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STUDY OF INSTABILITIES AND TRANSITION TO TURBULENCE

IN A LINEAR HALL ACCELERATOR

By R. V. Hess, J. Burlock, B. Sidney, P. Brockman,
and R. H. Weinstein

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IN A LINEAR HALL ACCELERATOR

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INTRODUCTION

Recently instabilities and transition to plasma turbulence (in the broad sense) in MHD accelerators and power converters have received much attention because of the increase in losses for operation of such devices. This paper presents a study of such mechanisms for a linear Hall accelerator with applied axial electric field and a radial magnetic field. The major purpose is to study the instabilities and the transition to turbulence in a well-defined crossed-field geometry. Special emphasis is given to the range of the operating conditions where the mean free path ranges from values smaller to values of the order or larger than the dominating wavelength of the instability and on the effect on transition to turbulence. This is one of the universal problems of plasma physics, which aside from having application to MHD power converters and a variety of plasma accelerators, has also application to thermonuclear fusion and studies of astrophysics and the magnetosphere.

Use has been made of cold cathodes as well as cathodes heated to thermionic emission temperatures capable of operation at large currents. The effect of injection of highly ionized plasmas is also under investigation, making use of hollow cathode arc injectors capable of injecting gaseous as well as alkali vapor plasmas (see appendix).

The effect of special magnetic field shaping which helps to combine magnetic nozzle effect with Hall current acceleration is not considered in the present experiments. However, a review is given of such configurations for high-power operation at higher pressures, than in the present experiments, where the instabilities are less pronounced, using experiments with a coaxial Hall current accelerator performed at this laboratory and at the AVCO Everett Research Laboratory (ref. 1). Recently, a modified configuration of such a device was used by G. Cann (ref. 2) to obtain very high specific impulse. Equally encouraging results from this laboratory will be reported by W. Grossmann, H. Hassan, and R. V. Hess at the forthcoming Electric Propulsion Conference of the AIAA.

The present study extends the preliminary study of transition by the authors given in reference 3. It complements the study of turbulence given by Janes in preliminary form in reference 4 and in greater detail at the present symposium (ref. 5). For a theoretical interpretation of the observed phenomenon an effort is made to draw on the experience obtained within recent years for a variety of different configurations of electric and magnetic fields. This

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approach which requires a review of the whole field will turn out to be fruitful since the observed transition to turbulence appears to have universal meaning.

APPARATUS

The configuration of the linear Hall accelerator is shown in figure 1. The two coils and iron center piece produce a uniform radial magnetic field between the electrodes. This configuration has the advantage of good accessibility for diagnostics. The axial magnetic field is negligible at radial fields up to 300 gauss. The anode and cathode are 2.6 inches I.D. and 1 inch long. The anode is made of copper and for the cathode aluminum was used or tungsten heated to thermionic emission temperatures. Since it was found that the essential instability phenomena and the transition to turbulence do not change with cathode material and heating (this has also been reported in other instability experiments) the majority of experiments reported here use the simpler aluminum cathode. Recent successful operation with currents up to 10 amperes using a specially designed thermionically emitting cathode have shown similar results.

The discharge is contained in a 3-inch-long annulus between the 2.4-inch I.D. boron nitride sleeve and the 1.5-inch O.D. center glass. Various probes can be inserted radially through the boron nitride sleeve or axially through the end caps. All probes are tungsten and are shielded up to the probe tip.

The hollow cathode arc jet for plasma injection is described in the appendix.

OPERATION AND MEASUREMENTS

Operation

The data presented were taken at 8 and 30 (and some at one) μ Hg in argon. The axial current was varied from 1 to 50 amperes, axial voltage from 50 to 300 volts, and radial magnetic field from 0 to 250 gauss.

Hall Current - Method of Measurement and Variation With Magnetic Field

Hall current measurements are taken with a coil around the discharge section. This coil measures the change in the axial magnetic field of the Hall current. The discharge is turned off and the coil voltage is measured on an integrating galvanometer. This integrated voltage is proportional to the change in axial magnetic field which is proportional to the Hall current. Variations of Hall current with magnetic field at 20 amperes axial current are given in figure 2.

Variation of Voltage and Electric Field With Magnetic Field

The variation of arc voltage with magnetic field is shown in figure 3 for an axial current of 20 amperes. Measurement of the voltage inside the accelerating region indicates an approximately linear distribution outside of the sheaths, thus an approximately constant electric field.

Variation of Voltage Oscillation Spectrum With Magnetic Field

Voltage oscillations are displayed on a panoramic analyzer and photographed. The oscillation spectrum varied with pressure as indicated but did not appear to vary significantly with axial current. An increase in the amplitudes of the disturbances was, however, observed with current. Figure 4 presents photographs of the frequency oscillation spectrum at three different magnetic fields, but at the same axial current. The photographs show that at each magnetic field there is a peak frequency with harmonics and background noise or turbulence; the peaks and harmonics are less sharp at the lower pressures. As the magnetic field is increased the peak broadens and moves to a higher frequency at the same time the background noise rises and masks the harmonics. Figure 5 presents peak frequency versus magnetic field at 20 amperes axial current. The 8-micron curve is cut off where the combination of rising noise and peak broadening make the peak frequency measurement inaccurate.

Use of the new high current heated cathode operating at 10 amperes has made it possible to extend the operation to higher magnetic fields and again a considerable retardation of peak frequency shift was shown with increasing magnetic field as the turbulent state was approached. The new heated cathode made it possible also to operate at 1μ Hg at currents up to 10 amps. A shift in frequency spectrum was observed also but the peaks were lower and broader than at higher pressures, with low-frequency regions which were not observed at the higher pressures making their appearance. This may be related to the comparatively low frequencies for fully turbulent conduction at magnetic fields of 500 gauss reported by Janes (ref. 5). A study is in progress to determine to what extent these changes are related to the higher magnetic fields and lower densities used by Janes or are influenced by the slightly different configuration.

Propagation and Shape of Instability and Transition to Turbulence

For weak magnetic fields of about 20 gauss, probes displaced in the azimuthal direction show the existence of a rotating disturbance in the direction of the drift motion across electric and magnetic fields with an $m = 1$ mode. Probes displaced over an increasing axial distance show a phase shift indicating that this wave also propagates in the axial direction from anode to cathode with about the same wavelength and velocity of propagation. The wavelength and the velocity of propagation of this screw instability are on the average 15 cm and 4×10^3 m/sec. High-speed photographs show a luminous front across the tube at an average angle of about 45° . The slope of this wave front agrees with the direction of axial screw motion observed by the probes. Figure 6 gives a plot

of phase shift versus axial probe separation for a magnetic field of 20 gauss. Janes (ref. 5) also observes a spiral structure for fully turbulent conduction at a magnetic field of 500 gauss which, as previously noted, has a lower rotational frequency. Further, Janes' spiral is not in an $m = 1$ mode but covers only $1/3$ or $1/2$ of the circumference. Experiments are in progress to determine if this change in spiral structure also occurs in this apparatus as fully turbulent conduction is approached.

Electric Field Oscillations - Method of Measurement and Variation With Magnetic Field

The same probes axially displaced which were used to measure the phase difference of the propagating wave give also the electric field oscillations. For this purpose the probes are displaced until the maximum voltage difference is reached which, divided by the distance, gives the amplitude of the oscillating electric field from which the rms electric field can be obtained. This is a simple procedure for weak magnetic fields when the wavelength or scale of the oscillations is known. For higher magnetic fields when the wave begins to break up into turbulence the scale length of the largest amplitude oscillation has to be found before the electric field oscillations can be determined. The latter is given by the correlation distance obtained from the phase measurements described in the preceding section; the correlation distance of scale is seen to decrease with increasing magnetic field. The measurements indicate that at large magnetic fields, the oscillating electric field becomes considerably higher than the applied d-c electric field. For example, at 8μ Hg, 112 gauss and 20 amperes axial current, the d-c electric field is about 20 volts per cm, while the rms amplitude of the electric field oscillation is almost 50 volts/cm. Figure 7 gives a plot of rms voltage difference versus probe spacing for three magnetic fields with constant pressure and current.

The correlation curve for rms oscillations of voltage difference approaches a finite ordinate with increasing probe distance for a magnetic field of 112 gauss, as the correlation approaches zero. This variation is in contrast to that used in velocity correlation measurements for fluid turbulence (e.g., ref. 6, p. 37), where zero coordinate is approached with zero correlation. The reason is that the correlation in reference 6 is based on the mean values of the product of oscillating velocities at two points. In the present paper a different technique is used for obtaining the correlation curve, which conveniently makes use of the rms value of the voltage difference between two probes. The rms value increases here as the mean product (which is contained in the rms voltage difference) decreases. A similar approach to correlation studies is given in a paper by Bingham, Chen, and Harris for a reflex arc (ref. 7).

It should be noted that the increase in rms voltage difference with probe distance is very small for the lowest value of magnetic field in figure 7, for which measurements of change in phase shift with probe distance indicate the existence of a well-correlated spiral. This suggests that the almost constant rms value beyond about $1/2$ -cm probe distance for a magnetic field of 112 gauss does not rule out the coexistence of correlation for a large-scale spiral structure; the existence of such a large structure actually appeared in the phase

measurements for a magnetic field not included in figure 6. It is planned to use frequency filtering in the determination of the rms voltage differences in order to clarify these issues.

Ion Saturation Current Oscillations

A Langmuir probe held in the ion saturation region with a constant d-c bias shows an increase of a-c ion current with increased magnetic field. This indicates an increase in the quantity $\rho'v'$ where ρ' is the magnitude of the ion density fluctuation, and v' is the magnitude of the thermal velocity (which is used to determine the ion density for negligible ion motion) combined with directed ion velocity. The amplitudes of oscillations in ion saturation current and of electric fields are sufficiently large to warrant a careful check concerning the effect of the ion velocity oscillations on the determination of the ion density oscillations.

DISCUSSION OF RESULTS

Interpretation of Change in Spectrum of Voltage Oscillations With

Magnetic Field and Pressure as Transition to Turbulence

The experimental results are first briefly reviewed. The sequence of spectrums shown in figure 4 shows the appearance of a single peak at low magnetic fields, with the emergence of harmonics and an increase in the amplitudes of random oscillations of turbulence at higher magnetic fields. In figure 5 where the shift of the peak frequency with magnetic field is presented it is indicated that at first the frequency shifts about proportionally to the magnetic field but is retarded as the higher magnetic fields are reached where the harmonics submerge in broadened peaks and the turbulence level has increased. The frequency shift is more rapid at lower pressures and with it the emergence of turbulence. Furthermore, the frequency shift is more or less independent of current.

In an effort to understand if these experiments constitute a true transition to turbulence an attempt was made to find if they are related to other experiments where transition to turbulence is known to occur. The present transition spectrum and its variation shows a striking resemblance with that observed by Akhmedov and Zaitsev (ref. 8) for the Lehnert-Kadomtsev instability (for an axial discharge in an axial magnetic field) when the axial magnetic field is increased beyond its critical value. This transition occurs at higher magnetic fields than in the present experiments but the spectrum shows a similar pattern of peaks and harmonics whose shift to higher frequencies is first proportional to the magnetic field and retarded with the emergence of turbulence. The similarities extend to the pronounced effect of pressure variation and the small effect of current variation on the frequency shift. It should be noted also that their experiments were performed in a much higher pressure range. The transition to turbulence observed in the present experiments thus seems to have a more universal meaning beyond the present immediate application. No

effort has been made in this paper to study the spectrum of fully developed turbulence.

A frequency shift with magnetic field is also shown by Lary, Meyerand, and Salz (ref. 9) for a linear Hall accelerator at pressures of 7μ Hg and of the order of 4μ Hg, but harmonics, retardation of peak frequency shift at high magnetic fields with peak broadening and emergence of random oscillations were not discussed. However, the sketch of the spectrum of voltage oscillations (the actual pictures are not shown) in reference 9, figure 7 (for 7μ Hg), actually seems to show a retardation of the frequency shift at higher magnetic fields with broadening of the peaks though the effect is not quite as clear as in the present experiments. An actual plot of peak frequency shift vs magnetic field is shown in figure 8 or reference 9. The increase is about linear but it is limited to magnetic fields below 70 gauss. The harmonics do not appear in the sketch. No experiments on the variation of the frequency shift with pressure were reported in reference 9. Thus it appears that while no transition to turbulence may have been discussed in reference 9 it actually may have occurred at higher magnetic fields and peak frequencies similar to those observed in the present experiments at about 8μ Hg. Various statements in reference 9, however, seem to indicate that that investigation concentrated on lower values of magnetic field, where the turbulent effects have not yet become significant. A collisionless theory is used in reference 9 where Landau damping of randomized oscillations due to two stream microinstabilities in crossed electric and magnetic fields reduces the collisional conductivity across the magnetic field without changing the basic collisional conduction process. This concept is based on a mechanism originally proposed by Bunemann (ref. 10) in the absence of magnetic fields. The mechanism will be contrasted with others which actually produce a more truly turbulent conduction process in subsequent sections.

Criteria for Transition to Turbulence

A criterion for transition to turbulence is that the oscillations begin to play an important part as compared to steady processes. The voltage oscillations are not well suited for such a comparison since local values are being compared to the voltage across the whole discharge. A comparison between rms values of the oscillating and steady values of the electric field have, however, more validity. Measurements discussed in a previous section indicate an rms value of the electric field of the same order or in excess of the steady electric field as the magnetic field increases; furthermore, a decrease in scale of disturbances with increasing magnetic field is shown. The existence of transition to turbulence is thus likely. This, of course, does not necessarily mean that the electric field oscillations are the most important parameters in the turbulence conduction process, where the mean square density fluctuations (which are also large in the present experiments) play an important role in defining the turbulent conduction process (for example, ref. 11).

Another criterion for transition to turbulence is the existence of harmonics followed by a broadening of the peaks with submerging of the harmonics into the increasing random noise level. The fact that the frequency of the broad peak barely shifts with increasing magnetic field together with a widening of the frequency spectrum is also an indication of transition to turbulence. As a

matter of fact, the appearance of the comparatively large rms values of the electric field together with the appearance of harmonics and broadening of the spectrum is an indication of a strong type of plasma turbulence where nonlinear interactions between large-amplitude waves or vortices can produce a decay into smaller waves or vortices. A more detailed discussion is given in a later section. It should be emphasized that all these criteria are actually for the transition to a turbulence containing large-amplitude fluctuations and that the existence of plasma turbulence itself (especially that given in the various small perturbation theories) does not necessarily depend on such strong effects, thus they present a safe limit.

Effect of Transition to Turbulence on Hall Current

As shown in figure 2, the Hall current first increases with magnetic field but decreases beyond a peak; this, of course, is also of interest for propulsion. As discussed in a previous paper by the authors (refs. 12 and 13), the peaking of the Hall current at constant axial currents can have various reasons, and the effect of ion slip or transition to turbulence seem to be the predominant ones. Briefly, the ratio of Hall current to axial current densities with the inclusion of ion slip is given by

$$\frac{j_{\theta}}{j_x} = \frac{\omega_e \tau_e}{1 + 2\omega_e \tau_e \omega_i \tau_i} \left(1 - \frac{n_e v_x}{j_x} \right)$$

For turbulent conduction it is approximately given by

$$\frac{j_{\theta}}{j_x} \approx \frac{1}{\alpha} \left(1 - \frac{n_e v_x}{j_x} \right)$$

where

$$\alpha = \frac{\langle (n - n_0)^2 \rangle}{n_0^2}$$

and is thus determined by the density fluctuations. This expression given in references 12 and 13 based on reference 11 applies to frequencies higher than ion cyclotron frequencies and the theory with proper inclusion of the directed and oscillatory motion of the ions has not yet been developed. The theory discussed in reference 11 further assumes that the density fluctuations are small compared with the ion density; furthermore, reference 11 assumes through the random phase approximation that the coupling between the higher modes is small. The relation of this assumption to turbulence is discussed in a later section. Furthermore, the preservation of the spiral structure of the instability as transition to turbulence occurs (though it appears not as well defined) must be considered in the description of the turbulence. Measurements of the ratio of

axial and Hall current densities give an approximate quantitative measure of the turbulent conduction. However, to understand the mechanism of the conduction process measurements of the correlations between electric field, density and current oscillations, which control this ratio are also required. Evaluation must also be made to see if the presence of a polarized spoke (having $m = 1$ mode or restricted width) requires description in terms of nonisotropic turbulence.

A comparison is made now of figures 3 and 4 for the variation of the spectrum of voltage oscillations with magnetic field and of figure 2 for the variation of Hall current with magnetic field. It is indicated that in the range of magnetic fields where the Hall current begins to deviate from its linear increase the peaks of the spectrum begin to broaden noticeably and the random noise level begins to increase. The predominance of turbulent-over ion slip effects requires that

$$\frac{1}{\alpha} < \frac{\omega_e \tau_e}{1 + 2\omega_e \tau_e \omega_i \tau_i}$$

It must be emphasized in this connection that the validity of including the ion-slip term does not only depend on the condition that $\omega_e \tau_e \omega_i \tau_i$ is appreciable (for large $\omega_e \tau_e$ this requires only a small value of $\omega_i \tau_i$) but it also requires that collisions between ions and neutrals can actually take place in the region of acceleration. At pressures of 8μ Hg the mean-free path for ion neutral collisions becomes of the order of the radius of the coaxial accelerator even taking into account the large charge exchange cross sections, and the ion-slip term begins to lose its meaning. As a result, it becomes necessary to use the equations for a fully ionized plasma including wall effects even if the plasma is only partially ionized. Thus it appears that at the lower pressures the effects of transition to turbulence will predominate in reduction of the Hall current with increasing magnetic field.

As previously noted, the ratio of j_θ/j_x gives an approximate measure of α . A calibration of Hall currents has not been performed on the present apparatus; however, earlier experiments performed at 15 and 30 microns (refs. 12 and 13) showed the maximum value of j_θ/j_x to be slightly less than 10. This implies that the minimum value of α is greater than 1/10. With increasing magnetic field and transition to turbulence, α increases but sufficiently high fields have not been used so far to determine an α corresponding to developed turbulence.

Effect of Transition on Variation of Electric With Magnetic Field

The variation of voltage at constant axial current with increasing magnetic field for operation at 8μ Hg in figure 3 shows a rather drastic change with the transition to turbulence at a magnetic field of about 80 gauss. (Note: Separate

measurements have indicated a linear drop in voltage in the region of plasma acceleration, outside of the sheaths, thus $E = \text{const}$). It even appears that before the transition to turbulence E is about proportional to B^2 and with the emergence of turbulence becomes proportional to B . Such behavior would be in agreement with a change from collisional to turbulent conduction predicted by theory. However, while it yields some confirmation, this indication alone would not have been sufficient since as pointed out in several papers by the authors (see also ref. 1) and by Chubb (ref. 14) from the NASA Lewis Laboratory since for a partially ionized plasma for certain variations of n_e and ν (collision frequency) with B , $E \propto B$ can be obtained without resorting to turbulence. The use of injection of highly ionized plasmas would also help to clear up this matter. In reference 9 it is emphasized that in the regime tested E/I varies approximately as B^2 in the regime tested. The highest magnetic field shown in the pertinent figure 6 of that reference is, however, only 70 gauss and up to this value a similar relation appears to hold for the present experiments.

Theory of Instabilities

Before discussing possible instability mechanisms a few remarks are made concerning the use of linearized analysis of instabilities for comparison with the experimentally determined phase velocities. A linearized analysis should hold for magnetic fields in the neighborhood of the critical value, where the instabilities begin to emerge. The existence of harmonics and the resultant distortion of the sinusoidal azimuthal wave (for example) and the increase of random noise indicates the emergence of nonlinear effects. The frequency increase in figure 5 persists through some of these conditions, before it is reduced as the turbulent level becomes more important. Thus it is doubted that the frequency increase with magnetic field can be explained purely on the basis of linearized instability theory; this is also indicated by the fact that on the average the disturbances are no longer growing. Support for such viewpoint exists in the experiments in reference 8 for a different type of electric-magnetic field configuration.

Among several recently proposed instabilities for Hall current devices and PIG discharges, none describe completely the phenomena observed in the present device. Two types of instabilities, however, could be modified to aid in the explanation of the present effects. One is an instability for MHD Hall current-generators originated by Velikhov (ref. 15) and put in more concise form by Tamor (ref. 16) which should also apply to Hall current accelerators. The instability is described in these references for a rectangular Hall device; however, it should be also applicable to a cylindrical geometry with axial and azimuthal current components. The origin of these instabilities is that sound waves moving against the direction of the current and relative to the center of mass motion may be amplified by the $j' \times B$ forces due to the perturbed current as the latter can be transmitted back to the perturbed plasma flow and cause growth of these perturbations. In the Velikhov-Tamor approach attention is first given to growth of perturbations in the axial direction, through $j' \times B$ forces; however, the possibility of growth of perturbations propagating against the Hall current is further included by Tamor. Proof of the existence of an

azimuthal $m = 1$ mode still has to be obtained through an extension of Tamor's approach to the present case, but such a mode should be possible in principle. Application of the concepts developed by Tamor suggests that for the present configuration the normal modes will not be purely compressional but a combination of compression and shear waves. Since the axial motion of the instability in the present tests is from anode to cathode, the theory applies here when the axial plasma velocity is larger than the speed of sound, i.e., for supersonic velocities. The sound waves moving against the azimuthal current component, i.e., in the direction of the electron drift would also be in agreement with the present results. Since the sound waves move relative to the plasma velocity, the increase in ion rotation with magnetic field (though the actual rotation is small) could explain at least qualitatively the increase in frequency with increasing magnetic field. However, it must be strongly emphasized that the theories by Velikhov and Tamor are developed for considerably higher pressures, where collisions play a dominant part and the applicability of the theory to low pressures must be established first. An attempt in this direction will be made in a later section.

Other instabilities which may contribute to the transition to turbulence in the Hall accelerator, in contrast to those in references 15 and 16, require nonuniformities in flow properties. One of particular interest is that found by Morse (ref. 17) which requires a negative density gradient in the direction of the electric field which would occur for ion acceleration. (It is an extension of the Hoh-Simon (refs. 18 and 19) instability which requires positive density gradients.) The instability is applied in reference 17 to a situation where the magnetic fields (across which diffusion occurs) are strong enough so that ion slip is needed to produce the relative motion of ions and electrons in the azimuthal direction; the motion of the ions in the direction of the electric field is thus relatively small. The propagation of the instability is against the direction of the Hall current at a phase velocity below the azimuthal drift of electrons due to coupling to the ions by space charge effects resulting in a polarized spoke with $m = 1$ mode (and modified for higher magnetic fields). For the range of operating conditions where ion-slip effects still exist in the present experiments order of magnitude agreement is found with the phase velocity obtained from Morse's results. The inclusion of axial propagation and with it spiral propagation into Morse's picture does not seem to offer conceptual difficulty. The electrons drifting in the axial direction (at a reduced mobility) would try to pull the azimuthal perturbation with them but the ions moving predominantly toward the cathode, for the large cyclotron radius chosen for acceleration, would reduce this motion by space charge effects. In the reference system of the ions the perturbation would move against the current or in the direction of the electrons; however, in the reference system of the accelerator it can still move down the accelerator from anode to cathode. For low densities or large mean-free path the effect of collisionless ion motion would more or less replace that of ion slip; the collisionless case is discussed further in a later section.

In Morse's paper the effect of ion oscillations which play an important part in the acoustic waves discussed by Velikhov-Tamor (refs. 15 and 16) is neglected. It should be pointed out briefly that for the low densities and high electron temperatures of the present experiments the so-called ion-acoustic waves may have different character in that their velocity is largely affected by

the electron thermal motion, which influences the ion thermal motion through space charge effects, and the ion mass. The instabilities by Morse as well as that by Simon and Hoh are not due to propagating waves, in contrast to the ion-acoustic waves. They are concerned with rotating periodic density perturbations modified here to include spiral perturbations. These perturbations lead to so-called drift and convective instabilities. It appears likely that coupling between the Velikhov-Tamor instabilities amplified by $j' \times B$ effects and the convective instabilities of the Morse type would occur. This is possible through space charge effects. However, since, as pointed out by Tamor (ref. 16), the instability restricted by Velikhov (ref. 15) to acoustic waves can also contain transverse shear waves, the coupling between the two types of instabilities may be even more direct. Further studies are also necessary to evaluate how the current perturbations of the Velikhov-Tamor approach can be incorporated in the convective instability approach. In most of the approaches temperature, conductivity, and ionizing perturbations are avoided. The question arises if the disturbance is not a simple arc spoke and not an instability, since it appears at weak magnetic fields. However, the disturbance does not exist at zero magnetic field, has an $m = 1$ mode at weak magnetic fields, and its amplitude increases considerably with magnetic field with subsequent transition to turbulence. The reason why the instability occurs at such weak magnetic fields is that in this crossed field geometry it is easy to buildup electric fields inside the plasma which drive the instabilities. More measurements are required to identify the various mechanisms such as, for example, studies using magnetic probes for the current instabilities. The spiral motion, however, could be obtained from various instability mechanisms.

The detailed structure of the spiral motion, of course, does not have to be the same for the instability and the transition to turbulence and for the fully developed turbulence at high magnetic fields reported by Janes (ref. 5) at this symposium. Whereas, in the present experiments, an $m = 1$ mode in the azimuthal direction was observed at weak magnetic fields, for the high magnetic fields used by Janes the disturbance covered only a much smaller part of the circumference (looking more like a spoke), together with a somewhat lower phase velocity of rotation. Careful studies are in progress to calculate the transition to turbulence at higher magnetic fields especially at the lower pressure of 1μ Hg. As previously indicated in this paper, the spectrum appears to be modified for experiments at 1μ Hg through the appearance of high-amplitude low-frequency regions which may be related to the low-frequency rotating spirals (with smaller width than the $m = 1$ mode) reported by Janes at this symposium. Note that a reduction in the rate of increase of rotation frequency with transition to turbulence was observed in the experiments. A reduction in the speed of rotation with the development of turbulence and increased "drag" on the perturbation may be possible. It has to be also investigated experimentally and theoretically in what sense a more confined spoke would offer minimum energy expenditure for the turbulent state. For the evaluation of these problems the effect of injection of well-ionized plasmas from the hollow cathode arc jet would obviously be of great interest.

The instability discussed by Lary, et al. (ref. 9) as previously noted, uses essentially the Landau damping of randomized oscillations to limit the drift velocity of the electrons in the azimuthal direction below the value

of E/B , without introducing a turbulence mechanism. The theory gives an azimuthal ion acoustic wave propagating approximately with the velocity of ion rotation, which is low for the present experiments, but it gives a qualitative explanation of the frequency shift. (No axial or spiral acoustic wave is included, which, however, should not be too difficult to do.) The theory, however, can not describe transition to turbulence across the magnetic field in the sense discussed here, since no added amplification as in convective instabilities (Morse) or through $j' \times B$ effects (Velikhov-Tamor) are considered. Finally, it should be noted that the effect of magnetic field variations near the electrode regions and of self-induced magnetic fields has been neglected and will be studied.

Mechanisms for Transition to Strong and Weak Turbulence at High and Low Densities

Since it was found that the spectrum of transition to turbulence for the present experiments for conduction in a cross-electric magnetic field configuration are related to that for diffusion for the Lehnert-Kadomtsev instability for axial electric magnetic fields, it is possible to discuss the transition to turbulence problems from a universal viewpoint. Comparison of recent results by Chen of crossed field and other experiments also suggest the universal character of the turbulence. Specific application to the present crossed field conduction requires certain modifications of the theories together with careful experiments to bring out the mechanisms of strong and weak turbulence and transition.

First the requirements for the production of strong turbulence are discussed. The existence of strong turbulence rests first of all on the condition that there are certain modes which are capable and have the time to grow to amplitudes which are of the order of the undisturbed or steady values. The fact that a certain growth has occurred does not yet mean that there exists a turbulent phenomenon, although even in the transition zone the diffusion or conduction process can be enhanced over the classical one. For the occurrence of strong turbulence itself the nonlinear effects connected with the production of large-amplitude disturbances have to produce disturbances of various sizes which can strongly interact with each other. These disturbances can be vortices of various scales or waves of various wavelengths. The vortex type of strong turbulence is the old hydrodynamic problem, the wave type of turbulence has recently received much attention as a fundamental problem of plasma physics in the United States and in Russia. Most studies on wave turbulence concern the so-called weak turbulence, where as stated, for example, in references 20 and 21 (some mathematical problems are discussed in greater depth in ref. 22) the growth or damping time of small oscillations considerably exceeds the time required for an appreciable change in phase shifts between the waves.¹ This weak turbulence allows for nonlinear processes of coalescence and decay of waves and the results of this interaction are represented by a solution of a Boltzmann

¹Note: while the basic problem is also treated in other papers, reference 20 has especially strong bearing here.

equation for waves rather than particles, whereby the molecular chaos used for particles appears to have its counterpart in the random phase approximation (used, for example, in ref. 11).² In reference 20 a strong wave interaction is also described referring to drift waves transverse to magnetic fields related to the convective Lehnert-Kadomtsev instability in axial electric and magnetic fields. It is stated that since all small drift perturbations have the same transverse phase velocity, perturbations of different wavelengths will retain their phase shifts for considerable times and the individual waves will interact fairly strongly. This makes it possible to regard such convection from the viewpoint of strong turbulence. This strong wave turbulence has similarities with the strong type of vortex turbulence, with some isotropy suppressed by the magnetic field, where neighboring vortices can interact with each other for a long time. The investigations of the high-frequency region of the spectrum and of the great variety of small-scale damping mechanisms available for plasmas are left to future studies of fully developed turbulence.

The difference between convective instabilities in high-density and low-density (rarefied) plasmas appears to be according to Kadomtsev (refs. 20 and 21) that at high densities the convective drift instabilities will yield strong vortex type of turbulence, whereas at low densities where the mean-free path is of the order or larger than the wavelength dimensions a drift wave type of strong turbulence could occur (note: other types of waves which may maintain their phase shifts for long times must also be considered).

The preservation of a spiral structure in the present case as the turbulent state is approached and the existence of correlations in preferred direction suggests that the turbulence develops while the main polarized instability structure remains more or less intact. The convective instability and transition mechanism discussed by Chen (ref. 24) actually for diffusion of a plasma in crossed electric and magnetic fields (for the Hoh-Simon instability including plasma rotation) also keeps the instability character intact. (Chen also indicates the universal character of the turbulence.) The references for turbulence discussed so far are concerned with diffusion rather than conduction processes; however, these two processes are evidently closely related. As a matter of fact, Hoh (ref. 18, p. 1187) in his discussion of a crossed field instability remarks that the perturbed azimuthal electric field, which together with the magnetic field causes the plasma to drift towards one electrode, enables the electrons to gain energy in the electric field at a much faster rate than collisional conduction. For strong turbulence, however, the conduction can no longer quantitatively be described by the process discussed in reference 11 although the process is still correct in principle.

Before discussing the effect at very low densities it should be noted that Velikhov (ref. 15) in investigating a turbulent conduction in a Hall current generator (which should also apply to accelerators) at high densities deals with random acoustic oscillations coupled to current oscillations through the

²For a discussion of this and other basic problems, see also the work by Litvak (ref. 23) where collisionless shocks with weak wave interaction are treated.

generalized Ohm's law. The turbulence discussed is of the weak type and does not include vorticity. It appears likely, however, that the turbulence in a high-density Hall device should be of the vortex type and, furthermore, strong. The reason for the vorticity lies first of all in the fact that according to Tamor (ref. 16) even the instabilities are likely to include shear waves. But, even more on general principles (of which Tamor's results may be a specific case), vorticity is very readily produced in the presence of $\mathbf{j} \times \mathbf{B}$ forces since only under special circumstances will they have a potential, i.e., $\nabla \times (\mathbf{j} \times \mathbf{B}) = 0$, as, for example, discussed in reference 25. Concerning the meaning of the Velikhov turbulence for low densities, it must be remembered that the $\mathbf{j} \times \mathbf{B}$ forces amplify acoustic waves which without these forces would be amplified by ion wave instabilities alone. However, to obtain strong turbulence, sound waves or shear waves would have to be found again which can interact strongly, which probably again would require drift waves coupled with the acoustic waves or with shear waves (Tamor), which may be related to drift waves.

A discussion of the effect of very low densities on the transition to turbulence is best introduced by distinguishing between the effects of ion acoustic waves and drift instabilities on the turbulence. This is done for crossed electric and magnetic fields by Chen (ref. 24), where the effect of ion inertia and of ion acoustic waves is included. The conclusion is reached, however, that the turbulence is mostly related to the drift instability. Various arguments have been used to support this view. The principal reason used by several authors such as, for example, Kadomtsev, Rosenbluth (in a variety of sources), Chen, and I. Bernstein (see also subsequent discussion) seems to be that the drift or convective instability is a macroscopic, hydromagnetic one, which can grow over a finite distance and like convective instabilities in fluid mechanics can yield strong turbulence. The turbulent diffusive or conductive transport caused by ion acoustic waves is, however, based on microscopic processes using a characteristic dimension for transport properties like, for example, the cyclotron radius. Kadomtsev in references 20 and 21, and references there, makes further distinction between very weak turbulence where only wave particle interactions (connected with many authors such as Drummond, Pines, Velikhov, and Vedenov) can take place and weak turbulence where also weak wave-wave interaction can occur. The very weak turbulence is based on the ion-acoustic waves whose growth depends on (resonant) trapping of particles in waves. For the weak turbulence Kadomtsev (refs. 20 and 21) uses the wave-wave interaction of drift waves (note: drift waves are not the only ones, for example, Litvak (ref. 23) uses compressional Alfvén waves for weak "turbulence" in a shock). The drift waves, as previously stated, can also interact strongly with each other and yield strong turbulence. Much of the preceding material is developed for the special example of the turbulence resulting from the Lehnert-Kadomtsev instability; however, in view of the universal character of the transition it should also have application here. It is shown by Kadomtsev (refs. 20 and 21), the problem is also treated for a complementary range by Kuckes (ref. 26), that as the density decreases and the mean-free path becomes larger than the wavelength the drift waves lose in importance and the very weak type of wave-particle-type turbulence remains, based on ion-acoustic instabilities. The latter problem is discussed in some detail by Rosenbluth (ref. 27, pp. 23-24). Thus the mechanism described by Lary, et al. (ref. 9) which does not use turbulent conduction in the present sense but a quasi-turbulent modification of

classical collision mechanisms could perhaps emerge at the very low densities. At very low densities other instability mechanisms like those described by Knauer (ref. 28) may also become of interest.

Finally, it is of great fundamental and practical importance to determine if there exist plasmas of high degree of ionization and/or low density for which transition to strong turbulent effects does not occur.¹ The cesium plasma experiments started by Rynn and D'Angelo (now performed by many others) suggest such a possibility. Experiments with highly ionized argon as well as cesium plasmas injected by hollow cathode arc jets are in progress or in preparation to investigate these effects, which are of great fundamental as well as practical importance. Experiments by Pinsley et al. from United Aircraft (ref. 29) making use of cesium contact ionization have apparently not included oscillation studies and are limited to low ion concentrations. However, contact ionization may provide one way to study the very low-density instabilities and their possible reduction, for comparatively quiescent plasma injection.

Instabilities for Hall Accelerators Using Magnetic Nozzle Effects

In view of the recent rather dramatic performance with an axially symmetric Hall accelerator, including magnetic nozzle effects in reference 2 and at this laboratory, possible instability problems of such configurations will be reviewed briefly. First the steady-flow plasma problems are defined. This type of Hall accelerator consists in essence of a diverging magnetic nozzle or magnetic mirror, which is crossed by current from a cathode situated on the axis and an annular anode. The interpretation of this Hall current acceleration as an enhanced magnetic mirror or magnetic nozzle effect (where the combination Hall and magnetic nozzle effects depends on configuration and operating conditions) was expressed in a slightly different form on pages 404 and 410 of reference 30 where this type of accelerator was first discussed. This view was also expressed in reference 31 (see also Hess' discussion remark) where important advances were reported on the effect of the magnetic field on the current path and in reference 1. For high performance the combination of Hall current and magnetic nozzle effect as well as the ionization is part of the optimization. Experimental and theoretical studies in references 32 and 33 in the configurations of references 1 and 30 to 33 indicated the need of preionization. This effect, however, seems properly combined with the others in the configuration of reference 2 and also in one used at this laboratory.

It should be pointed out that equally dramatic performance has been obtained by Ducati at Giannini and R. John at AVCO, Wilmington, with configurations using no external magnetic fields but magnetic nozzles due to self-magnetic fields, which however, require operation at much higher currents. Although this type of propulsion device does not require external magnets it would not seem to offer an advantage over a lower current arc with a light superconducting magnet (because of the much heavier power supply) where less electrode cooling is required and where the magnetic field shaping is possible. (For simulation of reentry of the solar wind, of course, the external magnet is of no concern.)

¹A decay of the onset of instabilities is also possible under these conditions.

The use of self-magnetic nozzles with magnetic pressure gradients for plasma acceleration was first reported in connection with cathode jets for high-current high-pressure arcs (ref. (34), section 94). The use of magnetic pressure gradients for steady plasma propulsion between extended electrodes was proposed several years ago by researchers at Los Alamos and by Hess for acceleration away from electrodes (ref. 30, fig. 2). The latter configuration has some common aspects with that used by Ducati and John who, in addition, used recent optimization techniques for arc jets operating at very high currents.

Concerning possible instabilities it should be first noted that magnetic nozzles produced by self-magnetic fields as in pinches produce their own plasma instabilities, so do some plasma guns using self-magnetic fields. Recent experiments with hollow cathode plasma injection into applied magnetic mirror or nozzles by Morse (ref. 17) for pressures of the order of 4μ Hg have also shown instabilities. Oscillation studies in a Hall accelerator of the type given in references 32 to 34 have shown instabilities (ref. 34) for low-pressure low-power operation, which include the Hoh-Simon mechanism in addition to the instabilities discussed herein. Oscillation measurements at comparatively high pressures (around 10 mm Hg) and high powers (40 kw), however, have shown so far no recognizable instabilities. The experiments performed by Cann (ref. 2) using applied magnetic fields apparently did not show instabilities to judge from their high performance. Similar conclusions can be drawn from the high-performance experiments with self-magnetic nozzles by Ducati and John.

One is faced with the pleasant task to study the full reasons for the efficiency of these devices, with special emphasis on the effect of higher flow velocities-pressure, -power and -ionization on instabilities and turbulence.

APPENDIX

THE HOLLOW CATHODE ARC JET FOR PLASMA INJECTION

The device is shown in figure 8 and consists of a tubular tantalum cathode (1/4-inch O.D. \times 0.020-inch wall \times 3 inch long) and a porous carbon ring anode ($1\frac{3}{4}$ -inch I.D. \times 1/2 inch long) whose plane is about 1/2 inch ahead of the cathode tip. The working fluid, argon, is injected both through the cathode tube and through the anode wall. (This latter gas source serves to reduce anode heating and arc spots.)

The electrode region is in a diverging magnetic field of about 250 gauss (at the anode), so that some acceleration of the plasma (using Hall currents interacting with the magnetic field) is achieved, as has been described elsewhere (ref. 1). The magnetic field also serves to confine the discharge to the forward tip of, or inside the cathode, thus reducing electrode erosion; when operating in a hollow cathode mode, the arc is generated at a plasma cathode, which is stable. Whereas conventional arc spots would tend to move on the cathode surface in the magnetic field, the stable plasma cathode represents a lower energy mode. The present device is distinguished from previous hollow cathode arcs investigated by us (ref. 35) in that the low-current—low-temperature mode has been bypassed and has entered the regime investigated by Lidsky, et al. (ref. 36).

A calorimetric study of the electrode cooling water indicates an arc efficiency of 62.6 percent at 2.2 kw (20 amps, 110 volts) and an efficiency of 46.5 percent at 3.2 kw (40 amps, 80 volts). Estimating the heat lost to the arc chamber walls as 572 watts (based on electrode temperature measurements, but neglecting back radiation from the plasma) and neglecting heat lost in vaporization of electrodes gives an overall arc jet efficiency of 36 percent at 2.2 kw and 28.6 percent at 3.2 kw. Both of these runs were made with a mass flow of 1.1 milligrams/sec of argon.

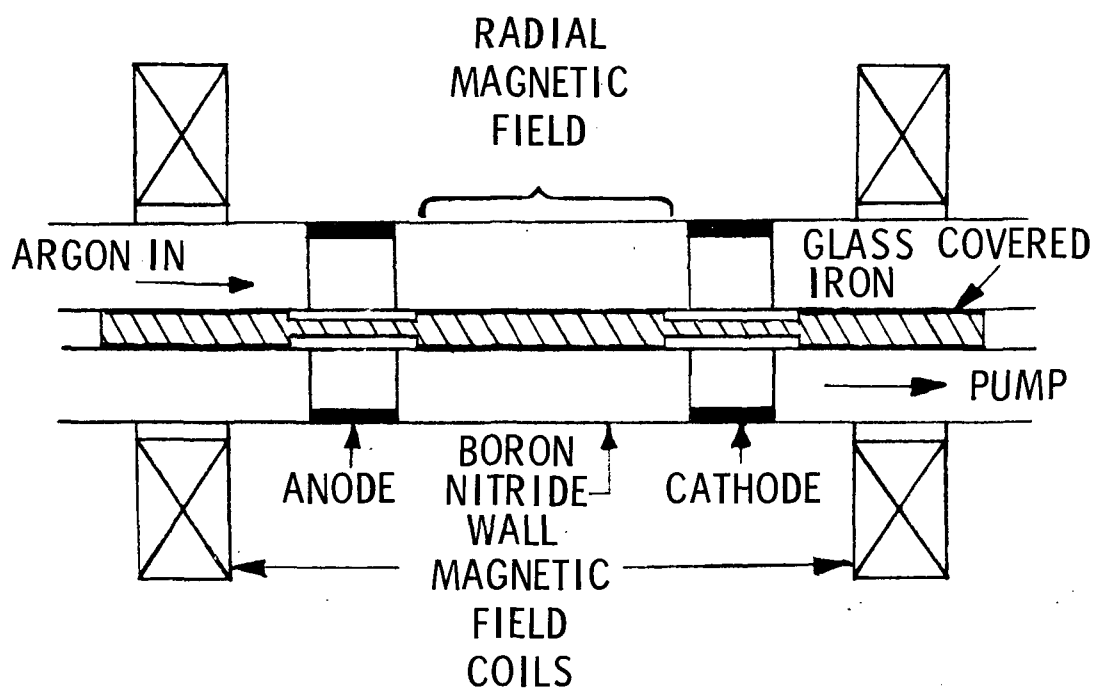
The present experiments are only preliminary and represent a far from ideal situation, due to excessive losses from the axial plasma beam impinging on the center body of the accelerator. A second, more compact arc jet, of slightly different geometry (fig. 9) will permit annular injection of plasma. In this configuration, both anode and cathode are axial, and the system is entirely closed, permitting more precise control of gas parameters; in particular, supersonic flow has been observed in a jet of this type operating in the 10 mm Hg range. Successful operation down to pressures of a few microns Hg has been achieved. Experiments are planned for a four-jet injection device (having a common anode with the Hall accelerator to reduce injection losses) which will provide a ring of plasma for the Hall accelerator. Operation with a variety of alkali metal vapors is also being studied.

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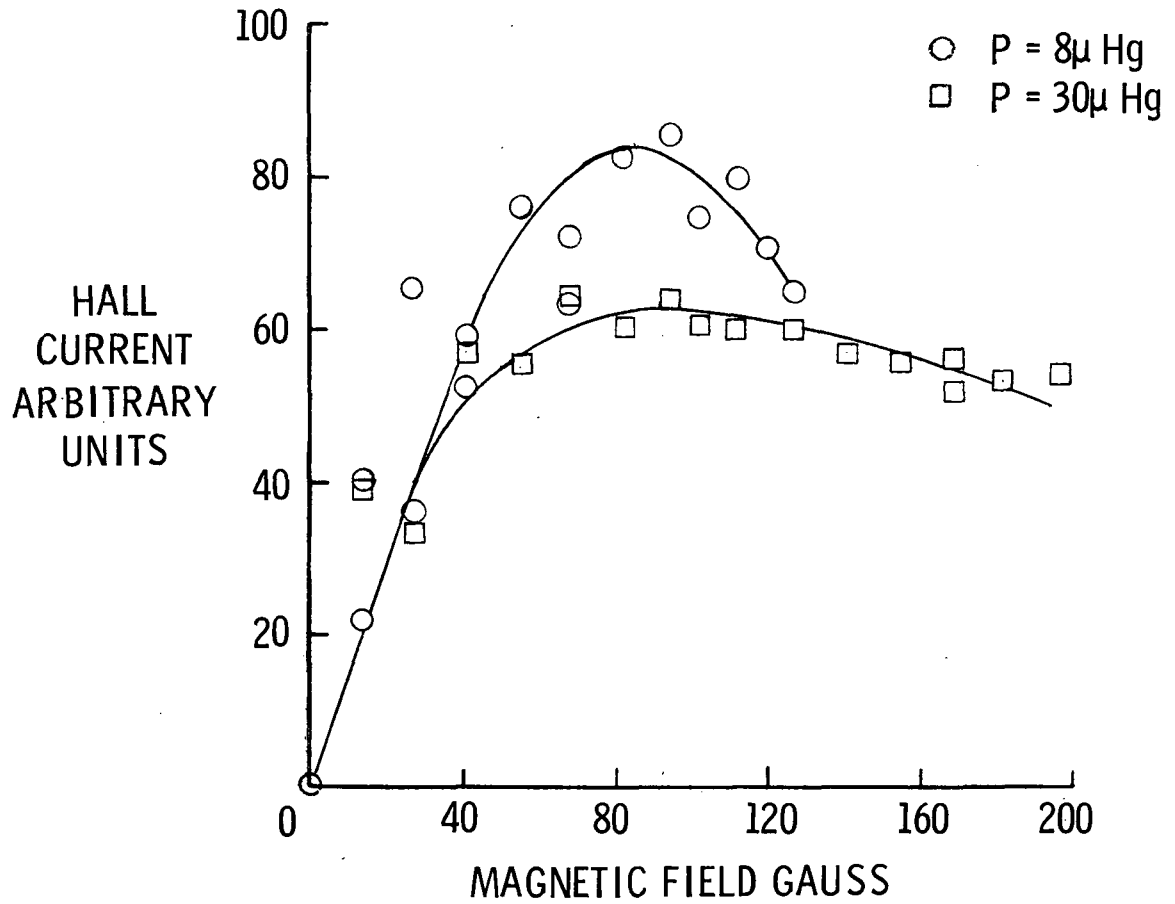
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Figure 1.- Linear Hall current accelerator.

$I_x = 20$ AMPERES



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Figure 2.- Hall current versus magnetic field.

$I_x = 20$ AMPERES

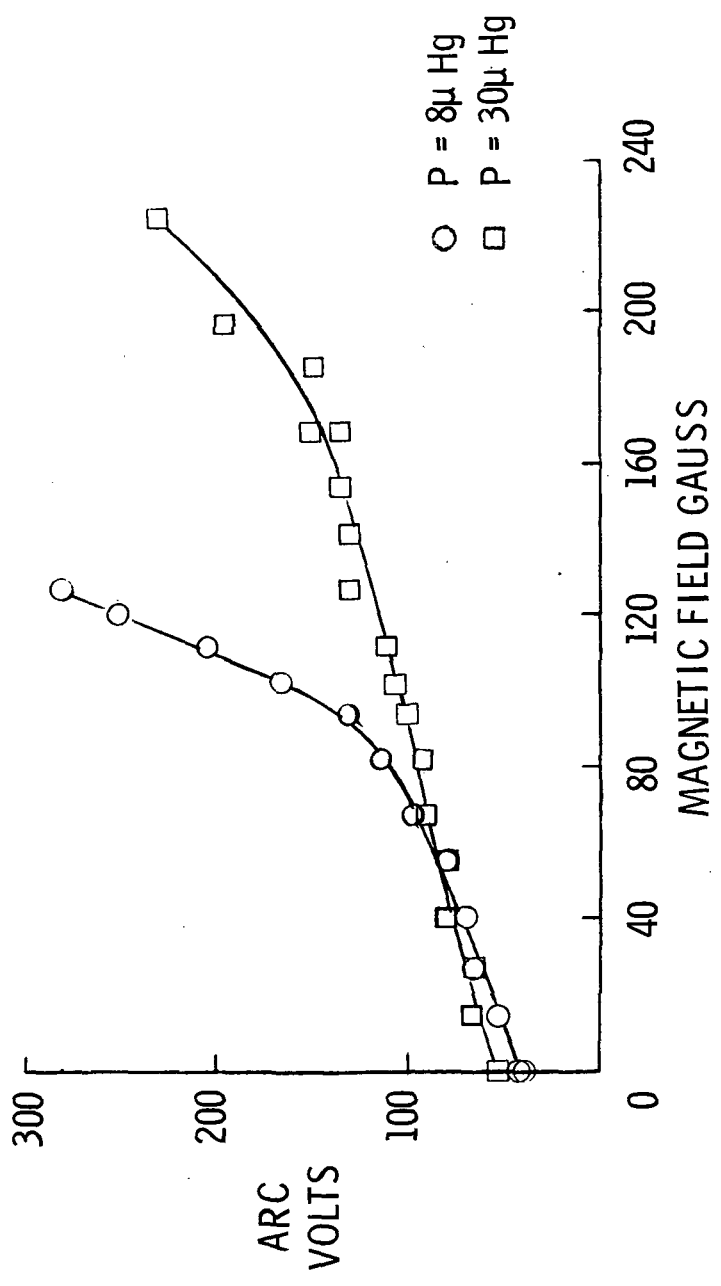
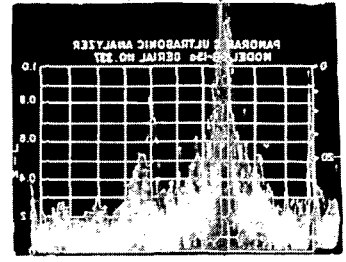


Figure 3.-- Arc voltage versus magnetic field.

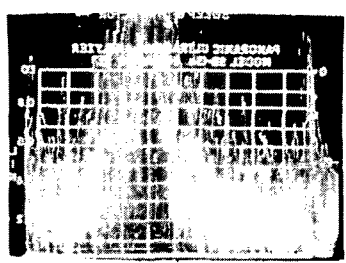
$P = 30\mu$
 $I_x = 20$ AMPERES



B = 14 GAUSS



B = 55 GAUSS



B = 140 GAUSS

100 50 0
Kc

NASA

Figure 4.- Frequency spectrum of voltage oscillation.