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SUMMARY

Expressions for nondimensional stress resultants and displacements are derived for multicell tanks of orthotropic construction loaded by internal pressure and axial compression, where the net axial load is assumed to be below the buckling value. The present analysis removes an incompatibility appearing in a previous solution for isotropic tanks. For isotropic tanks the axial stresses obtained from the present analysis differ significantly from previous results; however, all the other stresses are in complete agreement. The corrected axial stresses are presented in the form of curves.

INTRODUCTION

Multicell pressure vessels may find application as fuel and oxidizer tanks in large launch vehicles. This application represents a departure from the conventional circular-cylindrical-shell pressure vessels commonly used in such applications at the present time. A sketch of a cross section of an eight-lobe multicell tank composed of thin-walled partial circular cylindrical shells and radial webs is shown in figure 1. In general, the partial cylinders and radial webs may be of unstiffened, stiffened, or sandwich construction.

A primary motivation for considering the multicell tank is the efficient structural use of the radial webs. Whereas the separate slosh baffles required in conventional cylindrical tanks constitute largely parasitic weight, the radial webs in a multicell tank reduce fuel sloshing while providing additional structural strength. Preliminary studies (refs. 1 and 2) indicate that the use of multicell tanks in large launch vehicles can provide a significant weight saving over the use of conventional cylindrical tanks. This weight saving is due not only to the elimination of separate slosh baffles, but also to more shallow bulkheads resulting in lighter transition sections between tanks.

Expressions for the stresses and deformations in a pressurized multicell tank of isotropic construction are derived in reference 3. In that analysis, the resulting axial strains in the partial cylinders and radial webs are not compatible because the assumption is made that the axial stresses are uniformly distributed and equal in the partial cylinders and radial webs.

The purpose of the present paper is to present an analysis for multicell tanks of orthotropic construction loaded by internal pressure and axial compression. The multicell tank is assumed to be long enough so that a state of uniform strain exists over the cross section; thus, the strain incompatibility appearing in reference 3 does not appear in the present analysis. With linear cylindrical-shell theory used as the basic mathematical tool, nondimensional expressions for stress resultants and displacements are derived, and results obtained are compared with results from reference 3 for isotropic tanks.

SYMBOLS

a	radius of partial cylinders
$a_{11},a_{12},\ldots$	coefficients in equation (32)
h,h_w	thickness of homogeneous partial cylinders and radial webs, respectively
$k = \dfrac{D_\phi}{\left(a^2K_\phi\right)}$	
p	uniform internal pressure
v,w	displacement in ϕ-direction and normal displacement (positive inward), respectively, of the partial cylinders
x	axial coordinate
A_1,A_2,A_3	$\dfrac{K_\phi}{pa^2} C_1$, $\dfrac{K_\phi}{pa^2} C_2$, and $\dfrac{K_\phi}{pa} \epsilon_x$, respectively
B_1,B_2,B_3	constants in equation (32)
C_1,C_2	constants of integration in equations (22) and (23)
D_ϕ	hoop bending stiffness of partial cylinders
E_ϕ,E_x	elastic modulus of partial cylinders in hoop and axial directions, respectively
$E_{w,r},E_{w,x}$	elastic modulus of radial webs in radial and axial directions, respectively

$$K_{\phi} = \frac{E_{\phi} h}{\left(1 - \nu_x \nu_{\phi}\right)}$$

$$K_x = \frac{E_x h}{\left(1 - \nu_x \nu_{\phi}\right)}$$

$$K_{w,r} = \frac{E_{w,r} h_w}{\left(1 - \nu_{w,x} \nu_{w,r}\right)}$$

$$K_{w,x} = \frac{E_{w,x} h_w}{\left(1 - \nu_{w,x} \nu_{w,r}\right)}$$

$M_{\phi}, N_{\phi}, N_x, M_x$	hoop bending-moment, hoop membrane-stress, axial membrane-stress, and axial bending-moment resultants, respectively, in partial cylinders
$N_{w,r}, N_{w,x}$	stress resultant in radial webs in radial and axial directions, respectively
P_x	external axial compressive load
Q_{ϕ}	hoop transverse shear stress resultant in partial cylinders
α	half angle between radial webs
ϵ_{ϕ}	hoop strain in partial cylinders
$\epsilon_{w,r}$	radial strain in radial webs
ϵ_x	constant axial strain
θ	half arc angle of partial cylinders
ν_x, ν_{ϕ}	Poisson's ratios for partial cylinders associated with axial stress and hoop stress, respectively
$\nu_{w,x}, \nu_{w,r}$	Poisson's ratios for radial webs associated with axial stress and radial stress, respectively
ν	Poisson's ratio for isotropic tanks

ϕ hoop coordinate

χ_ϕ hoop-curvature change in partial cylinders

ANALYSIS

One lobe of the multicell tank studied is shown in figure 1. The thin-walled partial circular cylindrical shells and radial webs are of orthotropic

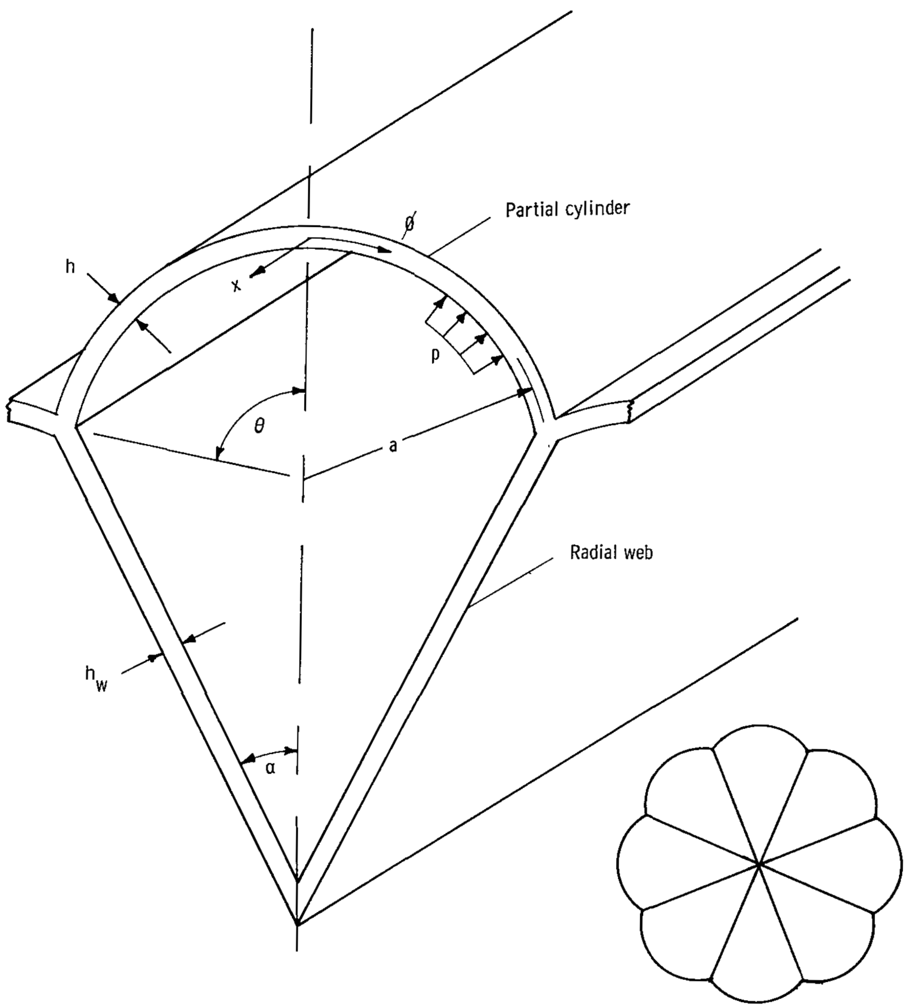

Figure 1.- One lobe of multicell tank studied and sketch of cross section of eight-lobe tank.

construction. Each lobe is loaded by uniform internal pressure and axial compression, where the net axial load is assumed to be below the buckling value.

The multicell tank is assumed long enough so that a state of uniform axial strain exists over the cross section. Then, the axial strain ϵ_x is constant, and the stress resultants acting on elements of the partial cylinders and radial webs do not vary in the axial direction. These stress and moment resultants are indicated on cylinder and web elements in figure 2.

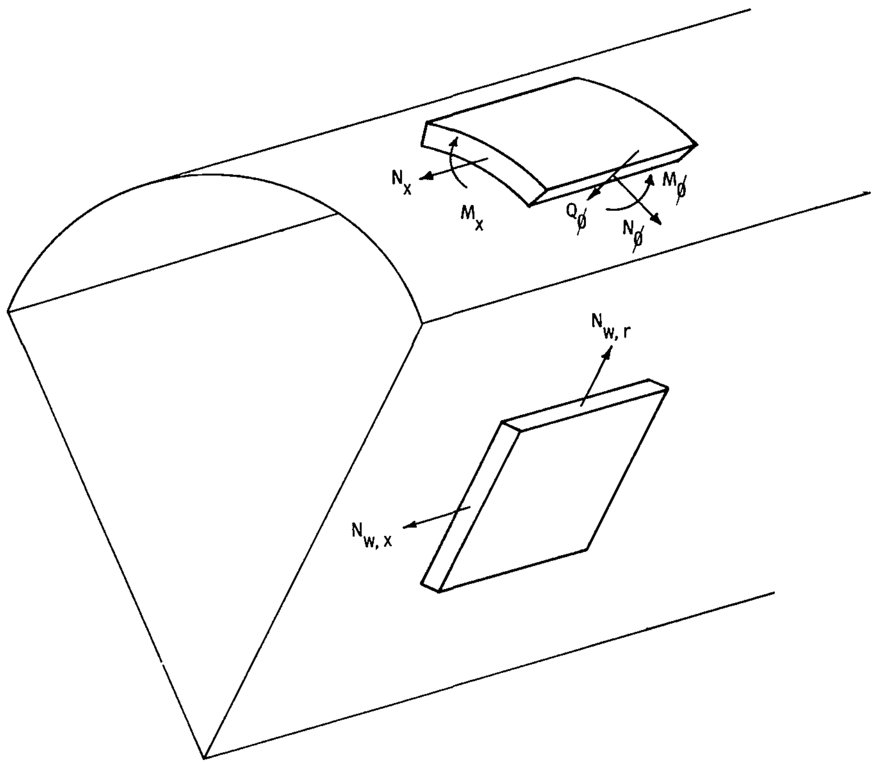

Figure 2.- Stress resultants acting on elements of partial cylinders and radial webs.

Equilibrium Equations

For constant axial strain, equilibrium equations for the partial cylinders are (see, for example, ref. 4):

Force equilibrium in the ϕ-direction:

$$\frac{dN_\phi}{d\phi} - Q_\phi = 0 \tag{1}$$

Force equilibrium in the normal direction:

$$\frac{dQ_\phi}{d\phi} + N_\phi - pa = 0 \tag{2}$$

Moment equilibrium:

$$\frac{dM_\phi}{d\phi} - aQ_\phi = 0 \tag{3}$$

Two additional equilibrium conditions are required for the radial stress resultants in the webs and the axial stress resultants. Force equilibrium at the junction of the partial cylinders and radial webs requires that

$$N_{w,r} = \left[2N_\phi \sin(\theta - \alpha) + 2Q_\phi \cos(\theta - \alpha)\right]_{\phi=\theta} \tag{4}$$

Overall force equilibrium in the axial direction gives

$$2a \int_0^\theta N_x \, d\phi + \frac{a \sin \theta}{\sin \alpha} N_{w,x} = pa^2 \left[\frac{\sin(\theta - \alpha)\sin \theta}{\sin \alpha} + \theta\right] - \frac{\alpha}{\pi} P_x \tag{5}$$

Stress-Resultant-Strain, Strain-Displacement,

and Reciprocal Equations

The homogeneous orthotropic stress-resultant-strain equations for the partial cylinders are:

$$N_\phi = K_\phi\left(\epsilon_\phi + \nu_x \epsilon_x\right) \tag{6}$$

$$N_x = K_x\left(\epsilon_x + \nu_\phi \epsilon_\phi\right) \tag{7}$$

and the moment-curvature equation is:

$$M_\phi = -D_\phi \chi_\phi \tag{8}$$

Similar expressions for the radial webs are:

$$N_{w,r} = K_{w,r}\left(\epsilon_{w,r} + \nu_{w,x} \epsilon_x\right) \tag{9}$$

and

$$N_{w,x} = K_{w,x}\left(\epsilon_x + \nu_{w,r}\epsilon_{w,r}\right) \tag{10}$$

When displacements are assumed to be appropriately small, the strain-displacement equations for the partial cylinders are (see, for example, ref. 4):

Hoop strain displacement

$$\epsilon_\phi = \frac{1}{a}\left(\frac{dv}{d\phi} - w\right) \tag{11}$$

Curvature displacement

$$\chi_\phi = \frac{1}{a^2}\left(\frac{dv}{d\phi} + \frac{d^2w}{d\phi^2}\right) \tag{12}$$

In addition to the preceding equations, the following reciprocal relationships must hold among the elastic constants as a result of assuming orthotropic construction:

$$\frac{\nu_x}{\nu_\phi} = \frac{K_x}{K_\phi} \tag{13}$$

$$\frac{\nu_{w,x}}{\nu_{w,r}} = \frac{K_{w,x}}{K_{w,r}} \tag{14}$$

Boundary and Symmetry Conditions

Three boundary conditions are to be imposed on the displacements in the partial cylinders at the junctions with the radial webs: (1) the rotation of a normal to the middle surface of the cylinder must be zero, that is, the cylinder is clamped, (2) the resultant displacement must be in the plane of the radial webs, and (3) the resultant displacement must equal the constant radial displacement of the webs. These conditions are expressed mathematically as follows:

$$\left(\frac{dw}{d\phi} + v\right)_{\phi=\theta} = 0 \tag{15}$$

$$\left[v \cos(\theta - \alpha) - w \sin(\theta - \alpha)\right]_{\phi=\theta} = 0 \tag{16}$$

$$\frac{a \sin \theta}{\sin \alpha} \epsilon_{w,r} = -\left[\frac{w}{\cos(\theta - \alpha)}\right]_{\phi=\theta} \tag{17}$$

where, equation (16) has been used to simplify equation (17). In addition, the symmetry of the problem is expressed by:

$$w(\phi) = w(-\phi) \tag{18}$$

$$v(\phi) = -v(-\phi) \tag{19}$$

Solution of Equations

Equations (1) to (12) constitute a system of twelve equations in twelve unknowns, that is, two displacements, v and w; six stress resultants, N_ϕ, N_x, M_ϕ, Q_ϕ, $N_{w,r}$, and $N_{w,x}$; and four strains, ϵ_x, ϵ_ϕ, $\epsilon_{w,r}$, and χ_ϕ. Using equation (3) to eliminate Q_ϕ from equations (1) and (2) and expanding the resulting equations for N_ϕ and M_ϕ in terms of v and w by using equations (6), (8), (11), and (12) gives the following sixth-order set of differential equations:

$$(1 + k)\frac{d^2v}{d\phi^2} + k \frac{d^3w}{d\phi^3} - \frac{dw}{d\phi} = 0 \tag{20}$$

$$k \frac{d^3v}{d\phi^3} - \frac{dv}{d\phi} + k \frac{d^4w}{d\phi^4} + w = a\nu_x\epsilon_x - \frac{pa^2}{K_\phi} \tag{21}$$

where

$$k = \frac{D_\phi}{a^2K_\phi}$$

Equations (20) and (21) are integrated to give:

$$v = C_1\left(\frac{2}{1 + k} \frac{\phi \sin \theta}{\theta} + \phi \cos \phi\right) + C_2 \sin \phi \tag{22}$$

$$w = C_1\left(\frac{2}{1 + k} \frac{\sin \theta}{\theta} + \frac{1 - k}{1 + k} \cos \phi - \phi \sin \phi\right) + C_2 \cos \phi - \frac{pa^2}{K_\phi} + a\nu_x\epsilon_x \tag{23}$$

where the conditions of symmetry given by equations (18) and (19) and the clamped condition given by equation (15) have been imposed on the displacements.

The constants C_1 and C_2 and the axial strain ϵ_x are determined by utilizing the boundary conditions (eqs. (16) and (17)) and the axial equilibrium condition (eq. (5)). In order to accomplish this task, it is convenient first to write the expressions for the six stress resultants in terms of the solution (eqs. (22) and (23)). The stress resultants can be expressed in terms of C_1, C_2, and ϵ_x by using the strain-displacement equations (eqs. (11) and (12)), the stress-strain equations (eqs. (6) to (10)), the reciprocal equations (eqs. (13) and (14)), the first differential equation of equilibrium (eq. (1)), and the junction equilibrium equation (eq. (4)). When the nondimensional constants $A_1 = \frac{K_\phi}{pa^2} C_1$, $A_2 = \frac{K_\phi}{pa^2} C_2$, and $A_3 = \frac{K_\phi}{pa} \epsilon_x$ are introduced, the nondimensional displacement and stress resultants are:

$$\frac{K_\phi}{pa^2} v = A_1\left(\frac{2}{1 + k} \frac{\phi \sin \theta}{\theta} + \phi \cos \phi\right) + A_2 \sin \phi \tag{24}$$

$$\frac{K_\phi}{pa^2} w = A_1\left(\frac{2}{1 + k} \frac{\sin \theta}{\theta} + \frac{1 - k}{1 + k} \cos \phi - \phi \sin \phi\right) + A_2 \cos \phi + \nu_x A_3 - 1 \tag{25}$$

$$\frac{N_\phi}{pa} = 1 + \frac{2k}{1 + k} A_1 \cos \phi \tag{26}$$

$$\frac{M_\phi}{pa^2} = \frac{2k}{1 + k}\left(\cos \phi - \frac{\sin \theta}{\theta}\right)A_1 \tag{27}$$

$$\frac{Q_\phi}{pa} = -\frac{2k}{1 + k} A_1 \sin \phi \tag{28}$$

$$\frac{N_{w,r}}{pa} = 2 \sin(\theta - \alpha) - \frac{4k}{1 + k} A_1 \sin \alpha \tag{29}$$

$$\frac{N_x}{pa} = \nu_x\left(1 + \frac{2k}{1 + k} A_1 \cos \phi\right) + \frac{K_x}{K_\phi}\left(1 - \nu_\phi \nu_x\right)A_3 \tag{30}$$

$$\frac{N_{w,x}}{pa} = \nu_{w,x}\left[2 \sin(\theta - \alpha) - \frac{4k}{1 + k} A_1 \sin \alpha\right] + \frac{K_{w,x}}{K_\phi}\left(1 - \nu_{w,r}\nu_{w,x}\right)A_3 \tag{31}$$

Evaluation of Remaining Constants

The constants A_1, A_2, and A_3 are determined by using the axial equilibrium condition given by equation (5), and the two remaining boundary conditions given by equations (16) and (17). Expanding these three equations in terms of A_1, A_2, and A_3 gives the following set of algebraic equations:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} \tag{32}$$

where

$$a_{11} = \frac{4k}{1 + k}\left(\nu_x - \nu_{w,x}\right)\sin \theta$$

$$a_{12} = 0$$

$$a_{13} = \left(1 - \nu_{w,x}\nu_{w,r}\right)\frac{K_{w,x}}{K_\phi}\frac{\sin \theta}{\sin \alpha} + 2\left(1 - \nu_x\nu_\phi\right)\frac{K_x}{K_\phi}\theta$$

$$a_{21} = \frac{2}{1 + k} \sin \theta + \theta \cos \theta - \tan(\theta - \alpha)\left(\frac{2}{1 + k} \frac{\sin \theta}{\theta} + \frac{1 - k}{1 + k} \cos \theta - \theta \sin \theta\right)$$

$$a_{22} = \frac{\sin \alpha}{\cos(\theta - \alpha)}$$

$$a_{23} = -\nu_x \tan(\theta - \alpha)$$

$$a_{31} = \frac{\sin \alpha}{\cos(\theta - \alpha)}\left[\frac{2}{(1 + k)\theta} + \frac{1 - k}{1 + k} \cot \theta - \theta\right] - \frac{4k}{1 + k} \frac{K_\phi}{K_{w,r}} \sin \alpha$$

$$a_{32} = \frac{\sin \alpha}{\tan \theta \cos(\theta - \alpha)}$$

$$a_{33} = \nu_x \frac{\sin \alpha}{\sin \theta \cos(\theta - \alpha)} - \nu_{w,x}$$

$$B_1 = \left(1 - 2\nu_{w,x}\right)\frac{\sin \theta \sin(\theta - \alpha)}{\sin \alpha} + \left(1 - 2\nu_x\right)\theta - \frac{\alpha}{\pi} \frac{P_x}{pa^2}$$

$$B_2 = -\tan(\theta - \alpha)$$

$$B_3 = \frac{\sin \alpha}{\sin \theta \cos(\theta - \alpha)} - 2 \frac{K_\phi}{K_{w,r}} \sin(\theta - \alpha)$$

Solving equation (32) for A_1, A_2, and A_3 successively yields,

$$A_1 = \frac{\left(\nu_x - \nu_{w,x}\right)\dfrac{\left(1 - 2\nu_{w,x}\right)\dfrac{\sin \theta \sin(\theta - \alpha)}{\sin \alpha} + \left(1 - 2\nu_x\right)\theta - \dfrac{\alpha}{\pi} \dfrac{P_x}{pa^2}}{\left(1 - \nu_{w,x}\nu_{w,r}\right)\dfrac{K_{w,x}}{K_\phi} \dfrac{\sin \theta}{\sin \alpha} + 2\left(1 - \nu_x\nu_\phi\right)\dfrac{K_x}{K_\phi} \theta} + 2 \dfrac{K_\phi}{K_{w,r}} \sin(\theta - \alpha) - 1}{\dfrac{\dfrac{4k}{1 + k}\left(\nu_x - \nu_{w,x}\right)^2 \sin \theta}{\left(1 - \nu_{w,x}\nu_{w,r}\right)\dfrac{K_{w,x}}{K_\phi} \dfrac{\sin \theta}{\sin \alpha} + 2\left(1 - \nu_x\nu_\phi\right)\dfrac{K_x}{K_\phi} \theta} + \cos \theta + \dfrac{\theta}{\sin \theta} - \dfrac{2}{1 + k} \dfrac{\sin \theta}{\theta} + \dfrac{4k}{1 + k} \dfrac{K_\phi}{K_{w,r}} \sin \alpha} \tag{33}$$

$$A_2 = \frac{\nu_x A_3 \sin(\theta - \alpha) + \left[\left(\frac{2}{1 + k}\frac{\sin\theta}{\theta} + \frac{1 - k}{1 + k}\cos\theta - \theta\sin\theta\right)\sin(\theta - \alpha) - \left(\frac{2}{1 + k}\sin\theta + \theta\cos\theta\right)\cos(\theta - \alpha)\right]A_1 - \sin(\theta - \alpha)}{\sin\alpha} \tag{34}$$

$$A_3 = \frac{\left(1 - 2\nu_{w,x}\right)\frac{\sin\theta\sin(\theta - \alpha)}{\sin\alpha} + \left(1 - 2\nu_x\right)\theta - \frac{\alpha}{\pi}\frac{P_x}{pa^2} - \frac{4k}{1 + k}\left(\nu_x - \nu_{w,x}\right)A_1\sin\theta}{\left(1 - \nu_{w,x}\nu_{w,r}\right)\frac{K_{w,x}}{K_\phi}\frac{\sin\theta}{\sin\alpha} + 2\left(1 - \nu_x\nu_\phi\right)\frac{K_x}{K_\phi}\theta} \tag{35}$$

For a given orthotropic tank, the displacements and stress resultants due to internal pressure and axial compression can now be determined from equations (24) to (31) and (33) to (35). Note that all of the stress resultants except the axial stress resultants contain only the constant A_1. Therefore, it is convenient to use the expression for A_3 (eq. (35)), to reformulate the axial stress resultants as follows:

$$\frac{N_x}{pa} = \nu_x\left(1 + \frac{2k}{1 + k}A_1\cos\phi\right) + \frac{\left(1 - 2\nu_{w,x}\right)\frac{\sin\theta\sin(\theta - \alpha)}{\sin\alpha} + \left(1 - 2\nu_x\right)\theta - \frac{\alpha}{\pi}\frac{P_x}{pa^2} - \frac{4k}{1 + k}\left(\nu_x - \nu_{w,x}\right)A_1\sin\theta}{\frac{E_{w,x}h_w}{E_x h}\frac{\sin\theta}{\sin\alpha} + 2\theta} \tag{36}$$

$$\frac{N_{w,x}}{pa} = \nu_{w,x}\left[2\sin(\theta - \alpha) - \frac{4k}{1 + k}A_1\sin\alpha\right] + \frac{\left(1 - 2\nu_{w,x}\right)\frac{\sin\theta\sin(\theta - \alpha)}{\sin\alpha} + \left(1 - 2\nu_x\right)\theta - \frac{\alpha}{\pi}\frac{P_x}{pa^2} - \frac{4k}{1 + k}\left(\nu_x - \nu_{w,x}\right)A_1\sin\theta}{\frac{\sin\theta}{\sin\alpha} + 2\frac{E_x h}{E_{w,x}h_w}\theta} \tag{37}$$

Now, all of the stress resultants can be determined for a given tank by using equations (26) to (29) for N_ϕ, M_ϕ, Q_ϕ, and $N_{w,r}$, and equations (36) and (37) for N_x and $N_{w,x}$, where A_1 is given by equation (33).

A specific case of interest is the homogeneous, isotropic tank. For this case it is convenient to write the stress resultants in terms of $\overline{A}_1 = \frac{2k}{1 + k}A_1$, where k reduces to $\frac{h^2}{12a^2}$, to give

$$\frac{N_\phi}{pa} = 1 + \overline{A}_1\cos\phi \tag{38}$$

$$\frac{M_\phi}{pa^2} = \left(\cos\phi - \frac{\sin\theta}{\theta}\right)\overline{A}_1 \tag{39}$$

$$\frac{Q_\phi}{pa} = -\bar{A}_1 \sin \phi \tag{40}$$

$$\frac{N_{w,r}}{pa} = 2 \sin(\theta - \alpha) - 2\bar{A}_1 \sin \alpha \tag{41}$$

$$\frac{N_x}{pa} = \nu(1 + \bar{A}_1 \cos \phi) + \frac{(1 - 2\nu)\left[\frac{\sin \theta \sin(\theta - \alpha)}{\sin \alpha} + \theta\right] - \frac{\alpha}{\pi}\frac{P_x}{pa^2}}{\frac{h_w}{h}\frac{\sin \theta}{\sin \alpha} + 2\theta} \tag{42}$$

$$\frac{N_{w,x}}{pa} = 2\nu\left[\sin(\theta - \alpha) - \bar{A}_1 \sin \alpha\right] + \frac{(1 - 2\nu)\left[\frac{\sin \theta \sin(\theta - \alpha)}{\sin \alpha} + \theta\right] - \frac{\alpha}{\pi}\frac{P_x}{pa^2}}{\frac{\sin \theta}{\sin \alpha} + 2\frac{h}{h_w}\theta} \tag{43}$$

where

$$\bar{A}_1 = \frac{\sin(\theta - \alpha) - \frac{1}{2}\frac{h_w}{h}}{\sin \alpha + \frac{h_w}{4h}\left(\frac{\theta}{\sin \theta} + \cos \theta\right) + \frac{1}{4k}\frac{h_w}{h}\left(\frac{\theta}{\sin \theta} + \cos \theta - \frac{2 \sin \theta}{\theta}\right)} \tag{44}$$

For reasonable values of $k = \frac{h^2}{12a^2}$, $\bar{A}_1$ is small and normally may be neglected in the expressions for the membrane stresses N_ϕ, $N_{w,r}$, N_x, and $N_{w,x}$.

A comparison of the present solution with that from reference 3 shows that the axial stress resultants N_x and $N_{w,x}$ differ; however, the stress resultants N_ϕ, M_ϕ, Q_ϕ, and $N_{w,r}$ are in complete agreement.

The present nondimensional axial stress resultants $\frac{N_x}{pa}$ and $\frac{N_{w,x}}{pa}$ due to internal pressure alone are plotted against $\theta - \alpha$ in figures 3, 4, and 5 for an eight-lobe isotropic tank with $\nu = \frac{1}{3}$, and $\frac{h_w}{h} = 1$, $\frac{1}{2}$, and 2, respectively. The terms containing $\bar{A}_1$ are found to be negligible when computing the stress resultants from equations (42) and (43) for these cases.

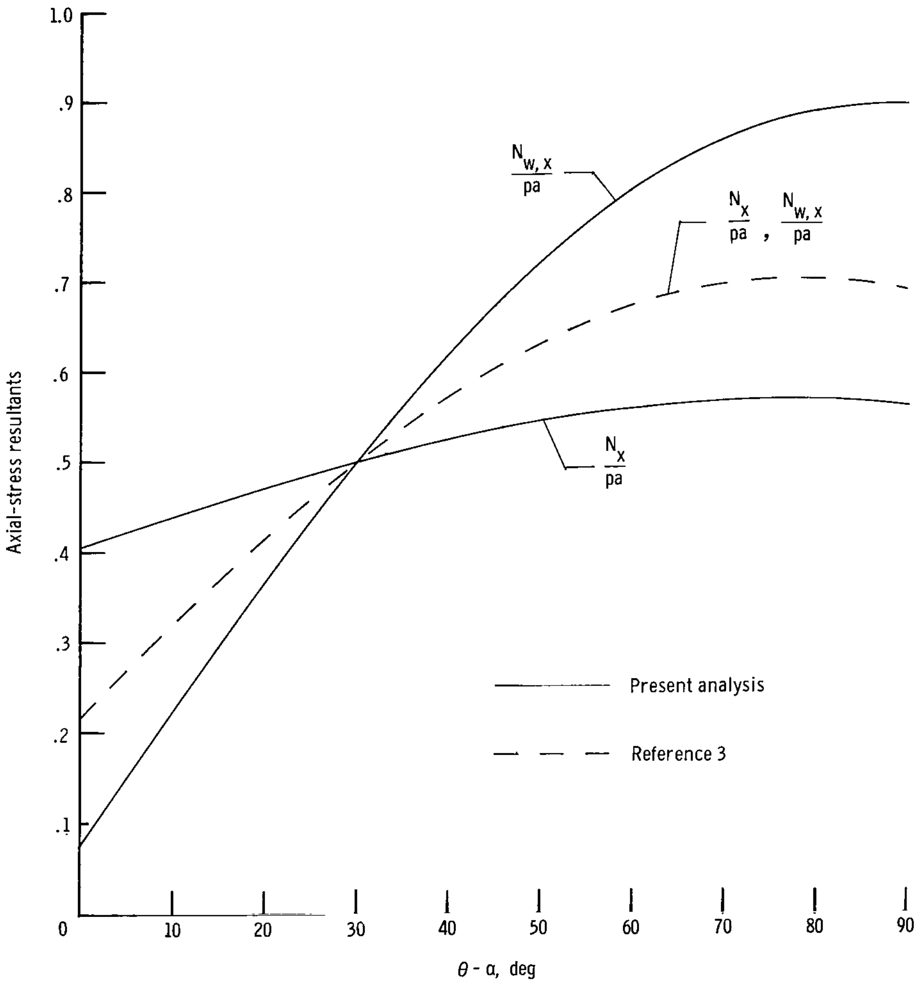

Figure 3.- Axial-stress resultants for a pressurized eight-lobe isotropic tank. $\frac{h_w}{h} = 1$.

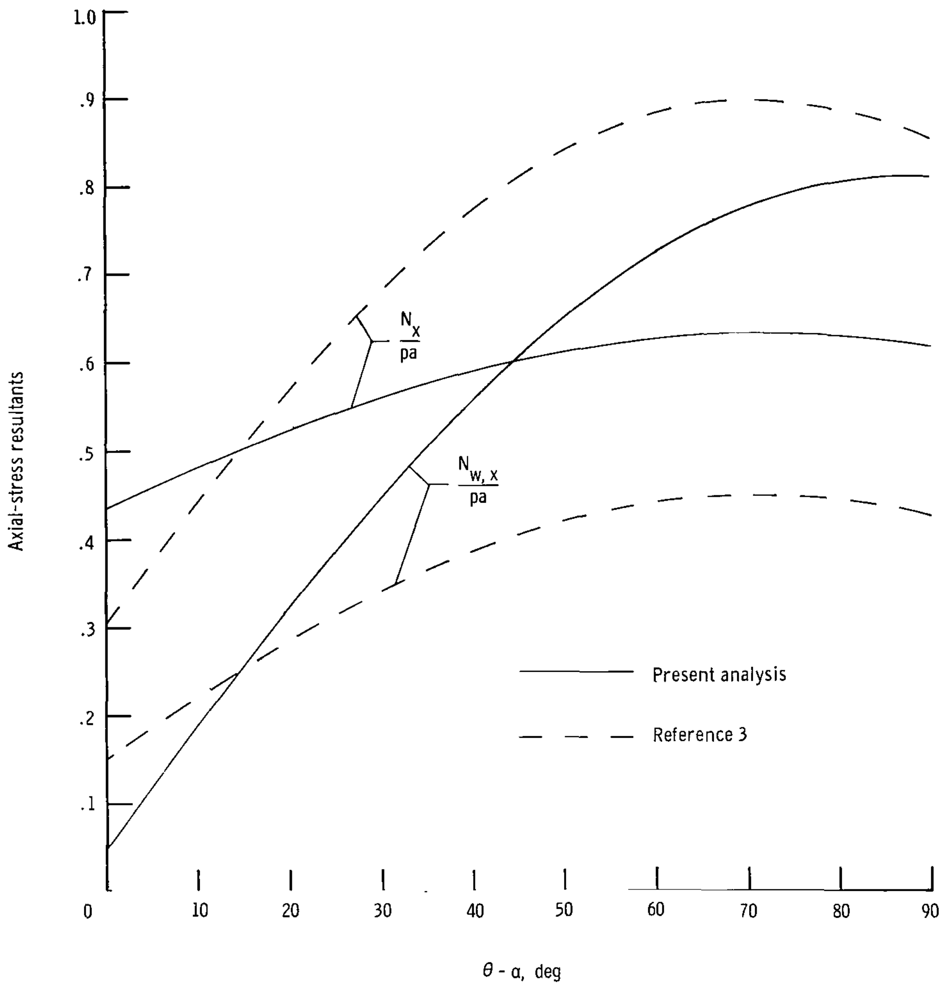

Figure 4.- Axial-stress resultants for a pressurized eight-lobe isotropic tank. $\frac{h_w}{h} = \frac{1}{2}$.

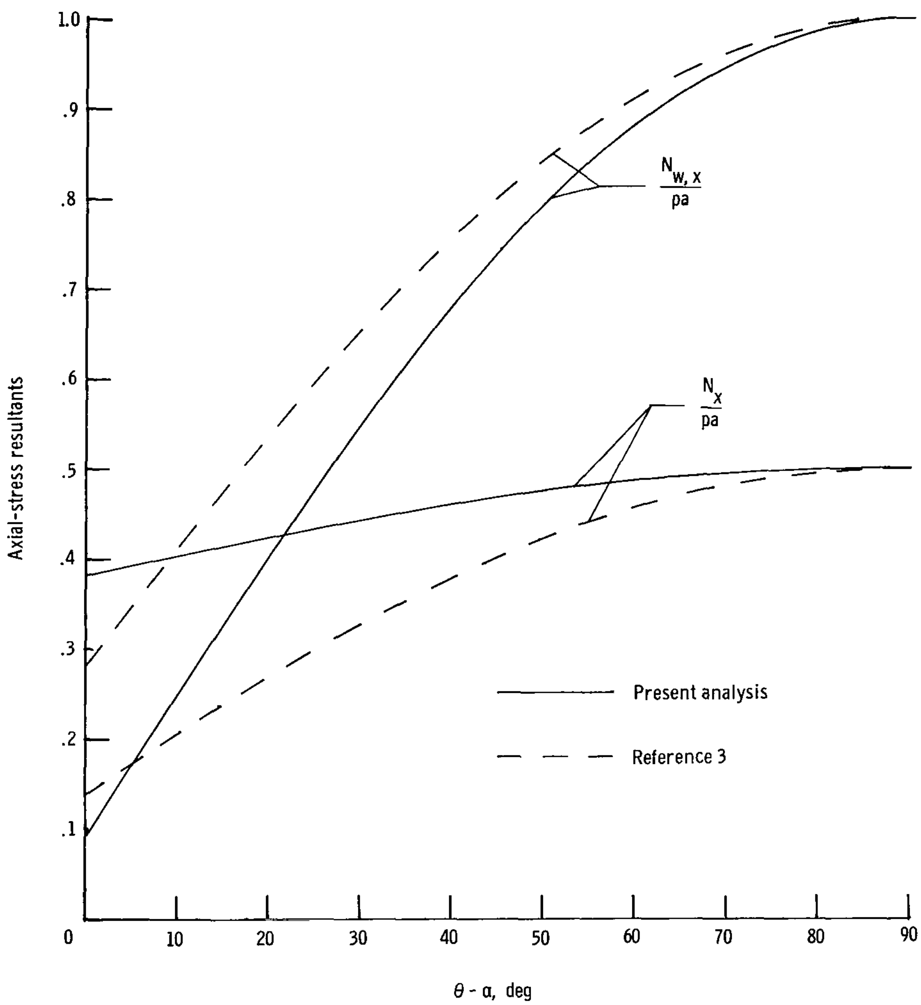

Figure 5.- Axial-stress resultants for a pressurized eight-lobe isotropic tank. $\frac{h_w}{h} = 2.$

RESULTS AND DISCUSSION

Expressions for nondimensional stress resultants and displacements are given for orthotropic tanks loaded by internal pressure and axial compression. Simplified expressions for the nondimensional stress resultants are also given for the case of isotropic construction. A comparison of the present results for isotropic construction with the results of reference 3 shows that the stress resultants, N_ϕ, M_ϕ, Q_ϕ, and $N_{w,r}$, are in agreement with the previous results. However, the axial stress resultants N_x and $N_{w,x}$, differ significantly from the results in reference 3.

In figures 3, 4, and 5 the present nondimensional axial stress resultants $\frac{N_x}{pa}$ and $\frac{N_{w,x}}{pa}$ due to internal pressure alone are plotted against $\theta - \alpha$ for an eight-lobe isotropic tank with $\nu = \frac{1}{3}$, $\frac{h_w}{h} = 1, \frac{1}{2}$, and 2, respectively. (It should be noted that $\theta - \alpha$ can not be greater than 90° due to geometric constraints, i.e., when $\theta - \alpha = 90°$, the partial cylinders are tangent to the radial webs at the junctions. When $\theta - \alpha = 0$, the multicell tank degenerates into a cylinder with radial webs.) For comparison, the axial stress resultants for the solution in reference 3, are also presented. Note that the nondimensional axial stress resultants from reference 3 imply that the axial stresses in the partial cylinders and radial webs are equal. When $\theta - \alpha = 0$ in figure 3, $\frac{N_x}{pa}$ from the present analysis is nearly twice the value given by reference 3, while $\frac{N_{w,x}}{pa}$ from the present analysis is less than half the value from reference 3. The cause of this large discrepancy in the axial stress resultants is easily recognized by comparing the expressions for the axial stress resultants (eqs. (42) and (43)) with the corresponding expression from reference 3 (eq. (38) in ref. 3). For $\nu = 0$ in equations (42) and (43) the axial stress resultants agree with the previous results. Therefore, the discrepancy between the present results and those from reference 3 is due to a Poisson's ratio effect which has been effectively neglected by the failure of the previous analysis to enforce compatibility of the axial strains.

CONCLUDING REMARKS

Expressions for nondimensional stress resultants and displacements are derived for multicell tanks of orthotropic construction loaded by internal pressure and axial compression. Simplified expressions for the nondimensional stress resultants are also given for the isotropic tank. The present analysis removes the axial-strain incompatibility appearing in a previous analysis for multicell tanks of isotropic construction. With this incompatibility removed, the axial stresses differ significantly from the previous results; however, all the other stresses are in complete agreement with the previous results.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., February 23, 1965.

REFERENCES

1. Blumrich, J. F.: Multi-Cell Structures Studied for Large Booster. Space/Aeron., vol. 39, no. 1, Jan. 1963, pp. 86-89.

2. Sterett, James B., Jr.: Shell Stability Problems in the Design of Large Space Vehicle Boosters. Collected Papers on Instability of Shell Structures - 1962. NASA TN D-1510, 1962, pp. 57-66.

3. Wilson, Paul E.; Dharmarajan, Sangiah; and Rogers, Phillip W.: Linear Bending Analysis of a Pressurized Multicell Shell Structure. GD/A 63-0968, ERR-AN-336, Gen. Dyn./Astronaut., Sept. 30, 1963.

4. Timoshenko, S.; and Woinowsky-Krieger, S.: Theory of Plates and Shells. Second ed., McGraw-Hill Book Co., Inc., 1959, pp. 512-513.

NASA-Langley, 1965 L-4135

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