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## MEASUREMENT RELATIONSHIPS FOR PEGASUS-TYPE METEOROID EXPERIMENTS

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**NASA**

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Huntsville, Alabama*

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ABSTRACT

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In Pegasus-type meteoroid measurement satellite experiments, where vehicle attitude information is available without information about the meteoroid closing velocity vector, the probable radiant and the puncture flux enhancement factor for sporadic meteors should be based on the assumption that velocity vectors for ambient meteoroids are isotropically distributed. Then, although half of the meteoroids which impact onto a flat transducer are more than 45 degrees from the normal, those within 45 degrees puncture three times more effectively than the others, and half of the puncturing meteoroids are less than 33.5 degrees from the normal when the earth is below the surface horizon. This distribution also implies that the flux of puncturing meteoroids is five times greater than the flux of incident meteoroids with mass equal to or greater than the mass of the almost-puncturing meteoroid with the average values of density, velocity, and impact angle.

The formulas for the celestial coordinates of the probable radiant of a puncturing meteoroid have been corrected for earth-shielding. This correction is important near the earth even under nominal circumstances; e.g., when the orbit height is 500 kilometers, the radius of the earth's disk (as seen from the vehicle) is 70 degrees. In that case, when the elevations of the earth's center are 0, 15, 30, 45, and 60 degrees, the probable radiants are deflected 5, 10, 20, 39, and 80 degrees, respectively, by the earth-shielding effect.



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## DEFINITION OF SYMBOLS

<u>Symbol</u>	<u>Definition</u>
$\bar{m}$	mass of a meteoroid which at mean density, mean velocity, and mean impact angle will just puncture a given sheet
$\beta_5$	logarithm of the puncture flux enhancement factor
$v$	meteoroid velocity in kilometers per second
$\rho_p$	meteoroid material density in grams per cubic centimeter
$\bar{v}, \bar{\rho}_p$	$\log \bar{v}$ and $\log \bar{\rho}_p$ represent the population means of $\log v$ and $\log \rho_p$
$m$	meteoroid mass in grams
$F_s$	flux of incident meteoroids with mass equal to or greater than $m$
$\beta_6$	value of $\log F_s$ when $m$ is one gram
$\beta_2$	slope of $\log F_s$ as a linear junction of $\log m$
$x$	angle of impact in radians with respect to the normal for a puncturing meteoroid
$p$	effective thickness of a distributed puncture transducer in centimeters
$t$	target subscript
$\rho_t$	target material density in grams per cubic centimeter
$H_t$	target hardness in Brinell units
$p_o$	crater depth in centimeters
$d$	projectile diameter in centimeters
$C_t$	bulk velocity of sound in kilometers per second
$E$	$10^{-6}$ x Young's modulus, kilograms per square centimeter
$\epsilon$	ductility, percent elongation in 2-inch gauge length at fracture

DEFINITION OF SYMBOLS (Continued)

<u>Symbol</u>	<u>Definition</u>
$\nu$	Poisson's ratio
$k_2$	a known function of target material parameters
$k_3$	a parameter depending on material and dynamic properties of the projectile or meteoroid, thought to be 3 for dense projectiles
$k_1$	an unknown function of meteoroid material parameters
$\emptyset$	meteoroid puncture flux
$\beta_n$	celestial latitude of radiant of surface normal
$\lambda_n$	celestial longitude of radiant of surface normal
$\beta_p$	celestial latitude of earth's center seen from the satellite
$\lambda_p$	celestial longitude of earth's center seen from the satellite
$\theta$	angular radius of the earth (meteoroid cross section) as seen from the satellite in radians
$r$	geocentric radial distance of the satellite in kilometers
$r_e$	radius of the earth in kilometers
$i$	inclination of meteoroid orbital plane to the ecliptic plane
$\beta_e$	celestial latitude of meteor radiant
$f$	cosmic weighting function for a photographic meteor
$h_m$	meteor height above sea level at the point of maximum brilliance
$P$	Öpik's probability that a meteoroid in a given orbit will encounter the earth during one revolution of the particle

DEFINITION OF SYMBOLS (Continued)

<u>Symbol</u>	<u>Definition</u>
$f_f$	reciprocal of the apparent fraction of the circle of celestial latitude through a meteor radiant
Z	meteor zenith angle in radians
s	constant scale factor to restore equality between the same size and the sample sum of the weighting functions for photographic meteors
$x_2$	angle of impact in radians with respect to the normal for a meteoroid impacting onto a surface
$\bar{m}_x$	mass of a meteoroid which at mean density, mean velocity, and angle of impact $x$ will just puncture a given sheet of material
$P_{mx}$	probability that the mass $m$ of a meteoroid is sufficient for puncture when the angle of impact is $x$
$R_1$	the fraction of the potentially puncturing meteoroids from the exposed hemisphere which are intercepted by the apparent segment of the earth
$R_2$	the apparent fraction of the earth's disk
D	angular separation of the center of the earth's disk from the surface zenith in radians
$\gamma$	surface zenith angle (radians) of probable radiant
M	angle at the center of the earth's disk between the meridian of the exposed hemisphere and the celestial meridian
$\beta_r$	celestial latitude of the probable radiant of the puncturing meteoroid
$\lambda_r$	celestial longitude of the probable radiant of the puncturing meteoroid
$\alpha$	half of the arc of the earth's disk (radians) subtended above the surface horizon
z	angle (radians) between the centroid of the apparent disk and the center of the earth
N	angle between celestial and surface meridians at the probable radiant



## TECHNICAL MEMORANDUM X-53255

### MEASUREMENT RELATIONSHIPS FOR PEGASUS-TYPE METEOROID EXPERIMENTS

#### SUMMARY

In Pegasus-type meteoroid measurement satellite experiments, where vehicle attitude information is available without information about the meteoroid closing velocity vector, the probable radiant and the puncture flux enhancement factor for sporadic meteors should be based on the assumption that velocity vectors for ambient meteoroids are isotropically distributed. Then, although half of the meteoroids which impact onto a flat transducer are more than 45 degrees from the normal, those within 45 degrees puncture three times more effectively than the others, and half of the puncturing meteoroids are less than 33.5 degrees from the normal when the earth is below the surface horizon. This distribution also implies that the flux of puncturing meteoroids is five times greater than the flux of incident meteoroids with mass equal to or greater than the mass of the almost-puncturing meteoroid with the average values of density, velocity, and impact angle.

The formulas for the celestial coordinates of the probable radiant of a puncturing meteoroid have been corrected for earth-shielding. This correction is important near the earth even under nominal circumstances; e.g., when the orbit height is 500 kilometers, the radius of the earth's effective disk (as seen from the vehicle) is 70 degrees. In that case, when the elevations of the earth's center are 0, 15, 30, 45, and 60 degrees the probable radiants are deflected 5, 10, 20, 39, and 80 degrees, respectively, by the earth-shielding effect.

#### I. INTRODUCTION

##### A. Purposes

The main purpose of this analysis is to show how attitude information for a Pegasus-type meteoroid puncture counting satellite can be used to infer the probable radiant of a puncturing meteoroid even when the earth partially obscures the celestial hemisphere to which the puncture transducer surface is exposed. A secondary purpose prerequisite to the main purpose is the derivation of the probability density function for the angle of impact of a puncturing meteoroid. A byproduct of the secondary purpose is a more accurate determination of the puncture flux enhancement factor; i.e., if a particular sheet of material is just puncturable by a meteoroid of mean density, mean velocity, mean impact angle, and mass  $\bar{m}$ , then the mean flux of puncturing meteoroids exceeds the mean flux of incident meteoroids with mass equal to or greater than  $\bar{m}$  by  $\beta_5$  order of magnitude [1]. The more accurate determination of  $\beta_5$  is a secondary purpose of this analysis.

## B. Methods and Assumptions

In most of this analysis, it is not necessary to know the true mean value of parameters such as the logarithms of the closing velocity and the material density  $\rho$  of the meteoroids, but only the variation with respect to the unknown true mean. All indicated logarithms are to base ten. The following assumptions seem appropriate:

1. All puncturing meteoroids are considered to enter cislunar space from outside the earth-moon system with appreciable velocity. Meteoroid geocentric velocity at any point in cislunar space is sufficiently higher than vehicle orbital velocity that it is essentially the closing velocity. The logarithm of velocity,  $v$ , when  $v$  is in kilometers per second, for the population of incident meteoroids, is essentially normally distributed with mean and standard deviation given by equations (1) and (2), respectively [1].

$$\log \bar{v} = \log 26.7 = 1.43, \quad (1)$$

$$\sigma_{\log v} = 0.18. \quad (2)$$

2. Most of the puncturing meteoroids in Pegasus-type experiments are dust balls instead of having the more substantial structure of meteorites. Although the mean density,  $\rho_p$ , for the population of dustball meteoroids is known only to within a factor two probable error, the logarithm of density,  $\rho_p$ , where the density is in grams per cubic centimeter, for the population of incident meteoroids, can be considered to be normally distributed with mean and standard deviation given by equations (3) and (4), respectively, [1].

$$\log \bar{\rho}_p = \log 0.44 \pm 0.30 = -0.35 \pm 0.30, \quad (3)$$

$$\sigma_{\log \rho_p} = 0.44. \quad (4)$$

3. It is assumed that the puncture transducers are exposed to a celestial hemisphere, and that the probable radiant of the velocity vector of a puncturing meteoroid is determined with sufficient accuracy by neglecting any systematic variations in meteoroid physical or statistical parameters over that part of the apparent celestial hemisphere which is not subtended by the earth. Most of the punctures are expected to be from sporadic meteors rather than from stream meteoroids [1]; but

the known occurrence of a major meteor stream is an exception to the isotropic flux which is otherwise assumed. If a puncture is concurrent with a well-defined meteor stream with unobstructed radiant within the median angle of deviation from the surface normal radiant, then the probable radiant should be considered to be that of the concurrent meteor stream.

4. At any particular distance from the surface of the earth, the mean flux,  $F_s$ , of meteoroids per unit area per unit time with mass equal to or greater than  $m$  grams is considered to be an exponential function of mass  $m$ ; i.e., [1],

$$F_s = 10^{\beta_6} m^{\beta_2} \quad (5)$$

where  $\beta_6$  is a constant. In a model for cislunar meteoroid impact recently suggested as a criterion for current purposes by Dalton [1],  $\beta_2$  is given as a function of distance from the surface of the earth with range  $-1.34 \leq \beta_2 \leq -1$  in cislunar space and with an effective value of  $-1.23$  for the orbit of the Pegasus-A meteoroid detection satellite. To determine the distribution of the angle of impact,  $x$ , for the population of puncturing meteoroids, the numerical value of  $\beta_2$  for the Pegasus-A mission will be assumed throughout cislunar space; i.e.,

$$\beta_2 = -1.23. \quad (6)$$

5. In Dalton's [2] model for the puncture of a homogeneous metallic wall by hypervelocity impact of a spherical projectile, the relation between the minimum sufficient mass,  $m$ , of the projectile, its velocity,  $v$ , and the angle of impact,  $x$ , can be expressed as

$$\log m = 1.9 + \log(p^3 \rho_t H_t) - (3/2)(\log v + \log \cos x) - \log \rho_p, \quad (7)$$

where  $\rho_p$  is the density of the projectile and where  $\rho_t$ ,  $H_t$ , and  $p$  are the density, hardness in Brinell units, and thickness in centimeters, respectively, for the punctured wall. Equation (7) is used in the present analysis only in the derivation of the distribution of the angle of impact,  $x$ , for the population of puncturing meteoroids.

6. Under contract number NAS7-219 to the Martin-Marietta Corporation for a survey and multivariate analysis of available laboratory hypervelocity impact data, Reismann, Donahue and Burkitt [3] found that in the highest of three velocity regimes the multiple correlation coefficient is nearly maximized (to 0.989) by representing the relation between projectile and target parameters for impact at normal incidence as follows:

$$p_0/d = 2.5(v/C_t)^{0.50} (E/E_t)^{1.31} (v/v_t)^{8.0} (c/\epsilon_t)^{0.43}, \quad (8)$$

where

$p_0$  = crater depth

$d$  = projectile diameter

$v$  = closing velocity in kilometers per second

$t$  = target subscript

$C_t$  = bulk velocity of sound in kilometers per second

$E = 10^{-6}$  x Young's modulus, kilograms per square centimeter

$\epsilon$  = ductility, percent elongation in 2-inch gauge length at fracture

$\nu$  = Poisson's ratio.

The exponents in equation (8) are the result of some rounding off of decimals in the published [3] values. The relation between the thickness,  $p$ , of a puncturable sheet and crater depth,  $p_0$ , in a thick target has been treated analytically by Andriankin and Stepanov [4] and by Herrmann and Jones [5] and stochastically by Dalton [2, 6]. The results are problematical; but for meteoroid impact, the following relation would seem to be the most appropriate assumption:

$$p = 1.59 p_0. \quad (9)$$

Dalton [1] showed that when equations (1), (3) and (9) are assumed for meteoroids, the compatibility of equations (7) and (8) for aluminum 2024-T3 implies that the mass,  $\bar{m}$ , - of a meteoroid with mean log density

( $\log \bar{\rho}_p = \log 0.44$ ), mean log velocity ( $\log \bar{v} = \log 26.7$ ), and mean impact angle ( $\bar{x}_2 = \pi/4$ ) - which will just puncture a particular metallic wall is

$$\bar{m} = 10^{10.985} (p C_t^{0.50} E_t^{1.31} v_t^{8.0} \epsilon_t^{0.43})^3. \quad (10)$$

It is assumed that equation (10) is superior to equation (7) for those purposes where it can be used; however, the exponent "3" and consequently also the exponent "10.985" in equation (10) are problematical for meteoroid impact, as well as the corresponding values in equation (7), and should be treated as unknown constants, say  $k_3$  and  $k_1$ , respectively, when incident flux parameters ( $\beta_6$  and  $\beta_2$  in equation (5)) are to be inferred from puncture flux parameters [7] (see Section II below).

## II. RELATION BETWEEN PARAMETERS FOR IMPACT AND PUNCTURE FLUXES

Corresponding to the meteoroid impact flux,  $F_s$ , in equation (5), there will be a puncture flux,  $\phi$ , per unit area per unit time, through a particular exposed wall or distributed puncture transducer, which can be represented by

$$\phi = 10^{\beta_6 + \beta_5} \bar{m}^{\beta_2}, \quad (11)$$

where  $\beta_6$  and  $\beta_2$  are the same constants used in equation (5) to express the incident (or impact) flux,  $F_s$ , the constant,  $\beta_5$ , is the logarithm of the puncture flux enhancement factor to be derived in Section IV, and  $\bar{m}$  is the nominally puncturable meteoroid mass which is now predicted by the improvised equation (10).

By the suggestion at the end of Section I about the problematical exponents in equation (10), an expression for  $\bar{m}$  alternative to equation (10) is

$$\bar{m} = 10^{k_1 + k_2 k_3} p^{k_3}, \quad (12)$$

where  $k_1$  and  $k_3$  are unknown functions of the projectile material and dynamic characteristics, to be determined for meteoroids by satellite experiments, and where  $k_2$  is a function of the target material parameters as follows:

$$k_2 = 0.50 \log C_t + 1.31 \log E_t + 8.0 \log v_t + 0.43 \log \epsilon_t. \quad (13)$$

By equations (11) and (12), one can write the following relation between  $\log \phi$  and  $(k_2 + \log p)$ , both of which (by equation (13)) have known values for each of the puncture transducers of different material and effective thickness in the Pegasus-type flight experiments:

$$\log \phi = (\beta_6 + \beta_5 + k_1\beta_2) + k_3\beta_2 (k_2 + \log p). \quad (14)$$

Dalton's [1] calculations for the Explorer XVI satellite imply -5.417 and -5.697 for  $\log \phi$  for the 1-mil and 2-mil puncture sensors, respectively, and -6.175 and -5.904 for  $(k_2 + \log p)$  for the same respective sensors. The 2-point solution to equation (14) for the Explorer XVI results would therefore be

$$\log \phi = -11.80 - 1.033 (k_2 + \log p). \quad (15)$$

The resulting slope in equation (15) would imply by equation (14) that, if 3 were the correct value of  $k_3$ , then the value for  $\beta_2$  would be -0.344. The value of  $\beta_2$  which Hawkins and Upton [8] found for photographic meteors is -1.34; and by Dalton's [1] model the effective value of  $\beta_2$  for the Explorer XVI orbit would be -1.20.

Extrapolation of the puncture results for Explorer XVI by equation (15) would indicate a higher puncture rate for thicker structures than one would want to anticipate for design purposes without verification by further Pegasus-type experiments. The results in equation (15) are not established with much confidence because the numbers of punctures (44 and 11 for the 1-mil and 2-mil sensors, respectively) are statistically small samples, especially when the extreme diversity of the population of incident meteoroids is considered. More pertinently, one should anticipate that the relation between  $\log \phi$  and  $(k_2 + \log p)$  in equation (14) may not be linear.

A sufficiently accurate value for  $\beta_5$  will be derived in Section IV; but this leaves more unknowns (4) in equation (14) than conditions (2) which can be applied toward their determination from Pegasus-type data. If values for  $\beta_6$  and  $\beta_2$  can be assumed from the statistics and physical theory of meteors, then the meteoroid impact parameters  $k_1$  and  $k_3$  can be found as constants (if equation (14) is linear) or as functions of  $(k_2 + \log p)$  (if equation (14) is nonlinear). But, because the impact parameters  $k_1$  and  $k_3$  are not known, it is not even possible to determine the value of  $\beta_6$  in equation (14) by assuming a value for  $\beta_2$ , or vice versa. Therefore, by equations (5), (11), (12), and (14), it follows that the nominally minimum sufficient mass for puncturing meteoroids cannot be determined from Pegasus-type data alone, unless the incident flux, cumulative with respect to mass, is known as a function of the mass.

### III. DISTRIBUTION OF ANGLE OF IMPACT FOR INCIDENT METEORIODS ISOTROPICALLY FROM AN APPARENT HEMISPHERE ONTO A PLAIN OR CONVEX SURFACE

Pegasus-type experiments give satellite attitude and punctured sensor identification at each meteoroid puncture event (see Reference 9 for a discussion of the Pegasus-A experiment). This information can be used to determine the celestial latitude,  $\beta_n$ , and celestial longitude,  $\lambda_n$ , of the radiant of the normal to the punctured surface, the corresponding celestial coordinates  $(\beta_p, \lambda_p)$  for the center of the earth (as seen from the punctured satellite), and the angular radius  $\theta$  of the earth's effective meteoroid encounter cross section seen from the satellite; i.e., approximately,

$$\theta = \sin^{-1} [(r_e + 100)/r], \quad (16)$$

where  $r$  is the geocentric radial distance of the satellite in kilometers and  $r_e$  is the radius of the earth in kilometers. Therefore, when the probability distribution of the radiants of incident meteoroids is assumed, and when the relation between the probability distributions of the angles of impact of incident meteoroids and of puncturing meteoroids is established (see Section V), the celestial coordinates  $(\beta_m, \lambda_m)$  of the probable radiant of the puncturing meteoroid can be determined as functions of the values of  $\beta_n$ ,  $\lambda_n$ ,  $\beta_p$ ,  $\lambda_p$ , and  $\theta$  at the time of puncture. In this manner, any large sample of punctures will give a distribution of probable radiants, although the resulting distribution may be somewhat more widely dispersed than the unknown population of true radiants.

The dispersion of the probable radiants of the puncturing meteoroids in Pegasus-type experiments has two spurious components because of some uncertainty in the satellite attitude information and because the meteoroid velocity vector in on-board coordinates is not measured but must be inferred probabilistically. Nevertheless, the resulting information is of considerable interest because the other three sources of such information are somewhat problematical and they involve different ranges of meteoroid mass than that for which Pegasus-type experiments have been designed. One such source of information is the interpretation of isophotes of zodiacal light from small dust particles in interplanetary space. Other inferences can be attempted from the distribution of the orbits of comets and from the effect of the combined action of several physical processes on the debris from disintegrating comets. More intensive studies of the distribution of meteoroid radiants have been based on the visual, photographic, and radio meter data; but several sources of observational bias complicate these analyses and may prejudice the results.

Without specific contrary information, one could assume that the distribution of the closing velocity vectors is isotropic and therefore that the probability density for meteor radiants is uniformly distributed over the exposed celestial hemisphere. Then the probability density function  $f(\beta_e)$  for the celestial latitude  $\beta_e$  of the radiants in the interval  $0 \leq \beta_e \leq \pi/2$  would be  $\cos \beta_e$ ; and, by integration, the mean (expected value) and median for  $\beta_e$  would be  $(\pi/2 - 1)$  and  $\pi/6$  radians (32.7 and 30.0 degrees), respectively.

Lovell's [10] results for the distribution of sporadic visual meteors, corrected for horizon zenith attraction and twilight effects, have been interpreted by Davison and Winslow [11] to indicate that approximately half of the sporadic meteoroids have orbital inclinations,  $i$ , within  $\pm 15$  degrees from the ecliptic. (Orbit inclination,  $i$ , in the interval  $0 \leq i \leq \pi/2$ , and radiant celestial latitude,  $\beta_e$ , are practically interchangeable here.) But the masses of the meteoroids had to be sufficient for duplicate visual observations (see Lovell's [10] description of Prentice's technique for visual meteors), and therefore several orders of magnitude higher than the largest mass one can practically expect to detect in Pegasus-type experiments. (By Dalton's [1] analysis, the 1.5, 8, and 16-mil puncture sensors on Pegasus-A are thought to be nominally puncturable by meteoroids of  $10^{-7.84}$ ,  $10^{-5.85}$ , and  $10^{-4.98}$  gram, respectively.) Then, by Hawkins [12] models for the distributions of meteoroids of cometary origin (most of those encountered in Pegasus-type experiments) and of asteroidal origin, the 2-observer visual meteor sample might be expected to have angles of inclination,  $i$ , distributed somewhat more like the asteroids. Allen [13] gives the arithmetic mean for the inclination of the orbits of the asteroids as approximately 10 degrees.

Assuming an isotropic distribution of the radiants of meteoroids (encountered by an observer in orbit about the sun) is not equivalent to assuming completely randomly distributed meteoroid orbits (with inclination,  $i$ , in the interval  $0 \leq i \leq \pi/2$ ). Briggs [14] indicates that  $\sin i$  would be the probability density function for the inclination,  $i$ , of completely random orbits (with all meteoroids considered, i.e., not just those detected by a moving observer). Then, by integration, the mean (expected value) of  $i$  would be one radian (57.3 degrees).

A completely random distribution of orbits may be indicated for long period comets; but the debris from the disintegration of comets may have orbits which are distributed less randomly. Davison and Winslow [15] describe this situation as follows: "The zodiacal dust particles are thought to be more highly concentrated in the plane of the ecliptic. They have two components of drift as they orbit the sun, one toward the plane of the ecliptic and one toward the sun [16]. The drift toward the plane of the ecliptic results from the attraction of



the planets. The drift or spiraling in toward the sun is a result of ... the Poynting-Robertson effect (Lovell [10] and Wyatt and Whipple [17])."

By establishing isophotes of the zodiacal light and interpreting them in relation to the Poynting-Robertson effect and data from observed meteors, Briggs [14] found that the probability density function  $f(i)$  for meteoroid orbit inclination,  $i$ , ( $0 \leq i \leq \pi/2$ ) is

$$\begin{aligned} f(i) &= (1.66 - 0.66i) \sin i, & \text{for aphelion} > 6.6 \text{ astronomical units} \\ &= 37e^{-6i} \sin i, & \text{for aphelion} \leq 6.6 \text{ astronomical units.} \end{aligned}$$

Then, by integration, the mean (expected value) of the inclination,  $i$ , of the orbit of any high-aphelion particles (whether or not encountered by a moving observer) would be  $1.66 - 0.66(\pi - 2) = 0.907$  radians (52 degrees). The corresponding mean  $i$  for low-aphelion particles would be  $12/37$  radians (18.6 degrees). In this analysis, Briggs indicated that particles in the 1 to 50 micron range play the dominant role in the zodiacal light; and Ehrlicke [18] says that, according to Takakubo, the radius of the particles most effective is about 20 microns. Also, by the relation which Briggs indicated between particle radius and material density, a spherical particle with a radius of 20 microns would have a mass of  $10^{-8.2}$  grams.

The author has not yet completed his analysis of the 285 photographic meteors common to the two tables of data for random samples of sporadic meteors published in 1958 and 1961 by Hawkins and Southworth [19, 20]. Some preliminary results were given in Reference 1; e.g., the sample range for meteoroid mass,  $m$ , in grams (above the atmosphere) is  $10^{-2.2} \leq m \leq 10^{1.0}$ , with a mean for  $\log m$  of  $-1.0$ . Figure 1 shows the cosmically weighted cumulative distribution of celestial latitude,  $\beta_e$ , with an arithmetic median  $\beta_e$  of 25 degrees for the positive values of  $\beta_e$ . Asterisk and circle points in Figure 1 represent positive and negative values of  $\beta_e$ , respectively. Figure 1 was obtained by weighting each meteor datum by

$$f = s h_m^{-2} v^{-3/2} p^{-1} f_f e^{-0.18Z} \quad (17)$$

where  $h_m$  is the meteor height above sea level at the point of maximum brilliance,  $v$  is the closing velocity,  $p$  is Öpik's (see Whipple [21]) probability that a meteoroid in a given orbit will encounter the earth during one revolution of the particle;  $f_f$  is the reciprocal of the apparent fraction of the circle of celestial latitude through the

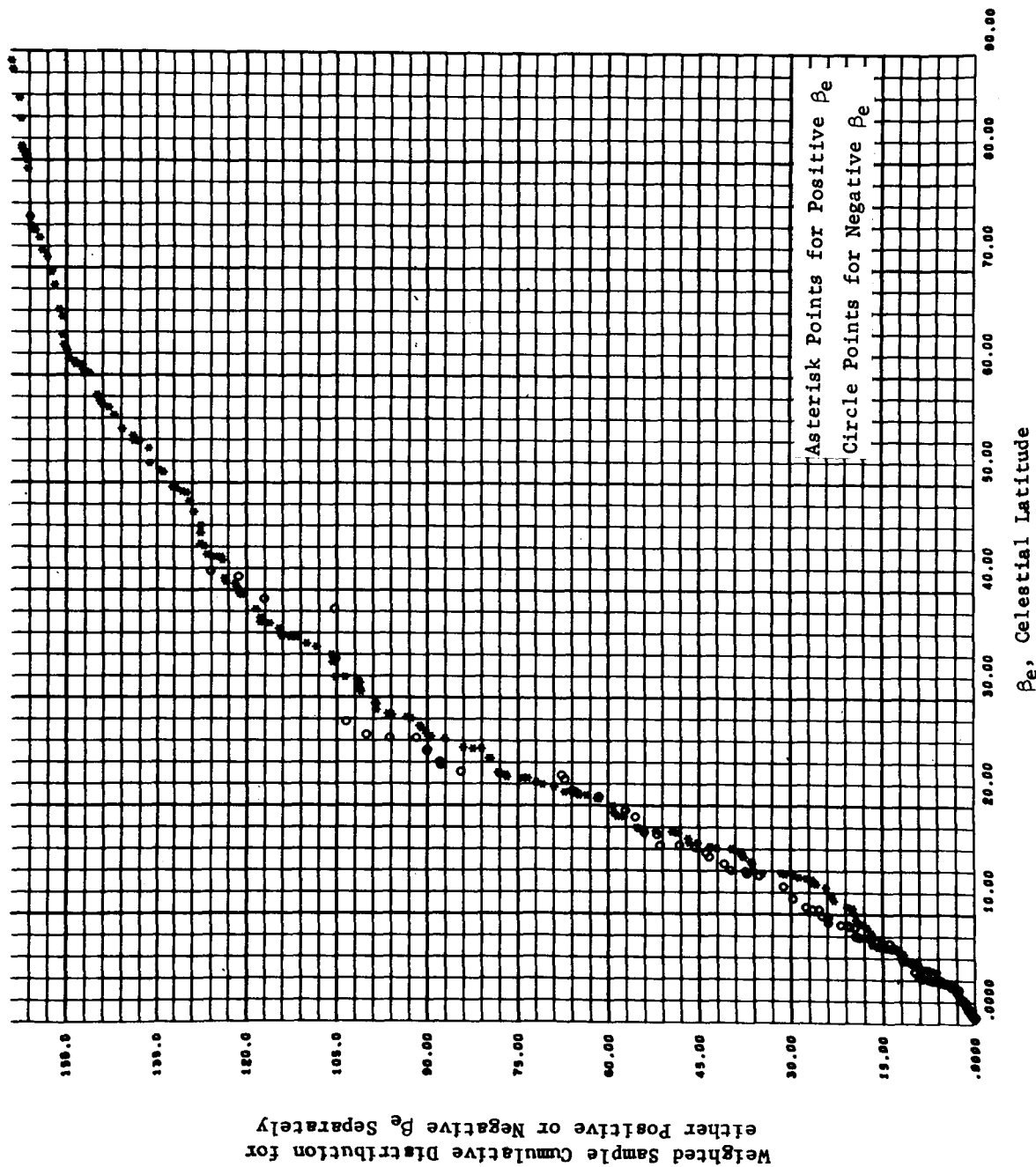


FIGURE 1. COSMICALLY WEIGHTED CUMULATIVE DISTRIBUTION OF CELESTIAL LATITUDE OF RADIANT FOR HAWKINS AND SOUTHWORTH'S [19,20] RANDOM SAMPLE OF SPORADIC PHOTOGRAPHIC METEORS

meteor radiant,  $e$  is the base of natural logarithms,  $Z$  is the meteor zenith angle in radians, and  $s$  is a constant scale factor which restores equality between the sample size and the sample sum of the weighting functions. The constant,  $-0.18$ , in the zenith angle weighting factor in equation (17) was chosen to minimize the sum of the squares of the differences between the cumulative distributions for positive and negative  $\beta_e$  below 42 degrees in Figure 1. (The zenith angle weighting factor is an approximate alternative to weighting with respect to meteor celestial longitude.) It is not yet known to what extent the results in Figure 1 may be changed by deleting Öpik's earth encounter probability,  $P$ , from the weighting function,  $f$  (equation (17)); but the corresponding results would be more appropriate for the present consideration (observer subject to the earth's orbital motion).

In an analysis of the distribution of sporadic radio meteor radiants corrected for antenna selectivity, Elford, Hawkins, and Southworth [22] show contour diagrams of radiant density over the celestial sphere, for each of the first eight months of 1962. Also they show corresponding results for the entire period both before and after correction for velocity selection effects. The monthly mean results, not corrected for velocity selection effects, show much shifting of the contour diagrams of radiant density from month to month. The mean results for the entire period undergo major changes when corrected for velocity selection effects; e.g., the concentration of radiants around the apex of the earth's way disappears, and the previously supposed scarcity of radiants near the antipex becomes questionable. Correction for velocity selection effects is important for the present purpose because the incident flux of radio meteors is proportional to the  $4\beta_2$  power of the velocity [22]; whereas, puncture flux through a given structure is thought to be proportional to the  $1.5\beta_2$  power of the velocity [1]. The parameter  $\beta_2$  is the slope of  $\log F_s$  as a function of  $\log m$ , and is thought to have an effective value between  $-1.34$  [8] and  $-1.0$  [22] for meteor data.

The main features of the preliminary results for the mean radiant distribution of sporadic radio meteors, corrected for velocity and other selection effects, are that radiants are more densely concentrated near the sun and antisun directions (about  $\pm 80$  degrees latitude from the apex of the earth's way) and that radiant density decreases by a factor of about three within about 25 degrees from the center of each of the two main concentrations. Elford, Hawkins, and Southworth [22] indicate that these results are for meteoroids with mass not less than  $1.3 \times 10^{-6}$  grams.

The dispersion in the sample of probable radiants from Pegasus-type experiments will have a spurious component due to any error in the distribution of radiants which must be assumed in determining each of the probable radiants. It would be prohibitively tedious to apply a distribution with intricate variations with respect to celestial coordinates

and season of the year. For any distribution with respect only to celestial latitude, it is not apparent from the discussion so far in this section that any improvement over the isotropic distribution could be found. Therefore, for simplicity and because the radiant activity within  $\pm 60$  degrees from the antipole is not yet established with corrections for velocity selection effects for radio meteors, any variations in the distribution of radiants with respect to celestial longitude will be ignored also in the rest of this analysis.

Particles incident isotropically from an exposed hemisphere onto an element of a plane surface give a  $\sin 2x_2$  probability density function for the angle of impact,  $x_2$ , with respect to the normal; i.e.,

$$f(x_2) = \sin 2x_2. \quad (18)$$

This result (equation (18)) is evident intuitively because that fraction of the celestial hemisphere which contributes radiants within the angular increment,  $dx_2$ , varies as  $\sin x_2$ , and the projection of the element of area onto the plane normal to any radiant direction varies as  $\cos x_2$ . Then, by integration, both the mean and the median for the angle of impact,  $x_2$ , for all meteoroids incident isotropically onto a plane surface, are equal to  $\pi/4$  radians.

#### IV. PUNCTURE FLUX ENHANCEMENT FACTOR

Equation (7) gives the logarithm of meteoroid mass,  $m$ , as a linear function of the logarithms of impact velocity,  $v$ , and meteoroid density,  $\rho_p$ . But, in Section I,  $\log v$  and  $\log \rho_p$  are described as essentially normally distributed and statistically independent random variables. Then  $\log m$  in equation (7) is a normally distributed random variable with mean and variance, by equations (1) through (4), as follows:

$$\log \bar{m}_x = -1.9 + \log(p^3 \rho_t H_t) - (3/2) \log \bar{v} - \log \bar{\rho}_p - (3/2) \log \cos x \quad (19)$$

$$\sigma_{\log m}^2 = (3/2)^2 \sigma_{\log v}^2 + \sigma_{\log \rho_p}^2 = (0.52)^2. \quad (20)$$

By taking the differential of equation (5), the number of meteoroids,  $-dF_s$ , impacting per square meter per second with masses between  $m$  and  $m + dm$  is

$$-dF_s = -10^{\beta_6} \beta_2 m^{\beta_2-1} dm. \quad (21)$$

Then, by equations (18) and (21), the flux of impacting meteoroids, within both an increment of mass  $dm$  and an increment of impact angle  $dx_2$ , is

$$-dF_s(\sin 2x_2) dx_2 = -10^{\beta_6} \beta_2 m^{\beta_2-1} \sin 2x_2 dx_2 dm. \quad (22)$$

By equation (22), the incremental number of punctures  $d\phi$ , per square meter per second, due to meteoroids in the increment of impact angle  $dx$  can be expressed by

$$d\phi = \left[ \int_0^{\infty} -10^{\beta_6} \beta_2 P_{mx} m^{\beta_2-1} dm \right] \sin 2x dx, \quad (23)$$

where  $P_{mx}$  is the probability that the mass,  $m$ , of a meteoroid, which is incident with an angle of impact,  $x$ , is sufficient for puncture (i.e.,  $P_{mx}$  is the cumulative distribution of the normally distributed variable  $\log m$  as defined by equations (7), (19), and (20)). The subscript has been dropped from  $x_2$  in equation (23) to indicate that the population of puncturing meteoroids is now considered rather than the population of impacting meteoroids.

The tangent line at the point  $P_{mx} = 1/2$  for the curve relating  $P_{mx}$  and  $\log m$  is a very close approximation to the curve in the interval  $0.25 \leq P_{mx} \leq 0.75$ , and its extension throughout the interval  $0 \leq P_{mx} \leq 1$  is close enough for the present purpose; i.e.,

$$\left. \begin{aligned} P_{mx} &= 0 & \text{for } m < \exp_{10} \left( \log \bar{m}_x - \frac{\sqrt{2\pi}}{2} \sigma_{\log m} \right) \\ &= 1 & \text{for } m > \exp_{10} \left( \log \bar{m}_x + \frac{\sqrt{2\pi}}{2} \sigma_{\log m} \right) \\ &= (2\pi)^{-1/2} \left( \frac{\log m - \log \bar{m}_x}{\sigma_{\log m}} \right) + \frac{1}{2}, & \text{otherwise} \end{aligned} \right\} \quad (24)$$

Therefore, by using the expressions from equation (24) in evaluating the integral in equation (23), it follows that the differential puncture flux,  $d\phi$ , corresponding to the differential impact angle,  $dx$ , is

$$d\phi = 10^{\beta_6} \left( \frac{1}{\sqrt{2\pi} \beta_2 \sigma_{\log m}} \right) \left( 10^{\frac{\sqrt{2\pi}}{2} \beta_2 \sigma_{\log m}} - 10^{-\frac{\sqrt{2\pi}}{2} \beta_2 \sigma_{\log m}} \right) \bar{m}_x^{\beta_2} \sin 2x dx. \quad (25)$$

In equation (19), by replacing  $\log \cos x$  with the mean of  $\log \cos x_2$ , say  $\log \cos \bar{x}_2$ , one also replaces  $\bar{m}_x$  (the mass of the nominally puncturing meteoroid at impact angle  $x$ ) with  $\bar{m}$  (the mass of the nominally puncturing meteoroid in equation (11)). Then, by equation (18),

$$\log \cos \bar{x}_2 = \int_0^{\pi/2} (\log \cos x_2) \sin 2x_2 dx_2 = - (1/2) \log e; \quad (26)$$

and, by equations (19) and (26),

$$\log \bar{m}_x = \log \bar{m} - (3/4) \log e - (3/2) \cos x. \quad (27)$$

One finds, by substituting the antilogarithm of the right side of equation (27) for  $\bar{m}_x$  in equation (25), and by using the numerical values of  $\beta_2$  and  $\sigma_{\log m}$  from equations (6) and (20), respectively, that

$$d\phi = 10^{\beta_6} \bar{m}^{\beta_2} (9.69 \sin 2x) (\cos x)^{1.84} dx. \quad (28)$$

Then, by integrating the right side of equation (28) over the interval  $0 \leq x \leq \pi/2$ ,

$$\phi = (5.05) 10^{\beta_6} \bar{m}^{\beta_2} = 10^{\beta_6 + 0.70} \bar{m}^{\beta_2}. \quad (29)$$

Therefore, by equations (5) and (29), the puncture flux enhancement factor is five, instead of four [1], and  $\beta_5$  in equation (11) is 0.70 instead of 0.60.

#### V. DISTRIBUTION OF ANGLE OF IMPACT FOR PUNCTURING METEORIODS ISOTROPICALLY FROM AN APPARENT HEMISPHERE ONTO A PLAIN OR CONVEX SURFACE

The probability density function  $f(x)$  of the angle of impact,  $x$ , for the population of meteoroids which actually puncture a given sheet of material exposed to an isotropic flux, is, by equations (28) and (29),

$$f(x) = (d\phi/dx)/\phi = 1.92(\cos x)^{1.84} \sin 2x. \quad (30)$$

Then, by integrating equation (30), the probability that a puncturing meteoroid had an angle of impact of not more than  $x$  radians from the normal is  $1 - (\cos x)^{3.84}$ . Therefore, the median and third quartile of impact angle for puncturing meteoroids are 33.5 and 45.8 degrees, respectively, from the normal.

#### VI. PROBABLE RADIANT FOR A PUNCTURING METEORIOD FROM A HEMISPHERE PARTIALLY OBSCURED BY THE EARTH

The fraction of the exposed celestial hemisphere which has the meteoroid radiants corresponding to a differential increment of impact angle,  $dx$ , varies as  $\sin x$ . Then, by equation (30), the number of meteoroids which puncture a hemispherically exposed flat transducer and which have radiants within a constant differential solid angle in the vicinity of a point on a meridian of the exposed hemisphere is proportional to  $(\cos x)^{2.84}$ .

The hemispherical mean radiant density for the puncturing meteoroids is not expected to differ appreciably from the radiant density corresponding to the median impact angle for puncturing meteoroids (for which  $(\cos x)^{2.84}$  equals  $2^{-(2.84/3.84)}$ ; i.e.,  $(1.67)^{-1}$ ).

The apparent disk of the earth with angular radius,  $\theta$ , in equation (16) subtends a solid angle of  $2\pi(1 - \cos \theta)$  steradians, (see Reference 23). Then, of the total number of meteoroids which would puncture, the fraction  $R_1$  obscured by the apparent fraction  $R_2$  of the earth's disk is sufficiently approximated by

$$R_1 = 1.67(\cos x)^{2.84} (1 - \cos \theta) R_2, \quad (31)$$

where  $x$  is the separation of the centroid of the apparent segment from the surface zenith.

The separation  $D$  of the center of the earth's disk  $(\beta_p, \lambda_p)$  from the surface zenith  $(\beta_n, \lambda_n)$  is related to the given coordinates through the formula for the haversine of  $D$  (see Reference 25):

$$\text{hav } D = \text{hav}(\beta_p - \beta_n) + \cos \beta_n \cos \beta_p \text{hav}(\lambda_n - \lambda_p). \quad (32)$$

By definition, the haversine and cosine of any angle  $A$  are related by

$$\text{hav } A = (1 - \cos A)/2. \quad (33)$$

Then, by equations (32) and (33),  $D$  is the smallest positive value in radians satisfying

$$\cos D = \cos(\beta_p - \beta_n) - [1 - \cos(\lambda_n - \lambda_p)] \cos \beta_n \cos \beta_p, \quad (34)$$

where  $D$  is in the interval  $0 \leq D \leq \pi$ .

When  $D \geq \pi/2 + \theta$ , the earth is below the surface horizon, both  $R_2$  and  $R_1$  in equation (31) vanish, and the probable radiant of a puncturing meteoroid is the surface zenith  $(\beta_n, \lambda_n)$ . But when  $D < \pi/2 + \theta$ , the probable radiant is on the earth's meridian (of the exposed hemisphere) and at an angular increment,  $\gamma$ , beyond the surface zenith, sufficiently approximated by

$$\left. \begin{aligned} \gamma &= \frac{xR_1}{1 - R_1} && \text{when } \frac{xR_1}{1 - R_1} \leq 4/3 \\ &= \theta + D/3 && \text{when } \frac{xR_1}{1 - R_1} > 4/3 \end{aligned} \right\} \quad (35)$$



When  $D \leq \pi/2 - \theta$ , the earth is entirely above the surface horizon,  $R_{\Sigma}$  becomes unity in equation (31), and  $D$  replaces  $x$  in both equations (31) and (35). But when  $D$  is in the interval  $\pi/2 - \theta < D < \pi/2 + \theta$ , only a segment of the earth's disk is above the surface horizon, the radius of the earth's disk to either horizon point of the periphery makes an angle  $\alpha$  with the meridian of the exposed hemisphere, and the centroid of the apparent segment of the earth's disk is displaced by an angle  $z$  from the earth's center. Then  $\alpha$  is a function of  $D$  and  $\theta$ ,  $z$  is a function of  $\alpha$  and  $\theta$ ,  $R_{\Sigma}$  in equation (31) is a function of  $\alpha$ , and  $x$  in equations (31) and (35) is a function of  $D$  and  $z$ . These relations are approximated sufficiently from the plane geometry for circular segments (see Reference 24) as follows:

$$\cos \alpha = (D - \pi/2)/\theta, \quad 0 < \alpha < \pi, \quad (36)$$

$$R_{\Sigma} = (\alpha - \sin \alpha \cos \alpha)/\pi, \quad (37)$$

$$z = (2\theta/3)(\sin^3 \alpha)/(\alpha - \sin \alpha \cos \alpha), \quad (38)$$

$$x = D - z. \quad (39)$$

Let  $M$  represent the angle at the center of the earth's disk between the meridian of the exposed hemisphere and the celestial meridian; and let the celestial coordinates of the probable radiant of the puncturing meteoroid be  $\beta_r$  and  $\lambda_r$ . Then the spherical triangles which are formed on the celestial sphere by the celestial pole and the great-circle arcs  $D$  and  $D + \gamma$  have the following solution (see Reference 25):

$$\text{hav } M = [\text{hav } (\pi/2 - \beta_n) - \text{hav } (\pi/2 - \beta_p - D)] \csc (\pi/2 - \beta_p) \csc D, \quad (40)$$

$$\text{hav } (\pi/2 - \beta_r) = \text{hav } (\pi/2 - \beta_p - D - \gamma) + \sin (\pi/2 - \beta_p) \sin (D + \gamma) \text{hav } M, \quad (41)$$

$$\text{hav } (\lambda_r - \lambda_n) \cos \beta_r \cos \beta_n = \text{hav } \gamma - \text{hav } (\beta_n - \beta_r). \quad (42)$$

Then, by equations (33), (40), and (41), the celestial latitude,  $\beta_r$ , of the probable radiant (in the interval  $-\pi/2 \leq \beta_r \leq \pi/2$ ) can be calculated from:

$$\sin \beta_r = \sin(\beta_p + D + \gamma) + \frac{\sin(D + \gamma)}{\sin D} [\sin \beta_n - \sin(\beta_p + D)], \quad (43)$$

where, by definition,  $-\pi/2 \leq \beta_r \leq \pi/2$ . Also, by equations (33) and (42), the celestial longitude,  $\lambda_r$ , of the probable radiant can be calculated from

$$\cos (\lambda_r - \lambda_n) = 1 + \frac{\cos \gamma - \cos(\beta_n - \beta_r)}{\cos \beta_r \cos \beta_n}. \quad (44)$$

In addition to equation (44) for  $\cos (\lambda_r - \lambda_n)$ , one must have a formula for  $\sin (\lambda_r - \lambda_n)$  to remove the quadrant ambiguity. Let  $N$  be the angle between the celestial and surface meridians at the probable radiant; i.e.,  $0 \leq N \leq \pi$ ,

$$\text{hav } N = [\text{hav } (\pi/2 - \beta_n) - \text{hav } (\pi/2 - \beta_r - \gamma)] \csc (\pi/2 - \beta_r) \csc \gamma. \quad (45)$$

Then, by equations (33) and (45),  $N$  can be calculated from

$$\cos N = 1 + \frac{\sin \beta_n - \sin(\beta_r + \gamma)}{\cos \beta_r \sin \gamma}, \quad 0 \leq N \leq \pi. \quad (46)$$

Therefore, by the law of sines,

$$\sin (\lambda_r - \lambda_n) = \frac{\sin \gamma \sin N}{\sin (\pi/2 - \beta_n)} = \frac{\sin \gamma \sin N}{\cos \beta_n}. \quad (47)$$

## VII. CONCLUSIONS AND RECOMMENDATIONS

In Pegasus-type meteoroid measurement satellite experiments, where vehicle attitude information is available without information about the meteoroid closing velocity vector, the probable radiant and the puncture flux enhancement factor for sporadic meteors should be based on the assumption that velocity vectors for ambient meteoroids are isotropically distributed. Then, although half of the meteoroids which impact into a flat transducer are more than 45 degrees from the normal, those within 45 degrees puncture three times more effectively than the others, and half of the puncturing meteoroids are less than 33.5 degrees from the normal when the earth is below the surface horizon.

The probability density functions  $f(x_2)$  and  $f(x)$  and the corresponding cumulative distributions for angles of impact  $x_2$  and  $x$  for incident and puncturing meteoroids, respectively, are as follows when the earth is below the surface horizon:

$$f(x_2) = \sin 2x_2 \quad (18)$$

$$\int_0^{x_2} f(x_2) dx_2 = (1 - \cos 2x_2)/2$$

$$f(x) = 1.92 (\cos x)^{1.84} \sin 2x \quad (30)$$

$$\int_0^x f(x) dx = 1 - (\cos x)^{3.84}.$$

The probability density functions in equations (18) and (30) above also imply that the flux  $\phi$  of puncturing meteoroids is five times greater than the flux  $F_g$  of incident meteoroids with mass  $m$  equal to or greater

than the mass  $\bar{m}$  of the almost-puncturing meteoroid with the average values of material density, velocity, and impact angle; i.e.,

$$F_s = 10^{\beta_6} \bar{m}^{\beta_2} \quad (5)$$

$$\phi = 10^{\beta_6 + 0.70} \bar{m}^{\beta_2}. \quad (29)$$

The Pegasus-type experiments give puncture flux,  $\phi$ , as a function of target thickness,  $p$ , and target material parameter,  $k_2$ , rather than as a function of the parameters  $\beta_2$  and  $\beta_6$  relating flux and mass in equations (5) and (29) above; i.e.,

$$\bar{m} = 10^{k_1} (10^{k_2} p)^{k_3} \quad (12)$$

$$\log \phi = (\beta_6 + 0.70 + k_1 \beta_2) + k_3 \beta_2 (k_2 + \log p) \quad (14)$$

where  $k_1$  and  $k_3$  depend on the unknown meteoroid material parameters, on the mechanics of impact, and possibly also on  $k_2$  and  $p$ , and where  $k_2$  is a known function of the known target material parameters as follows:

$$k_2 = 0.50 \log C_t + 1.31 \log E_t + 8.0 \log v_t + 0.43 \log \epsilon_t. \quad (13)$$

As observed from the site of puncture in a Pegasus-type experiment, the angular radius,  $\theta$ , of the earth's effective meteoroid encounter cross section is sufficiently approximated from:

$$\sin \theta = (r_e + 100)/r \quad (16)$$

where  $r$  is the geocentric radial distance of the satellite in kilometers and  $r_e$  is the radius of the earth in kilometers.

The separation  $D$  ( $0 \leq D \leq \pi$  radians) of the center of the earth's disk (at celestial coordinates  $\beta_p$  and  $\lambda_p$ ) from the surface zenith (at celestial coordinates  $\beta_n$  and  $\lambda_n$ ) is calculated from:

$$\cos D = \cos (\beta_p - \beta_n) - [1 - \cos (\lambda_n - \lambda_p)] \cos \beta_n \cos \beta_p. \quad (34)$$

When  $D \geq \pi/2 + \theta$  the earth is below the surface horizon and the probable radiant of a puncturing meteoroid is the surface zenith ( $\beta_n, \lambda_n$ ). But when  $D$  is in the interval  $\pi/2 - \theta < D < \pi/2 + \theta$ , a fraction  $R_2$  of the earth's disk with central angle  $2\alpha$  is apparent above the surface horizon and shields a fraction  $R_1$  of the meteoroids from the exposed celestial hemisphere. The centroid of the apparent segment is separated  $z$  radians from the center of the earth's disk and  $x$  radians from the surface zenith. Then the probable radiant is on the earth's meridian (of the exposed hemisphere) and at an increment  $\gamma$  radians beyond the surface zenith. The relations between these parameters are sufficiently approximated by:

$$\cos \alpha = (D - \pi/2)/\theta, \quad \theta < \alpha < \pi \quad (36)$$

$$R_2 = (\alpha - \sin \alpha \cos \alpha)/\pi \quad (37)$$

$$z = (2\theta \sin^3 \alpha)/(3\pi R_2) \quad (38)$$

$$x = D - z \quad (39)$$

$$R_1 = 1.67(\cos x)^{2.84} (1 - \cos \theta) R_2 \quad (31)$$

$$\gamma = \frac{xR_1}{1 - R_1}, \quad \text{when } \frac{xR_1}{1 - R_1} \leq 4/3 \quad (35)$$

$$= \theta + D/3 \quad \text{when } \frac{xR_1}{1 - R_1} > 4/3$$

Also, when  $D \leq \pi/2 - \theta$  the earth is entirely above the surface horizon,  $R_2$  becomes unity in equation (31) and  $D$  replaces  $x$  in both equations (31) and (35). Then, when at least part of the earth is apparent above the surface horizon ( $D < \pi/2 + \theta$ ), the celestial coordinates of the probable radiant ( $\beta_r, \lambda_r$ ) can be calculated from:

$$\sin \beta_r = \sin (\beta_p + D + \gamma) + \frac{\sin(D + \gamma)}{\sin D} [\sin \beta_n - \sin (\beta_p + D)]$$

$$-\frac{\pi}{2} \leq \beta_r \leq \frac{\pi}{2}, \quad (43)$$

$$\cos (\lambda_r - \lambda_n) = 1 + \frac{\cos \gamma - \cos (\beta_n - \beta_r)}{\cos \beta_r \cos \beta_n} \quad (44)$$

$$\sin (\lambda_r - \lambda_n) = \frac{\sin \gamma \sin N}{\cos \beta_n} \quad (47)$$

where

$$\cos N = 1 + \frac{\sin \beta_n - \sin(\beta_r + \gamma)}{\cos \beta_r \sin \gamma}, \quad 0 \leq N \leq \pi. \quad (46)$$

The formulas for the celestial coordinates of the probable radiant  $(\beta_r, \lambda_r)$  of a puncturing meteoroid have been corrected for earth-shielding. This correction is important near the earth even under nominal circumstances; e.g., when the orbit height is 500 kilometers, the radius of the earth's disk (as seen from the vehicle) is 70 degrees. In that case, when the elevations of the earth's center are 0, 15, 30, 45, and 60 degrees, the probable radiants are deflected 5, 10, 20, 39, and 80 degrees, respectively, by the earth-shielding effect.

Probable radiants in Pegasus-type experiments should be calculated from the formulas re-stated above in this section.

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MEASUREMENT RELATIONSHIPS FOR PEGASUS-TYPE  
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