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SUMMARY

The integral equations governing flux distributions for free-molecule flow through converging or diverging tubes and slots are developed under assumptions of uniform entering flux and diffuse particle reflection from the walls. The equations are solved on an IBM 7094 computer by applying an iterative method of solution. Specific cases studied in the analysis are tube length to radius ratios of 1/2, 1, 2, 4, 8, and 16; slot length to width ratios of 1/4, 1/2, 1, 2, 4, and 8; and, for each ratio, up to 14 variations of wall half-angle ranging from 75° to -60°. Wall and exit-aperture flux distributions and total and direct transmission probabilities are presented as functions of these parameters. Transmission probabilities for a wall half-angle of zero are compared with results of other investigations and show excellent agreement. Results are presented in tabular and graphic form, and reasonably accurate results for parametric values within the specified ranges may be determined by interpolation.

INTRODUCTION

Gas flow characteristics under conditions of free-molecule flow have been of interest since the early work of Knudsen and Clausing and such study has gained impetus in recent times primarily because of the rapid expansion of space technology. Applications of solutions of free-molecule flow include a wide range of problems, for example, the design of electric rocket thrustors, vacuum gages, and vacuum facilities (refs. 1 and 2). The subject of this report is free-molecule flow through converging or diverging tubes and converging or diverging two-dimensional slots.

Mathematical formulation of such free-molecule "internal" flow problems was first accomplished by Clausing for right-circular tubes. Later investigators rederived Clausing's equation by different techniques and also formulated the problem for other models (e.g., ref. 3).

The problem discussed herein is to determine the flux distribution along the walls of the tube or the slot for the case of random, free-molecular gas flow into the inlet.

Particle reflection from the walls is considered herein as in most other investigations, to be diffuse. This is an appropriate assumption from a physical viewpoint, if the particles are adsorbed and then evaporated from the wall.

Of course, the presence of an adsorbed layer of particles on the surface implies the possibility of surface diffusion. Evaluation of the effect of surface diffusion on flow requires knowledge of the diffusion coefficient and equilibrium adsorption data for the particular system considered. For example, it is shown in reference 4 that, for silver atoms effusing through molybdenum and nickel orifices, the total flow may be 30 percent greater than that calculated for vapor flow alone. For the effect to be of this magnitude, it was required that the length-radius ratio of the orifice be less than 1, while the radius was small ($R < 0.5$ mm). In order to keep the analysis herein independent of the gas or tube material, surface diffusion effects were not considered. A more detailed discussion of gas-surface phenomena is given in references 5 and 6.

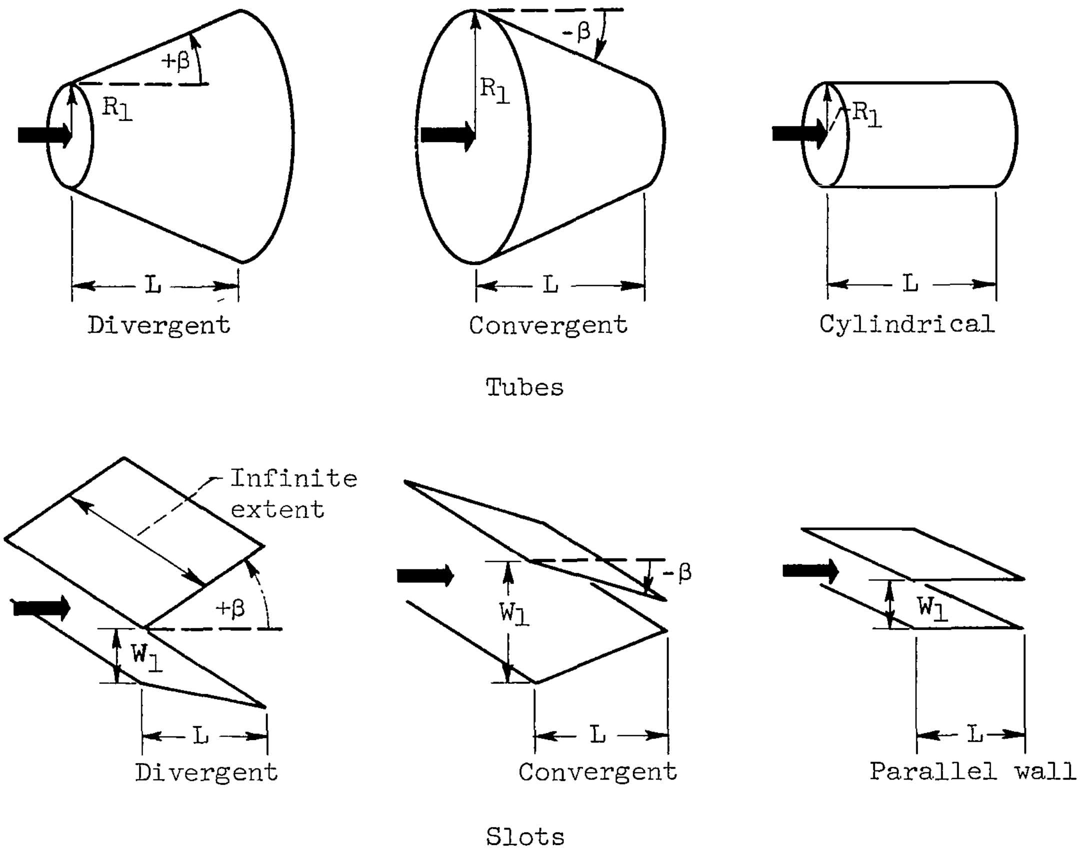
Once the wall flux distribution has been determined, the remaining flow characteristics are readily calculated. The general expression for the wall flux distribution is an integral equation, and solution in closed form is usually not possible without an additional assumption. An assumption that has been used for the right-circular-tube problem, for example, is that particle flux on the tube wall varies linearly with distance from the entrance (ref. 7). As mentioned in reference 7, however, the integral equation is amenable to direct integration on a digital computer, and no assumption regarding the wall flux distribution is required. An iterative method of solution, on a digital computer, is described in reference 8 for the problem of the right-circular tube and in reference 9 for diverging tubes with wall half-angles ranging from 0° to 22.5°. As discussed in reference 9, solutions obtained with the assumption of a linear wall flux variation for the right-circular tube are in approximate agreement with numerical solutions. This assumption, however, is no longer applicable for a tapered tube.

An iterative numerical method of solution that is similar to those reported in references 8 to 10 is applied herein to cover a much wider range of geometric variations than were available previously. Wall and exit-aperture flux distributions and total and direct transmission probabilities are presented in both tabular and graphic form. The computer program used to obtain these results is given in appendix C by Carl D. Bogart.

ANALYTIC RELATIONS

Configurations and Basic Flux Equations

The configurations investigated herein are shown in sketch (a) and figure 1.



(a)

Particle flow into the tube or slot is assumed to be uniform over the inlet and to have random distribution of direction. Explicit knowledge of the speed distribution function is not a requirement for this problem (see ref. 7). This implies that solutions are independent of the wall or particle temperature. Downstream of the exit, vacuum conditions are assumed so that there is no return flow from this region. Free-molecule, or Knudsen, flow is assumed in the region internal to the tube or slot; thus, particle-particle collisions are negligible and only particle-wall collisions need be considered. Particle reflection from the walls is assumed diffuse, that is, assumed to follow the cosine law. With these assumptions, the particle arrival rate at any point on the wall may be determined numerically on a digital computer.

The basic relation from which the particle flow behavior may be derived is (ref. 7)

$$dn_b = \frac{n_a \cos \theta_{ab} \cos \theta_{ba}}{\pi l_{ab}^2} dA_a \tag{1}$$

where

dn_b flux (particles per unit area per unit time) arriving at differential area dA_b from dA_a

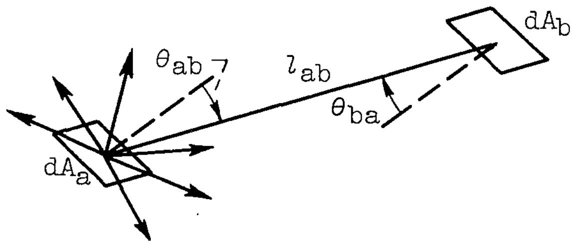
n_a flux leaving differential area dA_a

l_{ab} distance from a to b

θ_{ab} angle between l_{ab} and the normal to dA_a

θ_{ba} angle between l_{ab} and the normal to dA_b

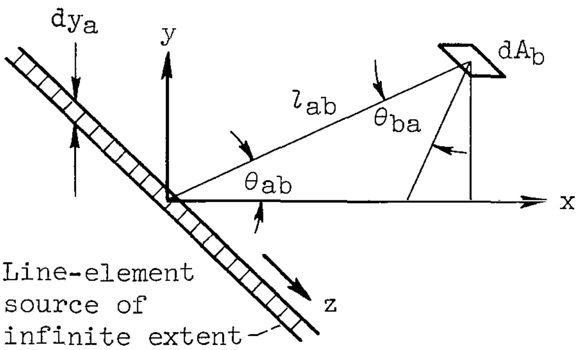
The geometric relations expressed in equation (1) are shown in sketch (b) and all symbols are defined in appendix A.



(b)

The total arrival rate n_b at dA_b is obtained by integrating equation (1) over all the source area A_a that contributes to the flux at dA_b.

If the configuration being examined is two-dimensional, that is, of infinite extent in the third dimension, such as shown in sketch (c), the particle flux relations can be developed from the line-source relation



(c)

$$dn_b = \frac{n_a \cos\theta_{ab} \cos\theta_{ba}}{2l_{ab}} dy_a \qquad (2)$$

where

dn_b flux (particles per unit area per unit time) arriving at the differential area dA_b from line element dy_a

n_a flux leaving line source dy_a

l_{ab} distance from b to line element

Equation (2) is, of course, obtained from equation (1) by the partial integration of dA_a over the line element from z equals negative infinity to positive infinity. The total arrival rate n_b at dA_b is obtained by integrating equation (2) over all line elements that contribute to the flux at dA_b.

The following consistent subscript notation is used in all subsequent developments:

Subscript	Surface or plane
1	Inlet plane
2, 3	Wall surface
4	Exit plane

Tube Configuration

The flow relations for tubes are given in the following paragraphs. Some identities helpful in reducing the general relations are given in appendix B.

Tube wall flux. - The flux $n_2(x_2)$ on the tube wall at any point (see fig. 1(a)) consists of particles arriving directly from the inlet plus particles arriving from the remainder of the wall. When the general flux equation (1) is employed, the flux at point 2 becomes

$$n_2(x_2) = \int_{A_1} n_1 \frac{\cos \theta_{12} \cos \theta_{21}}{\pi l_{12}^2} dA_1 + \int_{A_3} n_3(x_3) \frac{\cos \theta_{23} \cos \theta_{32}}{\pi l_{23}^2} dA_3 \tag{3}$$

Since n_1 is constant, the first term of equation (3) may be integrated directly. Expressed in dimensionless form, it becomes

$$\frac{1}{2\bar{r}_2} \left\{ \frac{\bar{x}_2^2 \sec^2\beta + 3\bar{x}_2 \tan \beta + 2}{\sqrt{\bar{x}_2^2 \sec^2\beta + 4\bar{r}_2}} - \bar{x}_2 \cos \beta - \bar{r}_2 \sin \beta \right\} \tag{4}$$

Barred quantities are nondimensional variables expressed as ratios to the inlet radius R_1.

Since $n_3(x_3)$ is variable over the area A_3 and is also equal to the unknown arrival rate $n_2(x_2)$ that is being sought, the second term of equation (3) cannot be completely integrated directly and must be finally determined by some other means. A numerical method that employs an iterative technique is used herein. First, however, since the flux $n_3(x_3)$ is independent of the angle α_3 (see fig. 1(a) and appendix B), integration with respect to α_3 can be carried out to yield

$$\int_{x_3} n_3(x_3) \frac{\cos \beta}{2r_3} \left\{ 1 - (x_3 - x_2) \sec \beta \frac{(x_3 - x_2)^2 \sec^2\beta + 6r_2r_3}{\left[(x_3 - x_2)^2 \sec^2\beta + 4r_2r_3\right]^{3/2}} \right\} dx_3 \tag{5}$$

The complete expression for $n_2(x_2)$ in nondimensional form is

$$\frac{n_2(\bar{x}_2)}{n_1} = \frac{1}{2\bar{r}_2} \left(\frac{\bar{x}_2^2 \sec^2\beta + 3\bar{x}_2 \tan \beta + 2}{\sqrt{\bar{x}_2^2 \sec^2\beta + 4\bar{r}_2}} - \bar{x}_2 \cos \beta - \bar{r}_2 \sin \beta \right)$$
$$+ \int_0^{\bar{L}} \frac{n_3(\bar{x}_3)}{n_1} \frac{\cos \beta}{2\bar{r}_3} \left\{ 1 - (\bar{x}_3 - \bar{x}_2) \sec \beta \frac{(\bar{x}_3 - \bar{x}_2)^2 \sec^2\beta + 6\bar{r}_2\bar{r}_3}{\left[(\bar{x}_3 - \bar{x}_2)^2 \sec^2\beta + 4\bar{r}_2\bar{r}_3\right]^{3/2}} \right\} d\bar{x}_3 \tag{6}$$

In this equation, as in subsequent ones, the half-angle β is positive for a divergent configuration and negative for a convergent configuration.

Tube exit-plane flux. - The flux at a point in the exit plane (see fig. 1(b)) consists of particles arriving directly from the inlet, plus particles arriving from the wall:

$$n_4(r_4) = \int_{A_1} n_1 \frac{\cos\theta_{14}\cos\theta_{41}}{\pi l_{14}^2} dA_1 + \int_{A_2} n_2(x_2) \frac{\cos\theta_{24}\cos\theta_{42}}{\pi l_{24}^2} dA_2 \tag{7}$$

Integration of equation (7) expressed in nondimensional form yields

$$\begin{aligned}\frac{n_4(\bar{r}_4)}{n_1} &= \frac{1}{2}\left[1 - \frac{\bar{L}^2 + \bar{r}_4^2 - 1}{\sqrt{\left(1 + \bar{L}^2 + \bar{r}_4^2\right)^2 - 4\bar{r}_4^2}}\right] \\ &+ 2\int_0^{\bar{L}} \frac{n_2(\bar{x}_2)}{n_1}\left(\frac{\bar{r}_2^2(\bar{L} - \bar{x}_2)\left[(\bar{L} - \bar{x}_2)^2 + \bar{r}_2^2 - \bar{r}_4^2\right] + \bar{r}_2(\bar{L} - \bar{x}_2)^2\left[(\bar{L} - \bar{x}_2)^2 + \bar{r}_2^2 - \bar{r}_4^2\right]\tan\beta}{\left\{\left[(\bar{L} - \bar{x}_2)^2 + \bar{r}_4^2 + \bar{r}_2^2\right]^2 - 4\bar{r}_2^2\bar{r}_4^2\right\}^{3/2}}\right) d\bar{x}_2\end{aligned} \tag{8}$$

The first term of equation (8) represents the flux of particles arriving directly from the inlet opening. The second term is the flux of particles that have experienced one or more collisions with the wall before arriving at a location $\bar{r}_4$ in the exit plane.

Tube transmission probability. - The fraction of particles incident upon the tube inlet plane that passes through the tube without colliding with the wall, that is, the direct transmission probability P_d, is obtained by integrating the direct flux portion of equation (8) over the exit area of the tube. The resulting expression for P_d is

$$P_d = \frac{1}{2}\left[1 + \bar{L}^2 + \bar{R}_L^2 - \sqrt{\left(1 + \bar{L}^2 + \bar{R}_L^2\right)^2 - 4\bar{R}_L^2}\right] \tag{9}$$

The total transmission probability P_t, or the fraction of particles incident upon the tube inlet area that eventually pass out through the downstream end of the tube, is determined by integrating the total flux n_4 over the exit area and dividing by the total inlet flux:

$$P_t = \int_0^{\bar{R}_L} \frac{n_4(\bar{r}_4)}{n_1} \bar{r}_4 \, d\bar{r}_4 \tag{10}$$

Slot Configuration

The flow relations for slots are given in the following paragraphs. Some identities helpful in reducing the general relations are given in appendix B.

Slot wall flux. - The basic line source relation (eq. (2)) is used to obtain the arrival flux at the wall due to direct flow from the inlet and flow from other elements of the opposite wall such as shown in figure 1(c):

$$n_2(x_2) = \int_{W_1} n_1 \frac{\cos\theta_{12}\cos\theta_{21}}{2l_{12}}\,dy_1 + \int_{\mathscr{L}} n_3(x_3)\frac{\cos\theta_{23}\cos\theta_{32}}{2l_{23}}\,d\mathscr{L} \tag{11}$$

It will be noted that in this configuration no flux arrives at point 2 from any other point on that same wall, since all such points are located at angles of 90° from point 2. After the proper geometric relations are substituted into equation (11) (see appendix B) and the indicated integrations are performed, the following nondimensional expression for the flux on the wall is obtained:

$$\frac{n_2(\bar{x}_2)}{n_1} = \frac{1}{2}\left\{1 - \cos\beta\left[\frac{\bar{x}_2\sec^2\beta + \tan\beta}{\sqrt{\bar{x}_2^2\sec^2\beta + 2\bar{x}_2\tan\beta + 1}}\right]\right\}$$

$$+ \frac{1}{2}\int_0^{\bar{L}} \frac{n_3(\bar{x}_3)}{n_1}\cos\beta\left\{\frac{1 + 2(\bar{x}_2 + \bar{x}_3)\tan\beta + 4\bar{x}_2\bar{x}_3\tan^2\beta}{\left[(\bar{x}_2 - \bar{x}_3)^2 + (1 + \bar{x}_2\tan\beta + \bar{x}_3\tan\beta)^2\right]^{3/2}}\right\}d\bar{x}_3 \tag{12}$$

The first term in brackets in equation (12) is the arrival rate at a point on the wall due to direct flux from the inlet. The second term in brackets represents the flux arriving from the opposite wall. Barred quantities are nondimensional variables expressed as ratios to the inlet slot width.

Slot exit-plane flux. - The flux at a point y_4 in the exit plane (fig. 1(d)) consists of contributions from the open end directly and from the two walls. The equation expressing these terms is (see appendix C)

$$\frac{n_4(\bar{y}_4)}{n_1} = \frac{1}{2}\left\{\frac{1 - 2\bar{y}_4}{\sqrt{4\bar{L}^2 + (1 - 2\bar{y}_4)^2}} + \frac{1 + 2\bar{y}_4}{\sqrt{4\bar{L}^2 + (1 + 2\bar{y}_4)^2}}\right\}$$

$$+ 2\int_0^{\bar{L}} \frac{n_2(\bar{x}_2)}{n_1}(\bar{L} - \bar{x}_2)\left\{\frac{\bar{W}_L + 2\bar{y}_4}{\left[4(\bar{L} - \bar{x}_2)^2 + (1 + 2\bar{y}_4 + 2\bar{x}_2\tan\beta)^2\right]^{3/2}} + \frac{\bar{W}_L - 2\bar{y}_4}{\left[4(\bar{L} - \bar{x}_2)^2 + (1 - 2\bar{y}_4 + 2\bar{x}_2\tan\beta)^2\right]^{3/2}}\right\}d\bar{x}_2 \tag{13}$$

The first term in brackets in equation (13) represents particle flux at a point $\bar{y}_4$ in the exit plane coming directly from the inlet without a wall collision. The second term in brackets represents flux of particles at point $\bar{y}_4$ that experience one or more wall collisions before arriving at that point.

Slot transmission probability. - The fraction of particles that are incident upon the slot inlet and pass through the slot without colliding with the wall is P_d and is obtained by integration of the first term of equation (13) over the exit area:

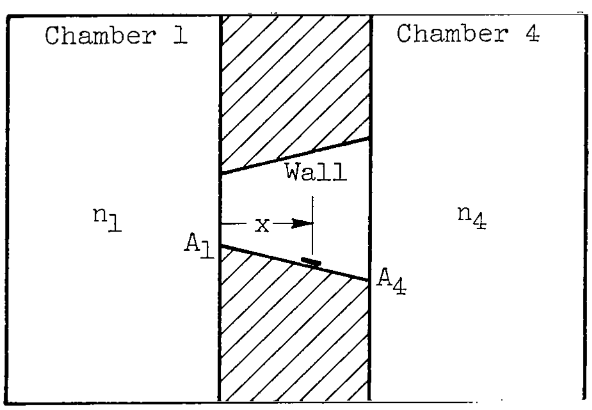
$$P_d = \frac{1}{2}\left[\sqrt{4\bar{L}^2 + (1 + \bar{W}_L)^2} - \sqrt{4\bar{L}^2 + (1 - \bar{W}_L)^2}\right] \tag{14}$$

The total transmission probability P_t is the integral of both terms of equation (13) over the exit area:

$$P_t = \int_{-\bar{W}_L/2}^{\bar{W}_L/2} \frac{n_4(\bar{y}_4)}{n_1}\, d\bar{y}_4 \tag{15}$$

Additional Relations

Wall fluxes. - As previously mentioned, the tube and slot wall flux relations, equations (6) and (12), may be used for either converging or diverging configurations by applying the appropriate sign to β, the wall half-angle. A relation between the wall flux values for flow through any particular configuration (sketch (d)) in the forward or reverse directions may also be developed from consideration of equilibrium requirements.



(d)

Consider the flow through the opening from each chamber (sketch (d)) independently with the assumption that the other chamber is at zero pressure. The flux at some point on the wall at a distance x from the inlet A_1 may be calculated first for flow from the left chamber and then for flow from the right chamber. Let these two wall fluxes be designated $(n_x)_{14}$ and $(n_x)_{41}$, respectively. Since these flows have been assumed to occur under free-molecule conditions, with no interaction of particles, the flows may be superimposed without affecting the separate solutions. The total arrival rate at the wall at point x is then $(n_x)_{14} + (n_x)_{41}$.

If the gas in the total enclosure is considered to be in a steady-state, the arrival rates n_1 and n_4, and indeed the arrival rates anywhere within the enclosure, must all be equal. Thus, it can be stated that the wall arrival rate at point x under these conditions is

$$n_1 = (n_x)_{14} + (n_x)_{41} \tag{16}$$

or

$$\frac{(n_x)_{41}}{n_1} = 1 - \frac{(n_x)_{14}}{n_1} \tag{17}$$

This relation, which does not depend on the shape of the opening, or passageway, provides a convenient tool for evaluation of reverse flow through a particular configuration, (e.g., tube or slot) once the flow in the forward direction has been calculated from equation (6) or (12).

Transmission probability. - Under steady-state conditions in the enclosure in sketch (d), there must be no net flow through the tube, or slot, connecting the two chambers. From this steady-state requirement, a relation between the transmission probabilities through the same configuration in the forward and reverse directions can be derived. Again, consider the flows from the two chambers to be occurring independently of each other. The particle flow from chamber 1 to chamber 4 is $n_1A_1(P_{14})_t$; the particle flow from chamber 4 to chamber 1 is $n_4A_4(P_{41})_t$, where $(P_{14})_t$ and $(P_{41})_t$ are the total transmission probabilities for the respective flows. When the flows are superimposed, the respective total transmissions are not affected. Under steady-state conditions then, since there must be no net flow,

$$n_1A_1(P_{14})_t = n_4A_4(P_{41})_t \tag{18}$$

But, also under steady-state conditions $n_1 = n_4$, so that equation (18) becomes

$$(P_{14})_t = \frac{A_4}{A_1} (P_{41})_t \tag{19}$$

As long as free-molecule flow conditions exist, the values of transmission probability, as defined herein, do not depend on flow rate. Equation (19) is a general relation, then, dependent only on configuration parameters.

Consideration of the direct flux relations (eqs. (9) and (14)) shows that a similar relation holds for the direct transmission probabilities as well:

$$(P_{14})_d = \frac{A_4}{A_1} (P_{41})_d \tag{20}$$

If, for example, the configuration variables for the tube are introduced into equation (19) or (20), the equation may be written as

$$(P_{14})_{\frac{L}{R_1},\beta} = \left(1 + \frac{L}{R_1} M\right)^2 (P_{41})_{\frac{L}{R_L},-\beta} \tag{21}$$

where

$$\frac{L}{R_L} = \frac{\frac{L}{R_1}}{1 + \frac{L}{R_1} M} \tag{22}$$

(see fig. 1(b)).

For the slot configuration equation (19) or (20) becomes

$$(P_{14})_{\frac{L}{W_1},\beta} = \left(1 + 2 \frac{L}{W_1} M\right) (P_{41})_{\frac{L}{W_L},-\beta} \tag{23}$$

where

$$\frac{L}{W_L} = \frac{\frac{L}{W_1}}{1 + 2 \frac{L}{W_1} M} \tag{24}$$

(see fig. 1(d)).

The transmission probabilities presented in the next section were explicitly determined for various configuration parameters and flow directions. By application of equation (21) or (23), values of the reverse-flow transmission probabilities may be readily determined.

METHOD OF SOLUTION

Numerical solutions for the preceding equations were obtained on an IBM 7094 computer. The FORTRAN program is given in appendix C.

Solution of the wall flux equations (eqs. (6) and (12)) was accomplished by application of an iterative procedure similar to that described in detail in reference 10. The procedure, as applied to equation (6), for example, consists in supplying an initial guess of the unknown function $n_3(\overline{x}_3)$ in the integral of equation (6). The equation is numerically integrated by using a combination of Simpson's rule and trapezoidal integration. New values of the function are thus produced that are used in turn, in the next iteration. Iteration proceeds in this manner until convergence is reached, that is, until the maximum change in any one of the final pointwise values, compared with its previous value, is less than some preassigned limit (for details see appendix C). For the solutions presented herein, the largest value used for this limit was 0.02 percent. Similar to the findings of reference 10, trial solutions in which widely differing initial guesses were employed always resulted in convergence to practically the same final answers; however, the rate of convergence differed. The simple initial guess of $n_3(\bar{x}_3) = 0$ generally yielded the most rapid rate of convergence.

After the wall flux relations are solved, the exit-plane equations (eqs. (7) and (13)) may be solved by direct numerical integration. Similarly, the total transmission probability equations (eqs. (10) and (15)) are amenable to direct numerical integration by use of the exit-plane flux values.

The accuracy of the results, of course, depends on the size of the increment used in the numerical integration of the various equations and, in general, improves as the increment size is made smaller. Smaller increments, however, increase machine computation time, particularly with respect to the wall flux equations. Consideration of both these factors led to the selection of varying increment sizes such that the total number of calculated points was approximately the same for all configurations.

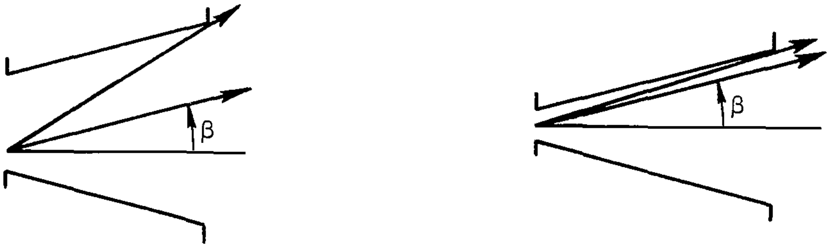
RESULTS AND DISCUSSION

Transmission Probabilities

The transmission probabilities calculated by equations (9), (10), (14), and (15) are presented in tables I and II. A comparison of the calculated total transmission probability for the straight tube and the parallel-walled slot (i.e., for $\beta = 0$) with that of previous investigations is shown in figure 2. The results of the present calculation by the iterative numerical technique are in agreement with those of reference 3. The transmission probabilities for $\beta = 0$ obtained in reference 3 were determined analytically under the assumption of a linear variation in wall flux. The numerical procedure used herein, as well as that used in reference 8, shows that this assumption is quite satisfactory for this case; however, it is not applicable when the wall half-angle departs from zero.

Total transmission probabilities for the convergent and the divergent tubes are shown in figure 3(a) for length to inlet-radius ratios up to 16. For the divergent tubes, the transmission probability appears to become asymptotic, especially for half-angles greater than about 20°, where the total transmission probability levels off at a length to inlet-radius ratio of about 10. The results that may be compared are in agreement with graphical results given in reference 9, which covers positive wall half-angles from 0° to 22.5°. Total transmission probabilities for the convergent and the divergent slots are shown in figure 3(b) for length to initial-width ratios up to 8. The behavior is qualitatively similar to that of the tubes.

The direct transmission probabilities P_d for the tubes and slots are shown in table II and figure 4. These probabilities rapidly approach limiting values as the length-radius ratios, or length-width ratios, approach infinity. The values $\sin^2\beta$ for the tube and $\sin\beta$ for the slot are the limits of equations (9) and (14), respectively. Physically, as the ratio of length to initial width (or radius) increases for a fixed wall half-angle, the percentage of approaching particles that can escape directly through the exit at angles greater than the wall half-angle β tends to decrease as shown in sketch (e).



(e)

The fraction of incident particles that experience one or more wall collisions before passing out of the tube, or slot, is simply equal to $P_t - P_d$. This fraction may be determined from the values given in tables I and II and is compared for the case where $\beta = 0$ with total and direct transmission probability values in figure 5. The $P_t - P_d$ curve passes through a maximum for this case as the length to inlet-radius ratio or length to inlet-width ratio increases. This behavior of the $P_t - P_d$ curve is to be expected if the total and the direct transmission probabilities each approach zero with increasing length-radius or length-width ratios as they do for zero or negative wall half-angles. For the divergent configurations, however, the total and the direct transmission probabilities do not approach zero (figs. 3 and 4) and a maximum in the $P_t - P_d$ curve does not generally occur.

Equations (21) to (24) may be used in connection with the data of tables I and II to obtain reverse-flow transmission probabilities for the various configurations. For example, consider a slot of length to inlet-width ratio of 1/2 and a wall half-angle equal to 45°. The reverse-flow-configuration parameters are a wall half-angle of 45° and from equation (24), a length to exit-width ratio of 1/4. From table I(b)

$$P_t = (P_{14})_{\frac{L}{W_1},\beta} = 0.986$$

For flow in the opposite direction, equation (23) is used to calculate P_t:

$$P_t = (P_{41})_{\frac{L}{W_L},-\beta} = 0.493$$

This value, calculated from equilibrium requirements, is identical with the value given in table I(b) that was calculated from the theoretical flow equations.

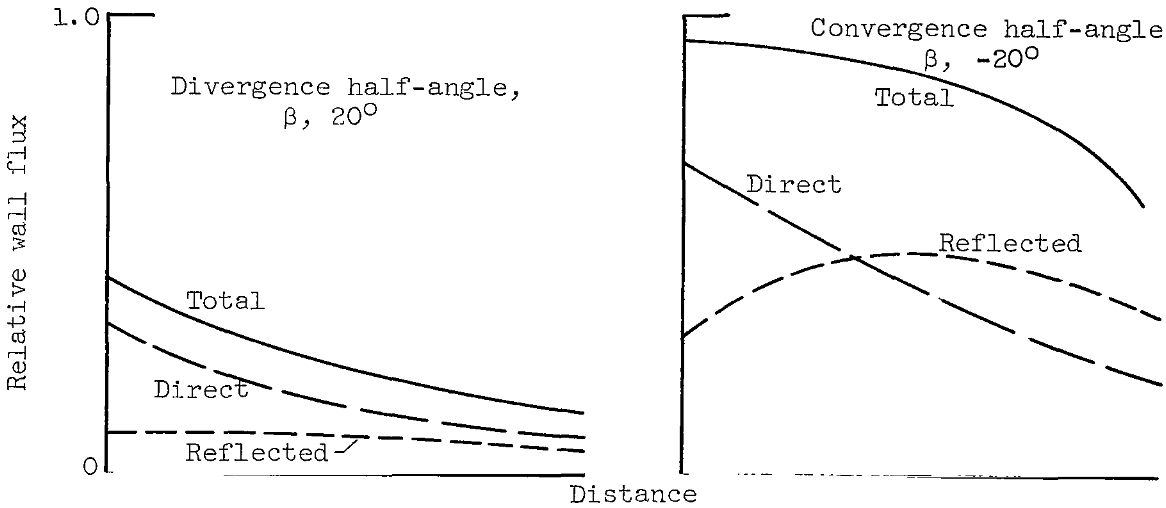
Wall Flux Distributions

Tubes. - Values of the incident flux or arrival rate at the wall of the convergent or divergent tube are given in table III(a). Several illustrative plots of data from this table are shown in figure 6. Wall flux varies nearly linearly along the tube length for $\beta = 0$, is concave downward for the converging tube, and is concave upward for the diverging tube.

Slots. - Wall flux values for the convergent or divergent two-dimensional slots are presented in table III(b), and several illustrative plots are shown in figure 7. The wall flux distributions for the slots follow qualitatively the same behavior as those for the tubes.

This behavior of the tube and slot wall flux distributions with varying wall half-angle is due to the varying magnitude of contributions received directly from the inlet and contributions from particles reflected from the walls.

The qualitative behavior of these components of the flux curves is shown in sketch (f) for a length to inlet-width ratio of 1 and wall half-angles of $\pm 20^{\circ}$.



(f)

The wall fluxes presented herein for both the tube and the slot configurations may be used in equation (17) to obtain flux values for reverse flow through each of the configurations. These correlations are not obvious in the plots of figure 7, since the dimensionless parameter length-radius or length-width ratio given thereon is expressed in terms of the flow entrance aperture. Equations (22) or (24) should be used along with equation (17) to determine the appropriate reverse-flow configurations.

The accuracy of the flow equation calculations may be checked by using some results given in table III. For example, length to inlet-radius ratios and wall half-angles of 1 and 45°, and 1/2 and -45°, respectively, result in similar tube configurations. The calculated values from table III(a) (rounded off to three places) are reproduced and summed for comparison in the following table:

Axial distance, x/L or 1 - x/L	Length to inlet-radius ratio, L/R_1		Sum of flux ratios
	1	1/2	
	Wall half-angle, β, deg		
	45	-45	
	Flux ratio, n_2/n_1		
0	0.166	0.834	1.000
.1	.135	.865	1.000
.2	.111	.889	1.000
.3	.093	.907	1.000
.4	.078	.922	1.000
.5	.066	.934	1.000
.6	.056	.944	1.000
.7	.048	.952	1.000
.8	.042	.958	1.000
.9	.036	.964	1.000
1.0	.031	.969	1.000

The equilibrium requirement of equation (17), that the sum should equal 1.000 at each point, is thus satisfied.

Exit-Plane Flux Distributions

The integrals in the exit-plane flux distribution equations (eqs. (8) and (13)) for both the tubes and the slots contain functions that become somewhat difficult to evaluate accurately by numerical methods at the exit plane near the wall. The problem may be overcome to a great extent by use of very small increments; however, practical considerations arise in deciding on an appropriate increment size. Results were determined to be fairly accurate over at least 95 percent of the exit opening. There is a possible error over the remaining 5 percent, and this possibility is indicated in the figures by the dashed portions of the curves near the walls.

Tubes. - Flux values across the exit plane at different radial distances from the tube axis are presented in table IV(a) for the convergent and the divergent tubes. Flux distributions across the exit plane of the cylindrical tubes of various lengths are shown in figure 8. The distributions tend to become flat with increasing length to radius ratio.

Flux distributions across the exit plane of tubes of various wall half-angle are shown in figure 9 for length to inlet-radius ratios of 0.5, 2, and 16. As would be expected from the transmission probability results, the magnitude of the curves decreases with increasing length to radius ratio.

Slots. - Flux values across the exit plane at different lateral distances from the centerline of the slot are given in table IV(b) for the convergent and the divergent slots. Flux distributions at the exit plane of the parallel-walled slots of various lengths are shown in figure 10, and several illustrative plots of the variations of exit-plane flux distribution with wall half-angle are shown in figure 11. As is true of the wall flux curves, the general behavior of the exit-plane curves is qualitatively similar for slots and tubes.

CONCLUDING REMARKS

The object of this report was to determine the flow characteristics of converging or diverging tubes and slots under conditions of free-molecule flow. An iterative numerical method of solution of the integral equations that describe the flow was employed. With this method, no a priori knowledge of the flux distribution along the wall was required, as it would be if closed-form analytic solutions were sought. Wall flux distributions, exit-plane flux distributions, and total and direct transmission probabilities were determined for a wide range of configuration length to radius (or, width) ratio and wall half-angle. Relations were also developed that may be used to determine the "reverse-flow" wall flux distribution and the transmission probability of a given configuration directly from the computed "forward-flow" results.

It was found that values of transmission probability for the configuration having a wall half-angle of zero were in agreement with values determined by other investigators. Some values presented for other configurations (i.e., wall half-angle not equal to zero) were found to be in agreement with graphic results of another investigation.

Wall flux distributions for the tubes and the slots were qualitatively similar to each other and were found to vary nearly linearly with distance for the zero wall half-angle configuration. For the divergent configurations the wall flux varied more sharply with distance near the entrance, while for convergent configurations it varied more sharply near the exit.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, February 11, 1964

APPENDIX A

SYMBOLS

A area

L length of configuration, measured normal to entrance plane

$\mathscr{L}$ length of slot wall, measured along wall

l length of line, see sketch (b)

M tangent of angle β of tube or slot

n flux or arrival rate, number/(unit area)(unit time)

P_d direct transmission probability, fraction of entering particles that exit downstream without a wall collision

P_t total transmission probability, fraction of entering particles that exit downstream

R_1 inlet radius of axisymmetric configuration

R_L exit radius of axisymmetric configuration

r radial distance

s length of surface normal from point on surface to centerline of configuration

t distance, for example, t_{ij}, distance from point i to end of s_j that is on centerline

W_L width of exit, two-dimensional configuration

W_1 width of inlet, two-dimensional configuration

x axial distance

y lateral distance

α position angle in cylindrical coordinate system

β wall half-angle of tube or slot, positive for diverging and negative for converging configuration

θ angle between surface normal and line l, see sketch (b)

Subscripts:

a,b general points

1 inlet plane

2,3 wall

4 exit plane

Superscript:

$(\bar{\ })$ dimensionless quantity for tube configuration when divided by R_1 and for slot configuration when divided by W_1

APPENDIX B

IDENTITIES FOR INTEGRATION OF EQUATIONS

The following identities for the tube configuration may be noted from figures 1(a) and (b):

$$dA_1 = r_1 \, dr_1 \, d\alpha_1$$

$$\cos \theta_{12} = \frac{x_2}{l_{12}}$$

$$\cos \theta_{21} = \frac{l_{12}^2 + s_2^2 - t_{12}^2}{2l_{12}s_2}$$

$$t_{12}^2 = r_1^2 + (x_2 + r_2 \tan \beta)^2$$

$$l_{12}^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos \alpha_1 + x_2^2$$

$$dA_3 = r_3 \, dx_3 \, d\alpha_3 \sec \beta$$

$$\cos \theta_{32} = \frac{l_{23}^2 + s_3^2 - t_{23}^2}{2l_{23}s_3}$$

$$\cos \theta_{23} = \frac{l_{23}^2 + s_2^2 - t_{32}^2}{2l_{23}s_2}$$

$$t_{32}^2 = r_3^2 + (x_3 - x_2 - r_2 \tan \beta)^2$$

$$t_{23}^2 = r_2^2 + (x_3 - x_2 + r_3 \tan \beta)^2$$

$$s_3 = r_3 \sec \beta$$

$$s_2 = r_2 \sec \beta$$

$$r_3 = R_1 + x_3 \tan \beta$$

$$r_2 = R_1 + x_2 \tan \beta$$

$$l_{23}^2 = r_2^2 + r_3^2 - 2r_2r_3 \cos \alpha_3 + (x_3 - x_2)^2$$

$$\cos \theta_{14} = \cos \theta_{41} = \frac{L}{l_{14}}$$

$$dA_2 = r_2 \, d\alpha_2 \, dx_2 \sec \beta$$

$$l_{14}^2 = r_1^2 + r_4^2 - 2r_1r_4 \cos(\alpha_1 - \alpha_4) + L^2$$

$$l_{24}^2 = r_2^2 + r_4^2 - 2r_2r_4 \cos \alpha_2 + (L - x_2)^2$$

$$\cos \theta_{42} = \frac{L - x_2}{l_{24}}$$

$$\cos \theta_{24} = \frac{l_{24}^2 + s_2^2 - t_{42}^2}{2l_{24}s_2}$$

$$t_{42}^2 = (L - x_2 - r_2 \tan \beta)^2 + r_4^2$$

The following identities for the slot configuration may be noted from figures 1(c) and (d):

$$d\mathscr{L} = dx_3 \sec \beta$$

$$\cos \theta_{12} = \frac{x_2}{l_{12}}$$

$$\cos \theta_{21} = \frac{l_{12}^2 + s_2^2 - t_{12}^2}{2l_{12}s_2}$$

$$s_2 = y_2 \sec \beta$$

$$t_{12}^2 = y_1^2 + (x_2 + y_2 \tan \beta)^2$$

$$l_{12}^2 = x_2^2 + (y_2 - y_1)^2$$

$$\cos \theta_{23} = \frac{l_{23}^2 + s_2^2 - t_{32}^2}{2l_{23}s_2}$$

$$\cos \theta_{32} = \frac{l_{23}^2 + s_3^2 - t_{23}^2}{2l_{23}s_3}$$

$$s_3 = -y_3 \sec \beta$$

$$y_2 = \frac{W_1}{2} + x_2 \tan \beta$$

$$y_3 = -\left(\frac{W_1}{2} + x_3 \tan \beta\right)$$

$$t_{32}^2 = y_3^2 + (x_3 - x_2 - y_2 \tan \beta)^2$$

$$t_{23}^2 = y_2^2 + (x_3 - x_2 + y_3 \tan \beta)^2$$

$$l_{23}^2 = (x_3 - x_2)^2 + (y_3 - y_2)^2$$

$$\cos \theta_{14} = \cos \theta_{41} = \frac{L}{l_{14}}$$

$$l_{14}^2 = L^2 + (y_1 - y_4)^2$$

$$\cos \theta_{42} = \frac{L - x_2}{l_{24}}$$

$$\cos \theta_{24} = \frac{l_{24}^2 + s_2^2 - t_{42}^2}{2 l_{24} s_2}$$

$$t_{42}^2 = y_4^2 + (L - x_2 - y_2 \tan \beta)^2$$

$$l_{24}^2 = (L - x_2)^2 + (y_2 - y_4)^2$$

$$W_L = W_1 + 2L \tan \beta$$

APPENDIX C

FORTRAN II CODE FOR TUBES AND SLOTS

by Carl D. Bogart

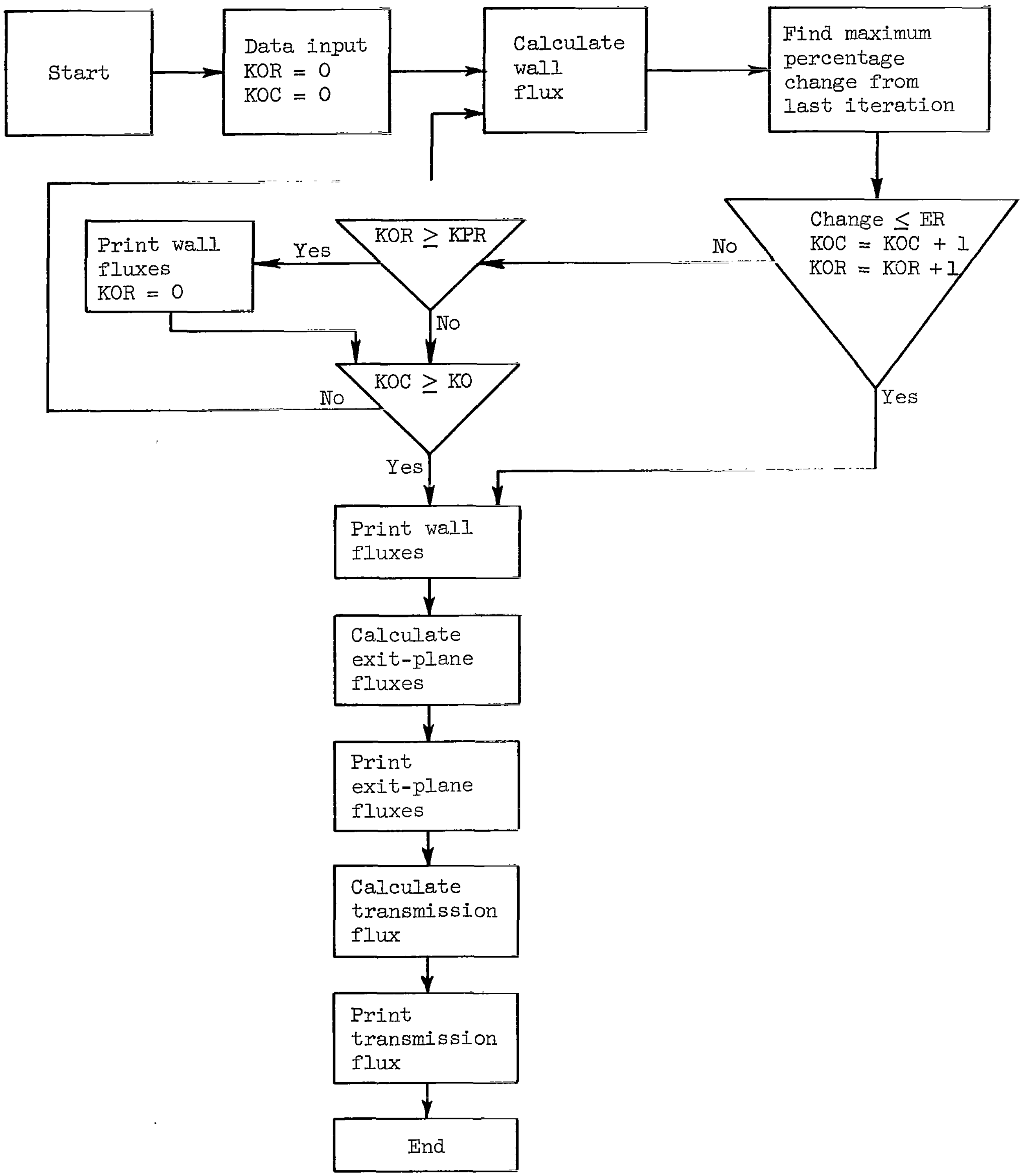
The FORTRAN II programs that calculate the wall and exit-plane flux distributions and the transmission probabilities follow essentially the same format for both the tube and the slot configurations. Thus, while both programs are included herein, only one set of control words, one set of problem specifications, one flow chart, and one set of sample data are given.

Control words:

IS	positive, set first guess equal to YO; zero or negative, read in first guess from binary cards
KO	maximum number of iterations on solution
KPR	frequency of intermediate print
M	number of heading cards

Problem specifications:

ER	maximum percentage change for convergence
H	reciprocal of mesh size for wall
HH	reciprocal of mesh size for exit plane
XLOR	length of tube or slot
XM	tan (beta)
YO	first guess at solution


Start
Data input
KOR = 0
KOC = 0
Calculate wall flux
Find maximum percentage change from last iteration
Change ≤ ER
KOC = KOC + 1
KOR = KOR + 1
No
KOR ≥ KPR
Yes
Print wall fluxes
KOR = 0
No
KOC ≥ KO
No
Yes
Yes
Print wall fluxes
Calculate exit-plane fluxes
Print exit-plane fluxes
Calculate transmission flux
Print transmission flux
End

FLOW CHART

PROGRAM LISTINGS

```
C     PROGRAM TO CALCULATE SIDEWALL AND EXIT PLANE FLUXES FOR SLOT
            COMMON Y,Z,W
            DIMENSION Y(1000),W(1000),Z(1000)
  1         READ INPUT TAPE 7,101,M,KO,KPR,IS
C     M=NUMBER OF HEADING CARDS
C     KO=MAXIMUM NUMBER OF ITERATIONS
C     KPR=FREQUENCY OF PRINT
C     IS=0,- CALL FIRST GUESS FROM BINARY CARDS
C     IS=+ SET FIRST GUESS=YO
            DO 2 J=1,M
            READ INPUT TAPE 7,104
  2         WRITE OUTPUT TAPE 6,104
            READ INPUT TAPE 7,100,XM,XLOR,H,ER,YO,HH
C     XM=TAN(BETA)
C     XLOR=L/W
C     H=1/MESH SIZE
C     ER=PERCENTAGE CHANGE FOR CONVERGENCE
C     YO=FIRST GUESS AT SOLUTION
C     HH=STEP-SIZE FOR END
            FLOR=4.*XLOR**2
            KOR=0
            KOC=0
            HX=1./H
            N=XLOR*H+1.5
            XS=XM*XM
            C=1.+XS
            SC=SQRTF(C)
            OSC=1./SC
            CA=.5*OSC
            TXM=2.*XM
            FXM=TXM**2
 21         DO 3 J=1,N
            XJ=J-1
            XJ=XJ/H
  3         Y(J)=YO+XJ*.5*(.5-YO)
  4         K=1
            X=0.
  5         DO 6 J=1,N
            XJ=J-1
            YY=XJ/H
            Z(J)=(1.+(X+YY)*TXM+FXM*X*YY)*Y(J)
  6         Z(J)=Z(J)/((X-YY)**2+(1.+XM*(X+YY))**2)**1.5
            S=0.
            SS=0.
            DO 7 J=2,N ,2
            S=S+Z(J)
  7         SS=SS+Z(J+1)
            SOM=(Z(1)-Z(N)+4.*S+2.*SS)*.33333333/H
            CON= SC-(XM+X*C)/SQRTF(X*X*C+TXM*X+1.)
            W(K)=CA*(CON+SOM)
            K=K+1
            X=X+HX
            IF(K-N) 5,5,8
  8         CK=0.
            DO 10 J=1,N
            A=ABSF(W(J)-Y(J))/W(J)
            IF(CK-A) 9,10,10
  9         CK=A
            K=J
 10         Y(J)=W(J)
            CK=CK*100.
```

```
      KOC=KOC+1
      KOR=KOR+1
      IF(KOR-KPR) 30,31,31
31    WRITE OUTPUT TAPE 6,102,KOC,K,CK
      WRITE OUTPUT TAPE 6,103,(Y(J),J=1,N)
      KOR=0
30    IF(ER-CK) 18,19,19
18    IF(KOC-KO) 4,19,19
19    WRITE OUTPUT TAPE 6,102,KOC,K,CK
      WRITE OUTPUT TAPE 6,105
      WRITE OUTPUT TAPE 6,103,(Y(J),J=1,N)
      UN=1.+TXM*XLOR
      HN=UN/HH*.5
      NN=HH+1.5
      K=1
      X=0.
39    TX=2.*X
      DO 40 J=1,N
      XJ=J-1
      YY=XJ/H
      A=(XLOR-YY)
      AA=4.*A*A
      B=1.+TXM*YY
      Z(J)=Y(J)*A*((UN+TX)/(AA+(B+TX)**2)**1.5+(UN-TX)/(AA+(B-TX)**2)
     1**1.5)
40    CONTINUE
      S=0.
      SS=0.
      DO 41 J=2,N,2
      S=S+Z(J)
41    SS=SS+Z(J+1)
      SOM=(Z(1)-Z(N)+4.*S+2.*SS)*.666666666/H
      CON=.5*((1.-TX)/SQRTF(FLOR+(1.-TX)**2)+(1.+TX)/SQRTF(FLOR+(1.+
     1TX)**2))
      W(K)=SOM+CON
      X=X+HN
      K=K+1
      IF(K-NN) 39,39,42
42    WRITE OUTPUT TAPE 6,106
      WRITE OUTPUT TAPE 6,103,(W(J),J=1,NN)
      S=0.
      SS=0.
      DO 43 J=2,NN,2
      S=S+W(J)
43    SS=SS+W(J+1)
      SOM=.666666666*(W(1)-W(NN)+4.*S+2.*SS)*HN
      WRITE OUTPUT TAPE 6,107,SOM
      CALL BCDUMP (Y(N),Y(1))
      GO TO 1
100   FORMAT(7F10.5)
101   FORMAT(14I5)
102   FORMAT(21H0ITERATION/K/EPSILON  2I6,F8.3)
103   FORMAT(1H08E15.6)
104   FORMAT(72H
     1                                                        )
105   FORMAT(22H0FLUX ON SIDE WALLS       )
106   FORMAT(16H0 FLUX OUT END    )
107   FORMAT(16H0TOTAL OUT END    F10.6)
      END
```

```
C     PROGRAM TO CALCULATE SIDEWALL AND EXIT PLANE FLUXES FOR TUBES
            COMMON Y,Z,W
            DIMENSION Y(2000),Z(2000),W(2000)
 1          READ INPUT TAPE 7,101,M,KO,KPR,IS
C     M=NUMBER OF HEADING CARDS
C     KO=MAXIMUM NUMBER OF ITERATIONS
C     KPR=FREQUENCY OF PRINT
C     IS=0,-,,CALL FIRST GUESS FROM BINARY CARDS
C     IS=+,SET FIRST GUESS=YO
            DO 7 J=1,M
            READ INPUT TAPE 7,104
 7          WRITE OUTPUT TAPE 6,104
            READ INPUT TAPE 7,100,XM,XLOR,H,FR,YO,HH
C     XM=TAN(BETA)
C     XLOR=L/R
C     H=1/MESH SIZE
C     FR=PERCENTAGE CHANGE FOR CONVERGENCE
C     YO=FIRST GUESS AT SOLUTION
C     HH=STEP-SIZE FOR END
            KOR=0
            C=1.+XM*XM
            SC=SQRTF(C)
            OSC=1./SC
            KOC=0
            N=XLOR*H+1.5
            IF(IS) 61,61,6
 61         CALL BCREAD(Y(N),Y(1))
            GO TO 8
 60         DO 99 J=1,N
 99         Y(J)=YO
 8          K=1
            NS=1
            XX=0.
 9          A=1.+XM*XX
            AA=.5/A
            DO 10 J=1,N
            XJ=J-1
            X=XJ/H
            B=1.+XM*X
            AL=ABSF(X-XX)
            Z(J)=AL*((AL*AL*C)+6.*A*B)
 10         Z(J)=Y(J)*(OSC-Z(J)/((AL*AL*C)+4.*A*B)**1.5)
            IF(NS) 11,12,12
 11         JJ=XX*H+1.5
            SOM=Z(1)+.5*Z(JJ-1)-Z(JJ)+.5*Z(JJ+1)-Z(N)
            GO TO 13
 12         SOM=Z(1)-Z(N)
 13         S=0.
            SS=0.
            DO 14 J=2,N,2
            S=S+Z(J)
 14         SS=SS+Z(J+1)
            SOM=(SOM+4.*S+2.*SS)/(3.*H)
            CON=((C*XX*XX)+3.*XM*XX+2.)/SQRTF((C*XX*XX)+4.*A)
     1      -(XX+XM*A)*OSC
            W(K)=(CON+SOM)*AA
            NS=-NS
            XK=K
            XX=XK/H
            K=K+1
            IF(K-N) 9,9,15
```

```
15        CK=0.
          DO 17 J=1,N
          A=ABSF(W(J)-Y(J))/W(J)
          IF(CK-A) 16,17,17
16        CK=A
          K=J
17        Y(J)=W(J)
          CK=CK*100.
          KOC=KOC+1
          KOR=KOR+1
          IF(KOR-KPR) 30,31,31
31        WRITE OUTPUT TAPE 6,102,KOC,K,CK
          WRITE OUTPUT TAPE 6,103,(Y(J),J=1,N)
          KOR=0
30        IF(ER-CK) 18,19,19
18        IF(KOC-KO) 8,19,19
19        WRITE OUTPUT TAPE 6,102,KOC,K,CK
          WRITE OUTPUT TAPE 6,105
          WRITE OUTPUT TAPE 6,103,(Y(J),J=1,N)
          XK=XM*XLOR+1.
          NN=HH+1.5
          HH=XK/HH
          K=1
          X=0.
112       DO 110 J=1,N
          XJ=J-1
          Q=XJ/H
          B=1.+XM*Q
          C=XLOR-Q
          Z(J)=Y(J)*B*C*((C*C+B*B+X*X)*XK-2.*B*X*X)
110       Z(J)=Z(J)/(X**4+2.*(C*C-B*B)*X*X+(C*C+B*B)**2)**1.5
          SOM=Z(1)-Z(N)
          S=0.
          SS=0.
          DO 111 J=2,N,2
          S=S+Z(J)
111       SS=SS+Z(J+1)
          SOM=(SOM+4.*S+2.*SS)*.666666666/H
          CON=.5*(1.-((XLOR**2-1.)+X*X)/SQRTF(X**4+2.* (XLOR**2-1.)*X*X+
     1    (XLOR**2+1.)**2))
          W(K)=SOM+CON
          X=X+HH
          K=K+1
          IF(K-NN) 112,112,113
113       WRITE OUTPUT TAPE 6,106
          WRITE OUTPUT TAPE 6,103,(W(J),J=1,NN)
          X=0.
          DO 114 J=1,NN
          Z(J)=W(J)*X
114       X=X+HH
          S=0.
          SS=0.
          DO 115 J=2,NN,2
          S=S+Z(J)
115       SS=SS+Z(J+1)
          SOM=HH*(Z(1)-Z(NN)+4.*S+2.*SS)*.666666666
          WRITE OUTPUT TAPE 6,107,SOM
          CALL BCDUMP(Y(N),Y(1))
          GO TO 1
100       FORMAT(7F10.5)
101       FORMAT(14I5)
```

```
102       FORMAT(21H0ITERATION/K/EPSILON   2I6,F8.3)
103       FORMAT(1H08F15.6)
104       FORMAT(72H
     1                                                  )
105       FORMAT(22H0FLUX ON SIDE WALLS         )
106       FORMAT(16H0 FLUX OUT END     )
107       FORMAT(16H0TOTAL OUT END     F10.6)
          END



*DATA
    1    40    41    1
   XM=.18,XLOR=2.,H=20.,ER=.01,YO=0.,HH=200.
 .18         2.0         20.         .01         0.          200.
```

REFERENCES

1. Goldin, Daniel S., and Norgren, Carl T.: Thrust Measurements of Colloidal Particles as an Indication of Particle Size and Thrustor Operation. AIAA Paper 63050-63.

2. Childs, J. Howard: Theoretical Performance of Reverse-Feed Cesium Ion Engines. NASA TN D-876, 1961.

3. Demarcus, W. C.: The Problem of Knudsen Flow, Rep. K-1302, Pts. I - VI, AEC, 1956-57.

4. Winterbottom, W. L., and Hirth, J. P.: The Diffusional Contribution to the Knudsen Cell Effusion Flux. Metals Res. Lab., Carnegie Inst. Tech., 1961.

5. Present, Richard D.: Kinetic Theory of Gases. McGraw Hill Book Co., Inc., 1958, p. 56.

6. Rogers, Milton: Gas-Surface Phenomena. AFOSR Rep. 2342, Apr. 1962.

7. Patterson, G. N.: A State-of-the-Art Survey of Some Aspects of the Mechanics of Rarefied Gases and Plasmas. ARL 62-353, May 1962.

8. Sparrow, E. M., Jonsson, V. K., and Lundgren, T. S.: Free-Molecule Tube Flow and Adiabatic Wall Temperatures. Jour. of Heat Transfer, ser. C, vol. 85, no. 2, May 1963, pp. 111-118.

9. Sparrow, E. M., and Jonsson, V. K.: Free-Molecule Flow and Convective-Radiative Energy Transport in a Tapered Tube or Conical Nozzle. AIAA Jour., vol. 1, no. 5, May 1963, pp. 1081-1087.

10. Richley, Edward A., and Bogart, Carl D.: Numerical Solutions of Knudsen Flow Entering a Circular Tube Through a Small Axial Orifice. NASA TN D-2115, 1964.

TABLE I. - TOTAL TRANSMISSION PROBABILITY

(a) Tubes

Length to inlet-radius ratio, L/R_1	Wall half-angle, β, deg													
	Diverging walls							Parallel walls	Converging walls					
	75	60	45	30	20	10	5	0	-5	-10	-20	-30	-45	-60
	Total transmission probability, P_t													
0.5	0.999	0.996	0.984	0.953	0.917	0.869	0.834	0.801	0.763	0.712	0.609	0.475	0.245	0.022
1	.999	.996	.979	.934	.874	.792	.730	.671	.606	.516	.347	.161	-----	-----
2	.999	.996	.976	.918	.834	.708	.608	.513	.408	.270	.062	-----	-----	-----
4	.999	.995	.975	.910	.807	.638	.495	.355	.204	.045	-----	-----	-----	-----
8	-----	-----	.975	.908	.795	.594	.410	.223	.044	-----	-----	-----	-----	-----
16	-----	-----	-----	.907	.791	.573	.359	.130	-----	-----	-----	-----	-----	-----

(b) Slots

Length to inlet-width ratio, L/W_1	Wall half-angle, β, deg													
	Diverging walls							Parallel walls	Converging walls					
	75	60	45	30	20	10	5	0	-5	-10	-20	-30	-45	-60
	Total transmission probability, P_t													
0.25	0.999	0.9975	0.990	0.973	0.953	0.927	0.908	0.889	0.868	0.838	0.775	0.686	0.493	0.149
.5	.999	.9970	.986	.958	.924	.876	.840	.804	.763	.704	.578	.394	-----	-----
1	.999	.9966	.983	.944	.891	.812	.748	.684	.607	.493	.237	-----	-----	-----
2	.999	.9964	.981	.935	.865	.750	.648	.541	.405	.188	-----	-----	-----	-----
4	.999	.996	.981	.930	.848	.701	.560	.398	.169	-----	-----	-----	-----	-----
8	-----	------	.981	.928	.839	.671	.495	.271	-----	-----	-----	-----	-----	-----

TABLE II. - DIRECT TRANSMISSION PROBABILITY

(a) Tubes

Length to inlet-radius ratio, L/R_1	Wall half-angle, β, deg													
	Diverging walls							Parallel walls	Converging walls					
	75	60	45	30	20	10	5	0	-5	-10	-20	-30	-45	-60
	Direct transmission probability, P_d													
0.5	0.966	0.910	0.849	0.780	0.727	0.673	0.639	0.610	0.578	0.538	0.460	0.362	0.191	0.0179
1	.955	.865	.764	.649	.563	.478	.426	.382	.337	.281	.184	.0843	-----	-----
2	.946	.823	.675	.510	.391	.282	.220	.172	.126	.0768	.0155	-----	-----	-----
4	.940	.790	.604	.399	.262	.149	.0934	.0557	.0265	.0046	-----	-----	-----	-----
8	-----	-----	.557	.330	.188	.0840	.0398	.0152	.0020	-----	-----	-----	-----	-----
16	-----	-----	-----	.292	.151	.0553	.0198	.00388	-----	-----	-----	-----	-----	-----

(b) Slots

Length to inlet-width ratio, L/W_1	Wall half-angle, β, deg													
	Diverging walls							Parallel walls	Converging walls					
	75	60	45	30	20	10	5	0	-5	-10	-20	-30	-45	-60
	Direct transmission probability, P_d													
0.25	0.983	0.954	0.921	0.883	0.853	0.820	0.799	0.781	0.761	0.733	0.678	0.602	0.437	0.134
.5	.977	.930	.874	.805	.750	.691	.652	.618	.581	.530	.429	.290	-----	-----
1	.972	.907	.822	.714	.625	.531	.469	.414	.356	.277	.124	-----	-----	-----
2	.969	.889	.777	.632	.512	.386	.306	.236	.163	.0678	-----	-----	-----	-----
4	-----	-----	.746	.574	.434	.290	.199	.123	.0446	-----	-----	-----	-----	-----
8	-----	-----	-----	.540	.389	.235	.141	.0623	-----	-----	-----	-----	-----	-----

TABLE III. - WALL FLUX DISTRIBUTIONS

(a) Tubes

Length to inlet-radius ratio, L/R_1	Axial distance, x/L	Wall half-angle, β, deg													
		Diverging walls							Parallel walls	Converging walls					
		75	60	45	30	20	10	5	0	-5	-10	-20	-30	-45	-60
		Flux ratio, n_2/n_1													
0.5	0	0.0176	0.0727	0.161	0.286	0.388	0.491	0.553	0.604	0.653	0.712	0.804	0.887	0.969	0.999
	.1	.0124	.0614	.145	.265	.367	.470	.532	.583	.633	.694	.789	.876	.964	.999
	.2	.00911	.0523	.130	.247	.346	.449	.511	.562	.613	.675	.773	.864	.958	.999
	.3	.00690	.0449	.118	.230	.327	.428	.490	.542	.593	.655	.756	.854	.952	.998
	.4	.00535	.0388	.106	.214	.308	.408	.470	.521	.572	.635	.738	.836	.944	.998
	.5	.00424	.0338	.0964	.197	.290	.388	.449	.500	.551	.614	.718	.819	.934	.997
	.6	.00341	.0296	.0876	.185	.273	.369	.429	.479	.530	.593	.698	.801	.922	.995
	.7	.00279	.0260	.0798	.172	.257	.350	.409	.458	.509	.571	.676	.781	.907	.992
	.8	.00231	.0230	.0727	.160	.242	.332	.389	.438	.487	.549	.653	.759	.889	.986
	.9	.00194	.0204	.0664	.149	.227	.314	.370	.417	.465	.526	.629	.734	.865	.972
	1.0	.00164	.0182	.0608	.139	.213	.297	.350	.396	.443	.503	.603	.706	.834	.926
1	0	0.0176	0.0734	0.166	0.302	0.420	0.541	0.614	0.674	0.732	0.798	0.896	0.967	-----	-----
	.1	.00913	.0530	.135	.264	.380	.502	.578	.640	.700	.772	.879	.960	-----	-----
	.2	.00537	.0396	.111	.232	.344	.465	.542	.605	.668	.744	.860	.952	-----	-----
	.3	.00343	.0303	.0927	.204	.311	.430	.506	.570	.635	.714	.838	.941	-----	-----
	.4	.00233	.0237	.0779	.179	.280	.396	.471	.535	.601	.682	.812	.928	-----	-----
	.5	.00166	.0189	.0659	.158	.253	.363	.437	.500	.566	.648	.784	.910	-----	-----
	.6	.00122	.0153	.0562	.140	.228	.332	.403	.465	.529	.611	.751	.888	-----	-----
	.7	.000924	.0126	.0482	.124	.205	.303	.371	.430	.492	.572	.713	.858	-----	-----
	.8	.000717	.0104	.0415	.109	.184	.275	.339	.395	.454	.532	.669	.818	-----	-----
	.9	.000568	.0088	.0360	.0969	.165	.249	.308	.360	.416	.489	.618	.761	-----	-----
	1.0	.000457	.00741	.0313	.0859	.148	.225	.278	.326	.377	.443	.560	.682	-----	-----
2	0	0.0176	0.0737	0.168	0.315	0.450	0.595	0.685	0.758	0.827	0.901	0.984	-----	-----	-----
	.1	.00537	.0399	.114	.247	.378	.529	.626	.707	.786	.875	.979	-----	-----	-----
	.2	.00233	.0241	.0811	.196	.318	.468	.569	.656	.744	.845	.972	-----	-----	-----
	.3	.00122	.0157	.0595	.157	.269	.412	.514	.604	.698	.811	.963	-----	-----	-----
	.4	.000722	.0108	.0449	.127	.227	.362	.461	.552	.650	.773	.951	-----	-----	-----
	.5	.000462	.00775	.0347	.104	.192	.316	.411	.500	.598	.728	.935	-----	-----	-----
	.6	.000314	.00576	.0273	.0854	.163	.275	.363	.448	.544	.677	.911	-----	-----	-----
	.7	.000223	.00440	.0218	.0708	.138	.237	.317	.396	.487	.618	.875	-----	-----	-----
	.8	.000164	.00343	.0177	.0590	.116	.203	.274	.344	.427	.549	.817	-----	-----	-----
	.9	.000124	.00273	.0145	.0492	.0982	.172	.233	.293	.364	.468	.715	-----	-----	-----
	1.0	.0000965	.00220	.012	.0412	.0824	.144	.194	.242	.298	.375	.522	-----	-----	-----
4	0	0.0176	0.0737	0.169	0.322	0.469	0.638	0.748	0.836	0.917	0.985	-----	-----	-----	-----
	.1	.00234	.0242	.0824	.204	.344	.526	.656	.769	.879	.977	-----	-----	-----	-----
	.2	.000723	.0109	.0463	.137	.257	.433	.572	.701	.834	.967	-----	-----	-----	-----
	.3	.000315	.00587	.0288	.0959	.195	.357	.495	.633	.786	.953	-----	-----	-----	-----
	.4	.000165	.00354	.0192	.0698	.151	.295	.426	.566	.733	.936	-----	-----	-----	-----
	.5	.0000976	.00231	.0134	.0522	.118	.242	.363	.500	.674	.912	-----	-----	-----	-----
	.6	.0000625	.00160	.00980	.0399	.0935	.198	.306	.433	.607	.878	-----	-----	-----	-----
	.7	.0000425	.00115	.00734	.0309	.0740	.161	.253	.366	.531	.828	-----	-----	-----	-----
	.8	.0000302	.000856	.00564	.0242	.0587	.129	.204	.298	.443	.747	-----	-----	-----	-----
	.9	.0000222	.000654	.00441	.0192	.0464	.101	.159	.230	.341	.605	-----	-----	-----	-----
	1.0	.0000168	.000511	.00349	.0152	.0364	.0770	.117	.163	.224	.323	-----	-----	-----	-----
8	0	---------	--------	0.170	0.325	0.478	0.665	0.793	0.899	0.983	-----	-----	-----	-----	-----
	.1	---------	--------	.047	.140	.270	.476	.650	.816	.966	-----	-----	-----	-----	-----
	.2	---------	--------	.020	.074	.168	.349	.533	.734	.945	-----	-----	-----	-----	-----
	.3	---------	--------	.010	.044	.112	.262	.437	.655	.920	-----	-----	-----	-----	-----
	.4	---------	--------	.0061	.029	.078	.200	.357	.577	.889	-----	-----	-----	-----	-----
	.5	---------	--------	.004	.020	.056	.153	.290	.500	.849	-----	-----	-----	-----	-----
	.6	---------	--------	.0027	.014	.042	.118	.231	.422	.795	-----	-----	-----	-----	-----
	.7	---------	--------	.0019	.010	.031	.090	.181	.344	.721	-----	-----	-----	-----	-----
	.8	---------	--------	.0014	.0079	.023	.067	.136	.265	.615	-----	-----	-----	-----	-----
	.9	---------	--------	.0011	.0060	.017	.049	.0959	.184	.452	-----	-----	-----	-----	-----
	1.0	---------	--------	.00085	.0045	.0129	.034	.0608	.101	.173	-----	-----	-----	-----	-----
16	0	---------	--------	-------	0.326	0.482	0.678	0.820	0.942	-----	-----	-----	-----	-----	-----
	.1	---------	--------	-------	.0750	.173	.375	.594	.847	-----	-----	-----	-----	-----	-----
	.2	---------	--------	-------	.0302	.0849	.232	.442	.757	-----	-----	-----	-----	-----	-----
	.3	---------	--------	-------	.0157	.0491	.155	.335	.670	-----	-----	-----	-----	-----	-----
	.4	---------	--------	-------	.00936	.0312	.108	.256	.584	-----	-----	-----	-----	-----	-----
	.5	---------	--------	-------	.00606	.0210	.0769	.195	.499	-----	-----	-----	-----	-----	-----
	.6	---------	--------	-------	.00415	.0147	.0554	.147	.413	-----	-----	-----	-----	-----	-----
	.7	---------	--------	-------	.00294	.0105	.0399	.108	.327	-----	-----	-----	-----	-----	-----
	.8	---------	--------	-------	.00213	.00753	.0282	.0764	.241	-----	-----	-----	-----	-----	-----
	.9	---------	--------	-------	.00157	.00540	.0192	.0494	.152	-----	-----	-----	-----	-----	-----
	1.0	---------	--------	-------	.00117	.00384	.0122	.0267	.0579	-----	-----	-----	-----	-----	-----

TABLE III. - Concluded. WALL FLUX DISTRIBUTIONS

(b) Slots

Length to inlet-width ratio, L/W_1	Axial distance, x/L	Wall half-angle, β, deg													
		Diverging walls							Parallel walls	Converging walls					
		75	60	45	30	20	10	5	0	-5	-10	-20	-30	-45	-60
		Flux ratio, n_2/n_1													
0.25	0	0.0175	0.0716	0.156	0.271	0.365	0.458	0.515	0.561	0.607	0.662	0.752	0.838	0.938	0.995
	.1	.0147	.0656	.147	.261	.353	.446	.503	.549	.595	.650	.741	.830	.933	.994
	.2	.0124	.0602	.139	.250	.342	.434	.491	.537	.583	.639	.731	.820	.927	.993
	.3	.0107	.0555	.132	.240	.331	.422	.478	.524	.571	.627	.719	.811	.920	.992
	.4	.00927	.0513	.125	.231	.320	.411	.466	.512	.558	.615	.708	.800	.913	.990
	.5	.00812	.0475	.118	.222	.309	.399	.454	.500	.546	.602	.696	.789	.904	.988
	.6	.00717	.0441	.112	.213	.299	.388	.442	.488	.533	.590	.683	.777	.894	.985
	.7	.00638	.0411	.106	.205	.289	.377	.431	.476	.521	.577	.670	.765	.883	.979
	.8	.00571	.0383	.101	.197	.279	.366	.419	.463	.508	.564	.657	.751	.871	.971
	.9	.00514	.0358	.0961	.189	.270	.355	.407	.451	.496	.551	.643	.737	.856	.956
	1.0	.00465	.0336	.0915	.182	.261	.344	.396	.439	.483	.538	.629	.722	.840	.927
0.5	0	0.0176	0.0724	0.160	0.285	0.389	0.495	0.560	0.613	0.666	0.730	0.831	0.924	-----	-----
	.1	.0125	.0611	.144	.264	.367	.473	.538	.591	.645	.710	.816	.915	-----	-----
	.2	.00931	.0522	.129	.245	.345	.450	.515	.569	.623	.690	.799	.904	-----	-----
	.3	.00721	.0450	.117	.227	.325	.428	.492	.546	.601	.668	.780	.891	-----	-----
	.4	.00574	.0392	.106	.211	.305	.406	.470	.523	.578	.645	.760	.876	-----	-----
	.5	.00469	.0344	.0960	.196	.287	.385	.447	.500	.554	.622	.737	.858	-----	-----
	.6	.00390	.0304	.0875	.182	.269	.364	.425	.477	.530	.597	.713	.837	-----	-----
	.7	.00329	.0271	.0800	.170	.253	.345	.404	.454	.506	.571	.686	.811	-----	-----
	.8	.00281	.0243	.0734	.158	.238	.326	.382	.431	.481	.545	.657	.780	-----	-----
	.9	.00246	.0218	.0675	.147	.223	.307	.362	.409	.457	.518	.625	.743	-----	-----
	1.0	.00213	.0198	.0622	.138	.210	.290	.342	.387	.433	.491	.592	.697	-----	-----
1	0	0.0176	0.0730	0.164	0.300	0.418	0.544	0.623	0.688	0.754	0.832	0.949	-----	-----	-----
	.1	.00932	.0528	.133	.261	.377	.503	.584	.653	.723	.808	.940	-----	-----	-----
	.2	.00576	.0398	.110	.227	.338	.464	.546	.616	.689	.780	.928	-----	-----	-----
	.3	.00391	.0310	.0917	.199	.304	.425	.507	.578	.653	.750	.914	-----	-----	-----
	.4	.00283	.0249	.0776	.175	.273	.389	.468	.539	.615	.715	.896	-----	-----	-----
	.5	.00214	.0204	.0664	.154	.245	.355	.431	.500	.575	.677	.874	-----	-----	-----
	.6	.00168	.0170	.0573	.137	.220	.323	.395	.461	.534	.635	.844	-----	-----	-----
	.7	.00135	.0144	.0500	.122	.198	.293	.360	.422	.491	.589	.804	-----	-----	-----
	.8	.00111	.0123	.0440	.109	.178	.265	.327	.384	.448	.538	.748	-----	-----	-----
	.9	.000934	.0107	.0389	.0975	.161	.240	.296	.347	.404	.483	.665	-----	-----	-----
	1.0	.000794	.00937	.0347	.0878	.146	.217	.267	.312	.360	.425	.545	-----	-----	-----
2	0	0.0176	0.0732	0.166	0.310	0.442	0.590	0.688	0.770	0.854	0.951	-----	-----	-----	-----
	.1	.00577	.0401	.112	.239	.367	.520	.625	.719	.817	.937	-----	-----	-----	-----
	.2	.00284	.0252	.0803	.188	.305	.455	.564	.665	.775	.919	-----	-----	-----	-----
	.3	.00169	.0173	.0602	.151	.255	.397	.505	.610	.730	.897	-----	-----	-----	-----
	.4	.00112	.0127	.0468	.123	.215	.345	.450	.555	.682	.871	-----	-----	-----	-----
	.5	.000800	.00968	.0374	.102	.183	.300	.398	.500	.629	.839	-----	-----	-----	-----
	.6	.000600	.00765	.0306	.0858	.156	.261	.349	.445	.572	.798	-----	-----	-----	-----
	.7	.000467	.00621	.0255	.0729	.134	.225	.304	.390	.509	.743	-----	-----	-----	-----
	.8	.000374	.00514	.0216	.0625	.115	.194	.261	.335	.439	.666	-----	-----	-----	-----
	.9	.000306	.00433	.0185	.0541	.0996	.166	.222	.281	.362	.547	-----	-----	-----	-----
	1.0	.000256	.00370	.0161	.0471	.0865	.142	.186	.230	.281	.358	-----	-----	-----	-----
4	0	0.0176	0.0733	0.167	0.315	0.457	0.624	0.740	0.843	0.949	-----	-----	-----	-----	-----
	.1	.00284	.0253	.0815	.195	.325	.502	.640	.773	.923	-----	-----	-----	-----	-----
	.2	.00112	.0128	.0482	.132	.240	.405	.549	.701	.892	-----	-----	-----	-----	-----
	.3	.000602	.00779	.0321	.0950	.183	.331	.471	.632	.857	-----	-----	-----	-----	-----
	.4	.000376	.00528	.0231	.0720	.144	.273	.403	.565	.817	-----	-----	-----	-----	-----
	.5	.000258	.00383	.0175	.0565	.116	.226	.343	.500	.770	-----	-----	-----	-----	-----
	.6	.000188	.00292	.0137	.0454	.0946	.187	.289	.433	.713	-----	-----	-----	-----	-----
	.7	.000143	.00230	.0111	.0372	.0778	.154	.240	.366	.641	-----	-----	-----	-----	-----
	.8	.000113	.00187	.00914	.0309	.0645	.127	.195	.298	.547	-----	-----	-----	-----	-----
	.9	.0000914	.00154	.00767	.0260	.0538	.103	.154	.226	.413	-----	-----	-----	-----	-----
	1.0	.0000755	.00130	.00653	.0221	.0451	.0829	.117	.156	.212	-----	-----	-----	-----	-----
8	0	---------	-------	0.167	0.317	0.464	0.644	0.776	0.899	-----	-----	-----	-----	-----	-----
	.1	---------	-------	.0486	.135	.251	.439	.613	.810	-----	-----	-----	-----	-----	-----
	.2	---------	-------	.0236	.0762	.159	.317	.492	.725	-----	-----	-----	-----	-----	-----
	.3	---------	-------	.0143	.0501	.111	.240	.400	.647	-----	-----	-----	-----	-----	-----
	.4	---------	-------	.00976	.0358	.0824	.187	.327	.571	-----	-----	-----	-----	-----	-----
	.5	---------	-------	.00714	.0270	.0634	.148	.267	.496	-----	-----	-----	-----	-----	-----
	.6	---------	-------	.00547	.0211	.0498	.117	.216	.422	-----	-----	-----	-----	-----	-----
	.7	---------	-------	.00433	.0168	.0397	.0932	.172	.347	-----	-----	-----	-----	-----	-----
	.8	---------	-------	.00352	.0137	.0320	.0733	.133	.269	-----	-----	-----	-----	-----	-----
	.9	---------	-------	.00232	.0113	.0261	.0569	.0977	.187	-----	-----	-----	-----	-----	-----
	1.0	---------	-------	.00246	.00949	.0213	.0436	.0673	.0990	-----	-----	-----	-----	-----	-----

TABLE IV. - EXIT-PLANE FLUX DISTRIBUTIONS

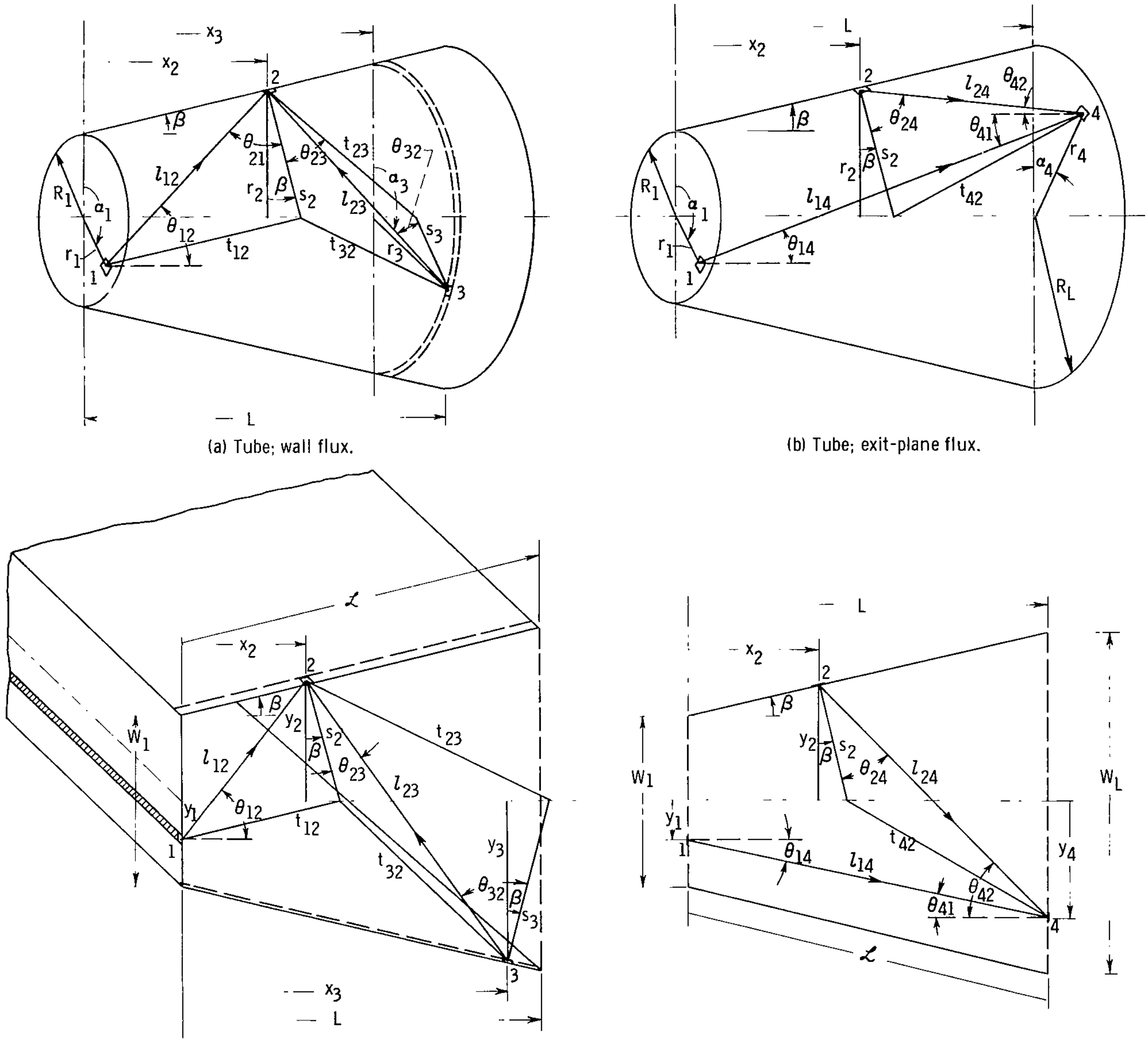
(a) Tubes

Length to inlet-radius ratio, L/R_1	Radial distance, r/R_L	Wall half-angle, β, deg: Diverging walls							Parallel walls	Converging walls					
		75	60	45	30	20	10	5	0	-5	-10	-20	-30	-45	-60
		Flux ratio, n_4/n_1													
0.5	0	0.802	0.810	0.824	0.846	0.865	0.884	0.896	0.906	0.916	0.927	0.947	0.965	0.986	0.997
	.1	.780	.802	.819	.843	.862	.882	.894	.904	.914	.927	.946	.965	.985	.997
	.2	.699	.773	.803	.832	.854	.876	.890	.901	.911	.924	.945	.964	.985	.997
	.3	.515	.717	.772	.813	.840	.866	.881	.894	.906	.920	.942	.963	.985	.996
	.4	.267	.619	.720	.782	.819	.851	.869	.883	.897	.913	.938	.961	.984	.996
	.5	.113	.474	.641	.735	.786	.828	.850	.868	.884	.903	.932	.958	.983	.996
	.6	.0513	.315	.531	.669	.740	.795	.824	.846	.867	.890	.924	.954	.982	.996
	.7	.0265	.193	.406	.582	.676	.750	.788	.816	.842	.871	.913	.948	.980	.996
	.8	.0152	.119	.293	.481	.597	.691	.740	.776	.809	.846	.898	.940	.978	.995
	.9	.00946	.0758	.206	.380	.506	.617	.676	.721	.763	.809	.875	.928	.974	.994
	.95	.00763	.0615	.172	.333	.458	.473	.637	.686	.732	.784	.859	.918	.971	.991
	1.0	.00461	.0331	.0903	.180	.259	.343	.395	.439	.483	.539	.631	.725	.842	.928
1	0	0.504	0.520	0.550	0.600	0.648	0.700	0.734	0.762	0.791	0.826	0.883	0.934	-----	-----
	.1	.450	.502	.541	.595	.644	.698	.732	.761	.790	.825	.883	.934	-----	-----
	.2	.303	.450	.513	.579	.634	.691	.726	.756	.786	.823	.881	.933	-----	-----
	.3	.153	.368	.468	.553	.616	.678	.717	.749	.780	.818	.878	.932	-----	-----
	.4	.0691	.274	.408	.517	.590	.661	.703	.737	.771	.812	.875	.930	-----	-----
	.5	.0329	.188	.340	.471	.558	.638	.685	.723	.759	.803	.870	.928	-----	-----
	.6	.0171	.125	.271	.419	.519	.610	.662	.704	.744	.791	.863	.924	-----	-----
	.7	.00971	.0823	.210	.363	.473	.575	.634	.680	.724	.776	.854	.919	-----	-----
	.8	.00590	.0552	.159	.307	.423	.534	.599	.650	.699	.756	.841	.912	-----	-----
	.9	.00378	.0377	.119	.253	.369	.486	.555	.611	.665	.728	.823	.902	-----	-----
	.95	.00308	.0313	.102	.226	.339	.456	.528	.586	.642	.709	.809	.894	-----	-----
	1.0	.00209	.0191	.0599	.133	.205	.286	.340	.386	.434	.495	.600	.708	-----	-----
2	0	0.204	0.220	0.252	0.313	0.380	0.463	0.522	0.576	0.633	0.706	0.827	-----	-----	-----
	.1	.164	.208	.246	.310	.378	.462	.521	.575	.632	.705	.827	-----	-----	-----
	.2	.0912	.176	.230	.301	.371	.457	.518	.572	.629	.703	.826	-----	-----	-----
	.3	.0426	.135	.205	.286	.360	.449	.511	.566	.625	.700	.823	-----	-----	-----
	.4	.0200	.0973	.176	.266	.346	.439	.502	.559	.619	.695	.820	-----	-----	-----
	.5	.0100	.0673	.146	.243	.328	.425	.491	.549	.610	.688	.816	-----	-----	-----
	.6	.00546	.0459	.118	.217	.306	.408	.476	.536	.599	.679	.810	-----	-----	-----
	.7	.00318	.0314	.0930	.191	.282	.387	.458	.520	.585	.667	.802	-----	-----	-----
	.8	.00196	.0217	.0724	.164	.255	.362	.435	.499	.567	.652	.791	-----	-----	-----
	.9	.00127	.0152	.0556	.138	.226	.333	.407	.472	.542	.630	.774	-----	-----	-----
	.95	.00104	.0128	.0484	.125	.209	.315	.389	.455	.525	.614	.756	-----	-----	-----
	1.0	.000762	.00867	.0314	.0797	.134	.206	.258	.307	.361	.434	.566	-----	-----	-----
4	0	0.0609	0.0694	0.0879	0.128	0.179	0.257	0.325	0.396	0.484	0.613	-----	-----	-----	-----
	.1	.0471	.0653	.0859	.126	.178	.256	.325	.396	.483	.613	-----	-----	-----	-----
	.2	.0251	.0548	.0802	.123	.175	.254	.322	.394	.481	.611	-----	-----	-----	-----
	.3	.0118	.0420	.0717	.117	.170	.250	.319	.390	.478	.608	-----	-----	-----	-----
	.4	.00564	.0304	.0619	.109	.164	.245	.314	.385	.474	.604	-----	-----	-----	-----
	.5	.00269	.0213	.0518	.101	.156	.238	.307	.379	.467	.598	-----	-----	-----	-----
	.6	.00159	.0148	.0423	.0908	.147	.229	.298	.371	.459	.591	-----	-----	-----	-----
	.7	.000933	.0103	.0339	.0805	.136	.218	.288	.360	.449	.581	-----	-----	-----	-----
	.8	.000579	.00719	.0268	.0700	.124	.205	.275	.347	.436	.568	-----	-----	-----	-----
	.9	.000377	.00510	.0209	.0596	.111	.190	.258	.329	.418	.546	-----	-----	-----	-----
	.95	.000308	.00431	.0183	.0544	.103	.180	.247	.318	.405	.509	-----	-----	-----	-----
	1.0	.000238	.00318	.0130	.0373	.0701	.122	.169	.219	.282	.379	-----	-----	-----	-----
8	0	--------	-------	0.0264	0.0431	0.0684	0.118	0.174	0.249	0.375	-----	-----	-----	-----	-----
	.1	--------	-------	.0258	.0427	.0681	.118	.173	.249	.375	-----	-----	-----	-----	-----
	.2	--------	-------	.0241	.0416	.0671	.117	.172	.248	.373	-----	-----	-----	-----	-----
	.3	--------	-------	.0217	.0397	.0654	.115	.170	.246	.371	-----	-----	-----	-----	-----
	.4	--------	-------	.0188	.0373	.0631	.112	.168	.243	.367	-----	-----	-----	-----	-----
	.5	--------	-------	.0159	.0344	.0602	.109	.164	.239	.363	-----	-----	-----	-----	-----
	.6	--------	-------	.0131	.0313	.0568	.106	.160	.234	.357	-----	-----	-----	-----	-----
	.7	--------	-------	.0106	.0279	.0529	.101	.154	.227	.349	-----	-----	-----	-----	-----
	.8	--------	-------	.00845	.0245	.0486	.0953	.148	.219	.339	-----	-----	-----	-----	-----
	.9	--------	-------	.00667	.0211	.0437	.0885	.139	.209	.316	-----	-----	-----	-----	-----
	.95	--------	-------	.00590	.0194	.0411	.0845	.134	.200	.283	-----	-----	-----	-----	-----
	1.0	--------	-------	.00446	.0141	.0292	.0590	.0931	.140	.219	-----	-----	-----	-----	-----
16	0	--------	-------	-------	0.0127	0.0221	0.0446	0.0782	0.145	-----	-----	-----	-----	-----	-----
	.1	--------	-------	-------	.0126	.0220	.0445	.0781	.145	-----	-----	-----	-----	-----	-----
	.2	--------	-------	-------	.0123	.0217	.0441	.0776	.144	-----	-----	-----	-----	-----	-----
	.3	--------	-------	-------	.0118	.0212	.0435	.0768	.143	-----	-----	-----	-----	-----	-----
	.4	--------	-------	-------	.0111	.0205	.0426	.0757	.141	-----	-----	-----	-----	-----	-----
	.5	--------	-------	-------	.0103	.0196	.0415	.0742	.139	-----	-----	-----	-----	-----	-----
	.6	--------	-------	-------	.00936	.0185	.0401	.0723	.136	-----	-----	-----	-----	-----	-----
	.7	--------	-------	-------	.00840	.0173	.0384	.0699	.132	-----	-----	-----	-----	-----	-----
	.8	--------	-------	-------	.00743	.0160	.0364	.0670	.128	-----	-----	-----	-----	-----	-----
	.9	--------	-------	-------	.00645	.0145	.0339	.0632	.122	-----	-----	-----	-----	-----	-----
	.95	--------	-------	-------	.00596	.0137	.0325	.0609	.117	-----	-----	-----	-----	-----	-----
	1.0	--------	-------	-------	.00457	.0101	.0233	.0430	.0824	-----	-----	-----	-----	-----	-----

TABLE IV. - Concluded. EXIT-PLANE FLUX DISTRIBUTIONS

(b) Slots

Length to inlet-width ratio L/W_1	Lateral distance, $\frac{y}{W_L/2}$	Wall half-angle, β, deg; Diverging walls							Parallel walls	Converging walls					
		75	60	45	30	20	10	5	0	-5	-10	-20	-30	-45	-60
		Flux ratio, n_4/n_1													
0.25	0	0.896	0.901	0.909	0.920	0.929	0.939	0.944	0.949	0.954	0.960	0.969	0.978	0.990	0.997
	.1	.877	.893	.904	.917	.927	.937	.943	.948	.953	.959	.969	.978	.990	.997
	.2	.805	.870	.890	.908	.921	.932	.939	.945	.950	.957	.967	.978	.989	.997
	.3	.627	.820	.864	.892	.909	.924	.932	.939	.945	.953	.965	.976	.989	.997
	.4	.361	.728	.818	.865	.890	.910	.921	.930	.938	.947	.961	.974	.988	.997
	.5	.175	.584	.744	.823	.861	.890	.905	.916	.927	.938	.956	.972	.988	.997
	.6	.0918	.414	.636	.761	.818	.860	.881	.897	.911	.926	.949	.968	.987	.997
	.7	.0541	.274	.507	.676	.759	.819	.848	.870	.889	.910	.939	.963	.985	.996
	.8	.0351	.183	.384	.575	.682	.763	.803	.832	.858	.886	.925	.956	.983	.996
	.9	.0244	.128	.288	.473	.594	.693	.745	.783	.817	.854	.906	.946	.980	.993
	.95	.0206	.109	.250	.425	.547	.653	.710	.752	.791	.833	.892	.939	.978	.985
	1.0	.0132	.0619	.138	.240	.322	.403	.453	.493	.533	.582	.664	.747	.851	.930
0.5	0	0.710	0.722	0.743	0.774	0.800	0.828	0.845	0.860	0.875	0.892	0.922	0.951	-----	-----
	.1	.652	.704	.733	.768	.797	.826	.844	.858	.873	.892	.922	.951	-----	-----
	.2	.480	.647	.705	.752	.786	.819	.838	.854	.870	.889	.920	.950	-----	-----
	.3	.279	.555	.656	.725	.768	.807	.829	.847	.864	.885	.918	.949	-----	-----
	.4	.149	.439	.589	.687	.743	.790	.816	.836	.856	.879	.915	.948	-----	-----
	.5	.0835	.326	.509	.638	.709	.767	.798	.822	.845	.871	.910	.945	-----	-----
	.6	.0505	.235	.425	.581	.668	.739	.776	.804	.830	.860	.904	.943	-----	-----
	.7	.0327	.170	.347	.518	.621	.705	.749	.782	.812	.847	.897	.939	-----	-----
	.8	.0225	.125	.279	.453	.568	.664	.716	.754	.790	.829	.887	.934	-----	-----
	.9	.0161	.9833	.223	.390	.510	.617	.675	.719	.761	.807	.873	.926	-----	-----
	.95	.0138	.0813	.199	.359	.479	.590	.651	.698	.742	.791	.863	.914	-----	-----
	1.0	.00987	.0526	.124	.223	.304	.385	.435	.475	.516	.567	.651	.737	-----	-----
1	0	0.452	0.470	0.502	0.554	0.603	0.657	0.692	0.723	0.755	0.795	0.864	-----	-----	-----
	.1	.384	.450	.493	.550	.600	.655	.691	.722	.754	.794	.864	-----	-----	-----
	.2	.245	.397	.467	.536	.592	.650	.687	.719	.751	.793	.863	-----	-----	-----
	.3	.137	.325	.428	.515	.578	.641	.680	.714	.747	.790	.861	-----	-----	-----
	.4	.0770	.252	.380	.487	.559	.629	.671	.706	.742	.785	.859	-----	-----	-----
	.5	.0457	.190	.329	.453	.536	.613	.659	.696	.734	.780	.856	-----	-----	-----
	.6	.0289	.142	.279	.416	.509	.594	.644	.684	.724	.772	.851	-----	-----	-----
	.7	.0192	.106	.233	.377	.478	.571	.625	.669	.711	.763	.845	-----	-----	-----
	.8	.0134	.0806	.193	.336	.443	.544	.603	.650	.695	.750	.837	-----	-----	-----
	.9	.00969	.0618	.158	.296	.405	.512	.575	.625	.674	.733	.825	-----	-----	-----
	.95	.00834	.0544	.143	.275	.384	.493	.557	.610	.661	.722	.803	-----	-----	-----
	1.0	.00645	.0391	.0991	.188	.263	.341	.391	.432	.475	.529	.624	-----	-----	-----
2	0	0.247	0.264	0.295	0.349	0.405	0.473	0.522	0.568	0.620	0.694	-----	-----	-----	-----
	.1	.204	.252	.290	.347	.403	.472	.521	.567	.619	.693	-----	-----	-----	-----
	.2	.127	.220	.275	.339	.398	.469	.519	.565	.617	.692	-----	-----	-----	-----
	.3	.0715	.180	.252	.327	.390	.463	.514	.562	.615	.690	-----	-----	-----	-----
	.4	.0411	.141	.226	.311	.379	.456	.509	.557	.611	.686	-----	-----	-----	-----
	.5	.0248	.108	.197	.291	.365	.446	.501	.550	.605	.682	-----	-----	-----	-----
	.6	.0159	.0819	.169	.270	.349	.434	.491	.542	.598	.676	-----	-----	-----	-----
	.7	.0107	.0623	.143	.247	.330	.420	.479	.532	.589	.669	-----	-----	-----	-----
	.8	.00746	.0478	.120	.223	.309	.403	.464	.519	.578	.659	-----	-----	-----	-----
	.9	.00541	.0370	.100	.199	.286	.382	.446	.502	.563	.646	-----	-----	-----	-----
	.95	.00466	.0327	.0913	.186	.273	.370	.435	.492	.553	.627	-----	-----	-----	-----
	1.0	.00379	.0255	.0691	.138	.201	.271	.318	.361	.409	.475	-----	-----	-----	-----
4	0	0.127	0.139	0.161	0.201	0.246	0.309	0.360	0.416	0.491	-----	-----	-----	-----	-----
	.1	.104	.133	.158	.200	.245	.308	.360	.415	.490	-----	-----	-----	-----	-----
	.2	.0650	.116	.150	.196	.243	.306	.358	.414	.489	-----	-----	-----	-----	-----
	.3	.0371	.0958	.139	.189	.238	.303	.356	.412	.487	-----	-----	-----	-----	-----
	.4	.0215	.0756	.125	.181	.232	.299	.352	.408	.484	-----	-----	-----	-----	-----
	.5	.0131	.0583	.110	.170	.225	.293	.348	.404	.480	-----	-----	-----	-----	-----
	.6	.00840	.0447	.0950	.159	.216	.286	.342	.399	.475	-----	-----	-----	-----	-----
	.7	.00565	.0342	.0812	.146	.205	.278	.334	.392	.469	-----	-----	-----	-----	-----
	.8	.00396	.0264	.0688	.133	.194	.268	.325	.384	.461	-----	-----	-----	-----	-----
	.9	.00287	.0206	.0579	.120	.181	.256	.314	.373	.440	-----	-----	-----	-----	-----
	.95	.00248	.0183	.0530	.113	.174	.249	.308	.364	.405	-----	-----	-----	-----	-----
	1.0	.00207	.0150	.0428	.0898	.135	.192	.234	.278	.334	-----	-----	-----	-----	-----
8	0	-------	------	0.0841	0.109	0.138	0.184	0.228	0.283	-----	-----	-----	-----	-----	-----
	.1	-------	------	.0827	.108	.138	.184	.228	.283	-----	-----	-----	-----	-----	-----
	.2	-------	------	.0788	.106	.137	.183	.227	.282	-----	-----	-----	-----	-----	-----
	.3	-------	------	.0730	.103	.134	.181	.225	.281	-----	-----	-----	-----	-----	-----
	.4	-------	------	.0659	.0984	.131	.179	.223	.279	-----	-----	-----	-----	-----	-----
	.5	-------	------	.0582	.0931	.127	.176	.221	.276	-----	-----	-----	-----	-----	-----
	.6	-------	------	.0507	.0872	.123	.172	.217	.273	-----	-----	-----	-----	-----	-----
	.7	-------	------	.0436	.0808	.117	.168	.213	.269	-----	-----	-----	-----	-----	-----
	.8	-------	------	.0372	.0741	.111	.163	.208	.264	-----	-----	-----	-----	-----	-----
	.9	-------	------	.0315	.0674	.105	.156	.202	.256	-----	-----	-----	-----	-----	-----
	.95	-------	------	.0290	.0639	.101	.153	.198	.241	-----	-----	-----	-----	-----	-----
	1.0	-------	------	.0245	.0532	.0827	.122	.156	.197	-----	-----	-----	-----	-----	-----



(a) Tube; wall flux.

(b) Tube; exit-plane flux.

(c) Slot; wall flux.

(d) Slot; exit-plane flux.

Figure 1. - Schematic illustration of configurations showing geometric relations involved in flux determinations.

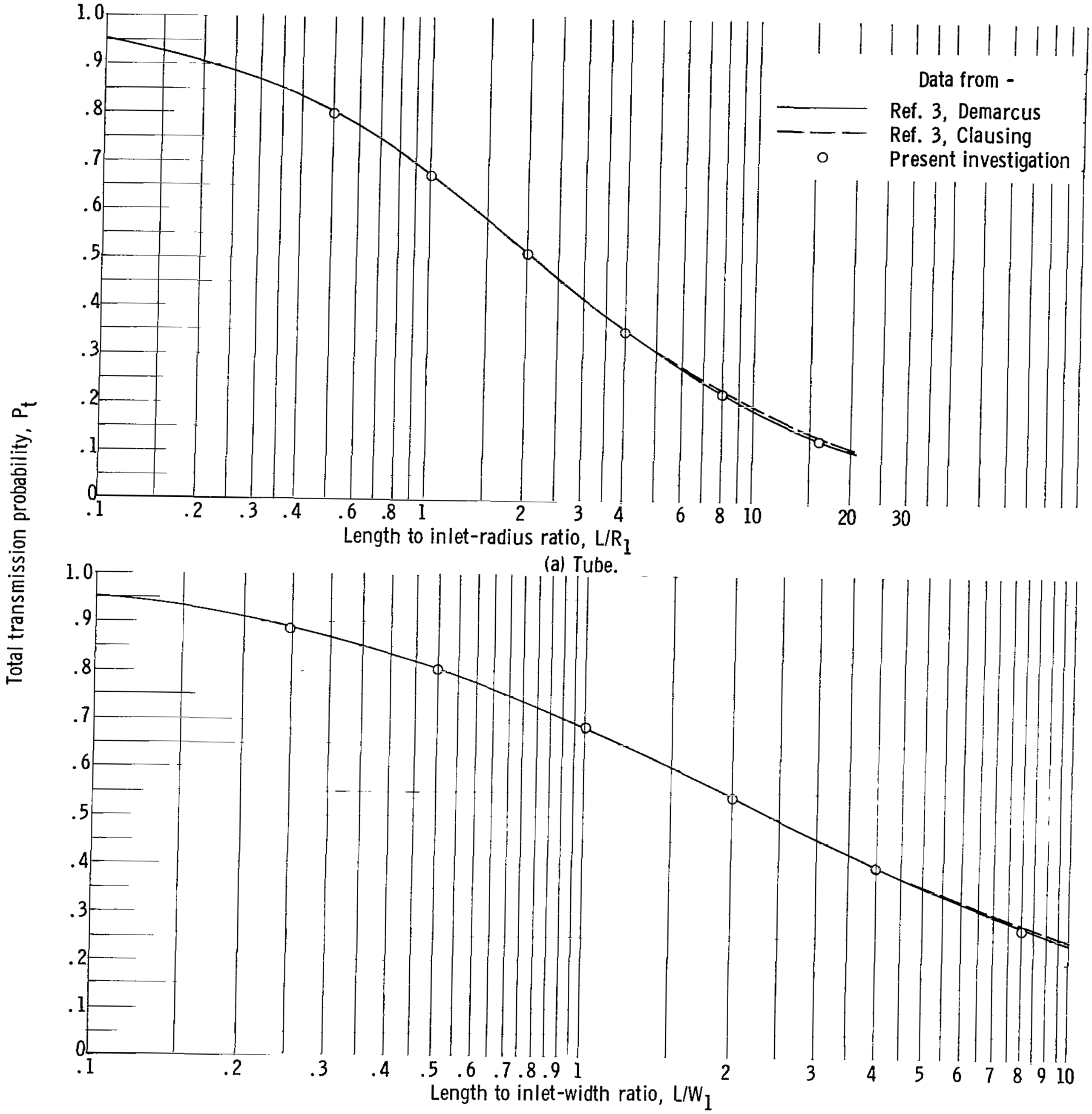
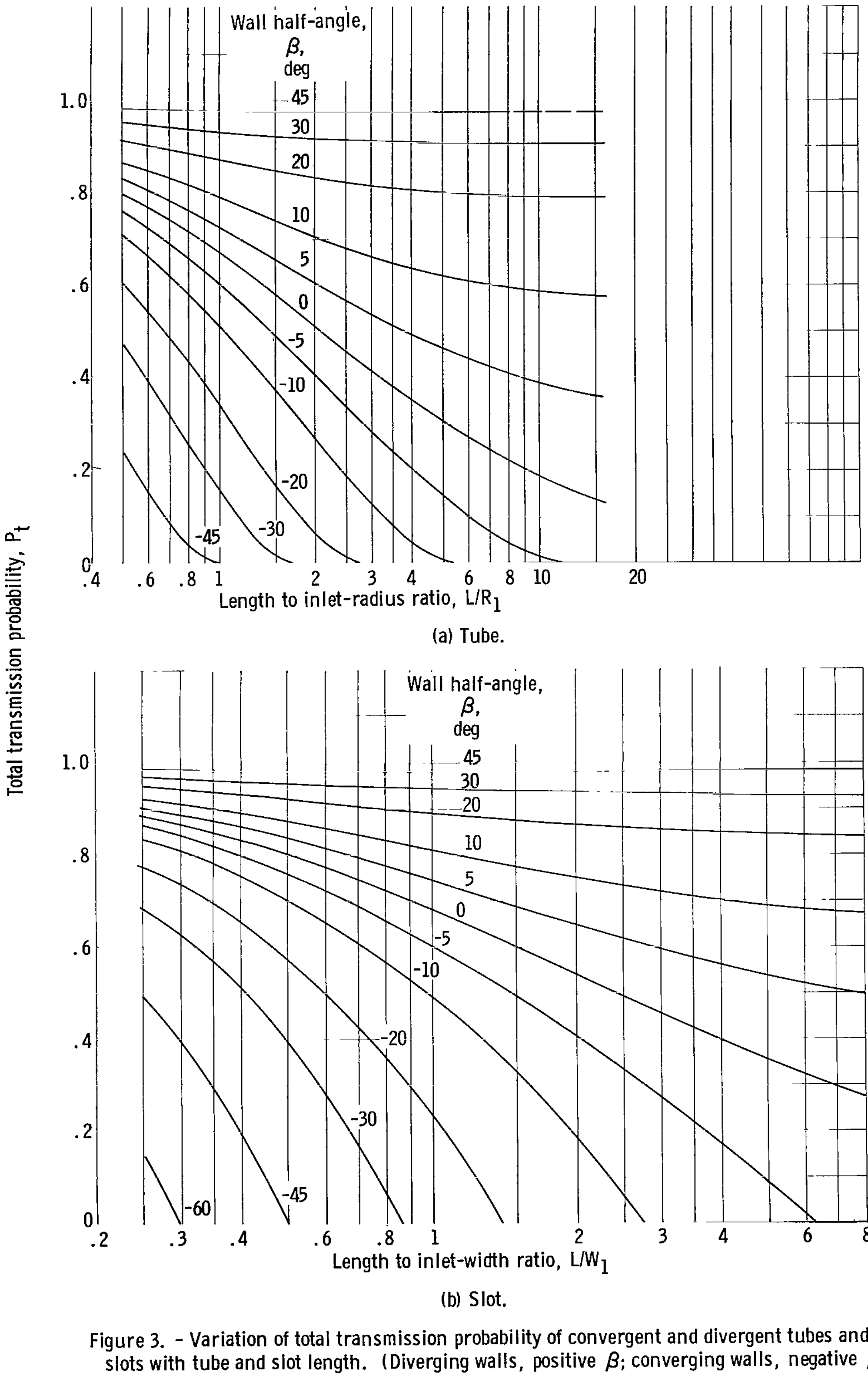


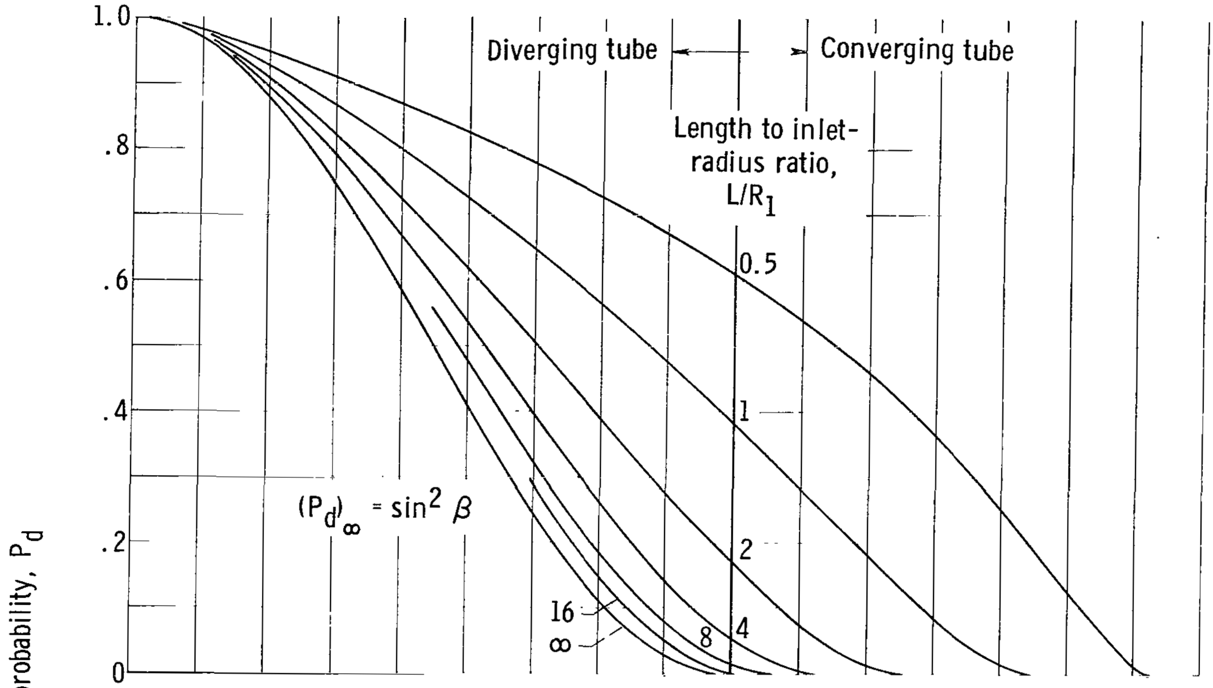
Figure 2. - Variation of total transmission probability with aperture length to inlet-radius or length to inlet-width ratio for a wall half-angle of zero.



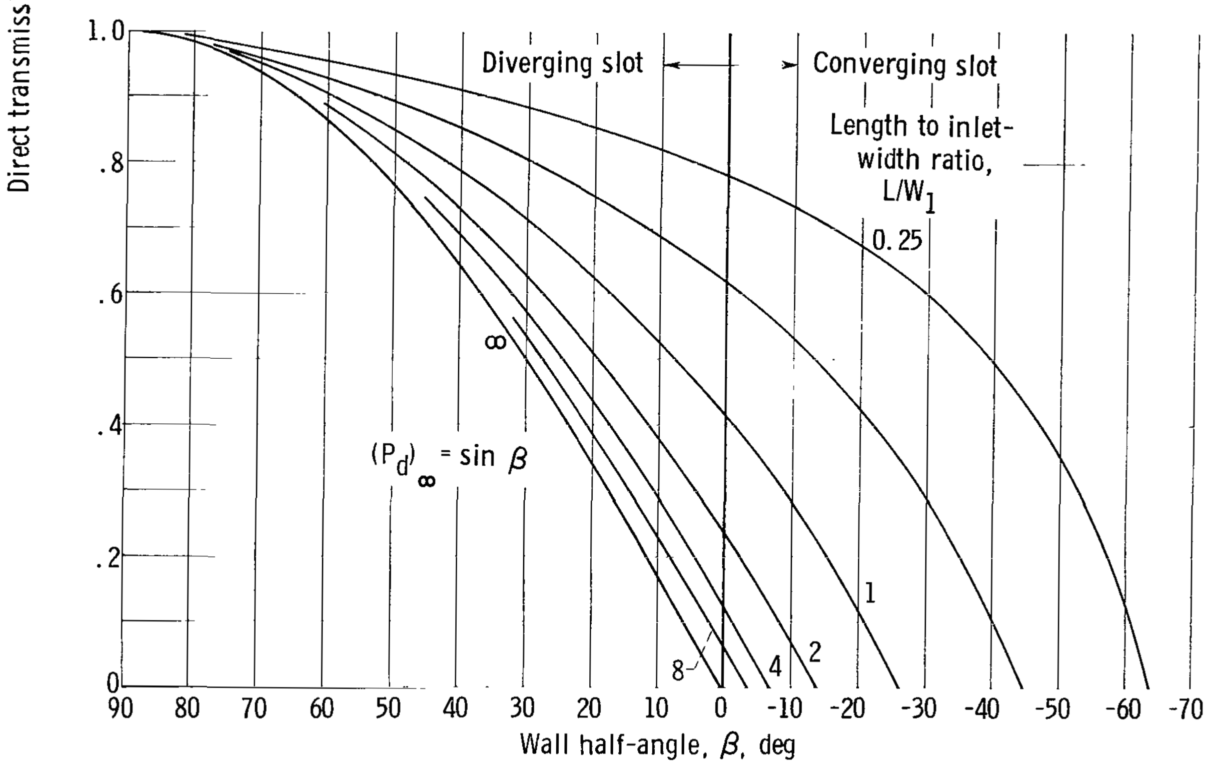
(a) Tube.

(b) Slot.

Figure 3. - Variation of total transmission probability of convergent and divergent tubes and slots with tube and slot length. (Diverging walls, positive β; converging walls, negative β.)



(a) Tube.



(b) Slot.

Figure 4. - Variation of direct transmission probability with wall half-angle.

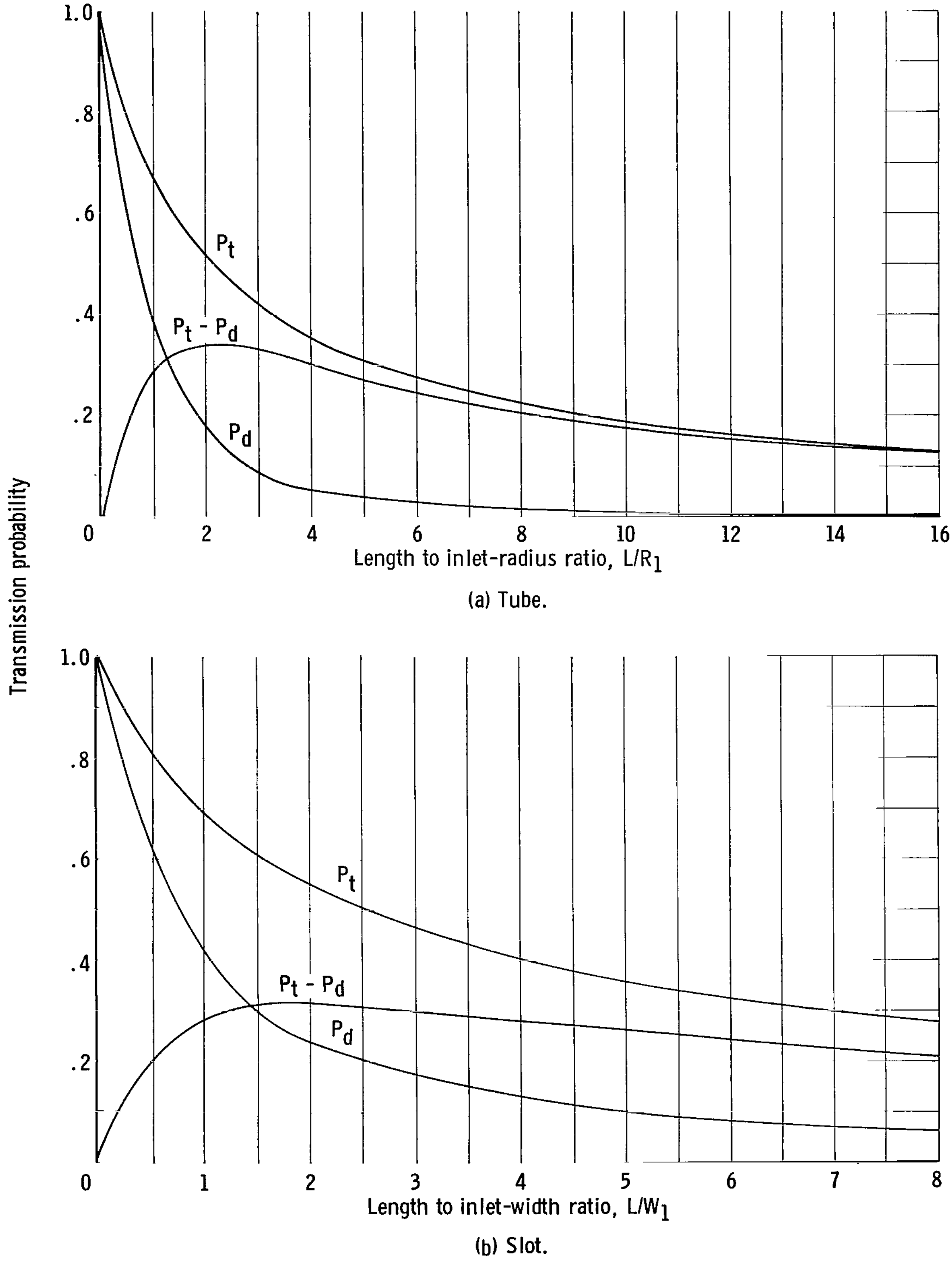
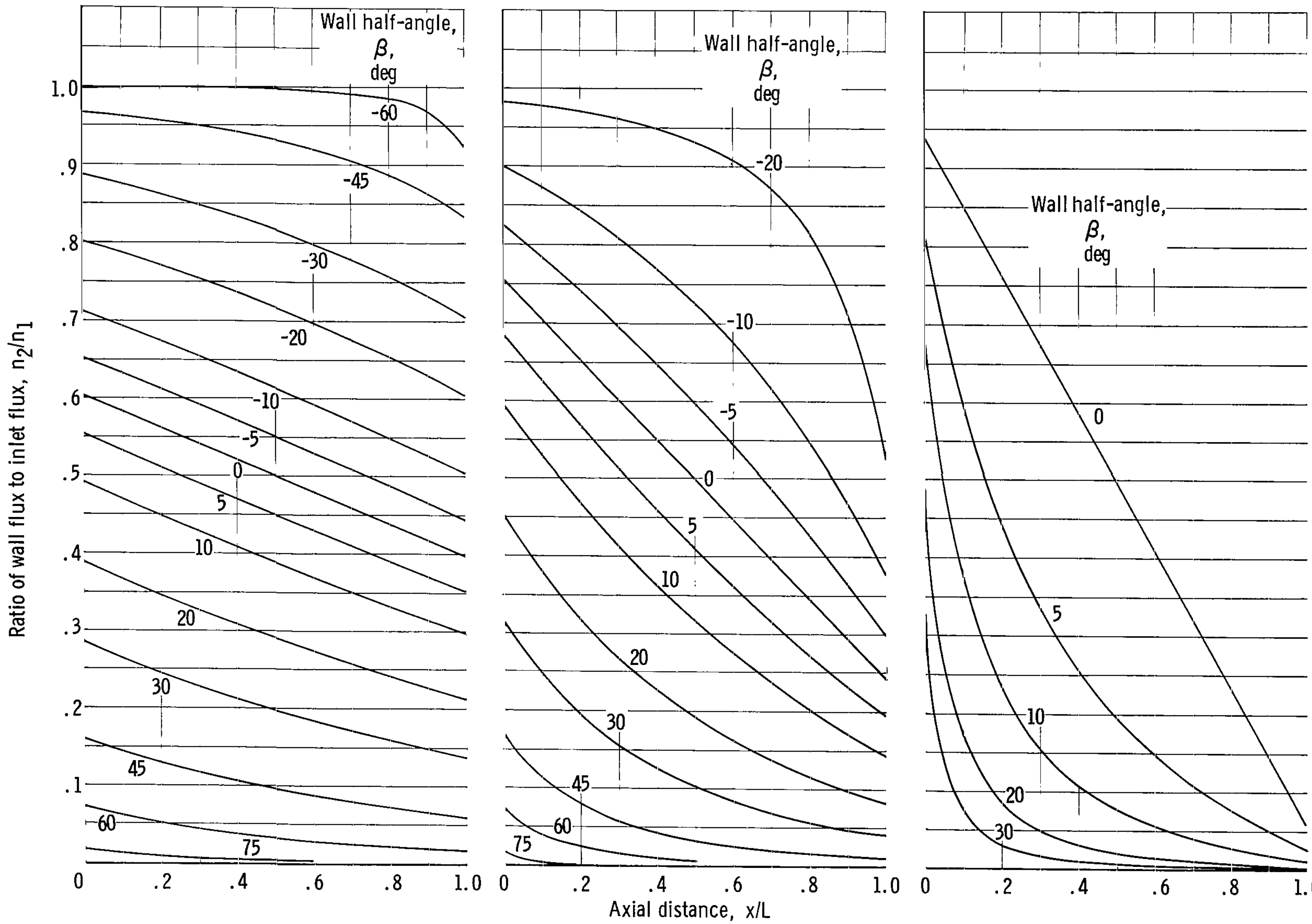


Figure 5. - Comparison of transmission probabilities through cylindrical tubes and parallel-walled slots.

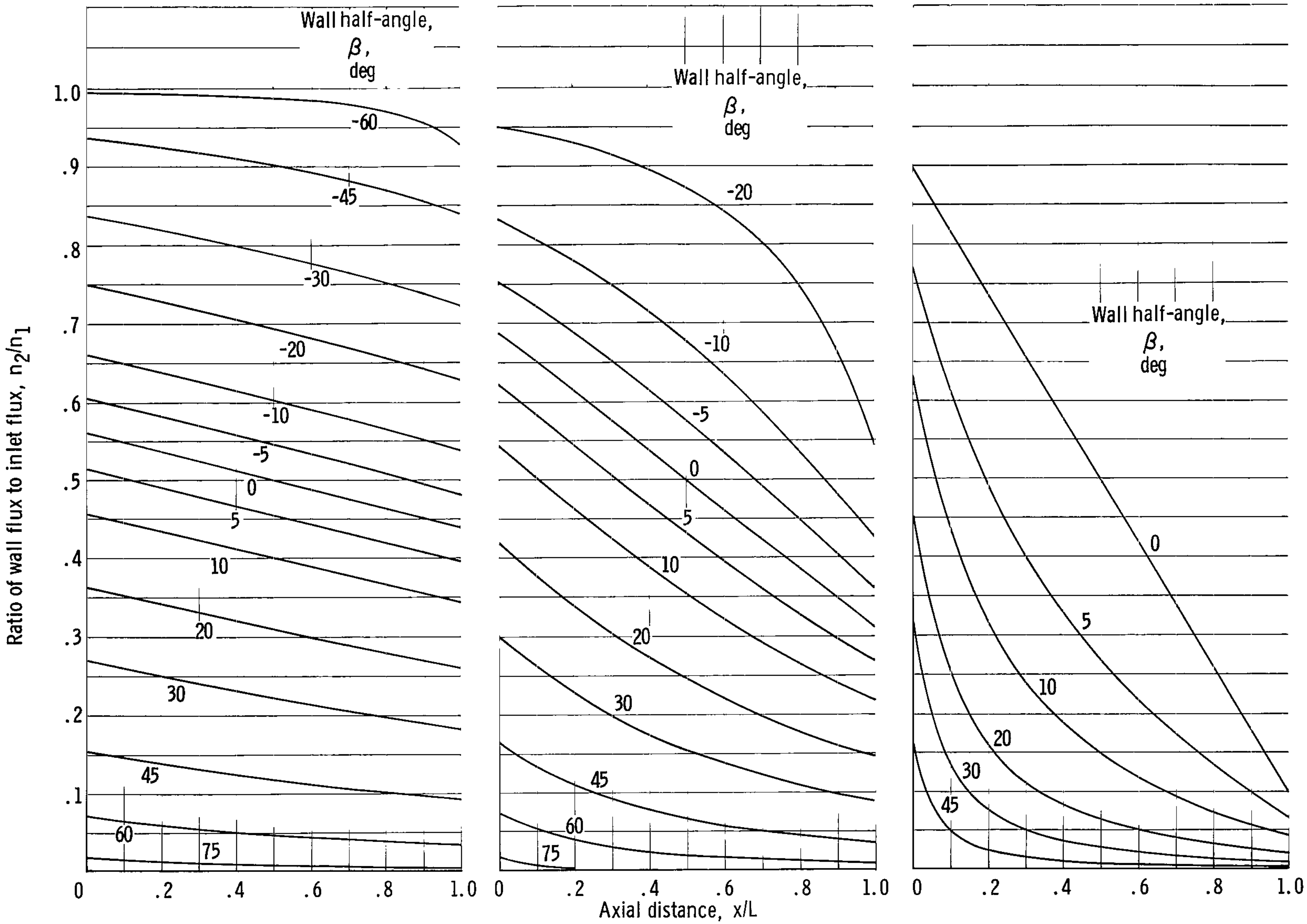


(a) Length to inlet-radius ratio, 0.5.

(b) Length to inlet-radius ratio, 2.

(c) Length to inlet-radius ratio, 16.

Figure 6. - Variation of flux along wall of convergent and divergent tubes. (Diverging walls; positive β; converging walls, negative β.)



(a) Length to inlet-width ratio, 0.25

(b) Length to inlet-width ratio, 1.

(c) Length to inlet-width ratio, 8.

Figure 7. - Variation of flux along wall of convergent and divergent slots. (Diverging walls, positive β; converging walls, negative β.)

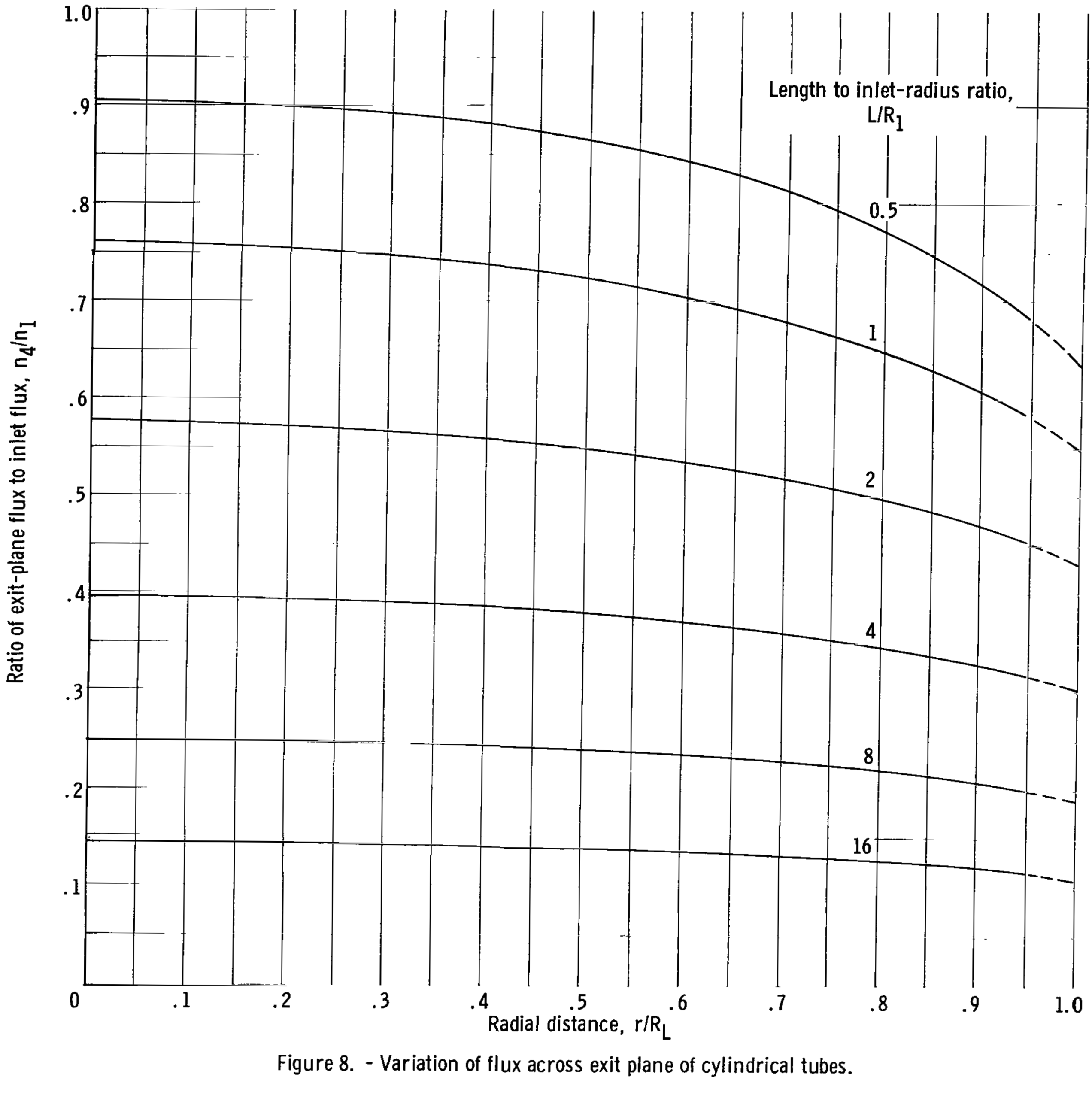
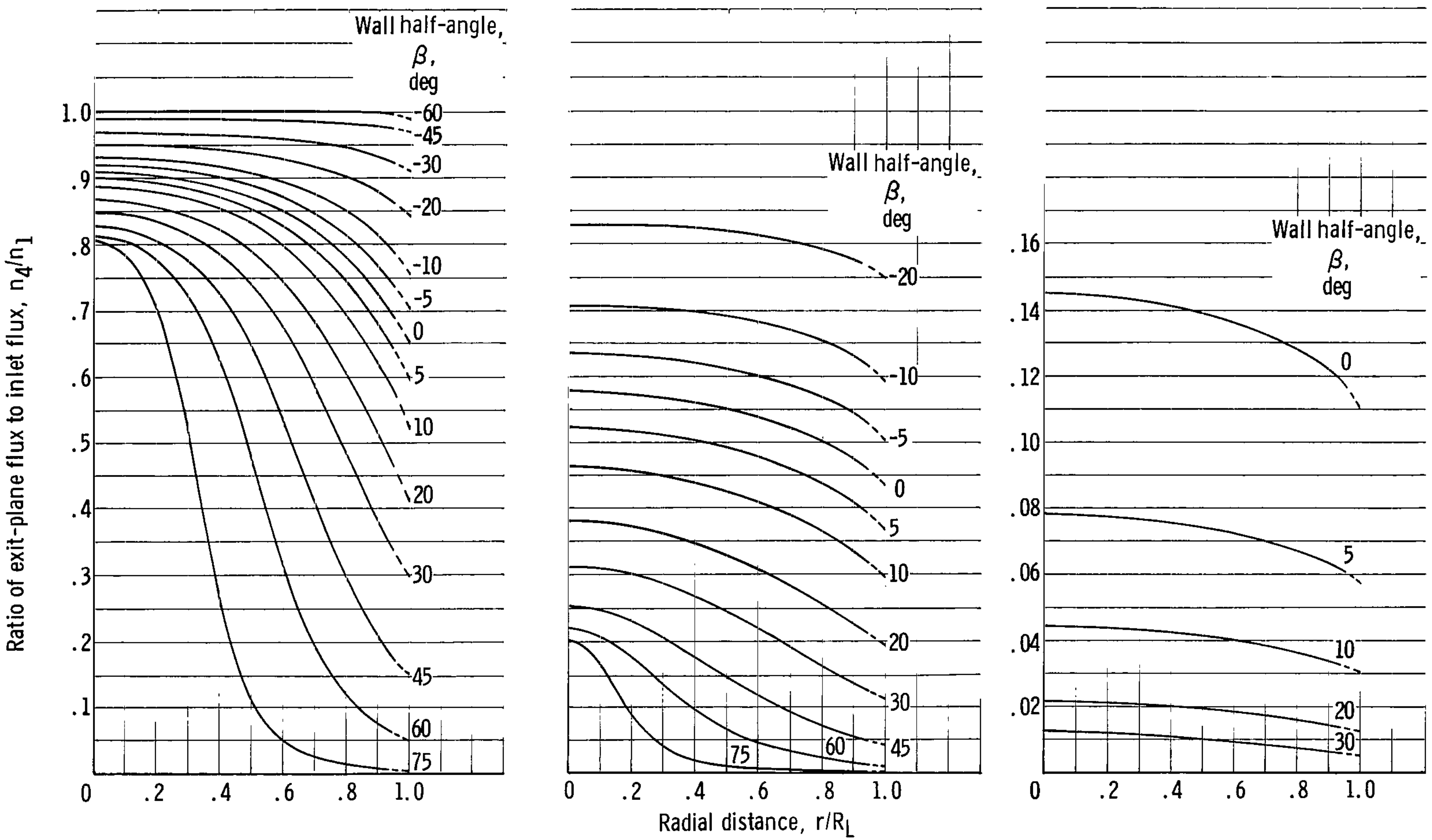


Figure 8. - Variation of flux across exit plane of cylindrical tubes.



(a) Length to inlet-radius ratio, 0.5.

(b) Length to inlet-radius ratio, 2.

(c) Length to inlet-radius ratio, 16.

Figure 9. - Variation of flux across exit plane of convergent and divergent tubes. (Diverging walls, positive β; converging walls, negative β.)

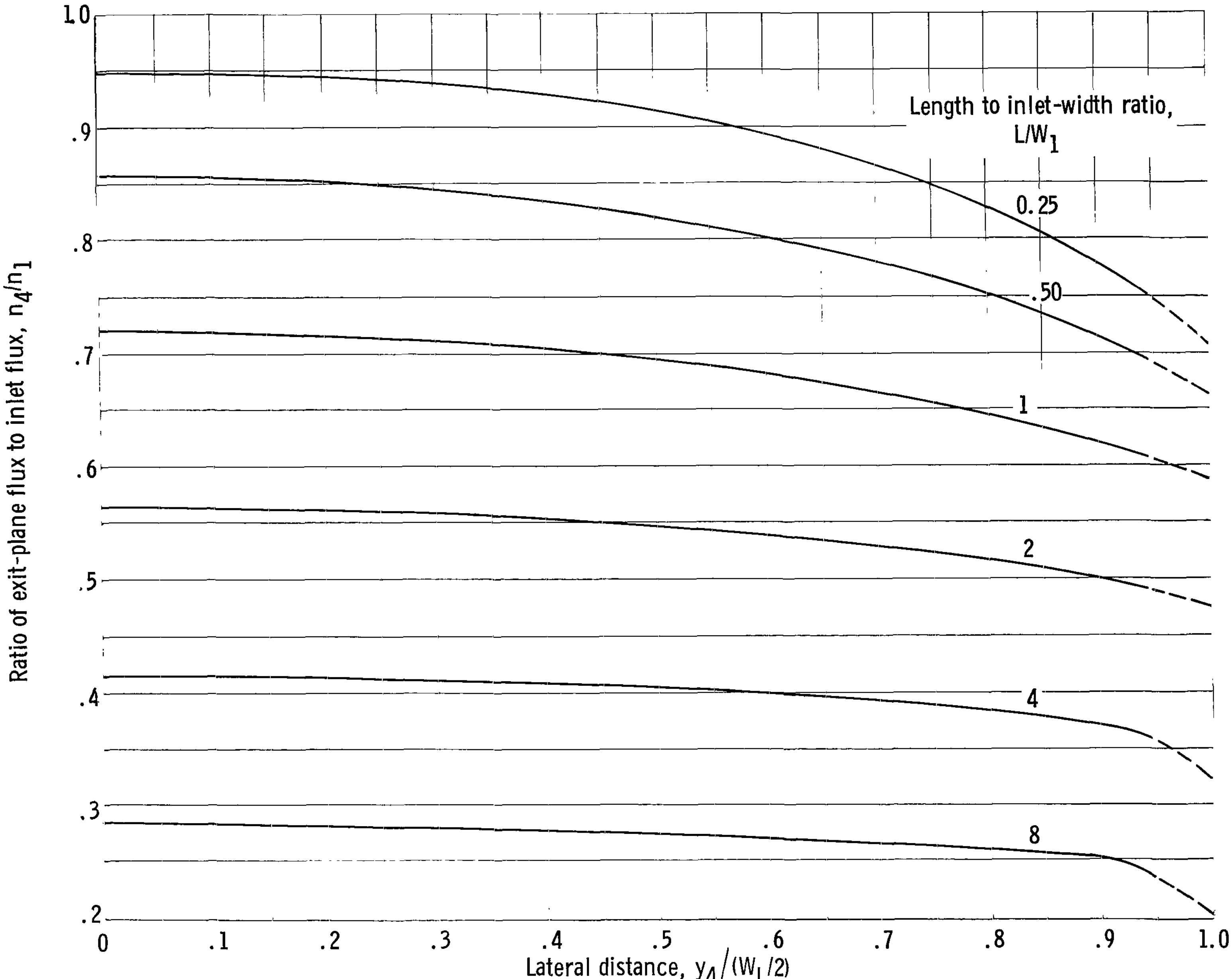


Figure 10. - Variation of flux across exit plane of parallel-walled slots.

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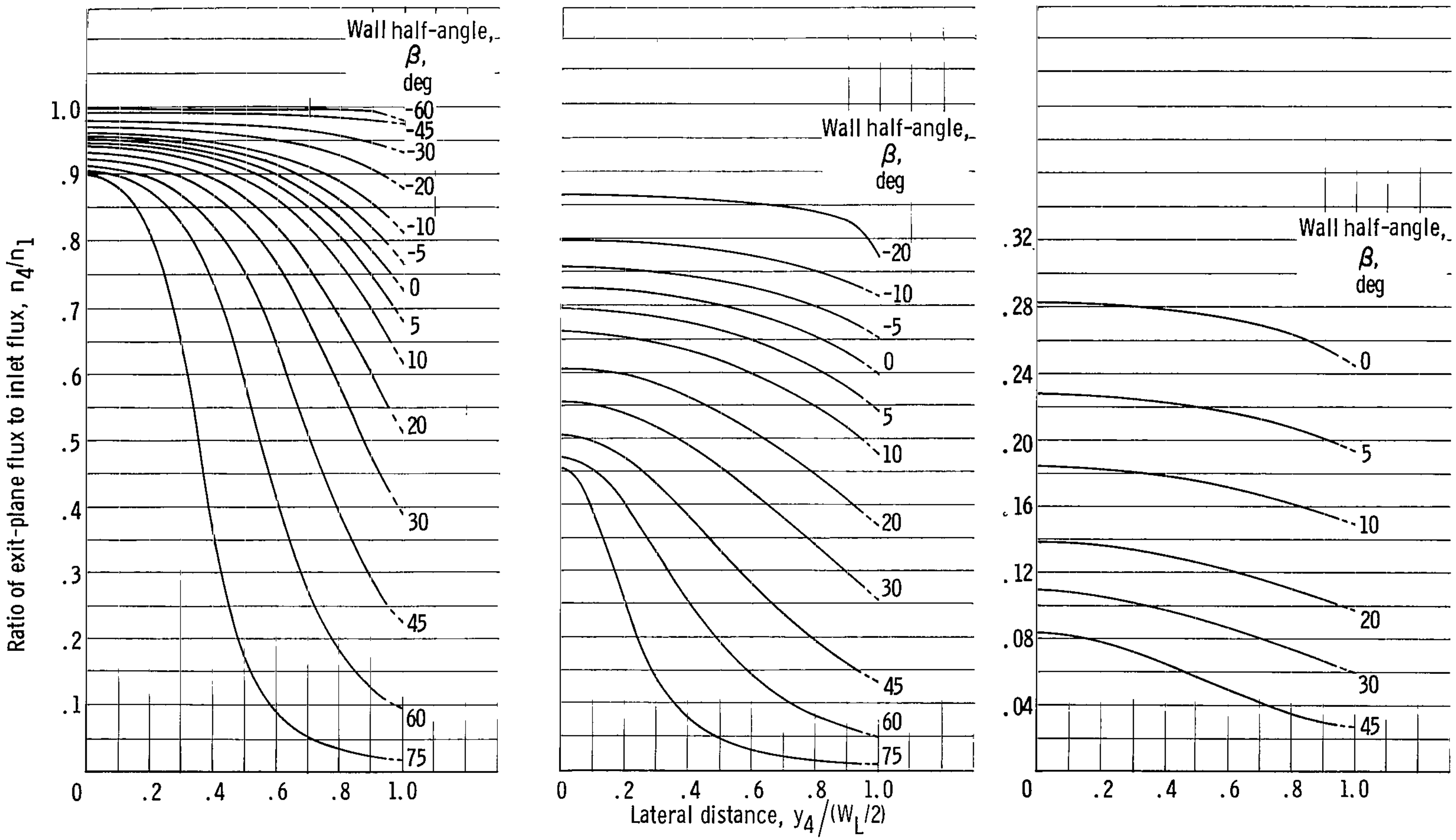

(a) Length to inlet-width ratio, 0.25.

(b) Length to inlet-width ratio, 1.

(c) Length to inlet-width ratio, 8.

Figure 11. - Variation of flux across exit plane of convergent and divergent slots. (Diverging walls, positive β; converging walls, negative β.)

2-17-85

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

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