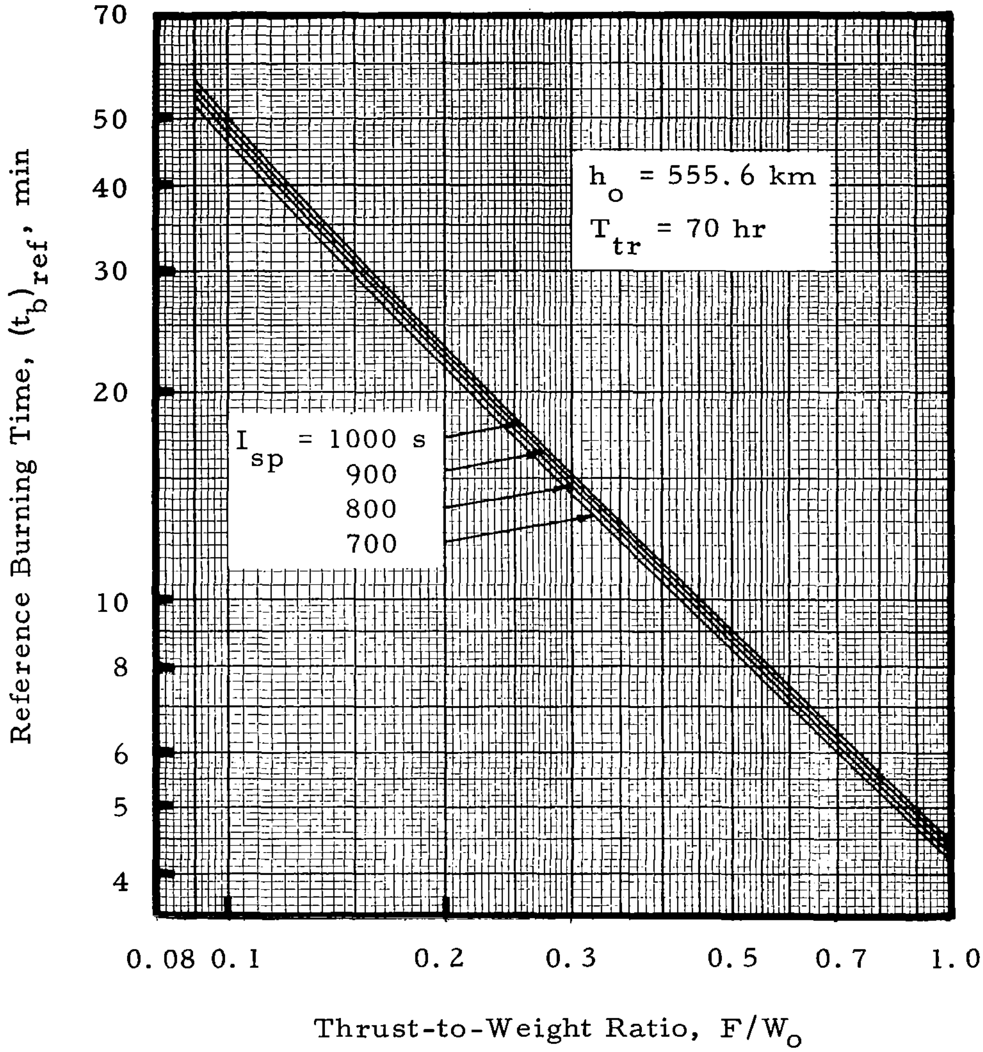

FIGURE 5. REFERENCE BURNING TIME TO INJECTION

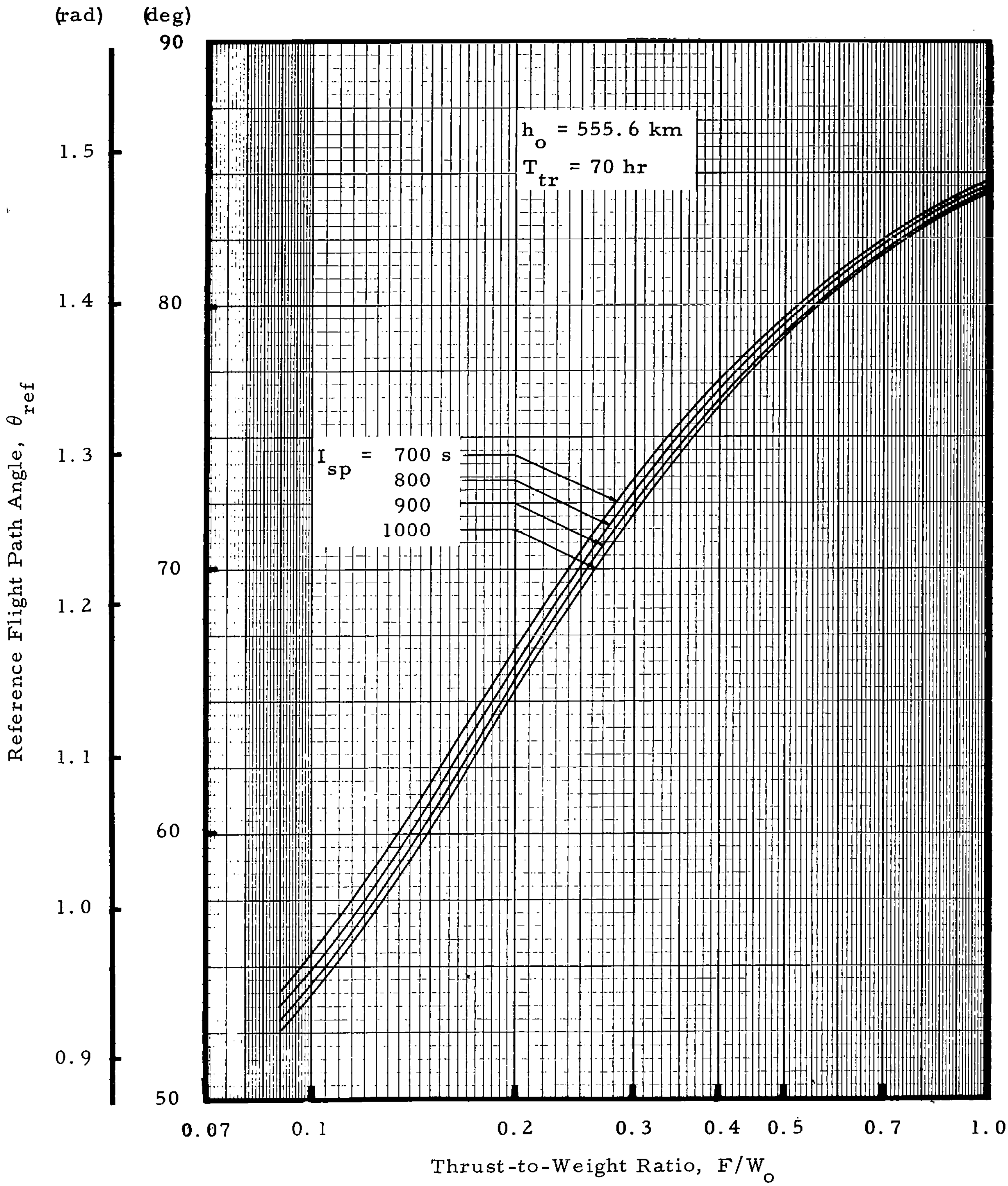

FIGURE 6. REFERENCE FLIGHT PATH ANGLE AT INJECTION

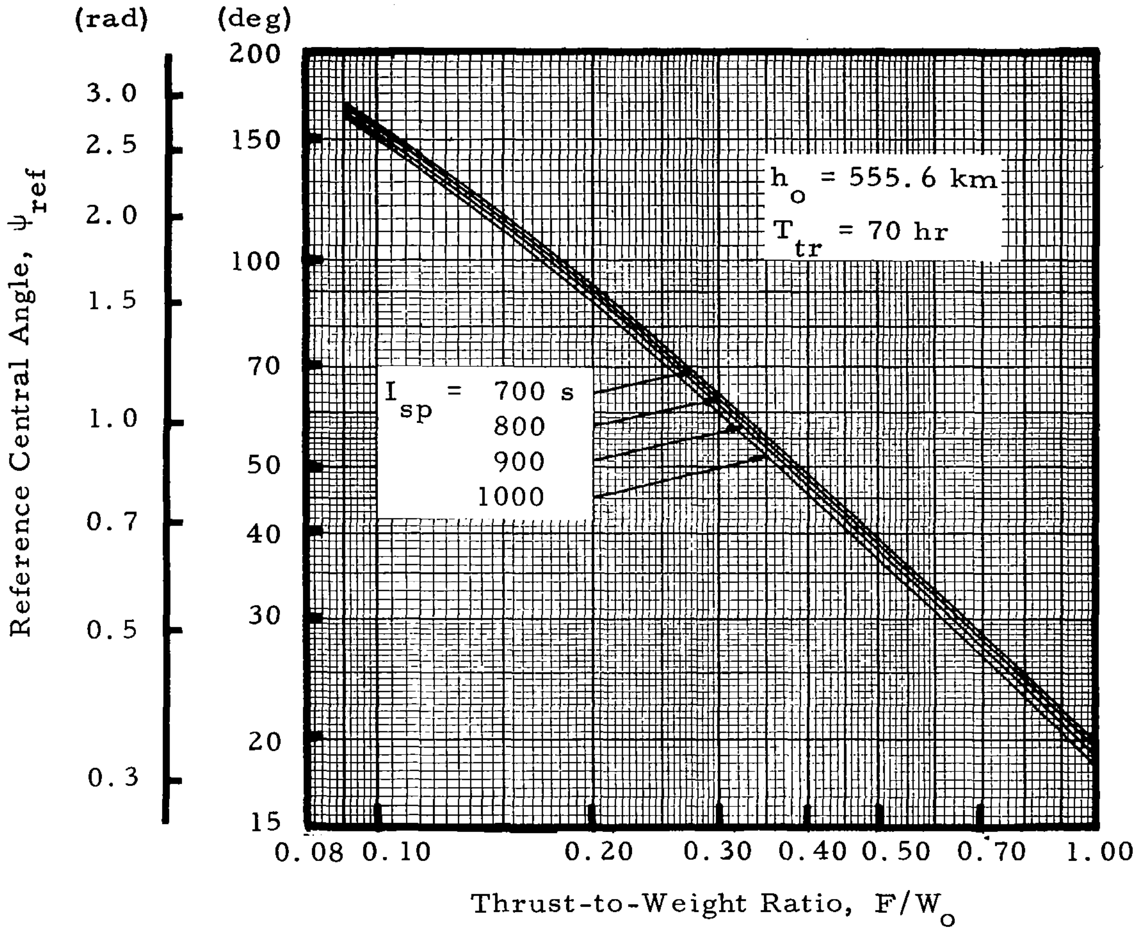

FIGURE 7. REFERENCE CENTRAL ANGLE AT INJECTION

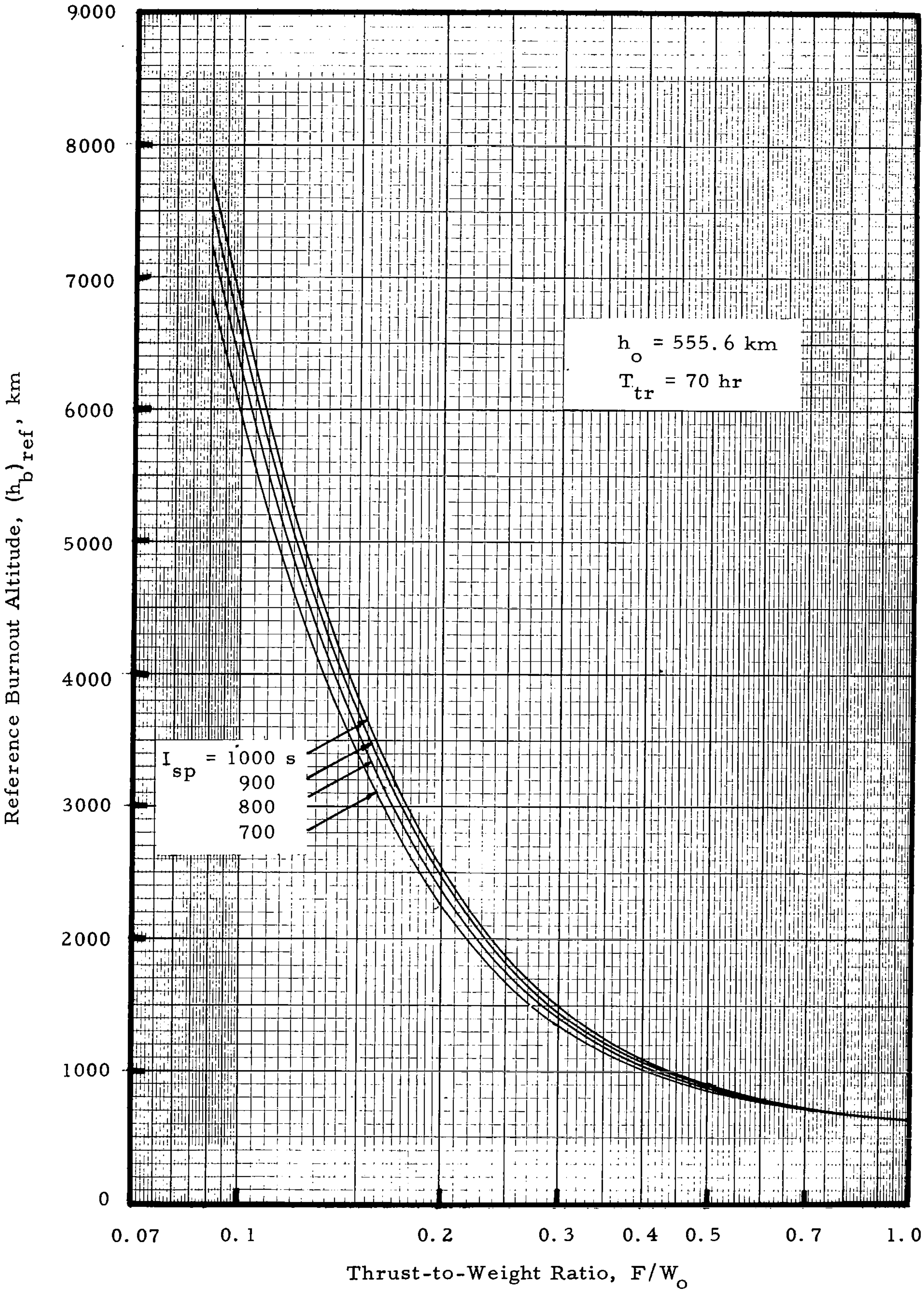

FIGURE 8. REFERENCE BURNOUT ALTITUDE

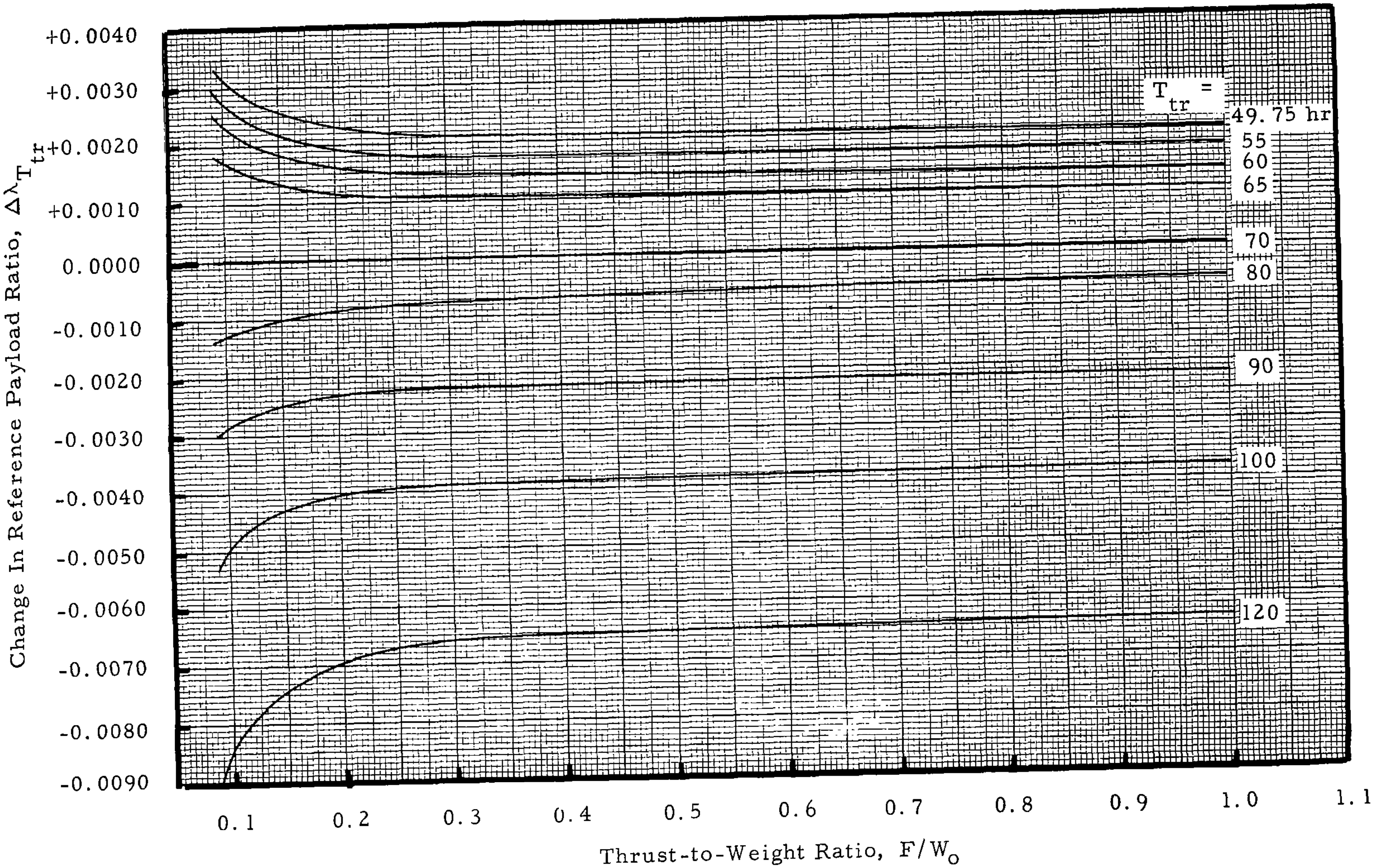

FIGURE 9. CHANGE IN REFERENCE PAYLOAD RATIO FOR NON-REFERENCE TRANSFER TIMES

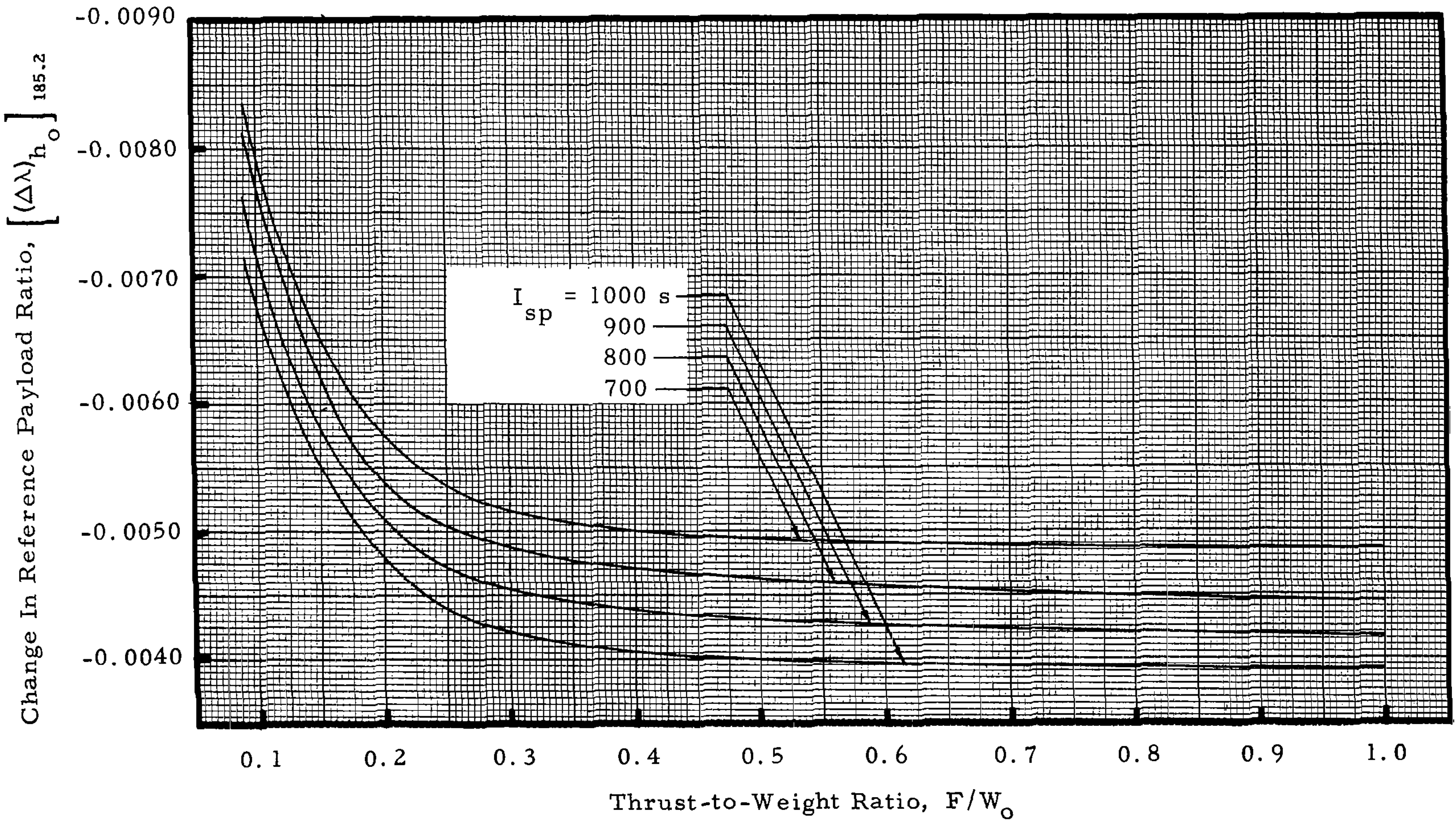

FIGURE 10. CHANGE IN REFERENCE PAYLOAD RATIO FOR INCREMENTAL ALTITUDE DECREASE OF 185.2 KM (100 N.M.)

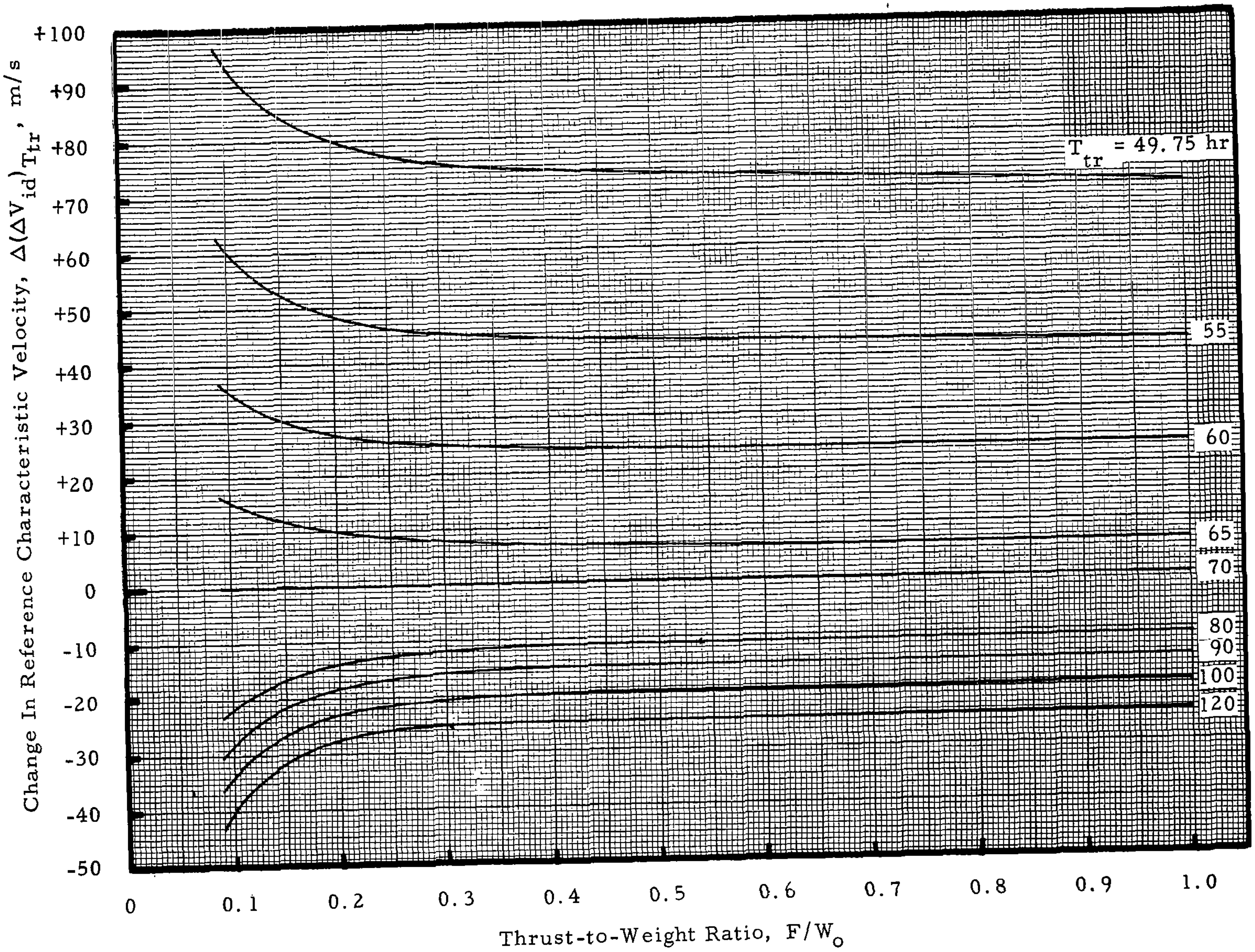

FIGURE 11. CHANGE IN REFERENCE CHARACTERISTIC VELOCITY FOR NON-REFERENCE TRANSFER TIMES

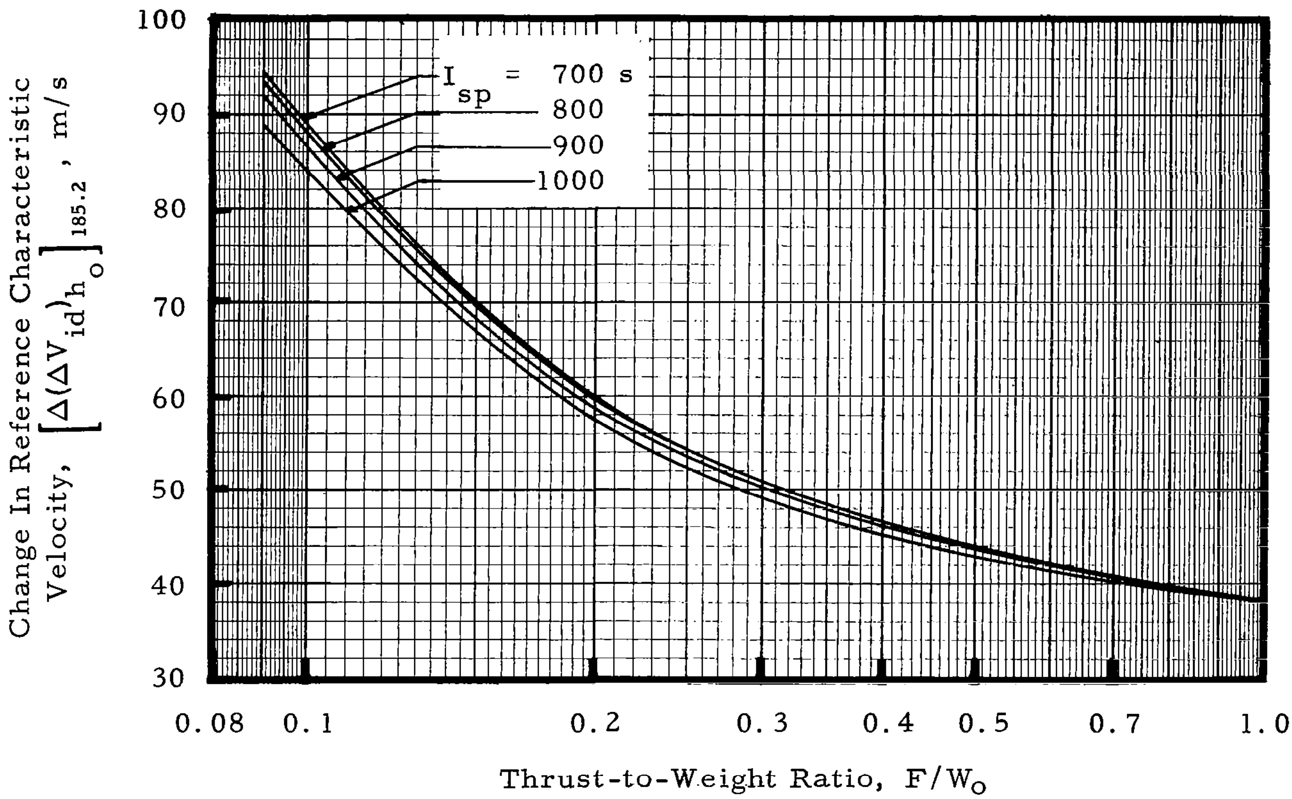

FIGURE 12. CHANGE IN REFERENCE CHARACTERISTIC VELOCITY FOR INCREMENTAL ALTITUDE DECREASE OF 185.2 KM (100 N.M.)

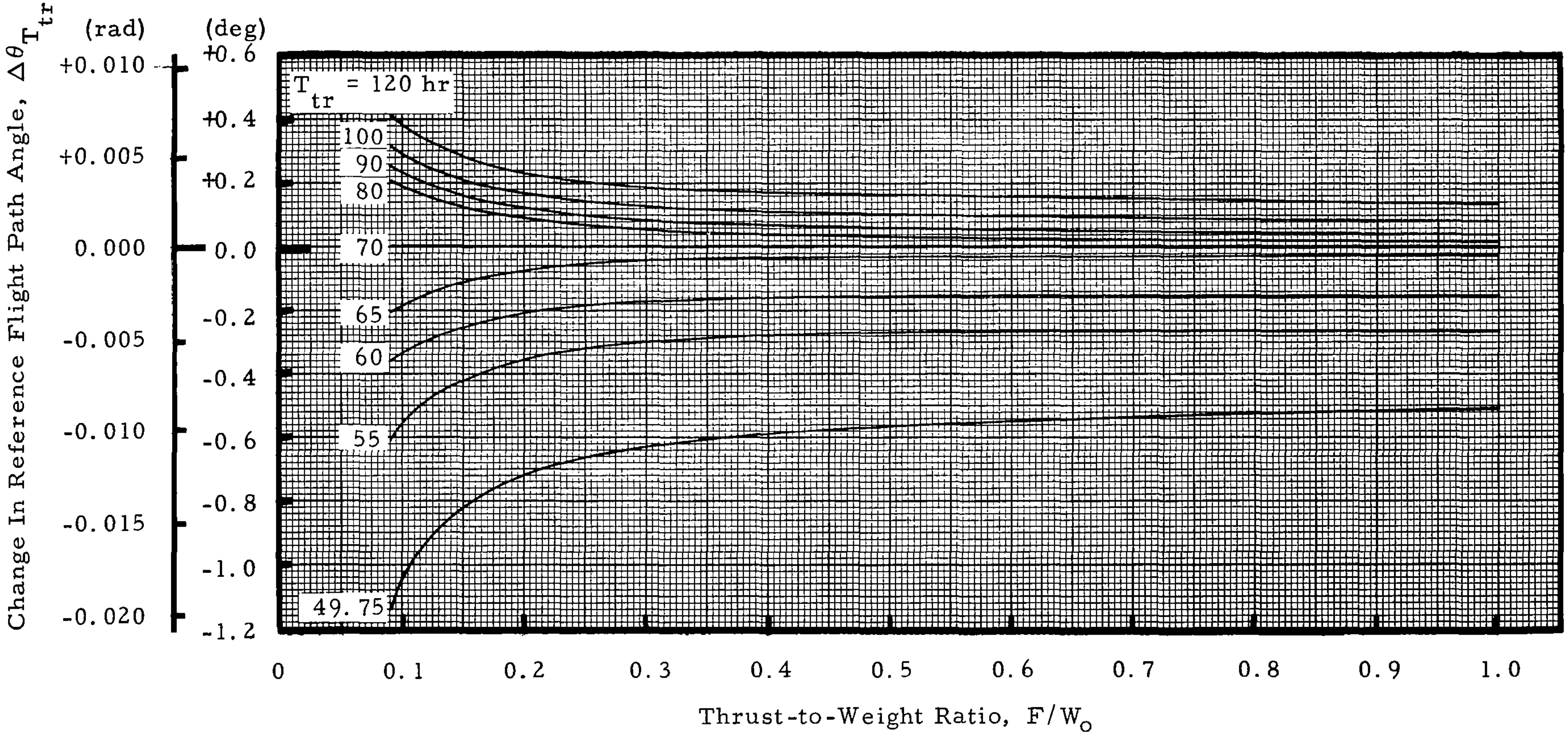

FIGURE 13. CHANGE IN REFERENCE FLIGHT PATH ANGLE FOR NON-REFERENCE TRANSFER TIMES

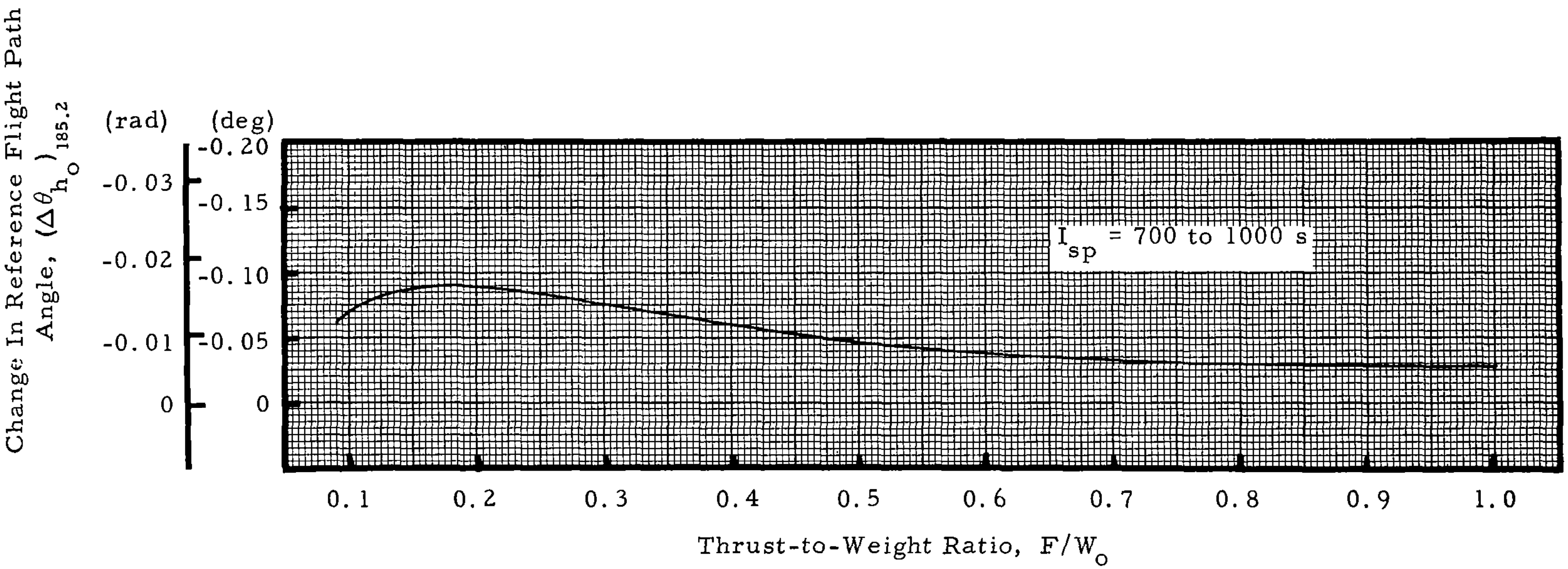

FIGURE 14. CHANGE IN REFERENCE FLIGHT PATH ANGLE FOR INCREMENTAL ALTITUDE DECREASE OF 185.2 KM (100 N. M.)

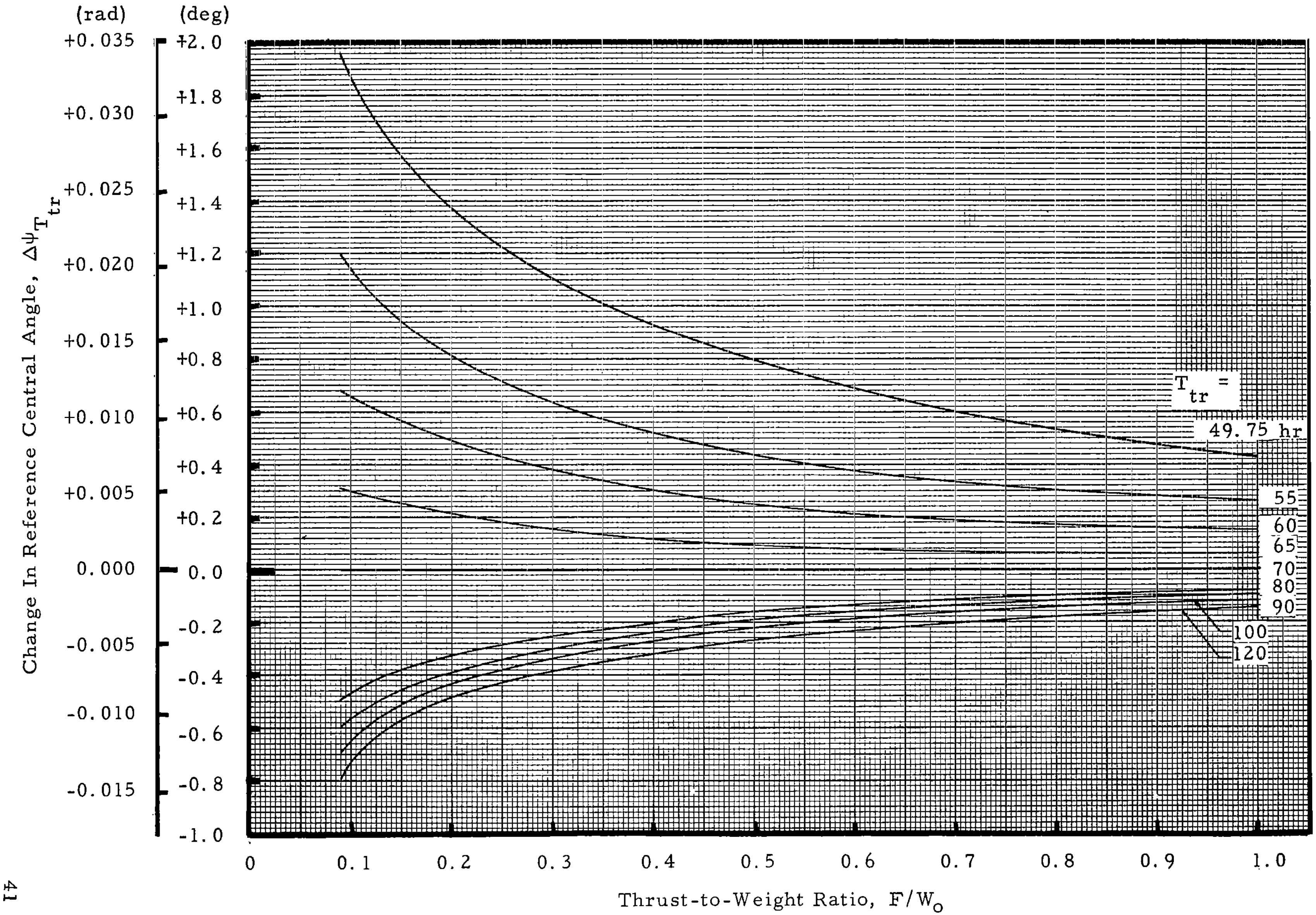

FIGURE 15. CHANGE IN REFERENCE CENTRAL ANGLE FOR NON-REFERENCE TRANSFER TIMES

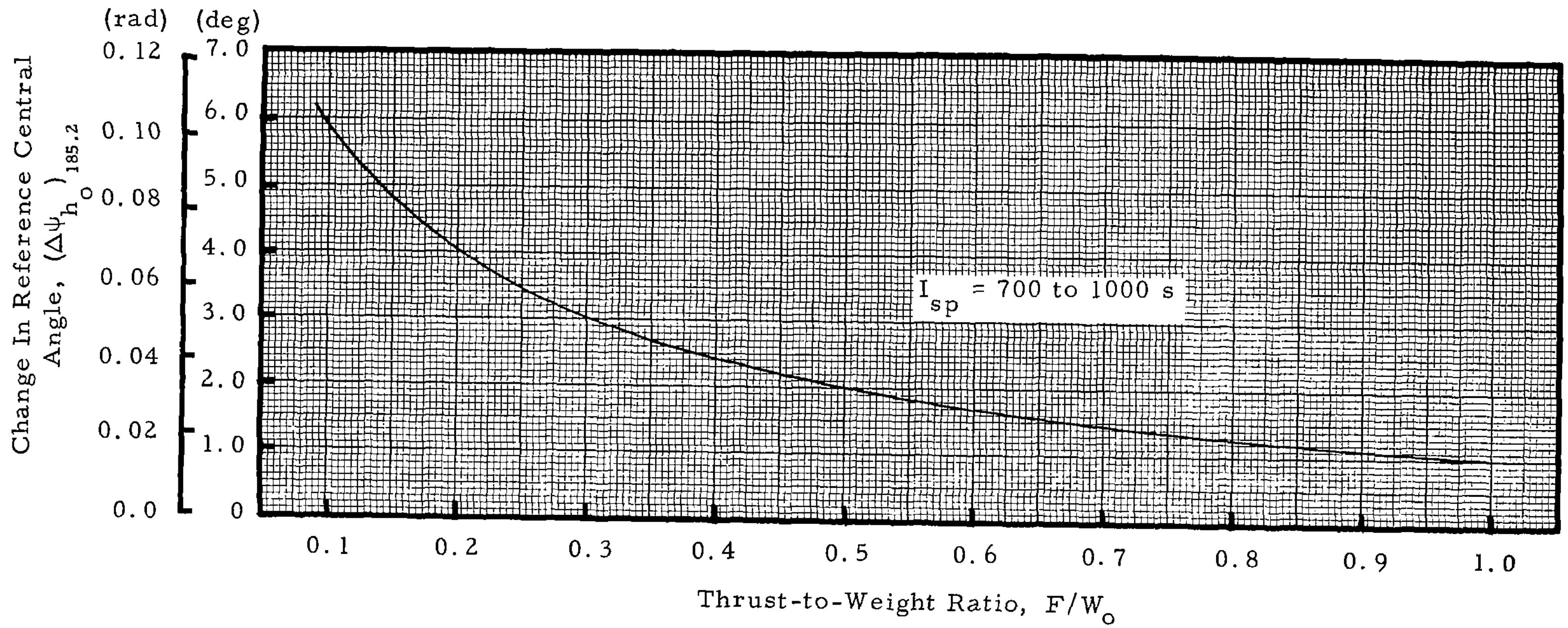

FIGURE 16. CHANGE IN REFERENCE CENTRAL ANGLE FOR INCREMENTAL ALTITUDE DECREASE OF 185.2 KM (100 N. M.)

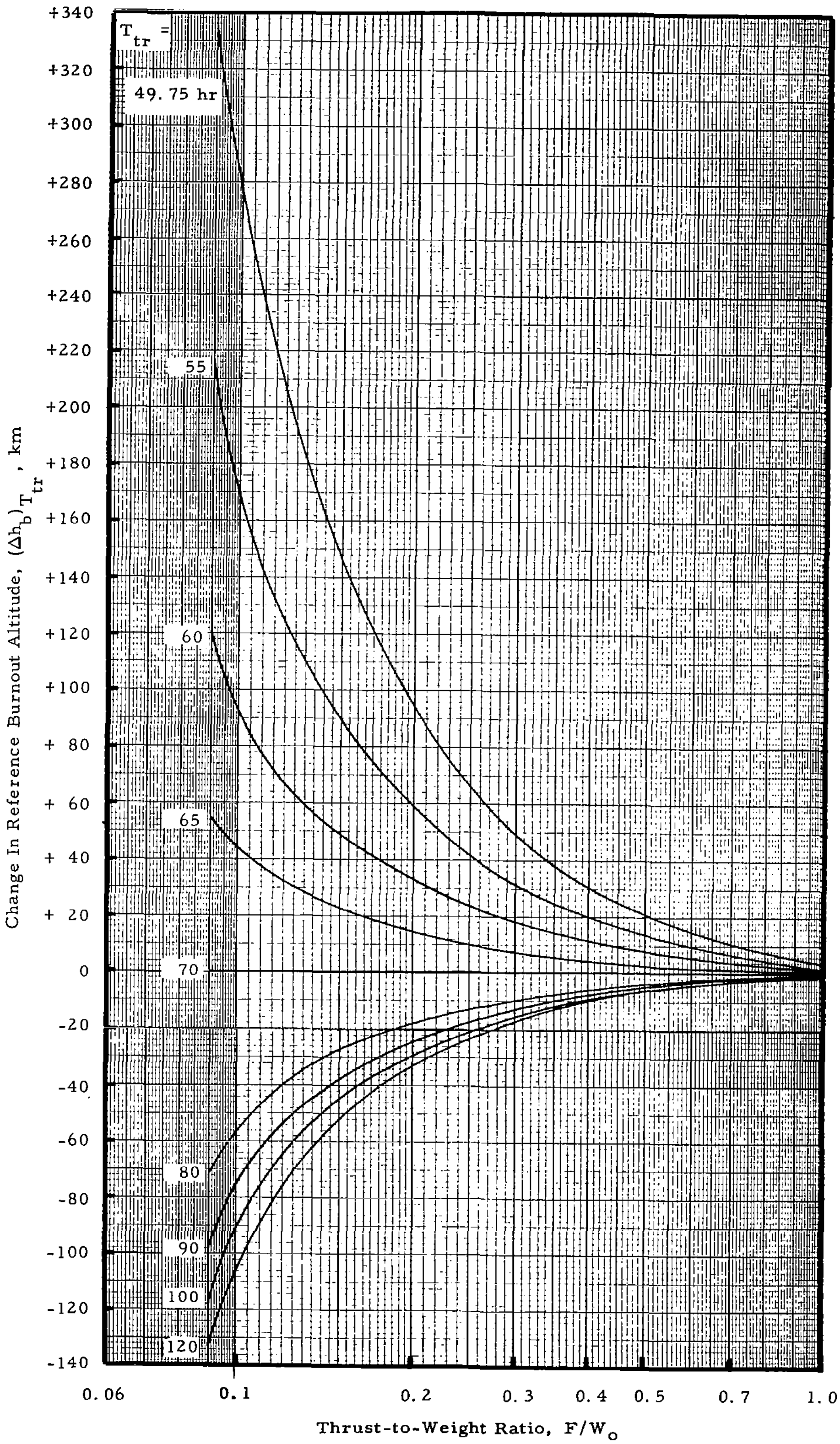

FIGURE 17. CHANGE IN REFERENCE BURNOUT ALTITUDE FOR NON-REFERENCE TRANSFER TIMES

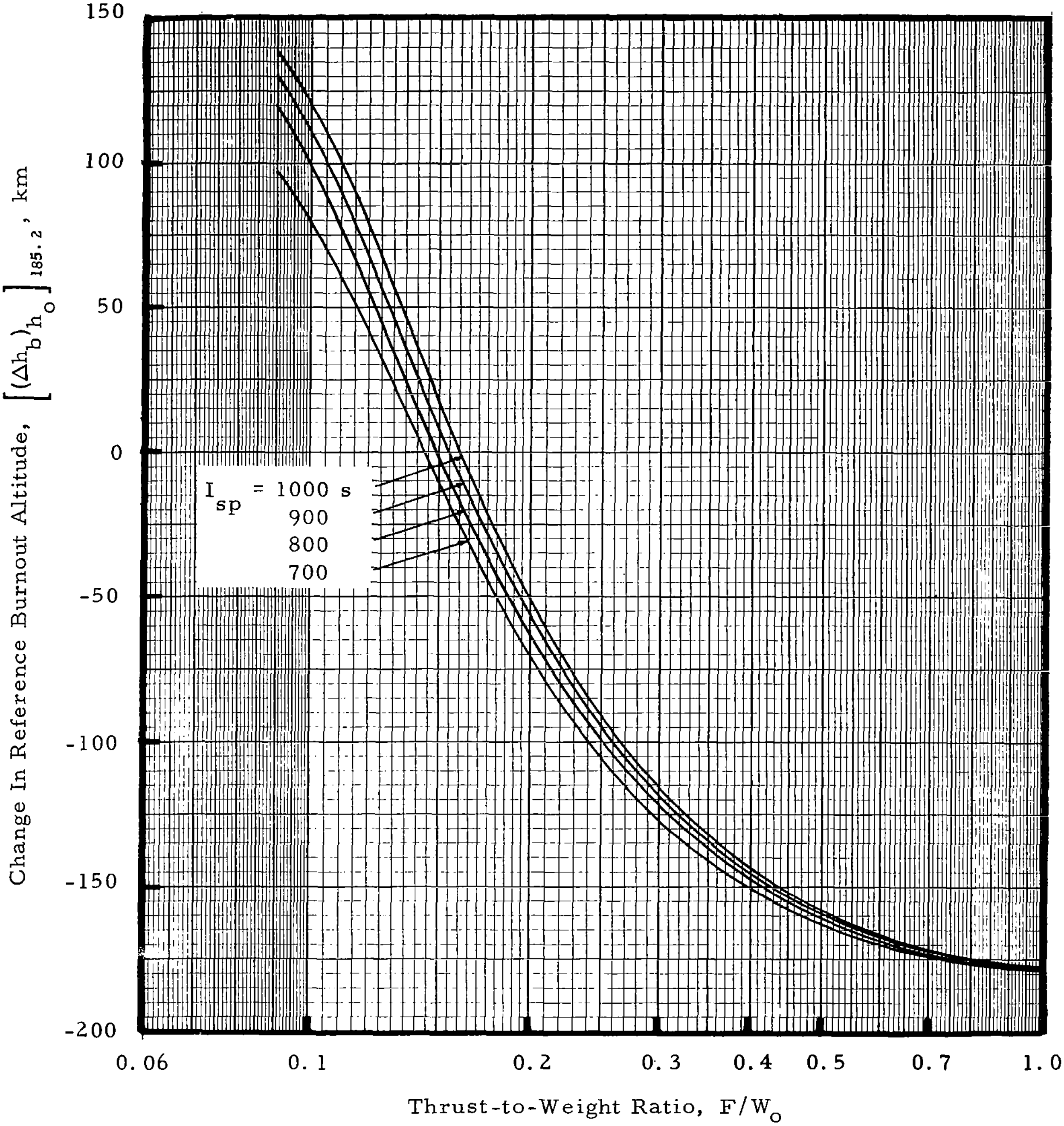

FIGURE 18. CHANGE IN REFERENCE BURNOUT ALTITUDE FOR INCREMENTAL ALTITUDE DECREASE OF 185.2 KM (100 N. M.)

Table 1. Data Assumed For Numerical Examples

Example Number	1	2	3	4	5
I_{sp} (sec)	735	980	750	755	885
F/W_o	0.20	0.40	0.30	0.45	0.10
h_o (km)	277.8	463.0	185.2	835.0	694.9
T_{tr} (hr)	85	78	110	57	60
A	0.20	0.09	0.05	0.12	0.08
B	0.15	0.15	0.12	0.22	0.07
C	0.00	0.02	0.03	0.00	0.00

Table 2. Comparison of Exact and Predicted Performance Parameters

Example Number	1			2			3			4			5				
Variable	Pred.	Actual	Per Cent Error	Pred.	Actual	Per Cent Error	Pred.	Actual	Per Cent Error	Pred.	Actual	Per Cent Error	Pred.	Actual	Per Cent Error	Maximum Error	Average Error
λ	0.5417	0.5412	0.09	0.6254	0.6255	0.02	0.5592	0.5598	0.11	0.5336	0.5338	0.04	0.6340	0.6386	0.72	0.72	0.19
ΔV_{id} (m/s)	3265	3269	0.12	3121	3120	0.03	3203	3205	0.06	3065	3056	0.29	3474	3484	0.29	0.29	0.16
ζ	0.364	0.364	0	0.278	0.277	0.37	0.3531	0.353	0	0.338	0.338	0	0.330	0.330	0	0.37	0.07
t_b(sec)	1337	1338	0.07	679.9	678.5	0.21	882.5	882.1	0.05	567.1	566.8	0.05	2920.5	2922.7	0.08	0.21	0.09
θ (rad)	1.1355	1.1439	0.73	1.3263	1.3258	0.04	1.2542	1.2538	0.03	1.3768	1.3783	0.11	0.9529	0.9517	0.13	0.73	0.21
ψ (rad)	1.6267	1.6258	0.06	0.8648	0.8687	0.45	1.1683	1.1641	0.36	0.6734	0.6800	0.97	2.6543	2.6477	0.25	0.97	0.42
h_b(km)	2177	2178	0.05	1013	1012	0.10	1115	1118	0.27	1180	1182	0.17	6432	6549	1.79	1.79	0.91

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