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A COMPARISON OF COMPONENTS

by Lloyd Carpenter

Goddard Space Flight Center
Greenbelt, Md.

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SUMMARY

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The components of the planetary perturbations in rectangular coordinates are given as series expansions in the well-known Hansen components. The results provide formulas for transforming a general theory in one set of components into the corresponding general theory in the other set of components by simple series multiplications. The results suggest a useful method of improving the convergence of successive approximations when computing perturbations in the rectangular coordinates.

AUTHOR

COMPUTATION OF GENERAL PLANETARY PERTURBATIONS, PART II A COMPARISON OF COMPONENTS*

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Musen's method of planetary perturbations (Reference 1) bears the greatest resemblance to the method of Hansen (Reference 2). The relations between the perturbations computed from the two methods have been studied extensively by the author, since Hansen's method has been used in many cases. A comparison of the integrands shows that the first-order results should be identical, term by term. There is no need to give this comparison here, however, since the main interest is in the higher order differences. These higher order results reveal the small differences in the coefficients and suggest a modification which has been used to improve the convergence of successive approximations in Musen's method.

In Hansen's form, the rectangular coordinates referred to the mean plane of the orbit at the fundamental epoch are given by

$$\bar{x} = r \cos(f + \sigma) \cos b ,$$

$$\bar{y} = r \sin(f + \sigma) \cos b ,$$

$$\bar{z} = r \sin b ,$$

using the notation of Clemence (Reference 3, page 317). Here, σ represents the reduction of the orbital longitude from the osculating plane to the mean plane. The approximation

$$\sigma = \Gamma - \frac{1}{2} s \frac{ds}{dv}$$

has been used by Hill (Reference 4) and Clemence (Reference 3). The other quantities are defined in terms of the elliptic elements and the perturbation components by the following set of formulas:

$$g = g_0 + n \left(t - t_0\right) ,$$

*In Part I of this report, a description of a program for computing Hansen's planetary perturbations was given. Examples were included for six minor planets.

$$\bar{\epsilon} - e \sin \bar{\epsilon} = g + n\delta z ,$$

$$\bar{r} = a(1 - e \cos \bar{\epsilon}) ,$$

$$r = \bar{r}(1 + \nu) ,$$

$$\bar{r} \cos f = a(\cos \bar{\epsilon} - e) ,$$

$$\bar{r} \sin f = a \sqrt{1 - e^2} \sin \bar{\epsilon} ,$$

$$\sin b = \frac{a}{\bar{r}} u .$$

Using these formulas,

$$\bar{x} = a(1 + \nu)\left[(\cos \bar{\epsilon} - e) \cos \sigma - \sqrt{1 - e^2} \sin \bar{\epsilon} \sin \sigma\right] \cos b ,$$

$$\bar{y} = a(1 + \nu)\left[\sqrt{1 - e^2} \sin \bar{\epsilon} \cos \sigma + (\cos \bar{\epsilon} - e) \sin \sigma\right] \cos b ,$$

$$\bar{z} = z(1 + \nu) u .$$

Writing Musen's formula for the perturbations in the form

$$\delta\mathbf{r} = \alpha\mathbf{r}_0 + \beta\left(\frac{1}{n} \frac{d\mathbf{r}_0}{dt}\right) + \gamma(a\mathbf{R}) ,$$

where

$$\mathbf{r}_0 = a\mathbf{P}(\cos \epsilon - e) + a\sqrt{1 - e^2}\, \mathbf{Q} \sin \epsilon ,$$

the corresponding components are

$$\bar{x} = a(1 + \alpha)(\cos \epsilon - e) - a\beta \frac{a}{r_0} \sin \epsilon ,$$

$$\bar{y} = a(1 + \alpha)\sqrt{1 - e^2} \sin \epsilon + a\beta \frac{a}{r_0} \sqrt{1 - e^2} \cos \epsilon ,$$

$$\bar{z} = a\gamma .$$

From the latitude components,

$$\boxed{\gamma = (1 + \nu) u}$$

without approximation.

Putting

$$\delta\epsilon = \bar{\epsilon} - \epsilon \qquad \text{and} \qquad r_0 = a(1 - e\cos\epsilon) ,$$

the remaining components yield

$$1 + \alpha = (1 + \nu)\cos b \left\{ \cos\sigma + \cos\sigma \frac{a}{r_0}(\cos\delta\epsilon - 1) \right.$$

$$\left. - \sin\sigma \sqrt{1 - e^2}\,\frac{a}{r_0}\left[\sin\delta\epsilon + \frac{e}{1 - e^2}\sin\epsilon\,(1 - e\cos\bar{\epsilon})\right]\right\}$$

and

$$\beta = (1 + \nu)\cos b \left\{ \cos\sigma \left[(1 - e\cos\epsilon)\sin\delta\epsilon + e\sin\epsilon\,(1 - \cos\delta\epsilon)\right]\right.$$

$$\left. + \sin\sigma \sqrt{1 - e^2}\left[\cos\delta\epsilon - \frac{e}{1 - e^2}(1 - e\cos\epsilon)\cos\bar{\epsilon} - \frac{e}{1 - e^2}(\cos\epsilon - e)\right]\right\}.$$

These formulas are exact. The remaining steps are to expand $\cos b$ in powers of u, $\cos\sigma$ and $\sin\sigma$ in powers of σ, and $\delta\epsilon$ in powers of $n\delta z$. In performing these expansions, only those terms which appear to be of significance will be kept. The results can easily be extended to include higher powers of the perturbations if necessary. The largest component is generally $n\delta z$, and the fifth power will be retained. The next largest component is ν, which appears here as a linear factor, so that no expansion in powers of ν is necessary. The third component, u, is generally much smaller. It enters through the factor $\cos b$, so that only the even powers of u occur. u^2 is kept here; but $u^2 n\delta z$, $u^2\nu$, and u^4 are dropped. Finally, σ is taken to be quite small so that σ^2, $\sigma \cdot n\delta z$, and $\sigma \cdot \nu$ are dropped. The fact that terms of the fifth order in $n\delta z$ may be important is illustrated in the case of the minor planet (1373) Cincinnati (Reference 5). This degree of approximation should be adequate for many cases of interest. Now,

$$\cos b = 1 - \frac{1}{2}\left(\frac{a}{r_0}\right)^2 u^2 ,$$

$$\cos\sigma = 1 ,$$

$$\sin\sigma = \sigma$$

and

$$1 + \alpha = (1 + \nu)\left[1 + \frac{a}{r_0}(\cos\delta\epsilon - 1)\right] - \frac{\sigma e}{\sqrt{1 - e^2}}\sin\epsilon - \frac{1}{2}\left(\frac{a}{r_0}\right)^2 u^2 ,$$

$$\beta = (1 + \nu)\left[\frac{r_0}{a} \sin \delta\epsilon + e \sin \epsilon \,(1 - \cos \delta\epsilon)\right] + \frac{\sigma}{\sqrt{1 - e^2}} \left(\frac{r_0}{a}\right)^2 .$$

From Kepler's equation

$$\delta\epsilon = n\delta z + e\,(\sin \bar{\epsilon} - \sin \epsilon) ,$$

which gives

$$\delta\epsilon = \frac{a}{r_0} n\delta z + \left[-\frac{e}{2}\left(\frac{a}{r_0}\right)^3 \sin \epsilon\right] n\delta z^2 + \left[\frac{e^2}{2}\left(\frac{a}{r_0}\right)^5 \sin^2 \epsilon - \frac{e}{6}\left(\frac{a}{r_0}\right)^4 \cos \epsilon\right] n\delta z^3$$

$$+ \left[-\frac{5e^3}{8}\left(\frac{a}{r_0}\right)^7 \sin^3 \epsilon + \frac{5e^2}{12}\left(\frac{a}{r_0}\right)^6 \cos \epsilon \sin \epsilon + \frac{e}{24}\left(\frac{a}{r_0}\right)^5 \sin \epsilon\right] n\delta z^4$$

$$+ \left[\frac{7e^4}{8}\left(\frac{a}{r_0}\right)^9 \sin^4 \epsilon - \frac{7e^3}{8}\left(\frac{a}{r_0}\right)^8 \cos \epsilon \sin^2 \epsilon + \frac{e^2}{12}\left(\frac{a}{r_0}\right)^7 \cos^2 \epsilon\right.$$

$$\left. - \frac{e^2}{8}\left(\frac{a}{r_0}\right)^7 \sin^2 \epsilon + \frac{e}{120}\left(\frac{a}{r_0}\right)^6 \cos \epsilon\right] n\delta z^5 + 0\,(n\delta z^6) .$$

Using the resulting expansions of $\cos \delta\epsilon$ and $\sin \delta\epsilon$,

$$\boxed{\begin{aligned} \alpha = \nu + (1 + \nu) \Bigg\{ & -\frac{1}{2}\left(\frac{a}{r_0}\right)^3 n\delta z^2 + \frac{e}{2}\left(\frac{a}{r_0}\right)^5 \sin \epsilon \, n\delta z^3 \\ & + \left[\frac{5(1 - e^2)}{8}\left(\frac{a}{r_0}\right)^7 - \frac{13}{12}\left(\frac{a}{r_0}\right)^6 + \frac{1}{2}\left(\frac{a}{r_0}\right)^5\right] n\delta z^4 \\ & + \left[-\frac{7e(1 - e^2)}{8}\left(\frac{a}{r_0}\right)^9 \sin \epsilon + \frac{5e}{4}\left(\frac{a}{r_0}\right)^8 \sin \epsilon - \frac{e}{2}\left(\frac{a}{r_0}\right)^7 \sin \epsilon\right] n\delta z^5 \Bigg\} \\ & - \frac{e}{\sqrt{1 - e^2}}\, \sigma \sin \epsilon - \frac{1}{2}\left(\frac{a}{r_0}\right)^2 u^2 \end{aligned}}$$

and

$$\boxed{\begin{aligned} \beta = (1 + \nu) \Bigg\{ & n\delta z - \frac{1}{6}\left(\frac{a}{r_0}\right)^3 n\delta z^3 + \left[\frac{e}{4}\left(\frac{a}{r_0}\right)^5 \sin \epsilon\right] n\delta z^4 \\ & + \left[\frac{3(1 - e^2)}{8}\left(\frac{a}{r_0}\right)^7 - \frac{2}{3}\left(\frac{a}{r_0}\right)^6 + \frac{3}{10}\left(\frac{a}{r_0}\right)^5 \sin \epsilon\right] n\delta z^5 \Bigg\} + \frac{\sigma}{\sqrt{1 - e^2}} \left(\frac{r_0}{a}\right)^2 . \end{aligned}}$$

Next $n\delta z$, ν, and u are given as expansions in α, β, and γ and the small quantity σ by inverting these expansions. The results are

$$\nu = \alpha + \frac{1}{2}\left(\frac{a}{r_0}\right)^3 \left(1 - \alpha + \alpha^2 - \alpha^3\right)\beta^2 - \frac{e}{2}\left(\frac{a}{r_0}\right)^5 \sin\epsilon \left(1 - 2\alpha + 3\alpha^2\right)\beta^3$$

$$+ \left[-\frac{5(1 - e^2)}{8}\left(\frac{a}{r_0}\right)^7 + \left(\frac{a}{r_0}\right)^6 - \frac{1}{2}\left(\frac{a}{r_0}\right)^5\right](1 - 3\alpha)\beta^4$$

$$+ \left[\frac{7e(1 - e^2)}{8}\left(\frac{a}{r_0}\right)^9 \sin\epsilon - e\left(\frac{a}{r_0}\right)^8 \sin\epsilon + \frac{e}{2}\left(\frac{a}{r_0}\right)^7 \sin\epsilon\right]\beta^5$$

$$- \frac{e}{\sqrt{1 - e^2}}\,\sigma\sin\epsilon - \frac{1}{2}\left(\frac{a}{r_0}\right)^2 \gamma^2 ,$$

$$n\delta z = \left(1 - \alpha + \alpha^2 - \alpha^3 + \alpha^4\right)\beta - \frac{1}{3}\left(\frac{a}{r_0}\right)^3 \left(1 - 3\alpha + 6\alpha^2\right)\beta^3 + \frac{e}{4}\left(\frac{a}{r_0}\right)^5 \sin\epsilon\,(1 - 4\alpha)\,\beta^4$$

$$+ \left[\frac{(1 - e^2)}{4}\left(\frac{a}{r_0}\right)^7 - \frac{1}{4}\left(\frac{a}{r_0}\right)^6 + \frac{1}{5}\left(\frac{a}{r_0}\right)^5\right]\beta^5 + \frac{\sigma}{\sqrt{1 - e^2}}\left(\frac{r_0}{a}\right)^2 ,$$

$$u = \left[1 - \alpha - \frac{1}{2}\left(\frac{a}{r_0}\right)^3 \beta^2 + \alpha^2\right]\gamma .$$

Terms of the third and higher orders in α and β factored by γ in the last equation have been dropped.

If the perturbations in Hansen's components are computed from mean elements, then certain terms in $n\delta z$ and u are zero. The corresponding terms in β and γ are of the second order.

It was mentioned before that the first-order results from the two methods are identical and therefore may be taken to represent either α, β, γ or ν, $n\delta z$, u. In either case,

$$\alpha - \nu = -\frac{1}{2}\left(\frac{a}{r_0}\right)^3 n\delta z^2 + \cdots ,$$

$$\beta - n\delta z = \nu \cdot n\delta z + \cdots ,$$

$$\gamma - u = \nu \cdot u ,$$

where the omitted terms are of the third order. When $n\delta z$ (or β) contains large terms resulting from near resonance, there will be a significant difference between α and ν in the constant terms and the terms with twice the long-period argument. The importance of this difference comes from the strong cross-action of these terms in the radius vector with the long-period terms in the longitude in the higher approximations. The convergence of successive approximations in Musen's method is considerably improved if the appropriate changes are made to these terms in α. If these changes are not made, they will appear in the next approximation anyway; but the new values of the long-period terms in β will be quite bad.

These considerations with respect to the long-period terms also should be taken into account in the application of other methods using rectangular coordinates.

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