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A METHOD FOR DETERMINING NODAL ARRIVAL TIMES AT THE MOON FROM PRECESSING NEAR-EARTH PARKING ORBITS HAVING VARIOUS INCLINATIONS

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SUMMARY

Some of the effects of precession of near-earth orbits on the timing of inplane launchings from such orbits for a nodal encounter with the moon are examined. A range of orbit inclinations from 18° to 38° is considered. From comparison with the nonprecessing orbit it is found that the orbit precession produces more frequent possible launch times and, in most cases, shorter intervals between possible launch times.

INTRODUCTION

If launch of a vehicle from a near-earth parking orbit and rendezvous or impact with the moon is desired, it is most efficient to launch in the plane of the parking orbit. In order to avoid the penalties associated with making a plane change the launch must be timed so that the vehicle will arrive in the vicinity of the moon at the same time that the moon arrives at the line of nodes of the lunar-orbit plane and the parking-orbit plane. As pointed out in reference 1, the precession of near-earth orbits about the earth's spin axis complicates the timing of such missions and must be accounted for in determining when the moon is in the proper position to allow a launch in the plane of the parking orbit.

In reference 1 the effects of parking-orbit precession on the velocity required to correct the lunar trajectory for more than anticipated delay times in the parking orbit are considered. In the present paper the effects of parking-orbit precession on the frequency of arrival of the moon at the line of nodes of the lunar-orbit plane and parking-orbit plane are considered. Various values of parking-orbit inclination are considered. It is assumed that the angular rate of precession of each parking orbit and the angular rate of the moon in its orbit are constants. The resulting equations are solved graphically and some numerical results are presented.

SYMBOLS

In this paper distances are measured in international nautical miles (1 international nautical mile = 1.852000 kilometers).

i_L	inclination of lunar orbit to earth's equator, deg
i_{LN}	inclination of lunar orbit at nodal point N, deg
i_S	inclination of parking orbit to earth's equator, deg
i_{SN}	inclination of parking orbit at nodal point N, deg
r	mean distance of parking orbit from center of earth, international nautical miles
R_E	equatorial radius of earth, 3444 international nautical miles
t	time, mean solar days
X_I,Y_I,Z_I	axes forming inertial geocentric equatorial coordinate system with X_I directed toward vernal equinox, Z_I directed along earth's axis of rotation, and Y_I completing usual orthogonal triad
α_L	right ascension of ascending node of lunar orbit on earth's equator, deg
α_{LP}	right ascension of moon's position in its orbit, deg
α_N	right ascension of line of nodes formed by lunar-orbit plane and parking-orbit plane, deg
α_S	right ascension of ascending node of parking orbit on earth's equator, deg
δ_{LP}	declination of moon's position in its orbit, deg
δ_N	declination of line of nodes formed by lunar-orbit plane and parking-orbit plane, deg
η_{LP}	angular position of moon in its orbit, measured from ascending node of lunar orbit on earth's equator, deg
ρ_{LS}	angle of intersection of parking orbit and lunar orbit at line of nodes, deg

Subscripts:

o initial conditions

1,2 ascending and descending node, respectively

A dot over a quantity indicates the time derivative of that quantity.

ANALYSIS

Figure 1 shows a schematic illustration of the geometrical system considered. The X_I,Y_I,Z_I axes form a geocentric equatorial inertial coordinate system with the X_I,Y_I axes lying in the earth's equator. The lunar-orbit plane LNB and the parking-orbit plane SNC are shown projected on a segment of an earth-centered unit sphere. The line ON is the line of nodes of planes LNB and SNC.

The problem is to determine how frequently the moon arrives at the nodal point N. According to the analysis of reference 2 the oblateness of the earth causes the plane of a near-earth orbit to precess about the earth's spin axis (Z_I axis in fig. 1) at a rate which depends on the orbital altitude and the inclination of the orbit to the earth's equator. In particular, reference 2 gives the following equation (expressed here in the notation of the present paper) for the precession rate:

$$\dot{\alpha}_S = -10.0\left(\frac{R_E}{r}\right)^{3.5} \cos i_S \tag{1}$$

in degrees per day for orbits of small eccentricity (0.05 or less). Thus, N will not remain fixed in space but will vary its position, depending on the precession of the parking orbit. It should be noted that the moon's orbital plane is not fixed in space but is subject to the perturbing effects of the earth and the sun. However, these effects have rather long periods and will be ignored herein. (For example, the inclination of the lunar orbit to the ecliptic varies periodically between $28^\circ35'$ and $18^\circ19'$ in 19.6 years.) It should also be noted that the assumption of a circular orbit for the moon eliminates the effects of the variation of the angular velocity of the moon. This variation amounts to about $\pm2^\circ$ per day from the mean value per excursion of the moon in its orbit.

It is apparent from figure 1 that, if the times can be found at which the right ascension of the moon is equal to the right ascension of N, the problem is solved. Figure 2 shows the unit-sphere projection of lunar- and parking-orbit planes and the various angles of interest. The approach taken for this investigation is to develop equations for the right ascension of N and the moon as a function of time, and by simultaneous solution to determine the times at which the equations are equal.

In figure 3 the spherical triangles LAN and SAN (fig. 3(a)) and LNP (fig. 3(b)) from figure 2 are shown. By use of the law of sines the following relations are obtained:

From triangle LAN:

$$\frac{\sin i_L}{\sin \delta_N} = \frac{\cos i_{LN}}{\sin(\alpha_N - \alpha_L)} \tag{2}$$

Similarly, from triangle SAN:

$$\frac{\sin i_S}{\sin \delta_N} = \frac{\cos i_{SN}}{\sin(\alpha_N - \alpha_S)} \tag{3}$$

From triangle LNP:

$$\cos i_{LN} = \frac{\cos i_L}{\cos \delta_N} \tag{4}$$

and from triangle SNP:

$$\cos i_{SN} = \frac{\cos i_S}{\cos \delta_N} \tag{5}$$

Substituting equations (4) and (5) into equations (2) and (3), respectively, and reducing yields the following equations:

$$\left.\begin{aligned} \tan \delta_N &= \tan i_L \sin(\alpha_N - \alpha_L) \\ \tan \delta_N &= \tan i_S \sin(\alpha_N - \alpha_S) \end{aligned}\right\} \tag{6}$$

Eliminating $\tan \delta_N$ and solving for $\tan \alpha_N$ yields

$$\tan \alpha_N = \frac{\tan i_L \sin \alpha_L - \tan i_S \sin \alpha_S}{\tan i_L \cos \alpha_L - \tan i_S \cos \alpha_S} \tag{7a}$$

If the precession rate of the parking orbit is assumed to be constant, equation (7a) can be written

$$\tan \alpha_N = \frac{\tan i_L \sin \alpha_L - \tan i_S \sin(\alpha_{S,o} - \dot{\alpha}_S t)}{\tan i_L \cos \alpha_L - \tan i_S \cos(\alpha_{S,o} - \dot{\alpha}_S t)} \tag{7b}$$

Equation (7b) gives the right ascension of the nodal point N as a function of time.

It is now necessary to express the right ascension of the moon as a function of time. From figure 4 it can be seen that

$$\alpha_{LP} = \alpha_L + \Delta\alpha_L \tag{8}$$

From triangle LMD

$$\tan \Delta\alpha_L = \tan \eta_{LP} \cos i_L \tag{9}$$

Then

$$\begin{aligned} \tan \alpha_{LP} &= \tan\left(\alpha_L + \Delta\alpha_L\right) \\ &= \frac{\tan \alpha_L + \tan \eta_{LP} \cos i_L}{1 - \tan \alpha_L \tan \eta_{LP} \cos i_L} \end{aligned} \tag{10}$$

Since the angular velocity of the moon in its orbit is assumed to be constant,

$$\eta_{LP} = \eta_{LP,o} + \dot{\eta}_{LP}t \tag{11}$$

and equation (10) becomes

$$\tan \alpha_{LP} = \frac{\tan \alpha_L + \cos i_L \tan\left(\eta_{LP,o} + \dot{\eta}_{LP}t\right)}{1 - \tan \alpha_L \cos i_L \tan\left(\eta_{LP,o} + \dot{\eta}_{LP}t\right)} \tag{12}$$

Thus equations (7b) and (12) are the desired equations for this analysis.

In order to determine the times at which equations (7b) and (12) are equal, a graphical solution is used. The equations are evaluated individually and plotted as a function of time. Points of intersection of the resulting curves are the desired solutions.

As a matter of interest, the angle between the lunar-orbit plane and the parking-orbit plane at the nodal point N is also determined. This angle ρ_{LS} is a measure of the plane-change maneuver required to place the rendezvous vehicle into the moon's orbital plane at N.

From triangle LSN in figure 3(a)

$$\cos \rho_{LS} = \cos i_L \cos i_S + \sin i_L \sin i_S \cos\left(\alpha_S - \alpha_L\right) \tag{13}$$

RESULTS AND DISCUSSION

The results of some numerical calculations using the previously discussed equations are shown in figure 5. These results are presented in table I. For these calculations, it was assumed that the inclination of the moon's orbit remained fixed at 28°. As indicated by the data presented in reference 3, this value corresponds to the inclination of the moon's orbit during the calendar years 1967 to 1971 to within ±1°. Values of inclination of the parking orbit of 18°, 26°, 28°, 30°, and 38° were chosen. A parking-orbit altitude of 228 international nautical miles was used (r = 3,672 international nautical miles for eq. (11)), resulting in a range of parking-orbit precession rates of 6.3° to 7.6° per day. The angular velocity of the moon is assumed to be 13.19° per day.

Figure 5 presents the variation of the right ascension of the line of nodes α_N, the right ascension of the moon α_{LP}, and the intersection angle ρ_{LS} for a 60-day period corresponding to approximately two revolutions of the moon about the earth. It should be noted that two values of α_N are plotted at any given time. These values of α_N correspond to each end of the line of nodes formed by the two orbital planes and are 180° apart. For the calculations given in table I, it is assumed that, initially, the moon is at its ascending node on the earth's equator $\left(\eta_{LP,o} = 0\right)$, that the ascending nodes of both the parking orbit and lunar orbit are coincident $\left(\alpha_{L,o} = \alpha_{S,o}\right)$, and that these ascending nodes are alined with the vernal equinox $\left(\alpha_{S,o} = \alpha_{L,o} = 0\right)$. It should be noted that these initial conditions place the moon in a position so that it is alined with the vernal equinox. These initial conditions are assumed for illustrative purposes only and, even though this situation may never occur, will not affect the generality of the results.

Examination of figure 5 shows that the nature of the variation of α_N with time changes markedly as the inclination of the parking orbit increases from values less than i_L to values greater than i_L. For values of $i_S < i_L$ (figs. 5(a) and (b)), the variation of α_N with time is seen to be cyclic in nature and, furthermore, to involve a periodic alteration in the slope of the curves. For values of $i_S > i_L$, figures 5(d) and (e) show that the variation of α_N with time always has a negative slope. For the special case of $i_S = i_L$, figure 5(c) shows that α_N varies linearly with time. This special case can be verified analytically by setting $i_S = i_L$ in equation 7(b) and differentiating with respect to time, remembering that $\dot{\alpha}_L = 0$. The result of this operation is

$$\dot{\alpha}_N = \frac{1}{2}\dot{\alpha}_S \tag{14}$$

It should be noted that since the moon's orbit is considered to be fixed and circular, the variation of α_{LP} in figure 5 is the same for all cases.

The time increment between arrivals of the moon at the line of nodes is plotted in figure 6 for each arrival and each parking-orbit inclination. Also shown is the time increment between arrivals which would occur if the parking orbit were not precessing $\left(\dot{\alpha}_S = 0\right)$. Figure 6 shows that the precession of the parking orbit results, for a majority of the calculations, in reduced time intervals between arrivals of the moon at the line of nodes when compared with the nonprecessing case. For the cases considered herein, a majority show a time increment between arrivals of 10 to 11 days. This increment compares with 13.9 days for the nonprecessing case. The longest and the shortest time increments are seen to be 21.5 days and 0.9 day, respectively. It will also be noted that the number of arrivals during the 60-day period considered increases from four for the nonprecessing case to a maximum of seven for $i_S = 30^\circ$. Thus, for the particular examples considered and the initial conditions assumed, the precession of the parking orbit results in more frequent possible launch times and, in most cases, shorter time intervals between the possible launch times. Figure 6 also indicates that parking-orbit inclinations greater than the lunar-orbit inclination are more favorable from the standpoint of launch frequency. In particular, for a parking-orbit inclination of 30°, seven possible launch times occur during the 60-day period considered. For parking-orbit inclinations of 28° $\left(i_S = i_L\right)$ and 38°, six possible launch times occur. This compares with five possible launch times for parking-orbit inclinations of 18° and 26°.

CONCLUDING REMARKS

In this investigation the effects of the precession of near-earth orbits due to the oblateness of the earth on the timing of launchings from such an orbit for a nodal arrival at the moon have been examined. The results of some representative calculations performed with the equations derived indicate that the precession of the parking orbit results in generally shorter time intervals between possible launchings and more frequent possible launch times during the 60-day period considered. The results also indicated that parking orbits with inclinations greater than the lunar-orbit inclination are more desirable since these orbits result in more frequent possible launch times and, in most cases, shorter time intervals between possible launchings than do parking orbits with inclinations less than that of the lunar orbit.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., April 28, 1964.

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1. Wells, William R.: The Influence of Precession of Earth Rendezvous Orbits on Lunar Mission Requirements. NASA TN D-1512, 1962.

2. King-Hele, D. G., and Gilmore, D. M. C.: The Effect of the Earth's Oblateness on the Orbit of a Near Satellite. Tech. Note No. G.W.475, British R.A.E., Oct. 1957.

3. Woolston, Donald S.: Declination, Radial Distance, and Phases of the Moon for the Years 1961 to 1971 for Use in Trajectory Considerations. NASA TN D-911, 1961.

TABLE I.- NODAL ARRIVAL TIMES, TIME INCREMENTS, AND INTERSECTION ANGLES

Nodal arrival	Calculated values for -																
	i_S of -															$\dot{\alpha}_S = 0$	
	18°			26°			28°			30°			38°				
	t, days	Δt, days	ρ_{LS}, deg	t, days	Δt, days	ρ_{LS}, deg	t, days	Δt, days	ρ_{LS}, deg	t, days	Δt, days	ρ_{LS}, deg	t, days	Δt, days	ρ_{LS}, deg	t, days	Δt, days
1	0.9	0.9	10.4	5.1	5.1	17.0	5.4	5.4	17.5	5.8	5.8	19.0	7.0	7.0	26.0	13.7	13.7
2	15.4	14.5	39.5	16.2	11.1	45.0	16.2	10.8	46.0	16.5	10.7	48.0	17.2	10.2	53.0	27.4	13.7
3	26.5	11.1	44.5	26.7	10.5	54.0	27.0	10.8	55.0	27.0	10.5	58.0	27.6	10.4	66.0	41.1	13.7
4	37.9	11.4	27.0	37.4	10.7	38.0	37.6	10.6	41.0	37.5	10.5	43.5	38.0	10.4	57.0	54.8	13.7
5	57.3	19.4	29.0	58.9	21.5	27.5	48.4	10.8	8.5	48.1	10.6	13.0	48.4	10.4	31.0		
6							59.4	11.0	27.0	52.2	4.1	3.5	56.2	7.8	11.5		
7										59.8	7.6	25.0					

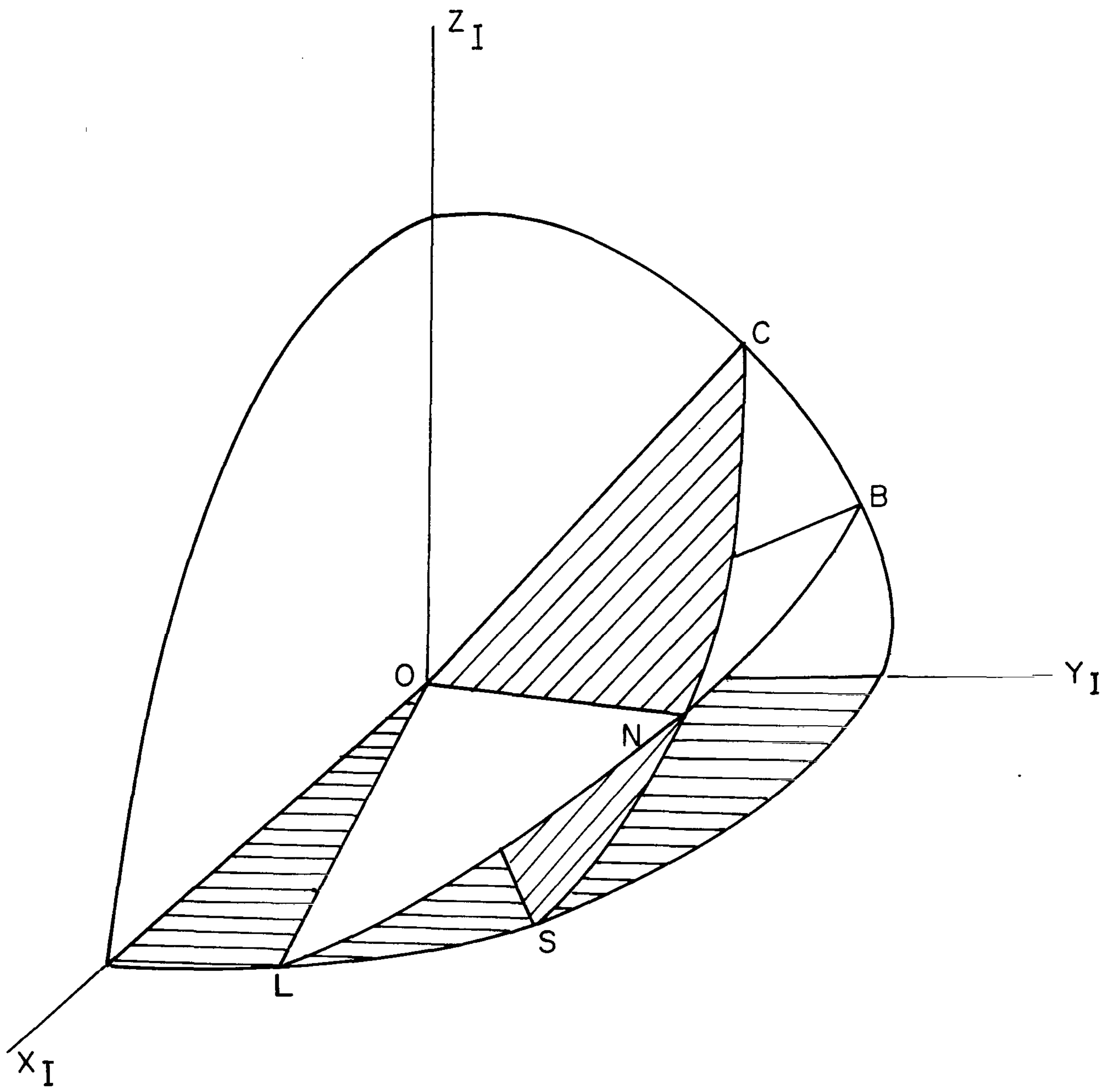


Figure 1.- Schematic illustration of coordinate system and orbital planes.

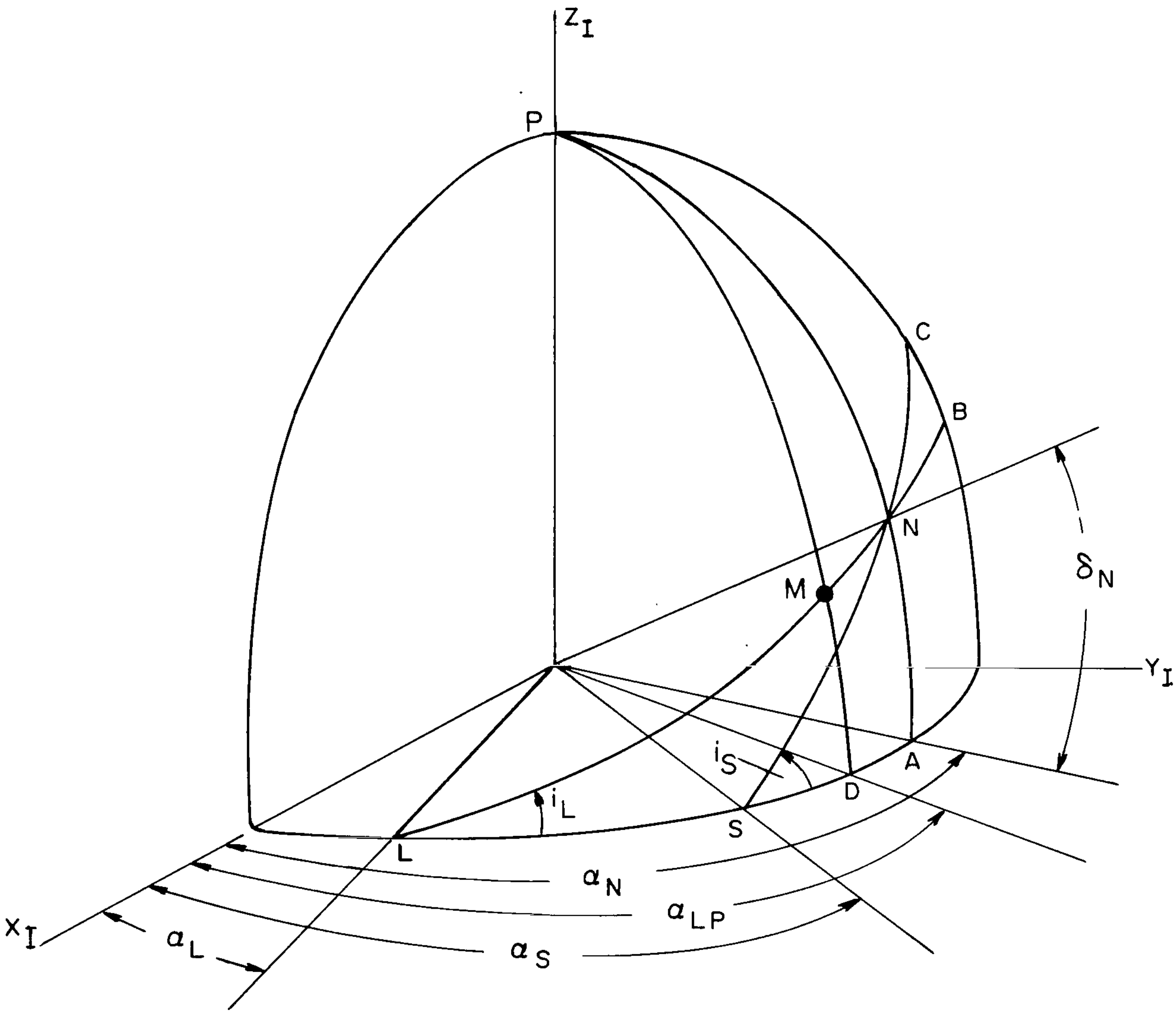
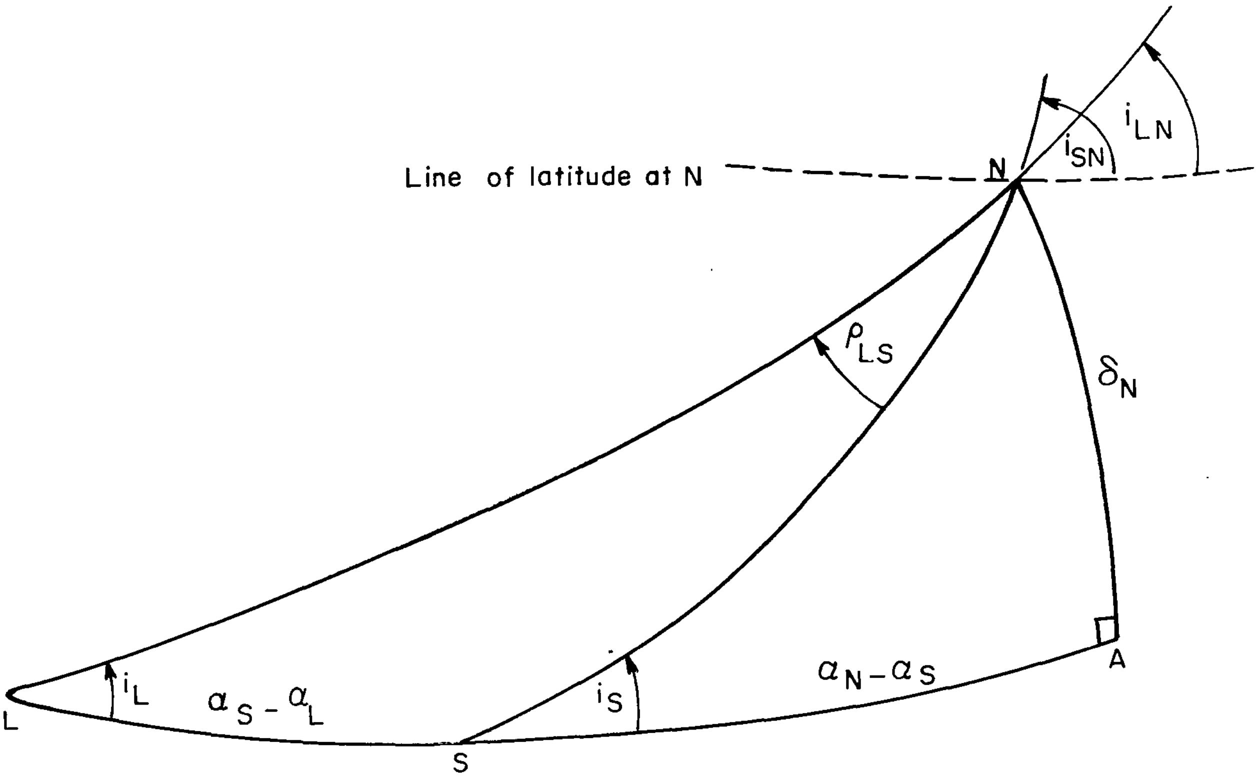
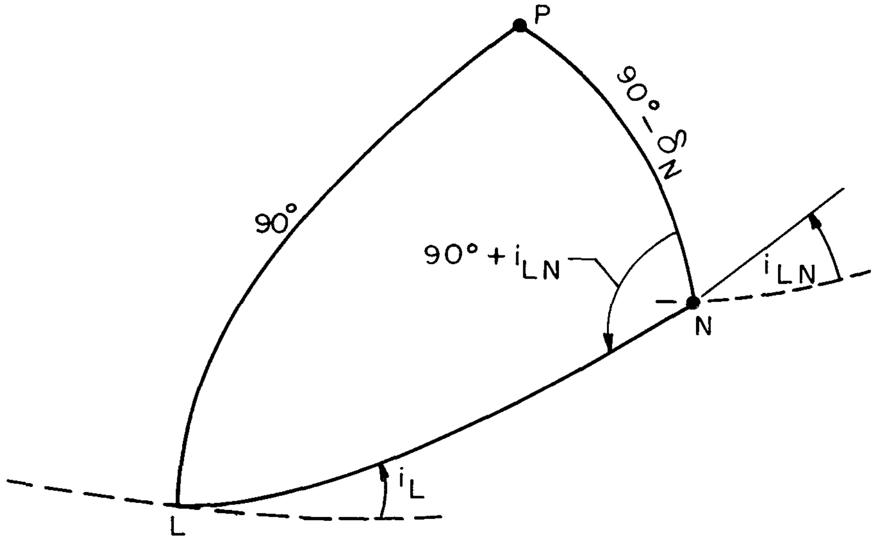


Figure 2.- Unit-sphere projection of orbital planes showing angles considered.



(a) Triangles LAN and SAN.



(b) Triangle LNP.

Figure 3.- Spherical triangles LAN, SAN, and LNP from figure 2.

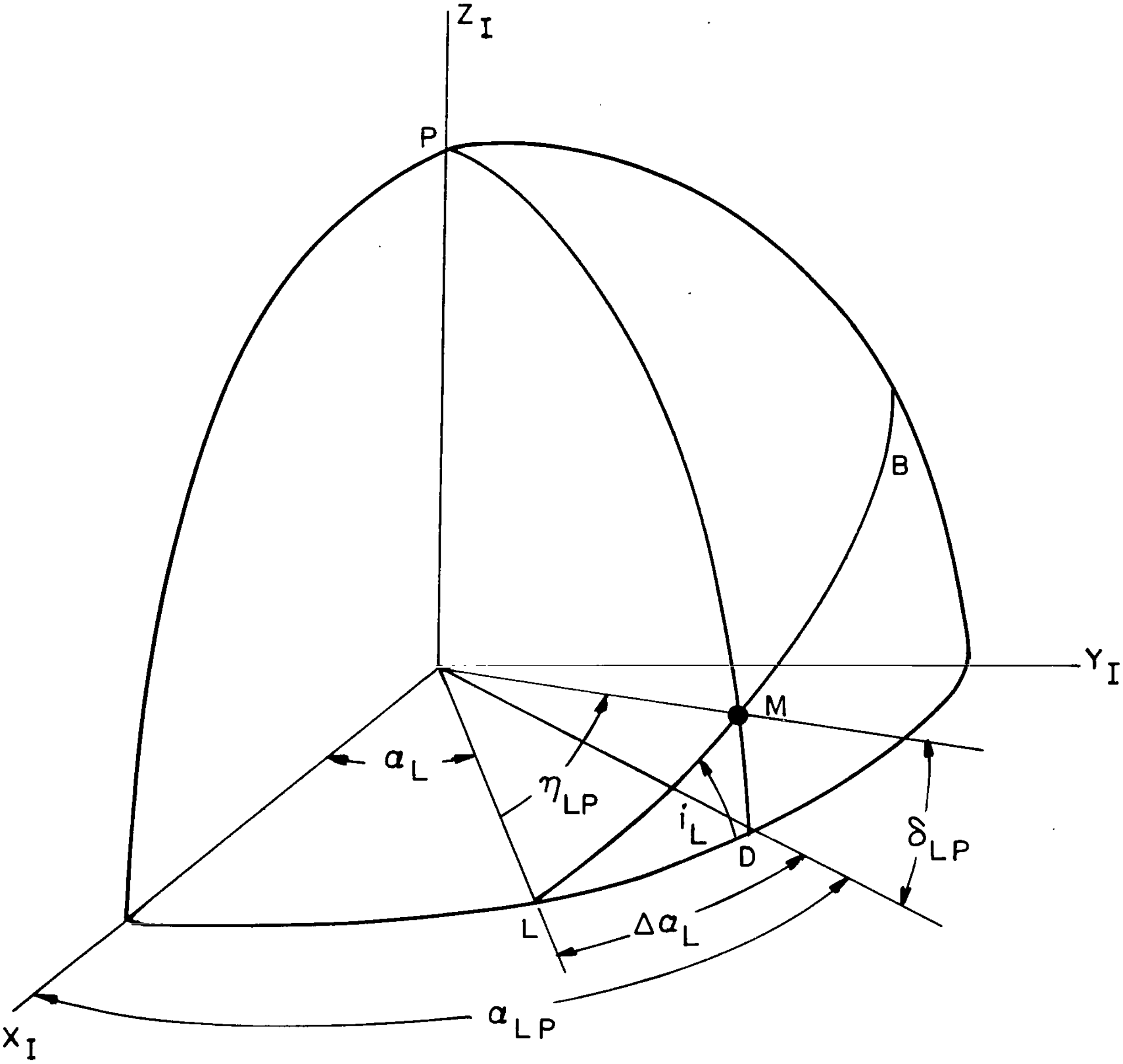
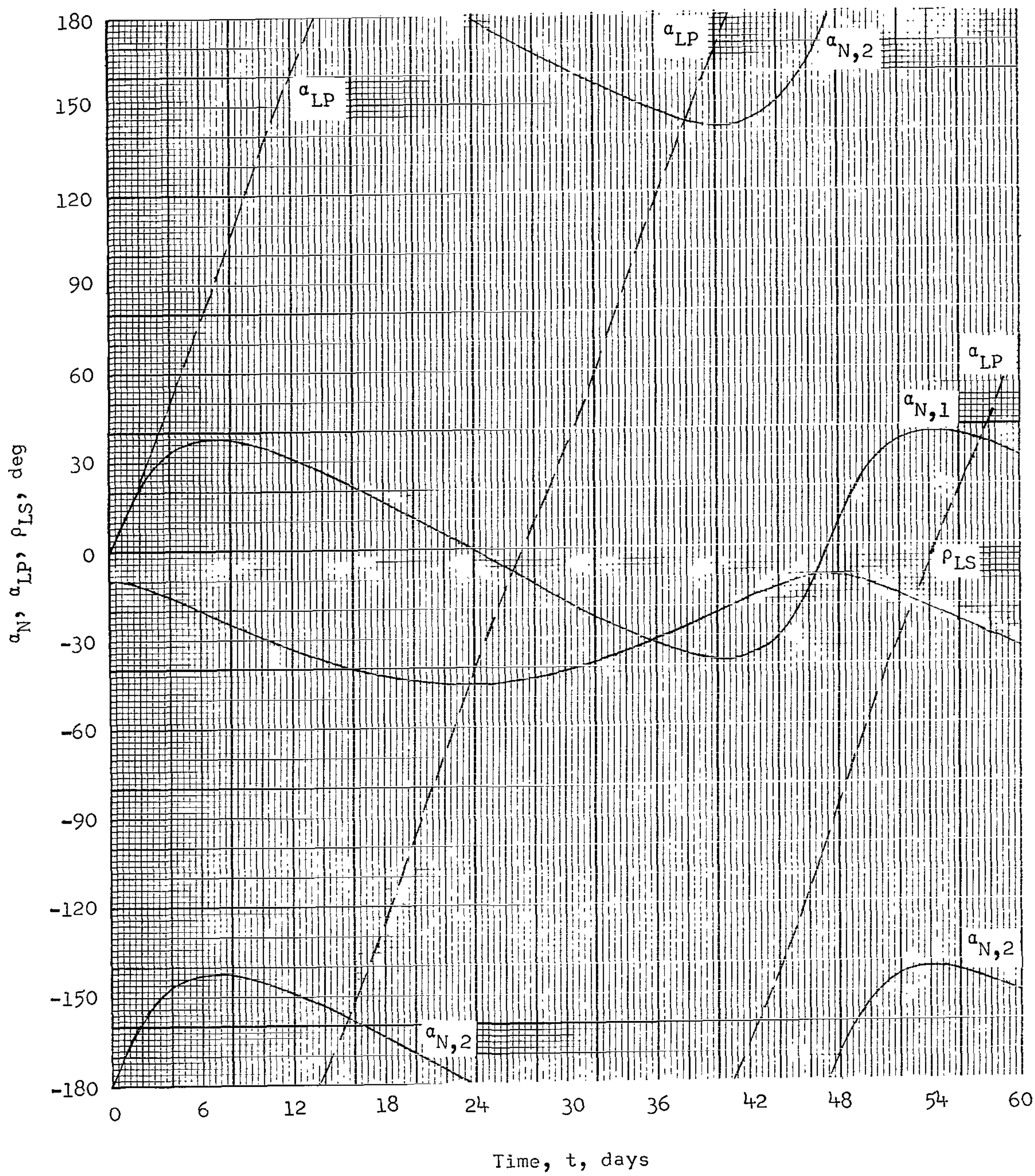
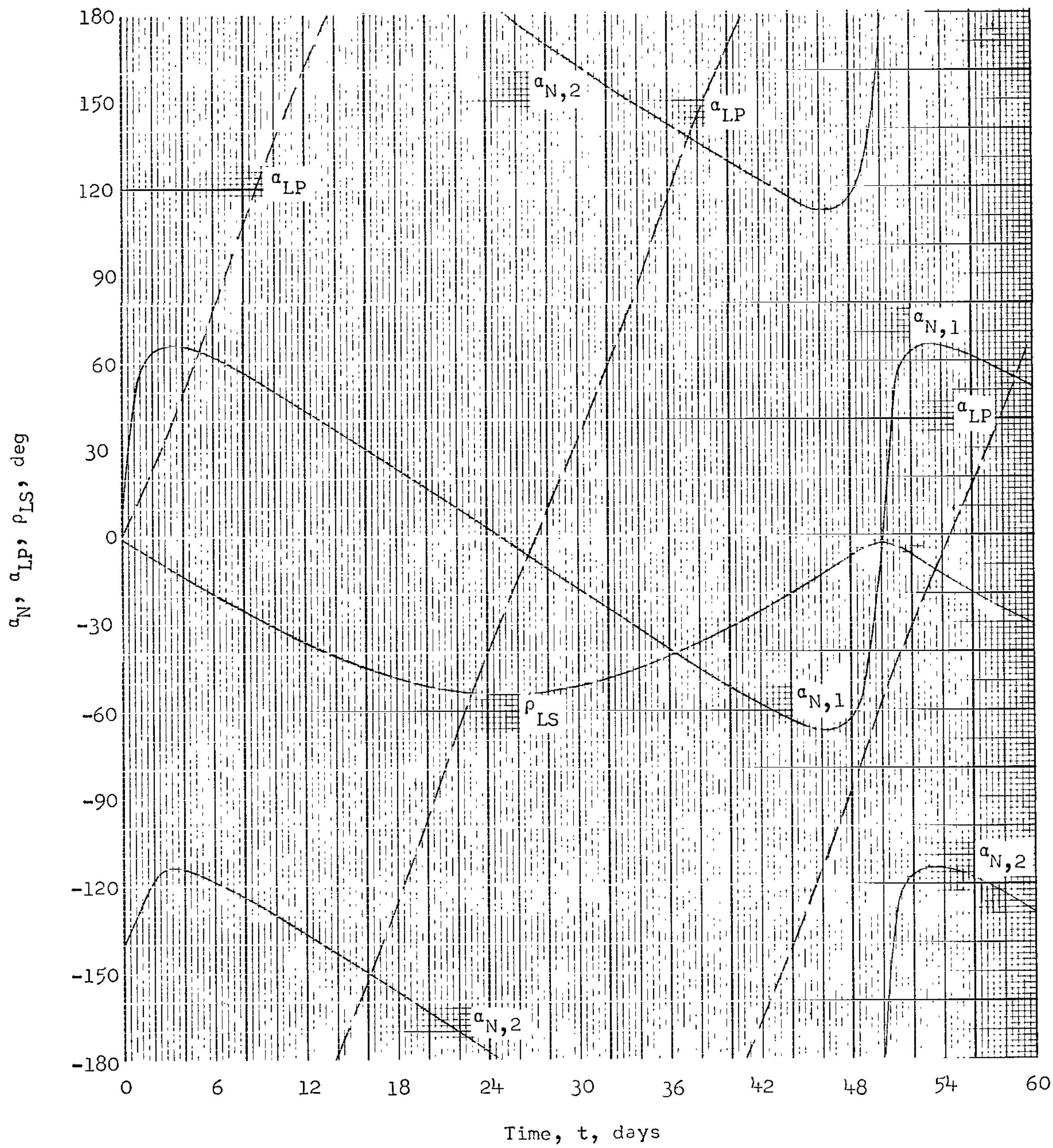


Figure 4.- Unit-sphere projection of lunar-orbit plane showing orientation of coordinate system.



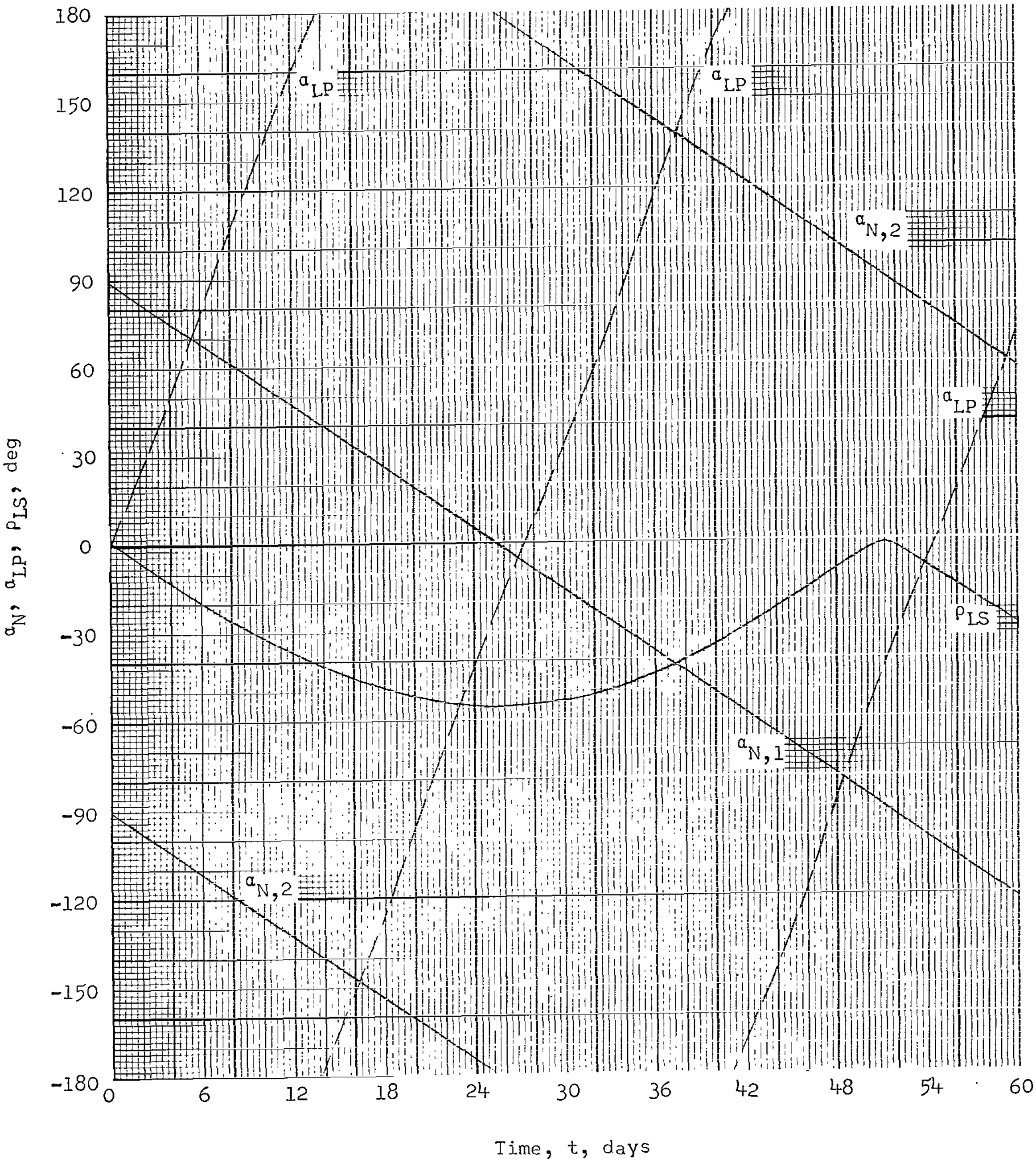
(a) $i_S = 18^o$.

Figure 5.- Variation with time of right ascension of nodal point α_N, moon's position α_{LP}, and intersection angle at line of nodes ρ_{LS} for various parking-orbit inclinations. $i_L = 28^o$.



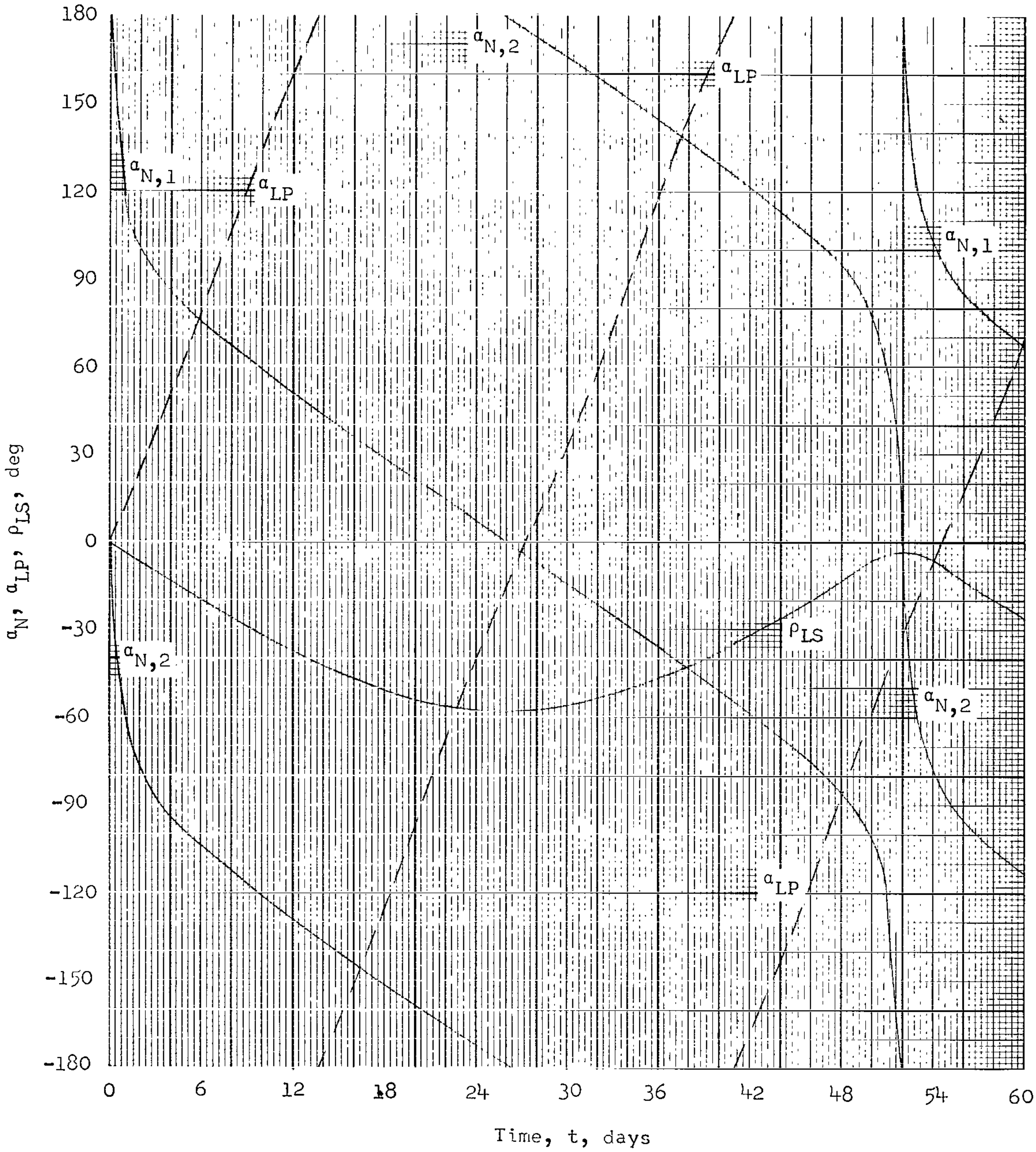
(b) $i_S = 26^{\circ}$.

Figure 5.- Continued.



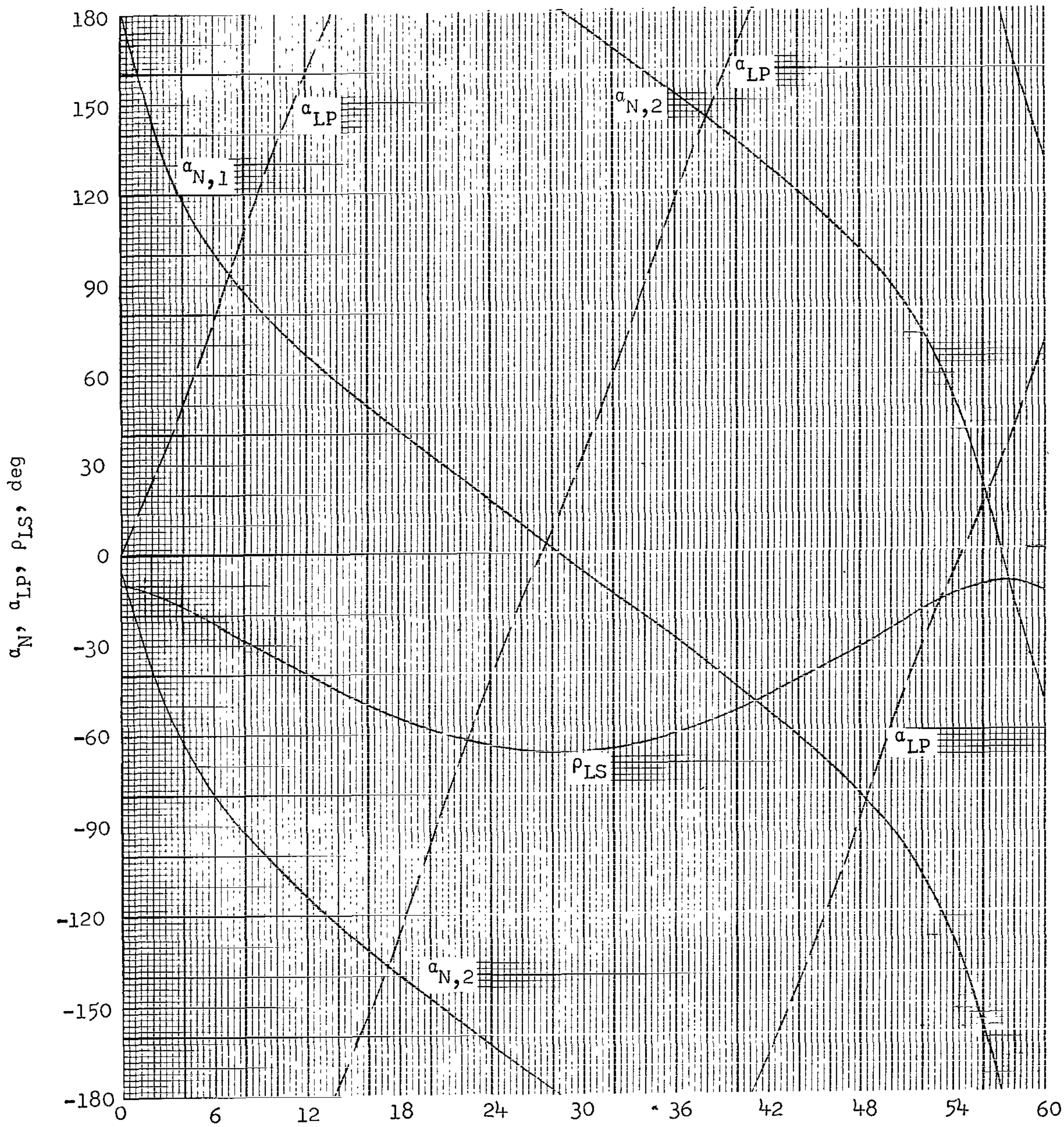
(c) $i_S = 28^\circ$.

Figure 5.- Continued.



(d) $i_S = 30^o$.

Figure 5.- Continued.



Time, t, days

(e) $i_S = 38^{\circ}$.

Figure 5.- Concluded.

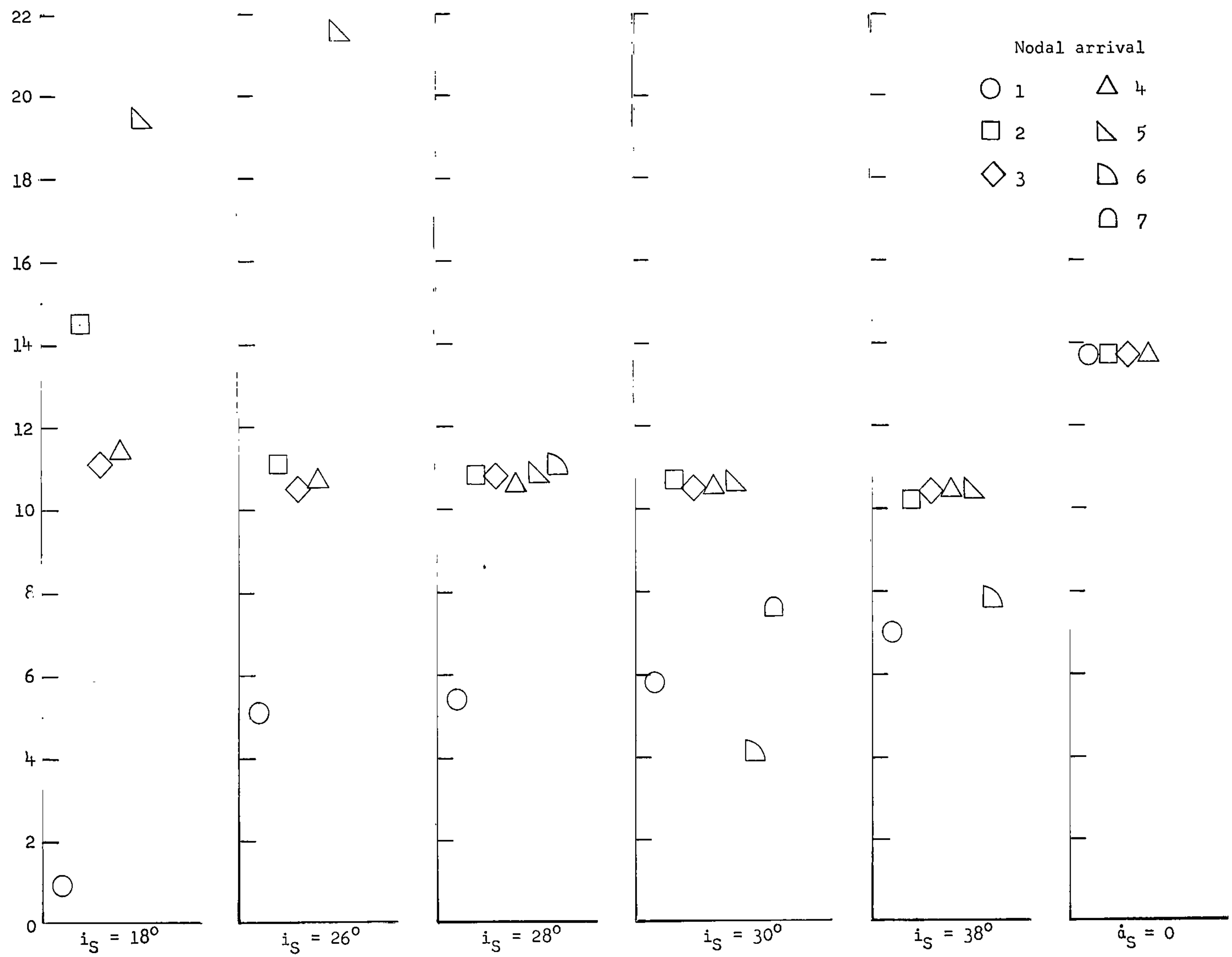


Figure 6.- Time increment between each successive arrival of moon at line of nodes for various parking-orbit inclinations.

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