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STABILITY OF HARD-CORE PINCH WITH

ANISOTROPIC PLASMA PRESSURE

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1. Introduction

The problem of stability of the hard-core pinch device has been the subject of both theoretical and experimental investigations during the recent years (Anderson, Baker et al 1958; Anderson, Furth et al 1958; Bickerton and Spalding 1962; Birdsall, Colgate and Furth 1959, 1960; Jukes 1961; Reynolds et al 1959; Tandon and Talwar, 1961, 1962; Taylor and Hopgood 1963; and Aitken et al 1964). The device consists of an annular plasma detached from the central core and the surrounding conducting wall due to opposite axial currents flowing in the core and the plasma. The interest in this device stems from the fact that according to ideal (infinite electrical conductivity) hydromagnetic theory, the system should be stable against all perturbations provided the axial return current through the plasma is less than or

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equal to the current in the central rod. On the other hand experiments (Aitken, 1964) have shown the presence of irreproducible magnetic probe signals indicative of fine scale instabilities in hard core geometry where the field distribution satisfying the stability criterion based on ideal hydromagnetic magnetic theory were set up.

There may be several causes for this disagreement between experiment and theory. The chief reason perhaps is that the idealizations demanded by the hydromagnetic theory are not fulfilled in the experiment. One, for example, is that the instability may be due to finite electrical conductivity of the plasma which allows a relative slippage between the lines of force and the fluid, thus removing the constraint imposed by infinite conductivity. This possibility has recently been explored by various investigators (Bickerton and Spalding, Tandon and Talwar, Taylor and Hopgood) who have shown that in the limit of vanishingly small electrical conducivity, the hard core pinch model is, in general, unstable. This gives one the feeling that the observed instability is due to finite conductivity, although a final answer can only be given when calculations are done with finite (and not vanishingly small) conductivity.

The purpose of the present paper is to investigate the instability of an idealized hard core pinch configuration, regarding the plasma pressure to be anisotropic rather than scalar. This happens in the case of a dilute plasma subject to a high magnetic field. In the simplest situation the plasma pressure is different in at least two directions viz parallel and perpendicular to the magnetic field. It is well-known that in case the plasma pressure is initially (before perturbation) isotropic in a static plasma, the MHD approximation gives the most pessimistic result regarding stability, i.e., if we obtain stability in the MHD approximation, the stability is certainly guaranteed in the more exact kinetic theory in the $(M/e) \rightarrow o$ limit as well as in the Chew, Goldberger and Low approximation (1956). If, on the other hand, the equilibrium state is characterized by an anisotropic pressure, new types of instability may arise - a typical example being the "mirror" and the "fire-hose" instability for plane waves in a homogenous plasma carrying a uniform magnetic field.

2. Formulation of the Problem

Consider a low density plasma contained between the two coaxial cylinders formed by the metallic core of radius $\Lambda_0 R$ and a regid perfectly conducting wall of radius $\Lambda_2 R$. Axial currents of strength J_c and J_p are respectively flowing in opposite direction through the central conductor and the plasma as a result of which the plasma will detach itself from the conducting central rod and the wall forming an equilibrium model of hard-core pinch as shown in Fig. 1. Let R and $\Lambda_1 R$ respective denote the radii of the inner and outer plasma-vacuum interfaces Λ_0, Λ_1 and Λ_2 are constants such that

$$\Lambda_0 < 1 \, , \quad \text{and} \quad \Lambda_2 > \Lambda_1 > 1 \tag{1}$$

we further assume that there is a strong axial magnetic field B_1 prevalent inside the plasma making the plasma pressure anisotropic. Let B_2 and B_3 be the axial magnetic fields in the inner and the outer vacuum regions respectively.

Assuming that there are only surface currents on the two plasma-vacuum interface, we can write for the equilibrium fields as

$$\begin{aligned}
&\left(0\hat{r}, \ \frac{2J_c}{\Lambda_0^2 R^2} r\,\hat{\theta}, \ B_2\hat{z}\right) && r \leq \Lambda_0 R \\
&\left(0\hat{r}, \ \frac{2J_c}{r}\hat{\theta}, \ B_2\hat{z}\right) && \Lambda_0 R < r < R \\
&\left(0\hat{r}, \ 0\hat{\theta}, \ B_1\hat{z}\right) && R < r < \Lambda_1 R \\
&\left(0\hat{r}, \ -\frac{2J_p}{r}\hat{\theta}, \ B_3\hat{z}\right) && \Lambda_1 R < r < \Lambda_2 R
\end{aligned} \tag{2}$$

We may express the magnetic fields in various regions in terms of the toroidal field B_b at the inner (r = R) plasma-vacuum interface. We thus have,

$$b_1 = \frac{B_1}{B_b}, \quad b2 = \frac{B_2}{B_b} \tag{3}$$

and

$$a = \frac{B_a}{B_b} = -\frac{J_p}{J_c} \Lambda_1 \tag{4}$$

here B_a denotes the toriodal field at the outer plasma-vacuum interface. In equilibrium the total stress (plasma plus magnetic pressure) should be continuous at each bounding surface. This gives

$$p_\perp + \frac{B_1^2}{8\pi} = \frac{B_2^2}{8\pi} + \frac{B_b^2}{8\pi} \qquad \text{at } r = R \tag{5}$$

and

$$p_\perp + \frac{B_1^2}{8\pi} = \frac{B_3^2}{8\pi} + \frac{B_a^2}{8\pi} \quad \text{at } r = \Lambda_1 R \tag{6}$$

It can readily be seen from equations (5) and (6) that the field parameters in equilibrium should satisfy the relation

$$1 + b_2^2 = a^2 + b_3^2 \tag{7}$$

Further, the equations governing the motion of the non-dissipative plasma are written as

$$\rho \frac{d\underline{v}}{dt} = - \nabla \cdot \overleftrightarrow{p} + \underline{j} \times \underline{B} \tag{8}$$

$$\frac{\partial \rho}{\partial t} = - \nabla \cdot (\rho \underline{v}) \tag{9}$$

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{v} \times \underline{B}) \tag{10}$$

$$\nabla \times \underline{B} = 4\pi \underline{j} \tag{11}$$

$$\nabla \cdot \underline{B} = 0 \tag{12}$$

(ρ, $\underline{v}$, $\underline{B}$ and $\underline{j}$ respectively denote the density, velocity magnetic field and current density vectors at a point). $\overleftrightarrow{p}$ the pressure tensor, which is diagonal in a local rectangular coordinate system with one axis along the magnetic field and invariant under rotation about $\underline{B}$ is given by

$$\overleftrightarrow{p} = p_{11}\, \underline{n}\,\underline{n} + p_{\perp}\left(\overleftrightarrow{1} - \underline{n}\,\underline{n}\right) \tag{13}$$

Here $\underline{n}$ is a unit vector pointing along the magnetic field and $\overleftrightarrow{1}$ signifies the unit dyadic p_{11} and $p_{\perp}$ represent the gas pressures along and perpendicular to the fiels respectively.

Neglecting the heat flow tensor, we further have the double adiabatic equations (Chew, Goldberger and Low 1956)

$$\frac{d}{dt}\left(\frac{p_{11} B^2}{\rho^3}\right) = 0 \tag{14}$$

$$\frac{d}{dt}\left(\frac{p_{\perp}}{\rho B}\right) = 0 \tag{15}$$

It can readily be seen from equation (8) that in the initial state p_{11} and $p_{\perp}$ are constants.

In the two vacuum regions, the equations to be satisfied are (11) and (12) with $\dot{\underline{j}} = 0$.

We study the stability of the static configuration characterized by the above equilibrium parameters and as depicted in Fig. 1. by imparting an infinitely small velocity perturbation $\underline{v}$ to the system. The various resulting perturbations, like the change in gas pressure, magnetic field, current density etc. are assumed to be of the form,

$$(\text{function of } r) \times \exp.\ \{ nt + i\,(m\,\theta + k\,z) \} \tag{16}$$

where n is the growth rate parameter, m the azimuthal number and k the wave number of the perturbation in the z-direction. A dispersion relation relating n, the parameter determining the stability, with other physical qualities is then obtained by adopting the usual normal mode analysis.with suitable boundary conditions.

3. Equations Governing the Perturbed State and Their Solutions

With the perturbations of the form (16) the linearized perturbation equations for the plasma can be written as

$$\rho n \underline{v} = -\nabla . \delta \overleftrightarrow{p} + \delta \underline{j} \times \underline{B}_1 \tag{17}$$

$$n \delta \rho = -\rho \nabla . \underline{v} \tag{18}$$

$$n \delta \underline{B} = (\underline{B}_1 . \nabla) \underline{v} - \underline{B}_1 \nabla . \underline{v} \tag{19}$$

$$\nabla \times \delta \underline{B} = 4\pi \delta \underline{j} \tag{20}$$

$$\nabla . \delta \underline{B} = 0 \tag{21}$$

$$n \delta p_{\parallel} = -p_{\parallel} \left(2ikv_z - \nabla . \underline{v} \right) \tag{22}$$

$$n \delta p_{\perp} = p_{\perp} \left(ikv_z - 2\nabla . \underline{v} \right) \tag{23}$$

where $\underline{v}$, $\delta p_{\parallel}$, $\delta p_{\perp}$, $\delta \rho$, $\delta \underline{j}$ and $\delta \underline{B}$ denote the perturbations, of first order of smallness, in various physical parameters.

The perturbed equations for the vacuum regions are

$$\nabla \times \delta \underline{B}^{(e)} = 0 \tag{24}$$

$$\nabla \cdot \delta \underline{B}^{(e)} = 0 \tag{25}$$

where $\delta \underline{B}^{(e)}$ represents the change in the magnetic field in the two vacuum regions.

(a) Solution for plasma region

Writing

$$\xi = \frac{\underline{v}}{n} \quad \text{and} \quad D = \partial/\partial r \tag{26}$$

we obtain from equations (17) to (23) that

$$\delta \underline{B} = B_1 \left[ik\xi_r , ik\xi_\theta , ik\xi_z - \nabla \cdot \underline{\xi} \right] \tag{27}$$

$$\xi = \left[A D\xi_z , \frac{im}{r} A \xi_z , \xi_z \right] \tag{28}$$

$$\nabla \cdot \underline{\xi} = - \frac{ik\xi_z}{p_\perp} \left(3p_\parallel - p_\perp + \frac{\rho n^2}{k^2} \right) \tag{29}$$

where

$$A = \frac{6p_\parallel - p_\perp + \frac{B_1^2}{4\pi} \frac{3p_\parallel}{p_\perp} + \frac{2\rho n^2}{k^2} \left(1 + \frac{B_1^2}{8\pi p_\perp} \right)}{ik \left[\frac{B_1^2}{4\pi} - p_\parallel + p_\perp + \frac{\rho n^2}{k^2} \right]} \tag{30}$$

and ξ_z satisfies the relation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \xi_z}{\partial r} \right) - \left(\beta^2 + \frac{m^2}{r^2} \right) \xi_z = 0 \tag{31}$$

with,

$$\beta^2 = \alpha^2 k^2 = k^2 \frac{\left(3p_\parallel + \frac{\rho n^2}{k^2} \right) \left(\frac{B_1^2}{4\pi} + p_\perp - p_\parallel + \frac{\rho n^2}{k^2} \right)}{6p_\parallel p_\perp - p_\perp^2 + \frac{B_1^2}{4\pi} 3p_\parallel + p_\perp \frac{2\rho n^2}{k^2} \left(1 + \frac{B_1^2}{8\pi p_\perp} \right)} \tag{32}$$

Solution of equation (30) is of the form

$$\xi_x = A_1 I_m(\beta r) + A_2 K_m(\beta r) \tag{33}$$

where A_1 and A_2 are arbitrary constants.

(b) Solution for vacuum regions

The change in magnetic field, $\delta \underline{B}^{(e)}$ inside the vacuum regions is given by equations (24) and (25). Thus $\delta \underline{B}^{(e)}$ is derivable from a scalar potential χ , satisfying the relation

$$\delta \underline{B}^{(e)} = \nabla \chi \tag{34}$$

This equation when combined with equation (25) leads to

$$\nabla^2 \chi = 0$$

So that

$$\chi = \left[C_1 I_m(kr) + C_2 K_m(kr)\right] \exp.\left[nt + i(m\theta + kz)\right] \tag{36}$$

Since there are two vacuum regions, separate boundary conditions are required for the two regions. The boundary conditions for the inner vacuum region are

$$\underline{N}.\underline{B} = 0 \qquad \text{at } r = \Lambda_0 R \tag{37}$$

and

$$[\![\underline{N}.\underline{B}]\!] = 0 \qquad \text{at } r = R + \xi_R^I$$

where $\underline{N}$ is the normal vector to the surface and ξ_R^{I} is the displacement at the inner-plasma vacuum interface.

Similarly the boundary conditions for the outer vacuum regions can be written as

$$\underline{N} \cdot \underline{B} = 0 \qquad \text{at} \quad r = \Lambda_2 R \tag{38}$$

and

$$[\![N \cdot \underline{B}]\!] = 0 \qquad \text{at} \quad r = \Lambda_1 R + \xi^{II}_{\Lambda_1 R}$$

where $\xi^{II}_{\Lambda_1 R}$ represents the displacement at the outer plasma vacuum interface.

With the help of boundary conditions (37) and (38), we write expressions for the change in the magnetic fields at the two plasma-vacuum interfaces as

$$\underline{\delta B}^{I} = \Big[ik\left(\frac{m}{x} B_b + B_2\right)\xi_R^{I}, \; -\frac{km}{x}\left(\frac{m}{x} B_b + B_2\right) Q_m(x, \Lambda_0 x)\, \xi_R^{I}, \; - k\left(\frac{m}{x} B_b + B_2\right) Q_m(x, \Lambda_0 x)\, \xi_R^{I} \Big] \tag{39}$$

and

$$\underline{\delta B}^{II} = \left[ik\left(\frac{m}{\Lambda_1 x} B_a + B_3\right) \xi^{II}_{\Lambda_1 R} \, , \, -\frac{km}{\Lambda_1 x}\left(\frac{m}{\Lambda_1 x} B_a + B_3\right) Q_m(\Lambda_1 x, \Lambda_2 x) \, \xi^{II}_{\Lambda_1 R} \, , \right.$$
$$\left. -k\left(\frac{m}{\Lambda_1 x} B_a + B_3\right) Q_m(\Lambda_1 x, \Lambda_2 x) \, \xi^{II}_{\Lambda_1 R} \right] \tag{40}$$

where

$$Q_m(p,q) = \frac{K'_m(q)\, I_m(p) - I'_m(q)\, K_m(q)}{K'_m(q)\, I'_m(p) - I'_m(q)\, K'_m(p)} \tag{41}$$

and

$$x = kR \tag{42}$$

Suffix I and II represent the parameters for the inner and outer plasma-vacuum interfaces respectively.

4. Dispersion Relation

In order to obtain the dispersion relation the pressure (kinetic + magnetic) balance condition must be satisfied at the two plasma - vacuum interfaces in the perturbed state. These conditions in the linearized form can be written as

$$\delta p_\perp + \frac{\underline{B}_1 . \delta \underline{B}}{4\pi} = \frac{\underline{B}^{I} . \delta \underline{B}^{I}}{4\pi} - \frac{B_b^2}{4\pi R} \xi_R^{I} \qquad \text{at } r = R$$

and

$$\delta p_\perp + \frac{\underline{B}_1 . \delta \underline{B}}{4\pi} = \frac{\underline{B}^{II} . \delta \underline{B}^{II}}{4\pi} - \frac{B_a^2}{4\pi \Lambda_1 R} \xi_{\Lambda_1 R}^{II} \qquad \text{at } r = \Lambda_1 R \tag{43}$$

Conditions (43) with the help of equations (2) - (4), (27) - (33) and (39) to (41) give the following characteristic equation relating n^2 with the various equilibrium parameters

$$F^2 + D_1 F + D_2 = 0 \tag{44}$$

where

$$n^2 = \frac{\left(\frac{B_b^2}{4\pi}\right)^2\left(3p_{\parallel} + \frac{\rho n^2}{k^2}\right)}{\left[3p_{\parallel}\left(2p_{\perp} + \frac{B_{\perp}^2}{4\pi}\right) - p_{\perp}^2 + 2\frac{\rho n^2}{k^2}\left(p_{\perp} + \frac{B_{\perp}^2}{8\pi}\right)\right]\left[\frac{B_{\perp}^2}{4\pi} + p_{\perp} - p_{\parallel} + \frac{\rho n^2}{k^2}\right]} \quad (45)$$

$$D_1 = -\left[\frac{x}{X_1} Q_m(y, \Lambda_1 y) + \frac{\Lambda_1 x}{X_2} R_m(y, \Lambda_1 y)\right] \quad (46)$$

$$D_2 = \frac{\Lambda_1 x^2}{X_1 X_2} P_m(y, \Lambda_1 y) \quad (47)$$

$$X_1 = \left[1 + \frac{(m + b_2 x)^2}{x} Q_m(x, \Lambda_0 x)\right] \quad (48)$$

$$X_2 = \left[a^2 + \frac{(am + b_3\Lambda_1 x)^2}{\Lambda_1 x} Q_m(\Lambda_1 x, \Lambda_2 x)\right] \quad (49)$$

$$R_m(\gamma, \Lambda_1\gamma) = \frac{I_m'(\gamma) K_m(\Lambda_1\gamma) - K_m'(\gamma) I_m(\Lambda_1\gamma)}{I_m'(\gamma) K_m'(\Lambda_1\gamma) - K_m'(\gamma) I_m'(\Lambda_1\gamma)} \qquad (50)$$

$$P_m(\gamma, \Lambda_1\gamma) = \frac{I_m(\gamma) K_m(\Lambda_1\gamma) - K_m(\gamma) I_m(\Lambda_1\gamma)}{I_m'(\gamma) K_m'(\Lambda_1\gamma) - K_m'(\gamma) I_m'(\Lambda_1\gamma)} \qquad (51)$$

with $\gamma = \beta R$ and the functions Q_m are given by the relation (41).

5. Neutral Stability Curves

The dispersion equation (44) has to be solved for the parameter n^2 in order to decide whether the configuration is stable or not. For any set of given physical parameters we have to incorporate the equation (45) with equation (44) while evaluating n^2. The equations (44) and (45) are extremely unwieldy and we shall be content with investigating the question of marginal stability ($n = o$) for anisymmetric perturbations ($m = o$). The eigenvalues n^2 for a non-dissipative, static plasma configuration (as is under investigation) are known to be real (positive or negative) so that the neutral stability curves define the transition from stable to unstable regime.

Numerical calculations for the roots $F_{1,2}$ of the quadratic equation (44) were done on computer taking $\Lambda_0 = 0.5$, $\Lambda_1 = 2$, $\Lambda_2 = 4$ and $b_2 = b_1$, $b_3 = \sqrt{1 - a^2}$. With α^2 as positive (equation 32) one root of the quadratic equation was plotted against x (=kR) to obtain neutral stability curves in a variety of cases. (figures (2) - (5)). In each figure curves (a), (b), (c) are for $\alpha = 0.2$, 0.4 and 0.6 respectively. Figures (2) and (3) are for $b_2 = b_1 = o$ and $a^2 = 0.1$ and 1.0 whereas $b_2 = b_1 = 1$, $a^2 = 0.1$, and 1.0 are the respective values for figures (4) and (5). Results were also obtained regarding α^2 in equation (32) as negative and are plotted in figures (6) - (9) taking $b_2 = b_1 = o$, and $b_2 = b_1 = 1$ for same set of values for $|\alpha|$ and a^2.

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ABSTRACT

The stability of a hard-core pinch model, characterized by an anisotropic pressure, is investigated using the usual normal mode technique. The dispersion formula is obtained with the "double-adiabatic" hydromagnetic equations and discussed for marginal stability for axisymmetric perturbations.

CAPTIONS TO FIGURES

Fig. 1 Equilibrium model of hard-core pinch.

Fig. 2 Neutral stability curves for $b_2 = b_1 = 0$, and $b_3{}^2 = 0.9$, and $\Lambda_o = 0.5$, $\Lambda_1 = 2$, $\Lambda_2 = 4$. Curve a, b, c respectively correspond to $\alpha = 0.2$, 0.4, and 0.6.

Fig. 3 Neutral stability curves for $b_2 = b_1 = b_3 = 0$, $\Lambda_o = 0.5$, $\Lambda_1 = 2$, and $\Lambda_2 = 4$. Curves a, b, c respectively correspond to $\alpha = 0.2$, 0.4, and 0.6.

Fig. 4 Neutral stability curves for $b_2 = b_1 = 1$, $b_3{}^2 = 1.9$ and $\Lambda_o = 0.5$, $\Lambda_1 = 2$, $\Lambda_2 = 4$. Curves a, b, c respectively correspond to $\alpha = 0.2$, 0.4, and 0.6.

Fig. 5 Neutral stability curves for $b_2 = b_1 = b_3 = 1$, and $\Lambda_o = 0.5$, $\Lambda_1 = 2$, $\Lambda_2 = 4$. Curves a, b, c respectively correspond to $\alpha = 0.2$, 0.4, and 0.6.

Fig. 6 Neutral stability curves for $b_2 = b_1 = 0$, $b_3{}^2 = 0.9$ with $\Lambda_o = 0.5$, $\Lambda_1 = 2$, $\Lambda_2 = 4$. Curves a, b, c respectively correspond to $\alpha = .2i$, $.4i$, $0.6i$.

Fig. 7 Neutral stability curves for $b_2 = b_1 = b_3 = 0$ with $\Lambda_o = 0.5$, $\Lambda_1 = 2$, $\Lambda_2 = 4$. Curves a, b, c respectively correspond to $\alpha = .2i$, $0.4i$, $0.6i$.

Fig. 8 Neutral stability curves for $b_2 = b_1 = 1$, $b_3{}^2 = 1.9$ with $\Lambda_o = 0.5$, $\Lambda_1 = 2$, $\Lambda_2 = 4$. Curves a, b, c respectively correspond to $\alpha = .2i$, $0.4i$, $0.6i$.

Fig. 9 Neutral stability curves for $b_2 = b_1 = b_3 = 1$, with $\Lambda_o = 0.5$, $\Lambda_1 = 2$, $\Lambda_2 = 4$. Curves a, b, c respectively correspond to $\alpha = 0.2i, 0.4i, 0.6i$.

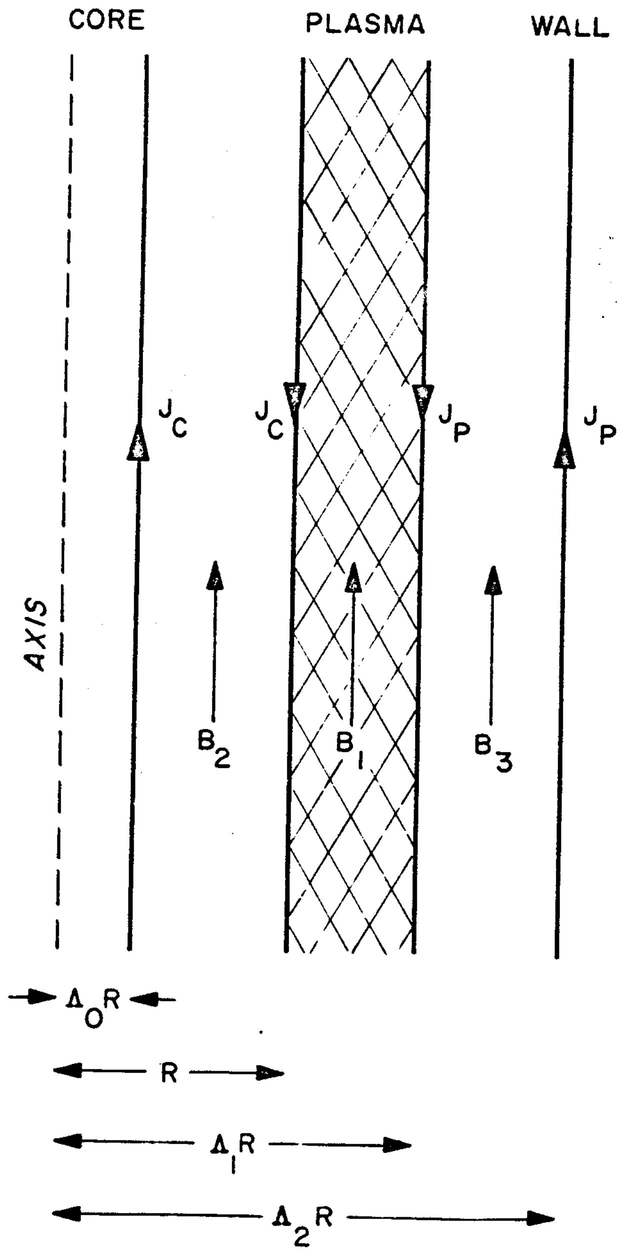


Figure 1

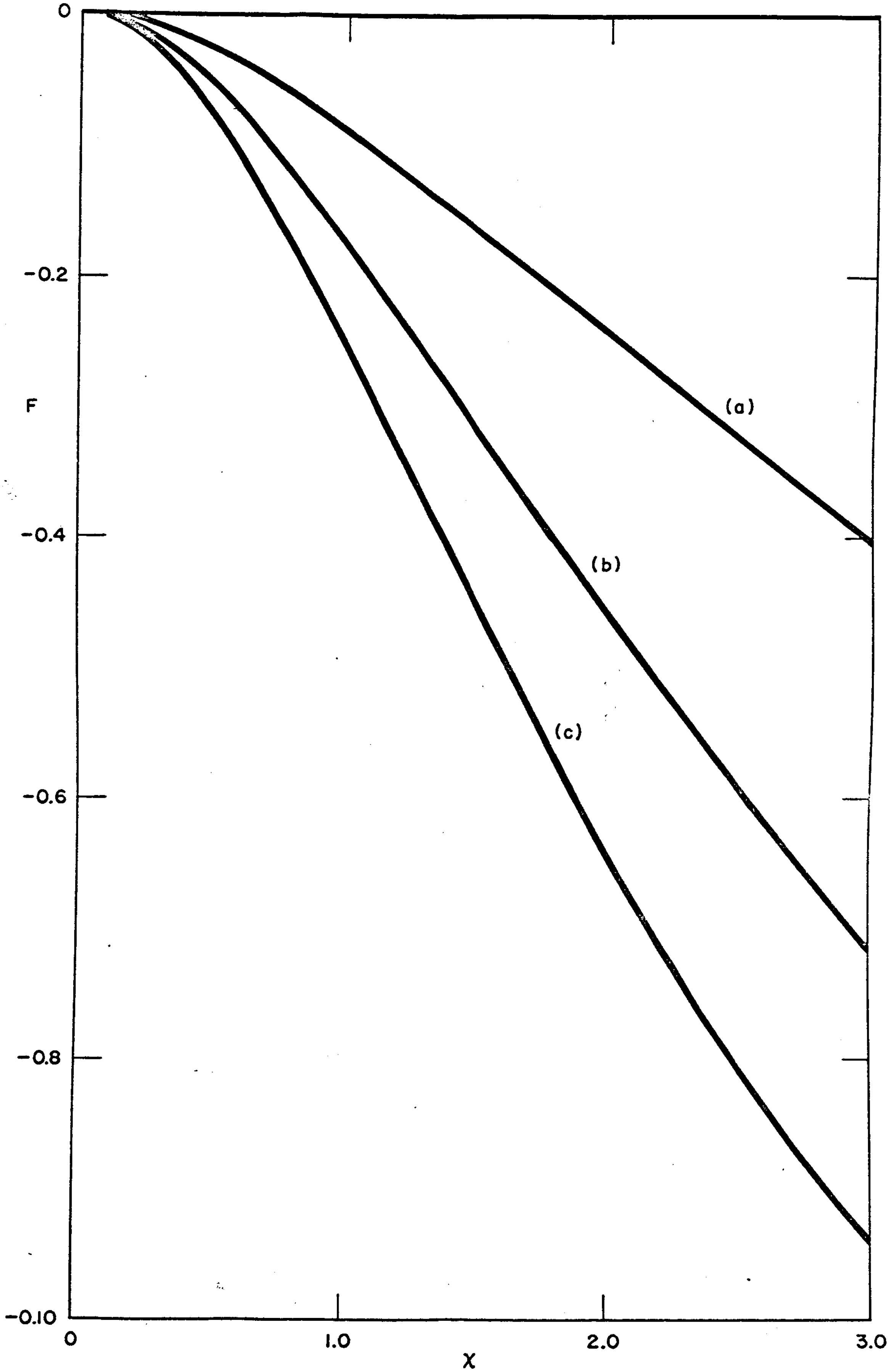
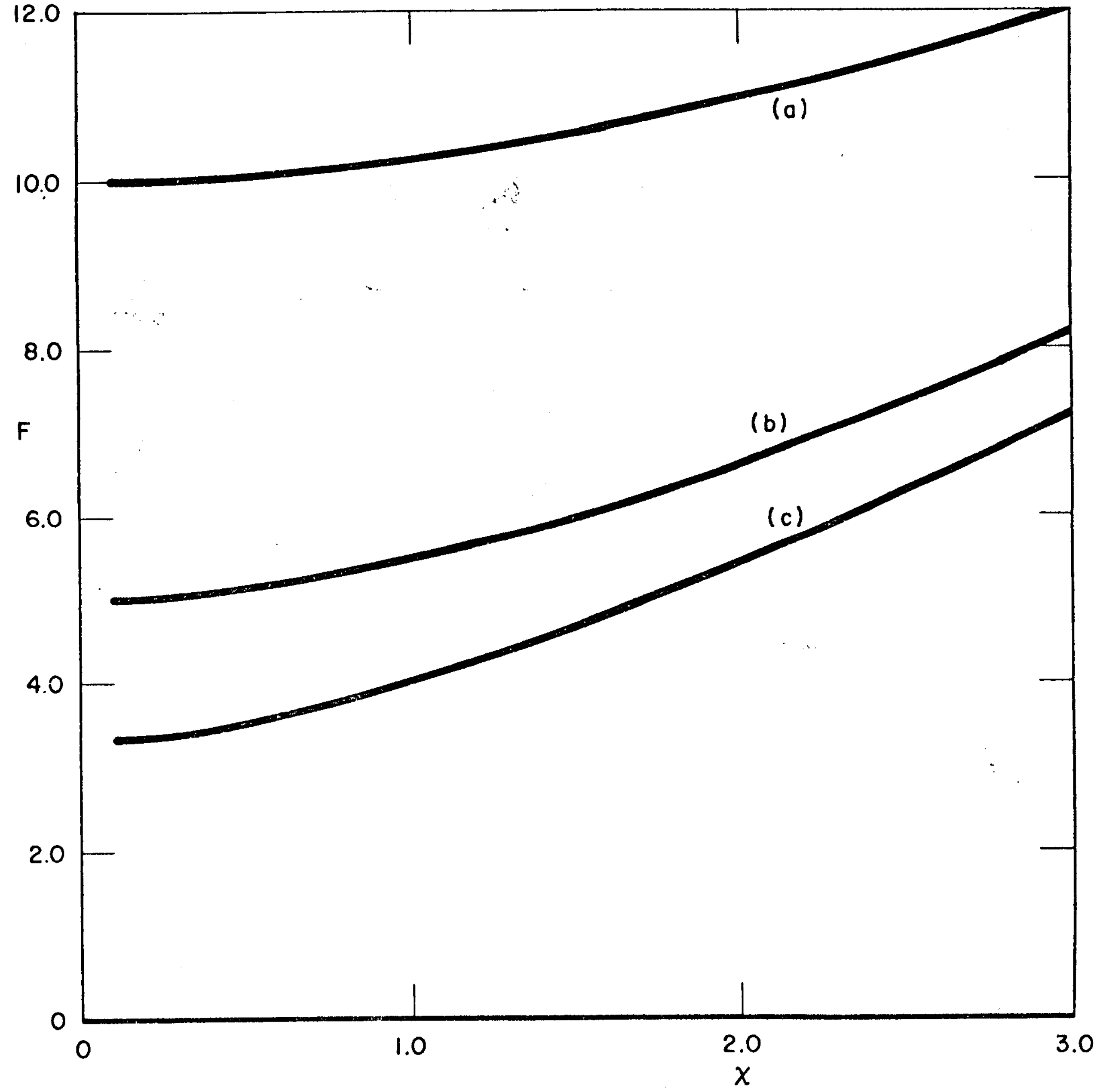


Figure 2



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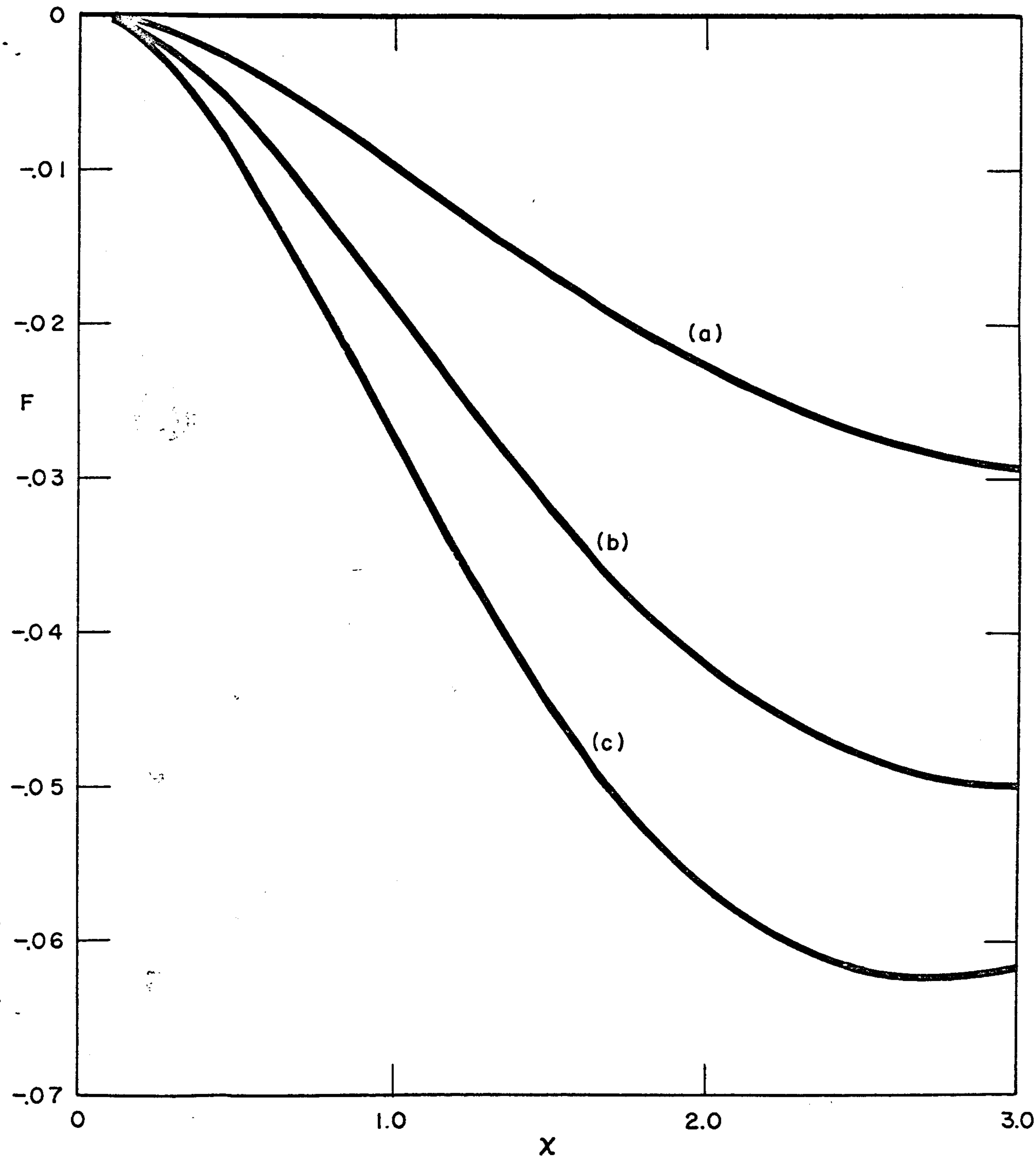


Figure 4

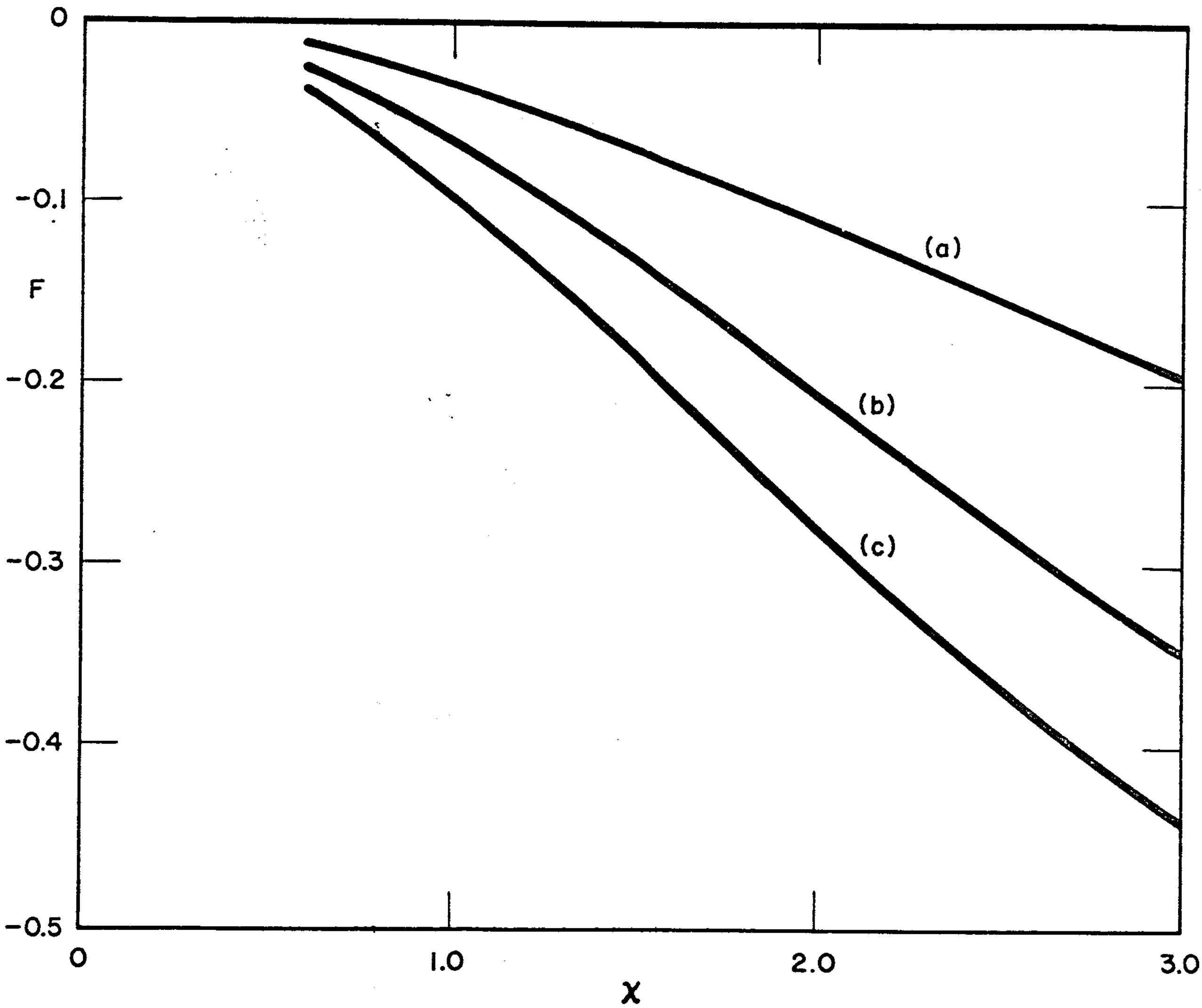
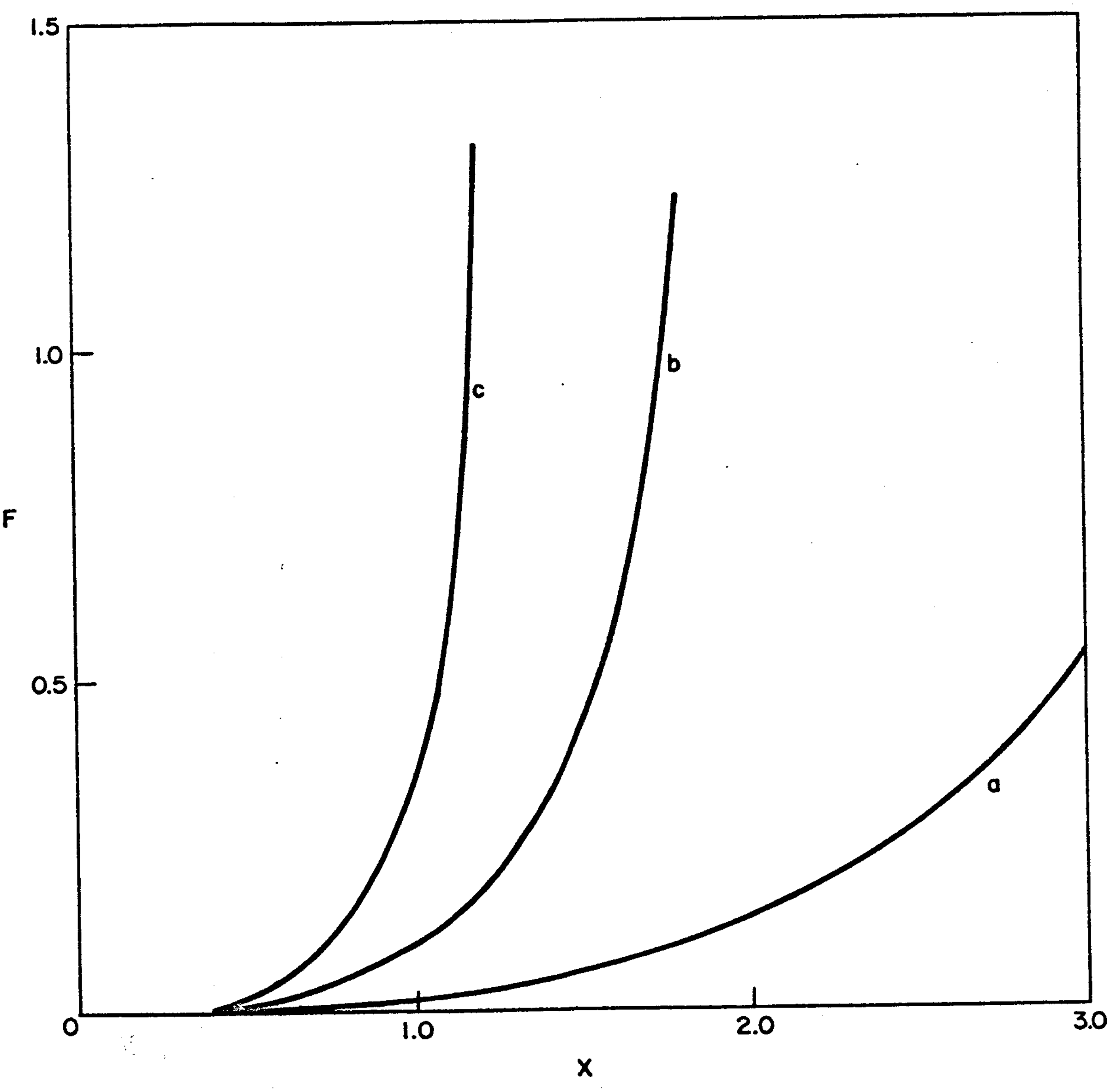


Figure 5



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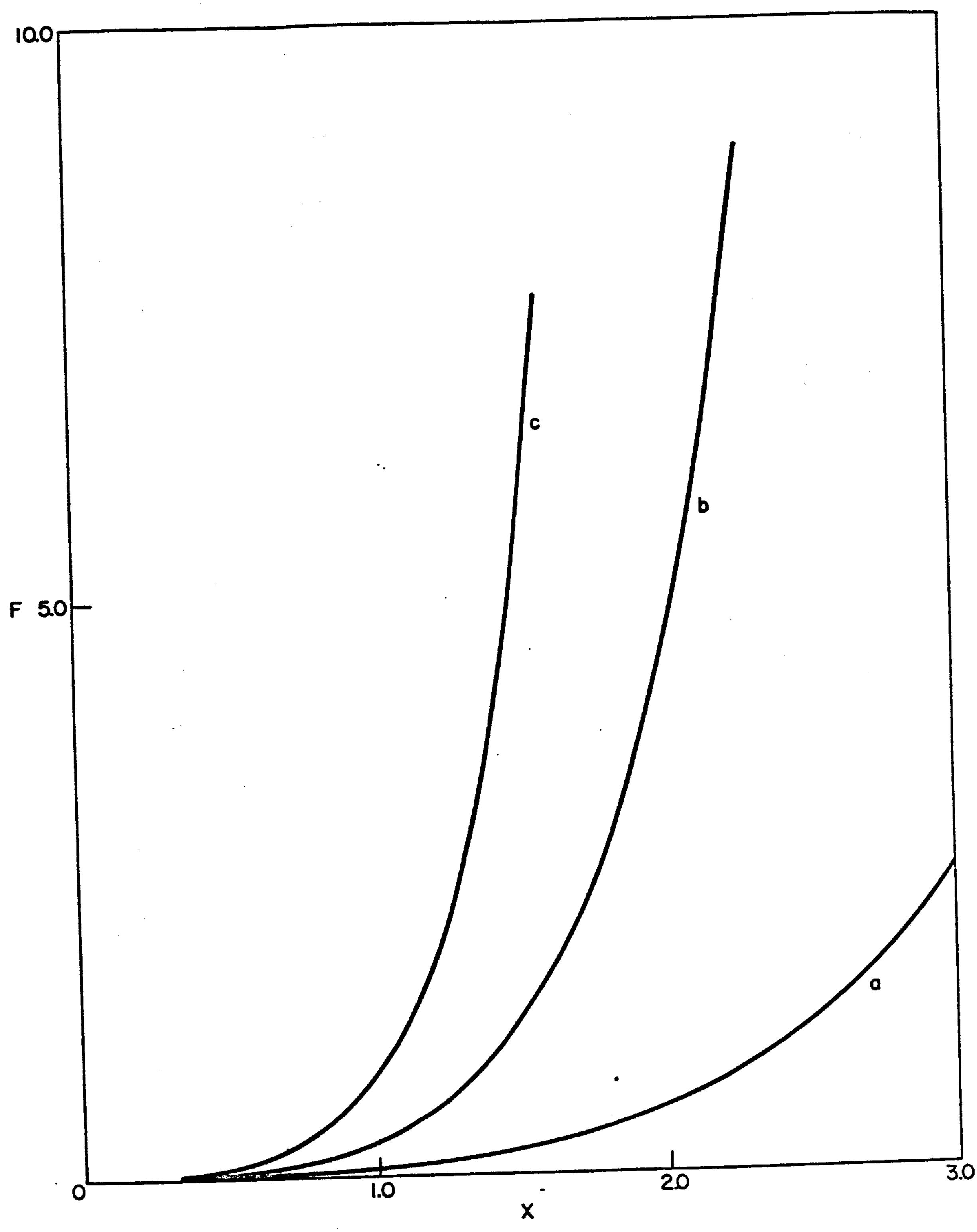

Figure 7

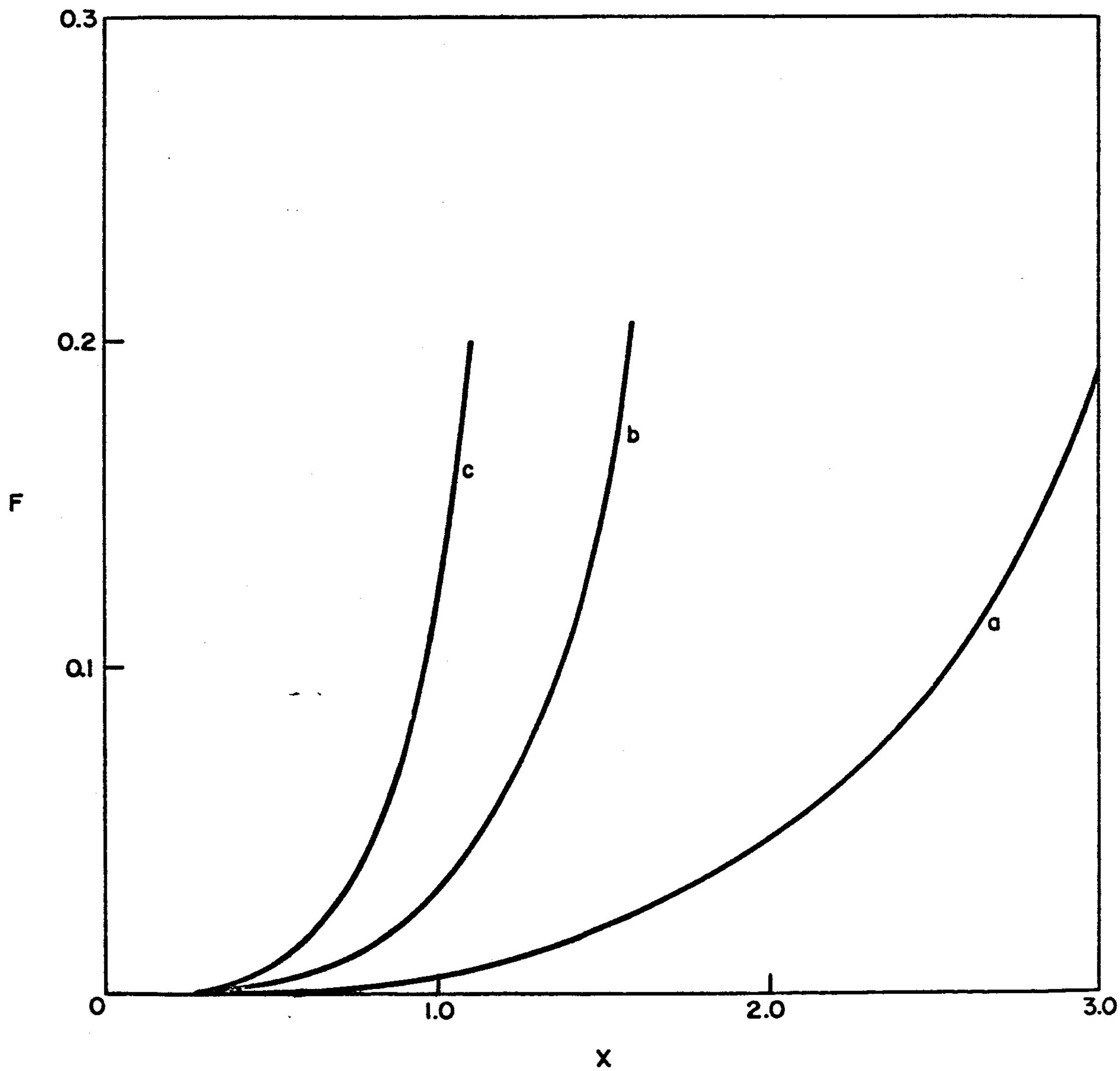

Figure 8

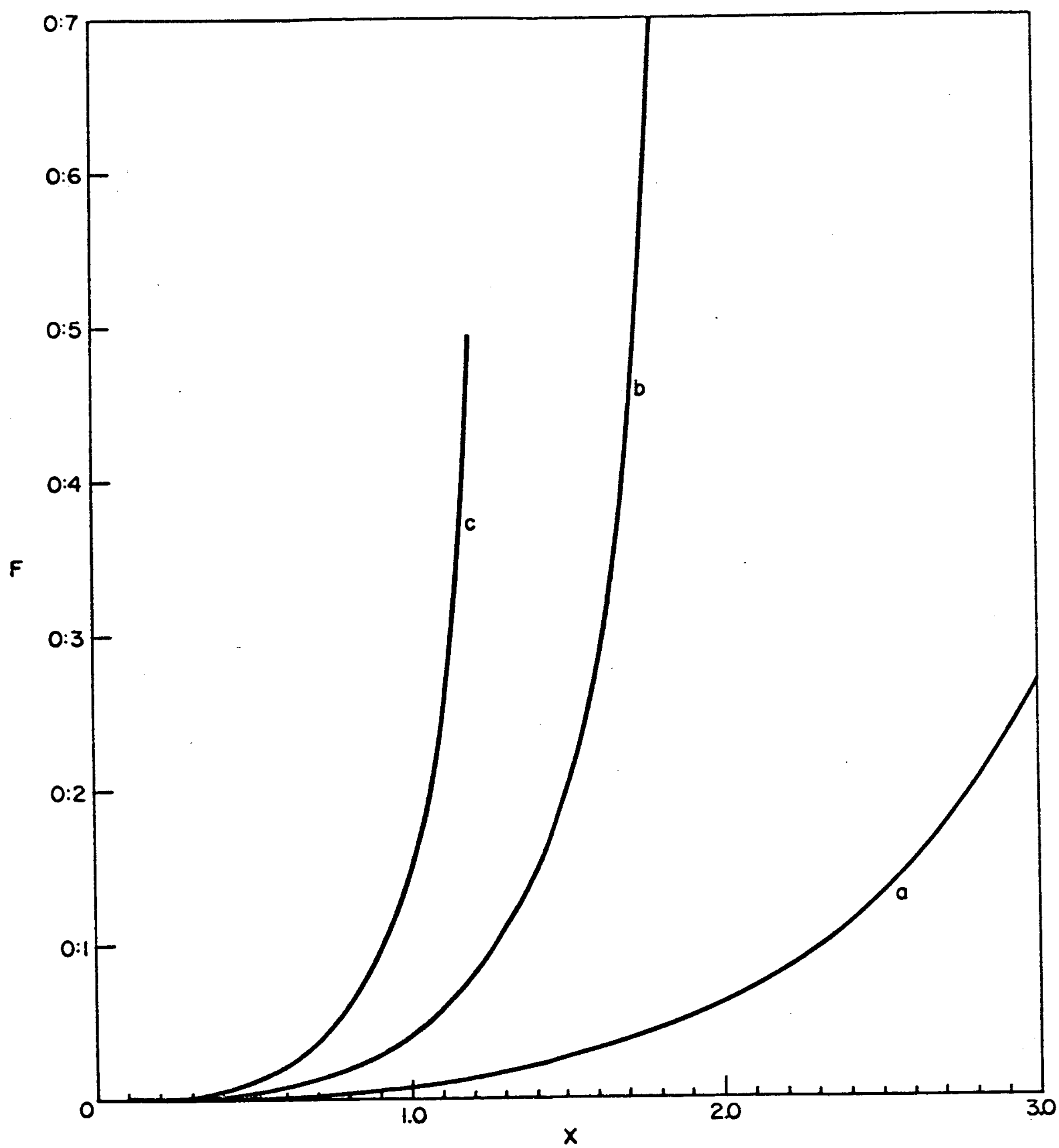

Figure 9